STAGE DISCHARGE ANALYSIS OF DHOND RIVER USING ADAPTIVE NEURO-FUZZY INFERENCE SYSTEM (ANFIS)

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CERTIFICATE

This is to certify that project report titled STAGE DISCHRGE ANALYSIS OF DHOND RIVER USING ANFIS submitted by Kamal Chawla M.Sc. 2nd year, Dept. of Mathematics, IIT Roorkee, has done a bonafide work under my guidance during the period 15th May 2015 to 30th June 2015 at National Institute of Hydrology, Roorkee.

The study under this project, has been completed successfully as per the objectives.

14/10/15

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DECLARATION

I declare that the summer internship project report titled STAGE DISCHRGE ANALYSIS OF DHOND RIVER USING ANFIS is my own work conducted under the supervision of Dr. Rama Mehta at the National Institute of Hydrology, Roorkee. I worked for 45 days under Dr. Rama Mehta at NIH. I further declare that to the best of my knowledge, the report does not contain any part of any work which has been submitted for the award of any degree either in this institute or any other institute without proper citation.

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INTRODUCTION

Stream flow information is important for effective and reliable planning and management of various water resources activities and the assessment, management and control of water resources can be effective if accurate and continuous information on river-flow is available. Generally a network of river gauging stations provides continuous information on river stage and sparse information of corresponding discharges.

Thus, the continuous discharge data corresponding to observed gauge can be obtained by developing a stage discharge relationship and using this relationship to convert the recorded stages into corresponding discharges. This relationship is determined by correlating measurements of discharge with the corresponding observations of stage. However, under certain conditions (flatter gradients and constricted channels) the discharge for a flood on a rising stage differs from that on the falling stage. This phenomenon is called hysteresis and results in a looped Stage–discharge curve (Tawfik et al., 1997) for floods with different stage–discharge relations for rising and falling water stages. Rating curve development approaches can be categorized into three main groups: the single curve approach, the rising and falling approach, and the Jone's approach (Tawfik et al., 1997). DeGagne et al. (1996) documented the process of developing a decision support system for the analysis and use of stage–discharge rating curve while other possible models have been proposed by Gawne and Simonovic (1994) and Yu (2000).

The functional relationship between stage and discharge is complex and cannot always be captured by these traditional modeling techniques (Bhattacharya and Solomatine, 2005). In the real world, stage and discharge relationship do not exhibit simple structure and are difficult to analyze and model accurately. Therefore, it seems necessary that soft computing methods e.g. artificial neural network (ANN) and fuzzy logic, which are suited to complex non-linear models, be used for the analysis. There are several applications of ANNs in stage–discharge modeling. Jain and Chalisgaonkar (2000) used three layer feed forward ANNs to establish stage-discharge relationship. Bhattacharya and Solomatine (2005) have found that the predictive accuracy of ANN model is superior than the traditional rating curves. The effectiveness of an ANN with a radial basis function was explored by Sudheer and Jain (2003). The ANN based approaches have also provided promising results in modeling loop rating curves (Jain and Chalisgaonkar, 2000; Sudheer and Jain, 2003).

The purpose of this study is to investigate and explore the potential of an alternate soft computing technique for stage discharge modeling based on fuzzy logic. The ability of fuzzy logic to model nonlinear events makes it even more important to investigate its ability to model stage discharge relationship. Uncertainty in conventional gauge–discharge rating curves involves a variety of components such as measurement noise, inadequacy of the model, insufficiency of river flow conditions, etc. Fuzzy logic based modeling approach has a significant potential to tackle the uncertainty problem in this field and to model nonlinear functions of arbitrary complexity. Other advantage of fuzzy logic is its flexibility and tolerance to imprecise data (Zadeh, 1999).

Fuzzy rule based modeling is a qualitative modeling scheme where the system behavior is described using a natural language (Sugeno and Yasukawa, 1993). The transparency of the fuzzy rules provides explicit qualitative and quantitative insights into the physical behavior of the system (Coppolaet al., 2002). The application of fuzzy logic as a modeling tool in the field of water resources is a relatively new concept although some studies have been carried out to a limited extent and these studies have generated considerable enthusiasm. Fuzzy rule based modeling has been attempted in water resources management, reservoir operation, flood forecasting and other areas of water resources analysis (Bardossy and Duckstein, 2002; Fontane et al., 1997; Kindler,1992; Mujumdar. and Sasikumar, 2002; Panigrahi and Mujumdar, 2000; Sasikumar and Mujumdar, 1998; Deka and Chandramouli, 2003; Lohani et al., 2005). This paper is concerned with the application of an emerging, powerful soft computing technique fuzzy logic to stage discharge rating curves.

STUDY AREA

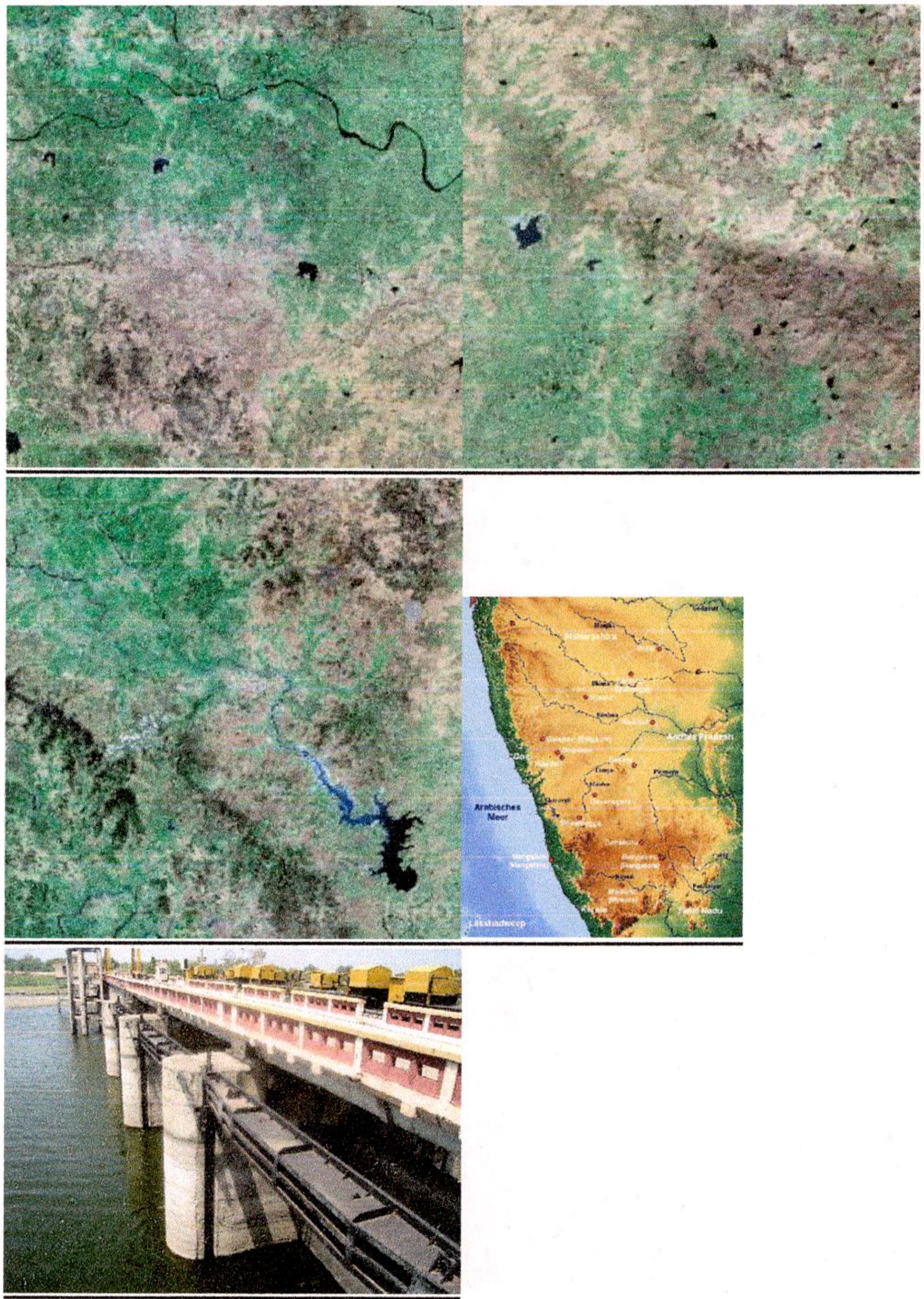


Fig.: Dhond HO –upper Bhima basin

Dhond is a distributory of Bhima river which is a part of upper BHIMA basin in Maharashtra. Earlier, it was in Princely state of bhima. The Upper Bhima Basin (UBB) covers an area of 14,700 km^2 with altitude decreasing from 1,100 m in the eastern side to 450 m towards the west. The important river in the basin is Bhima, a tributary of the river Krishna. River Bhima originates in Sahyadri, the mountain range running from North to South along West coast of India and flows in eastern direction finally joining the Krishna. The river basin receives most of the rains from South- West monsoon that lasts for a period of four and half months (June to mid-October). The precipitation pattern in UBB is very uneven while in the western ridge areas the annual rainfall is about 6,000 mm and it declines to 450–600 mm in eastern part. It is an important catchment in the context of serving intersectorial demands including drinking water supply. There are six major and more than 30 medium reservoirs in the catchment with gross storage capacity of 7,800 Mm^3(Million Cubic meter).

DATA ANALYSIS

- **DATA USED**

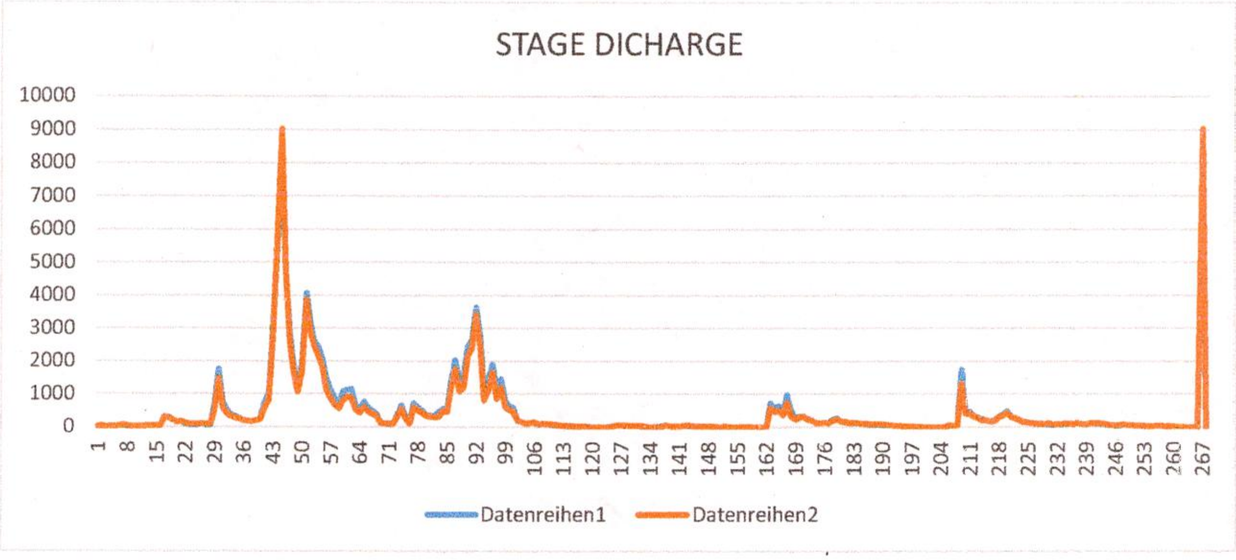
In this present study, we have collected monthly data of stage and discharge of dhond ho from 1973 to 2004 from national institute of hydrology(NIH)., the discharge is in m^3/s and stage is in m^3

Here we use to compare the original discharge and conventional discharge of the data, conventional discharge can be calculated by most commonly used power equation

$$Q = a *(H-H_0)^b$$

Where Q is the discharge, H is the stage, H_0 is the initial stage and a and b are calibration coefficients. a is the discharge when the effective depth of flow (h-a) is equal to 1; b is the slope of the rating curve (on logarithmic paper).

Here a= 100.976 and b=1.885



METHODS AND WORKING RULE

Overview of fuzzy logic

The classical theory of crisp sets can describe only the membership or non-membership of an item to a set. While, fuzzy logic is based on the theory of fuzzy sets which relates to classes of objects with unsharp boundaries in which membership is a matter of degree. In this approach, the classical notion of binary membership in a set has been modified to include partial membership ranging between 0 and 1 (Zadeh, 1965). The membership function is described by an arbitrary curve suitable from the point of view of simplicity, convenience, speed, and efficiency. A sharp set is a sub set of a fuzzy set where the membership function can take only the values 0 and 1. The range of the model input values, which are judged necessary for the description of the situation, can be portioned into fuzzy sets. The process of formulating the mapping from a given input to an output using fuzzy logic is called the fuzzy inference (Jang, 1993). The basic structure of any fuzzy inference system is a model that maps characteristics of input data to input membership functions, input membership function to rules, rules to a set of output characteristics, output characteristics to output membership functions, and the output membership function to a single-valued output or a decision associated with the output (Jang et al., 2002). In rule based fuzzy systems, the relationships between variables are represented by means of fuzzy if-then rules e.g. "If antecedent proposition then consequent proposition". Depending on the particular structure of the consequent proposition, three main types of fuzzy models are distinguished as: (1) Linguistic (Mamdani Type) fuzzy model (Zadeh, 1973; Mamdani, 1977)

(2) Fuzzy relational model (Pedrycz, 1984; Yi and Chung,1993) (3) Takagi–Sugeno (TS) fuzzy model (Takagi and Sugeno,1985).

A major distinction can be made between the linguistic model, which has fuzzy sets in both antecedents and consequents of the rules, and the TS model, where the consequents are (crisp) functions of the input variables. Fuzzy relational models can be regarded as an extension of linguistic models, which allow for different degrees of association between the antecedent and the consequent linguistic terms. In this work, the TS fuzzy model is employed to develop stage discharge rating curve. These models are relatively easy to identify and their structure can be readily analyzed(Lohani et al., 2005).

RATE DISCHARGE EQUATION

Accurate estimation of discharge in rivers is essential for hydrological and hydraulic analysis. Since discharge measurements are time-consuming and expensive, most discharge records are derived from a functional relationship between stage and discharge Q(y), referred to as a stage-discharge or rating curve. Typically, the relationship is established by statistical regression analysis from discharge measurements at different stages over a period of time. The relationship between the stage, hi and the corresponding discharge, Qi is often expressed in the forms:

$$\log(Qi)= A+B \log(hi)$$

$$Qi=C\ hi^{D}$$

Where A,B and C,D represent the parameters of the relationship. It is usual practice to measure the goodness of fit of the relationship using a criterion such as coefficient of determination, R2. This can often be very high, giving a false impression that estimated discharge have high precision (Clarke, 1999). In reality, the stage-discharge relationship may be affected by a number of inherent uncertainties.Both the measurement and the natural uncertainties may lead to a significant scatter in the relationship, and the use of a single-valued curve can cause underestimation and/ or overestimation of discharges. The uncertainties might be substantial if only a few discharge measurements are available and a single-valued curve is used for interpolation and extrapolation of discharges. The uncertainties due to incorrect discharges can cause potentially large errors, influencing flood forecasting, and statistical estimation of flood flows for design and decisions to promote flood defense schemes (Samuels's et.al. 2002).

The fuzzy set theory-based method provides an alternative means of treating the uncertainty in a stage-discharge relationship. The method allows integration of information of different quality into the modeling and evaluation process (Schulz & Huwe, 1999) and has been used for the analysis of uncertainties due to lack of knowledge (El-Baroudy & Simonovic, 2006) and scarcity of data (Guyonnet et al., 2003). It is also simpler to treat the stage-discharge relationship in a non-probabilistic framework using fuzzy sets, as uncertainties and imprecision are explicitly represented by vaguely defined boundaries and no assumptions on error dependencies have to be made. The aim of this paper is to demonstrate an application of fuzzy set theory- based methods for the analysis and propagation of uncertainties due to a scattered stage-discharge relationship. The fuzzy sub-clustering approach, based on the fuzzy set theory, is used for the representation of stage discharge relationship.

ADAPTIVE NEURO-FUZZY INFERENCE SYSTEM (ANFIS)-GRID BASE PARTITIONING

The Adaptive Neuro-Fuzzy Inference System (ANFIS), first introduced by Jang (1993), is a universal approximator and, as such, is capable of approximating any real continuous function on a compact set to any degree of accuracy (Jang et al., 1997). Thus, in parameter estimation, where the given data are such that the system associates measurable system variables with an internal system parameter, a functional mapping may be constructed by ANFIS that approximates the process of estimation of the internal system parameter.

The ANFIS is functionally equivalent to fuzzy inference systems (Jang et al., 1997). Below, the hybrid learning algorithm (Jang et al., 1997; Drake, 2000), which combines gradient descent and the least-squares method, is introduced and the issue of how the equivalent fuzzy

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(Scientist)

inference system can be rapidly trained and adapted with this algorithm is discussed.

As a simple example, a fuzzy inference system with two inputs x and y and one output z is assumed. The first-order Sugeno fuzzy model, a typical rule set with two fuzzy if-Then rules can be expressed as:

Rule 1: IF x is A1 AND y is B1 THEN $f_1 = p_1 x + q_1 y + r_1$
Rule 2: IF x is A2 AND y is B2 THEN $f_2 = p_2 x + q_2 y + r_2$

The resulting Sugeno fuzzy reasoning system is shown in Fig.1. Here, the output z is the weighted average of the individual rules outputs and is itself a crisp value. Nodes at the same layer have similar functions. The output of the ith node in layer l is denoted as O1,i.

Layer 1: Every node *i* in this layer is an adaptive node with node function

$O_{1,i} = \mu A_i(x)$ for *i* = 1, 2, or

$O_{1,i} = \mu B_{i-2}(y)$ for *i* = 3, 4

Where x (or y) is the input to the ith node and A_i (or B_{i-2}) is a linguistic label (such as "low" or "high") associated with this node. In words, $O_{1,i}$ is the membership grade of a fuzzy set A (=A_1, A_2, B_1, or B_2)and it specifies the degree to which the given input x (or y) satisfies the quantifier *A*. The membership function for *A* and *B* are generally described by generalized bell functions, e.g.:

$$\mu A_i(x) = \frac{1}{1 + [(x - c_i)/a_i]^{2b_i}}$$

Where $\{a_i, b_i, c_i\}$ is the parameter set. As the values of these parameters change, the bell-shaped function varies accordingly, thus exhibiting various forms of membership functions on linguistic label A_i. In fact, any continuous and piecewise differentiable functions, such as commonly used triangular-shaped membership functions, are also qualified candidates for node functions in this layer (Jang, 1993). Parameters in this layer are referred to as premise parameters. The outputs of this layer are the membership values of the premise part.

Layer 2: This layer consists of the nodes labeled II which multiply incoming signals and send the product out. For instance,

$$O_{2,i} = w_i = \mu A_i(x) \mu B_i(y) \qquad i = 1,\ 2 \qquad (4)$$

Each node output represents the firing strength of a rule.

Layer 3: In this layer, the nodes labeled N calculate the ratio of the rule's firing strength to the sum of all rules' firing strengths

$$O_{3,i} = \overline{w}_i = \frac{w_i}{w_1 + w_2} \qquad i = 1, 2 \qquad (5)$$

The outputs of this layer are called normalized firing strengths.

Layer 4: This layer's nodes are adaptive with node functions

$$O_{4,i} = \overline{w}_i f_i = \overline{w}_i (p_i x + q_i y + r_i)$$
(6)

Where $\overline{w}_i$ is the output of layer 3, and $\{p_i, q_i, r_i\}$are the parameter set. Parameters of this layer are referred to as consequent parameters.

Layer 5: This layer's single fixed node labeled Σ computes the final output as the summation of all incoming signals

$$O_{5,i} = \sum_{i=1} \overline{w}_i f_i = \frac{\sum_i w_i f_i}{\sum_i w_i} \qquad (7)$$

Thus, an adaptive network which is functionally equivalent to a Sugeno first-order fuzzy inference system is created. More information on ANFIS can be found in Jang (1993).

MODEL DEVELOPMENT

A Fuzzy Grid partitioning model has been developed using MATLAB Tool FUZZY. The optimum parameters for minimum error is given in TABLE-1 using one input and one output.

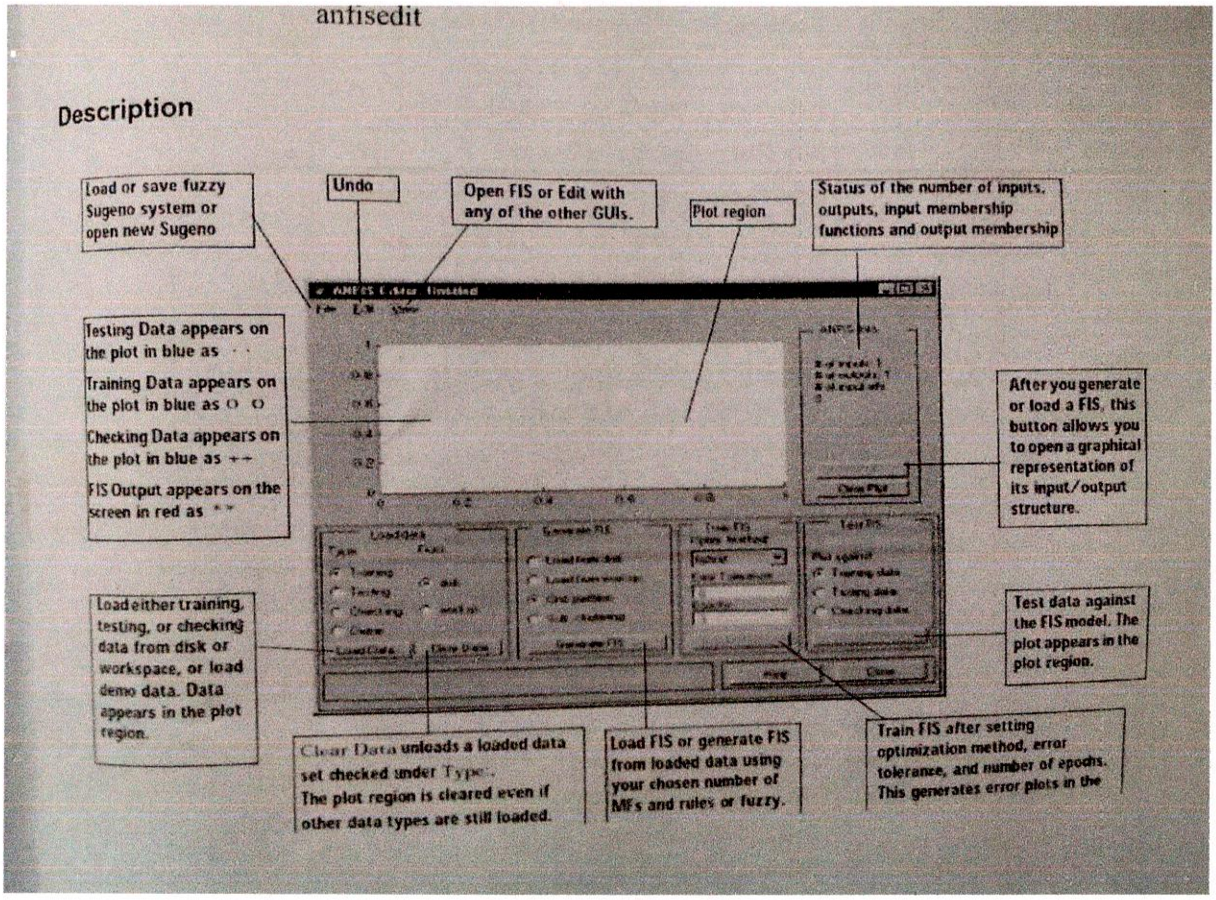


Fig1: ANFIS window

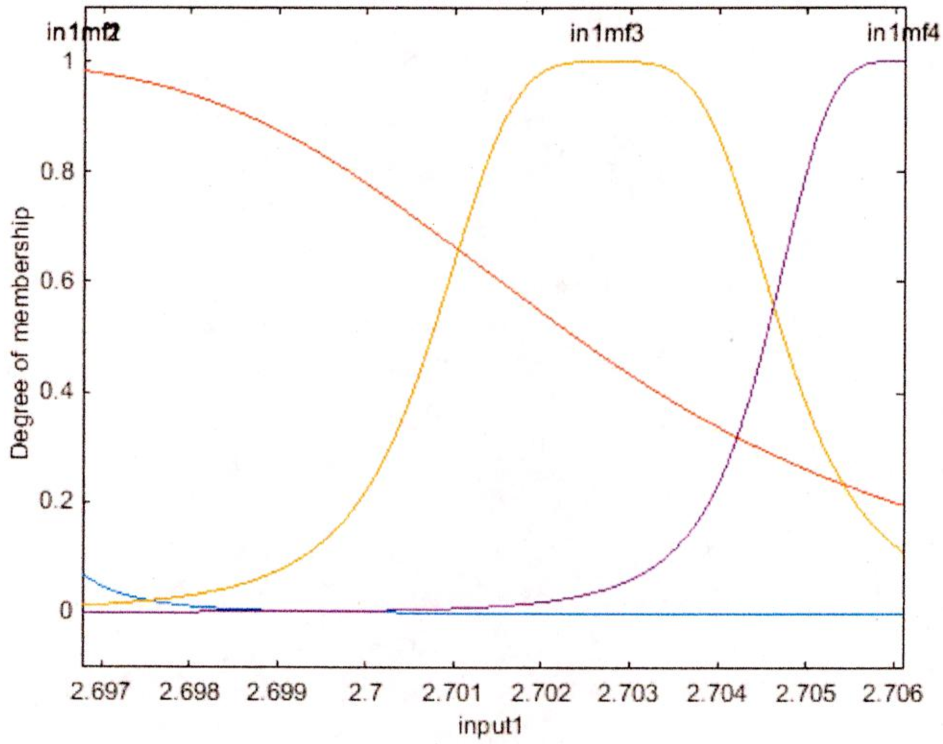


FIG 2: MEMBERSHIP FUNCTION (GBELL SHAPE)

Membership function: No. of MF = 4,

Shape is gbell.

Out of ANFIS is either constant or linear

Here we use linear output

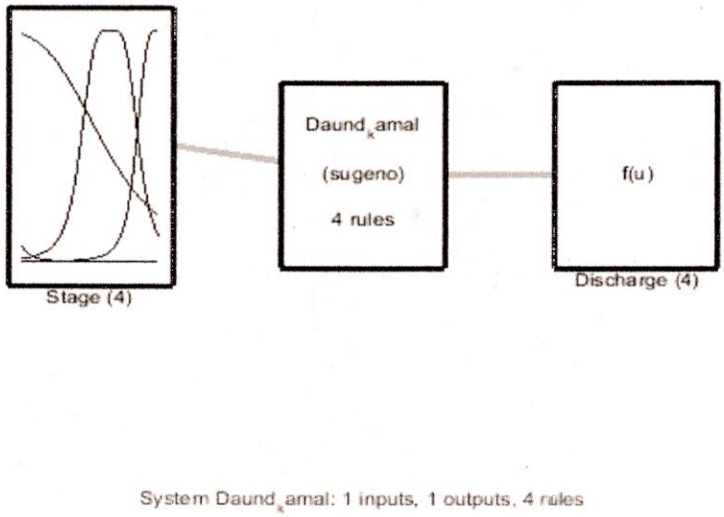


FIG 3: Fuzzy Logic design for ANFIS _ Grid Method WITH SINGLE INPUT AND SINGLE OUTPUT

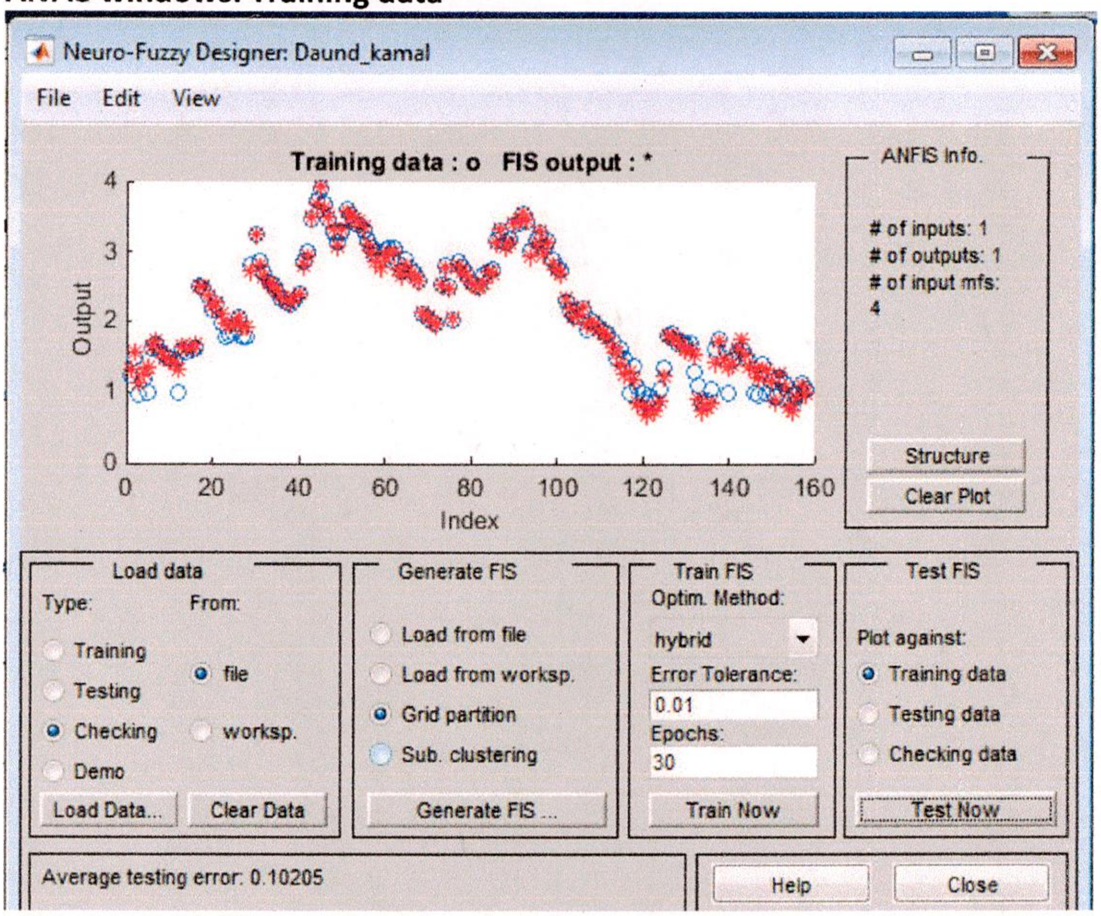
TABLE 1

Number of nodes	20
Number of non-linear parameters	12
Number of linear parameters	8
Total number of parameters	20
Number of training data pairs	154
Number of checking data pair	109
Fuzzy rules	4
Training error	0.102053
Checking error	0.08792222

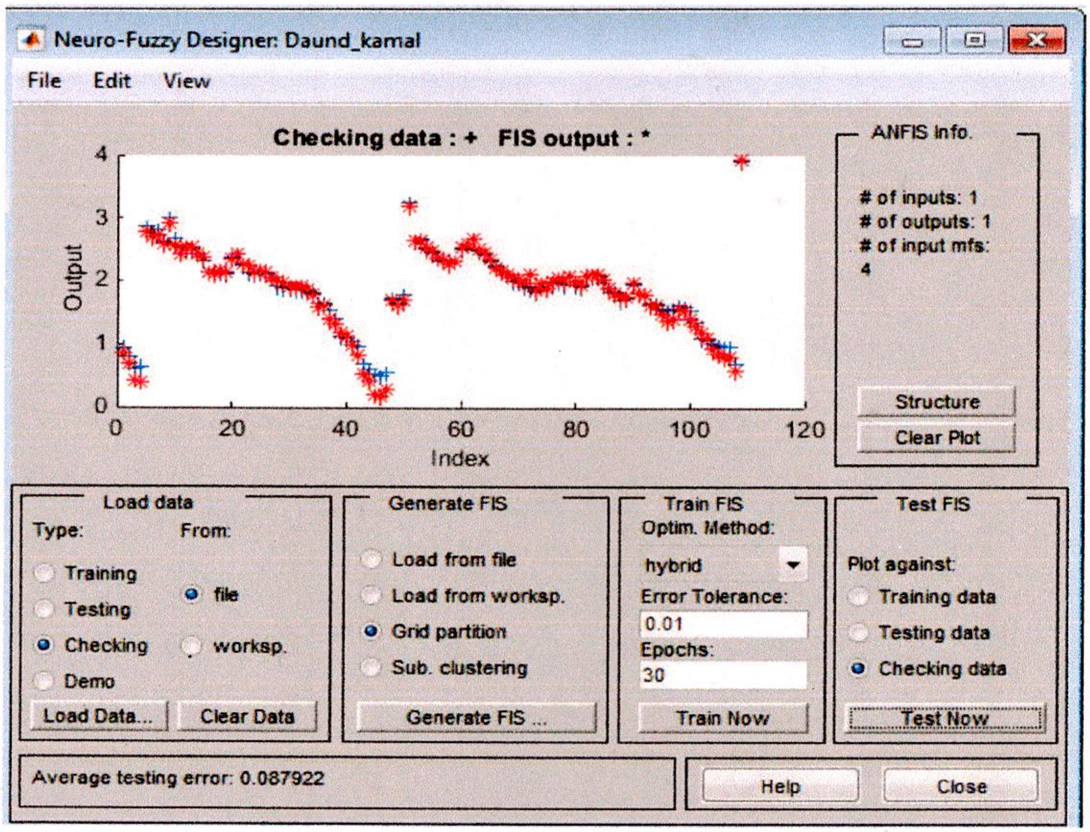
RESULT ANALYSIS

70% of the observations and validation has only 30%.and which is further divided into training 15% and testing 15%. It gives the results as comparative graphs between observed and modeled output.

ANFIS windows: Training data



ANFIS windows: Checking data



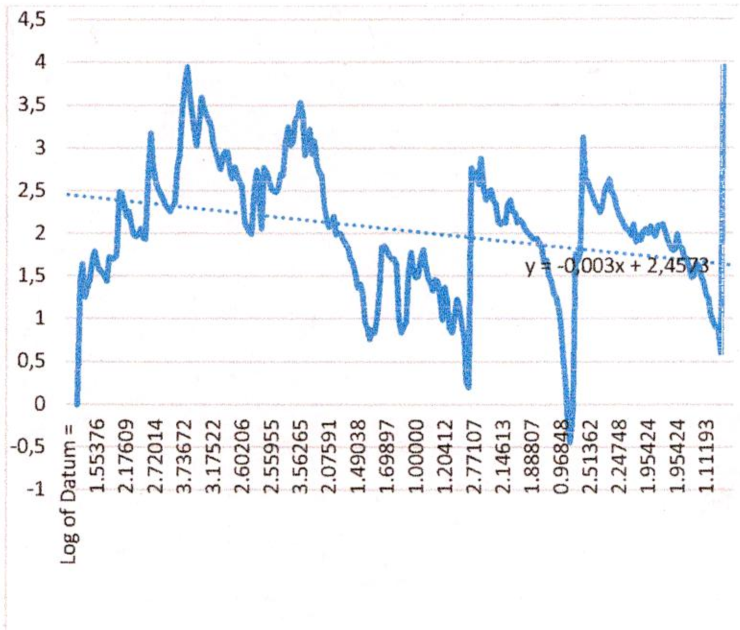


FIG: GRAPH BETWEEN CONVENTIONAL AND OBSERVED DATA WITH REGRESSION LINE

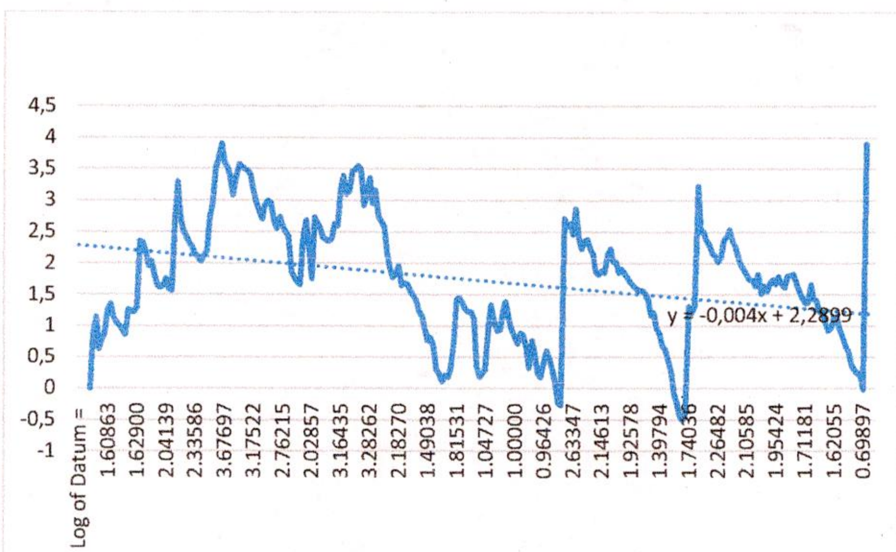


FIG: GRAPH BETWEEN OBSERVED DISCHARGE AND ANFIS GRID WITH REGRESSION LINE

By plotting above graphs we can find the regression coefficients a and b, after which we can easily solve the stage discherge equation and observe the relationship between the given data and conventional data.

By above analysis we can plot the above data on the basis of three techniques we used viz, conventional method, anfis grid and observed discharge at base 10

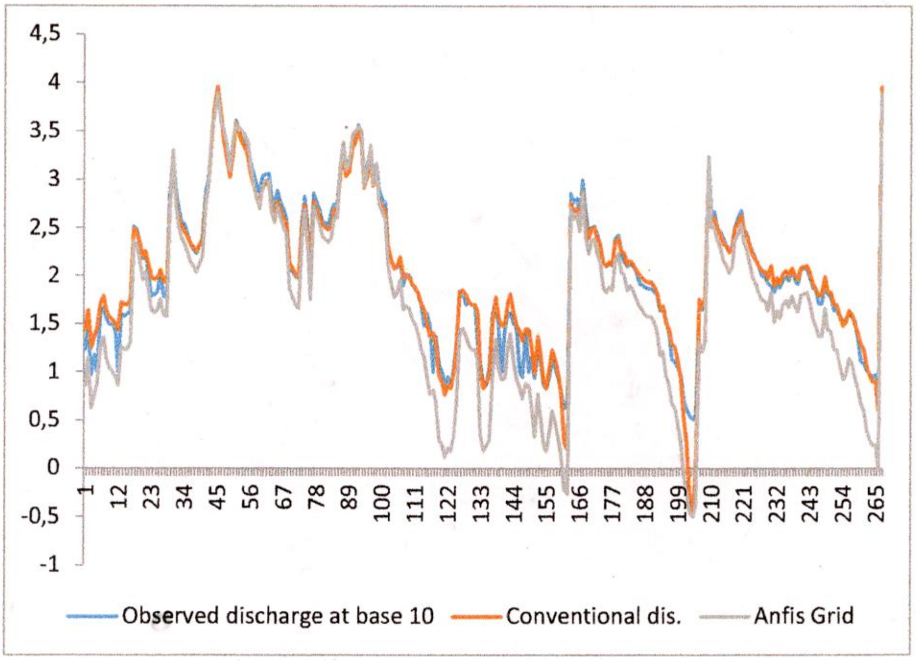


FIG: GRAPH BETWEEN OBSERVED, CONVENTIONAL AND ANFIS GRID

CONCLUSION

After plotting we can show that the relative error is minimum and hence we can conclude that ANFIS technique is the best feasible technique to analyze stage discharge data for any given discharge medium

REFERENCES

1. Optimal Operation of a Multi-Purpose Reservoir Using Neuro-Fuzzy Technique
   Rama Mehta · Sharad K. Jain
2. Discharge Modelling using Adaptive Neuro - Fuzzy Inference System **Dinesh C. S. Bisht* and Ashok Jangid** Department of Applied Sciences and Humanities ITM, University, Gurgaon, India E-mail: dcsbisht@gmail.com*, ashjangid@gmail.com****
3. Takagi–Sugeno fuzzy inference system for modeling stage–discharge relationship
   A.K. Lohani a,*, N.K. Goel b, K.K.S. Bhatia a
   a National Institute of Hydrology, Jalvigyan Bhawan, Roorkee 247667, India
4. A Journal on National Workshop on Advanced Computing in Hydrology And Its applications (ASCTHA-2014),
   By Dr. Rama Mehta

5. Optimal operations of a Multipurpose Resourvior using Neuro-Fuzzy Techniques; Rama Mehta,Trans IIM , Accepted ,2009

APPENDIX

- **DESCRIPTIVE STATISTICAL ANALYSIS**

ORIGINAL DISCHARGE	STAGE	CONVENTIONAL DISCHARGE	LOG DISCHARGE	LOG STAGE	LOG CON DIS	ANFIS
16.93	497.94	0.00000	1.22866	2.69718	1.43678	0.86080
30	498.08	43.53789	1.47712	2.69730	1.63887	1.14110
9.23	497.84	17.95152	0.96520	2.69709	1.25410	0.62680
14.94	497.9	23.36236	1.17435	2.69714	1.36852	0.77090
10	497.95	28.37840	1.00000	2.69719	1.45299	0.88250
40.61	498.14	51.54992	1.60863	2.69735	1.71223	1.24690
45.84	498.21	61.69546	1.66124	2.69741	1.79025	1.36090
35.79	498.08	43.53789	1.55376	2.69730	1.63887	1.14110
31.87	498.03	37.34859	1.50338	2.69726	1.57227	1.04670
30.61	498.01	34.99792	1.48586	2.69724	1.54404	1.00720
27.3	497.98	31.60679	1.43616	2.69721	1.49978	0.94610
10	497.94	27.33862	1.00000	2.69718	1.43678	0.86080
40.3	498.15	52.94685	1.60531	2.69736	1.72384	1.26380
37.84	498.13	50.17053	1.57795	2.69734	1.70045	1.22980
40	498.13	50.17053	1.60206	2.69734	1.70045	1.22980
42.56	498.17	55.79326	1.62900	2.69738	1.74658	1.29700
325.7	499.25	308.98847	2.51282	2.69832	2.48994	2.35980
311.8	499.2	293.09576	2.49388	2.69827	2.46701	2.32930
230	498.97	225.09275	2.36173	2.69807	2.35236	2.17410
150	498.71	158.44852	2.17609	2.69785	2.19989	1.96070

180	498.8	180.27645	2.25527	2.69793	2.25594	2.04010
95.27	498.58	129.26585	1.97896	2.69773	2.11148	1.83340
60.85	498.42	97.20292	1.78426	2.69760	1.98768	1.65230
63.17	498.39	91.66998	1.80051	2.69757	1.96223	1.61470
69.35	498.42	97.20292	1.84105	2.69760	1.98768	1.65230
110	498.51	114.71140	2.04139	2.69767	2.05961	1.75790
61.13	498.37	88.06605	1.78625	2.69755	1.94481	1.58900
59.86	498.36	86.28955	1.77714	2.69754	1.93596	1.57590
658.3	499.79	505.45335	2.81842	2.69879	2.70368	2.64000
1772	501.63	1503.45902	3.24846	2.70038	3.17709	3.30160
750	500.03	607.13912	2.87506	2.69900	2.78329	2.74480
524.98	499.53	405.22284	2.72014	2.69856	2.60769	2.51470
370	499.34	338.58577	2.56820	2.69840	2.52967	2.41240
348.4	499.2	293.09576	2.54208	2.69827	2.46701	2.32930
270	499.09	259.52273	2.43136	2.69818	2.41418	2.25830
216.7	498.94	216.84539	2.33586	2.69805	2.33615	2.15180
191.2	498.87	198.16484	2.28149	2.69799	2.29703	2.09760
171.6	498.8	180.27645	2.23452	2.69793	2.25594	2.04010
204.3	498.9	206.07405	2.31027	2.69801	2.31402	2.12120
250	499	233.48449	2.39794	2.69810	2.36826	2.19590
751.8	499.98	585.23423	2.87610	2.69895	2.76733	2.72370
1030	500.5	831.38666	3.01284	2.69940	2.91980	2.93090
2950	503.13	2676.73135	3.46982	2.70168	3.42760	3.51440
5454	505.9	5653.42060	3.73672	2.70406	3.75231	3.65910
8210	508.28	9020.86866	3.91434	2.70610	3.95525	3.90210
4753	505.2	4804.04987	3.67697	2.70346	3.68161	3.60430
2860	503.05	2606.23250	3.45637	2.70161	3.41601	3.50960
1869	501.9	1691.27555	3.27161	2.70062	3.22821	3.36580

1368	500.94	1070.99249	3.13609	2.69979	3.02979	3.08940
1935	501.96	1734.41936	3.28668	2.70067	3.23915	3.37830
4076	504.37	3881.50643	3.61023	2.70275	3.58900	3.56820
3306	503.45	2967.53973	3.51930	2.70196	3.47240	3.53080
2637	502.9	2476.43136	3.42111	2.70148	3.39383	3.49960
2460	502.53	2169.60755	3.39094	2.70116	3.33638	3.46640
2080	502.08	1822.23572	3.31806	2.70077	3.26060	3.40140
1497	501.06	1141.25796	3.17522	2.69989	3.05738	3.13010
1150	500.6	883.34085	3.06070	2.69949	2.94613	2.96810
883.9	500.19	679.76733	2.94640	2.69914	2.83236	2.81050
736.5	499.94	567.98266	2.86717	2.69892	2.75434	2.70650
1088	500.49	826.27262	3.03663	2.69940	2.91712	2.92710
1130	500.67	920.58717	3.05308	2.69955	2.96406	2.99380
1158	500.62	893.90895	3.06371	2.69951	2.95129	2.97550
689.9	499.85	530.05421	2.83879	2.69884	2.72432	2.66700
532.6	499.62	438.74144	2.72640	2.69864	2.64221	2.55980
770.8	500.01	598.33182	2.88694	2.69898	2.77694	2.73640
578.3	499.72	477.44720	2.76215	2.69873	2.67893	2.60770
506.3	499.53	405.22284	2.70441	2.69856	2.60769	2.51470
400	499.39	355.57679	2.60206	2.69844	2.55093	2.44040
127.3	498.61	135.75271	2.10483	2.69776	2.13275	1.86420
111.2	498.53	118.78651	2.04610	2.69769	2.07477	1.78000
98.08	498.46	104.81646	1.99158	2.69763	2.02043	1.70040
95.26	498.43	99.08103	1.97891	2.69760	1.99599	1.66450
360	499.28	318.71299	2.55630	2.69834	2.50340	2.37770
670	499.9	550.97366	2.82607	2.69888	2.74113	2.68910
328.4	499.18	286.84910	2.51640	2.69826	2.45765	2.31680
106.8	498.51	114.71140	2.02857	2.69767	2.05961	1.75790

728.5	500	593.95084	2.86243	2.69897	2.77375	2.73220
608.1	499.79	505.45335	2.78398	2.69879	2.70368	2.64000
500.86	499.62	438.74144	2.69972	2.69864	2.64221	2.55980
362.7	499.35	341.95272	2.55955	2.69841	2.53397	2.41810
360	499.29	321.98591	2.55630	2.69835	2.50784	2.38360
323.7	499.24	305.77842	2.51014	2.69831	2.48541	2.35380
472.4	499.32	331.89882	2.67431	2.69838	2.52101	2.40100
557.8	499.76	493.35895	2.74648	2.69876	2.69316	2.62630
571.7	499.72	477.44720	2.75717	2.69873	2.67893	2.60770
1460	501.16	1201.41100	3.16435	2.69998	3.07969	3.16300
2048	502.01	1770.76203	3.31133	2.70071	3.24816	3.38830
1430	500.98	1094.18136	3.15534	2.69982	3.03909	3.10310
1558	501.17	1207.50604	3.19257	2.69999	3.08189	3.16620
2436	502.5	2145.56604	3.38668	2.70114	3.33154	3.46300
2637	502.82	2408.47838	3.42111	2.70141	3.38174	3.49350
3653	503.93	3430.04183	3.56265	2.70237	3.53530	3.55110
2760	502.88	2459.35991	3.44091	2.70146	3.39082	3.49810
1079	500.48	821.17339	3.03302	2.69939	2.91443	2.92330
1438	501	1105.86321	3.15776	2.69984	3.04370	3.10990
1917	501.85	1655.71241	3.28262	2.70057	3.21898	3.35490
1117	500.57	867.59939	3.04805	2.69946	2.93832	2.95700
1460	501.16	1201.41100	3.16435	2.69998	3.07969	3.16300
790.1	500.06	620.46330	2.89768	2.69902	2.79272	2.75730
630	499.86	534.20768	2.79934	2.69885	2.72771	2.67150
581.2	499.69	465.67423	2.76433	2.69870	2.66808	2.59350
197.7	498.9	206.07405	2.29601	2.69801	2.31402	2.12120
149.6	498.67	149.17268	2.17493	2.69781	2.17369	1.92320
119.1	498.53	118.78651	2.07591	2.69769	2.07477	1.78000

122.3	498.57	127.13673	2.08743	2.69773	2.10427	1.82300
152.3	498.71	158.44852	2.18270	2.69785	2.19989	1.96070
80	498.42	97.20292	1.90309	2.69760	1.98768	1.65230
105.8	498.45	102.88782	2.02449	2.69762	2.01236	1.68860
102.6	498.43	99.08103	2.01115	2.69760	1.99599	1.66450
83.68	498.35	84.53006	1.92262	2.69753	1.92701	1.56270
73.88	498.3	75.98848	1.86853	2.69749	1.88075	1.49440
66.84	498.25	67.87540	1.82504	2.69745	1.83171	1.42200
53.43	498.14	51.54992	1.72779	2.69735	1.71223	1.24690
30	498.09	44.82908	1.47712	2.69731	1.65156	1.15930
40.9	497.99	32.71914	1.61172	2.69722	1.51480	0.96670
30.93	497.9	23.36236	1.49038	2.69714	1.36852	0.77090
10	497.92	25.31383	1.00000	2.69716	1.40336	0.81640
23.39	497.87	20.57341	1.36903	2.69712	1.31331	0.70030
12.01	497.72	9.16455	1.07954	2.69699	0.96211	0.30080
9.75	497.7	7.96972	0.98900	2.69697	0.90144	0.24100
7	497.66	5.81681	0.84510	2.69693	0.76468	0.11630
8.89	497.69	7.40176	0.94890	2.69696	0.86934	0.21050
7	497.68	6.85356	0.84510	2.69695	0.83592	0.17950
11.12	497.73	9.79126	1.04610	2.69699	0.99084	0.33010
22.67	497.89	22.41423	1.35545	2.69713	1.35052	0.74760
65.36	498.26	69.46360	1.81531	2.69746	1.84176	1.43680
67.59	498.27	71.06903	1.82988	2.69746	1.85168	1.45140
50	498.22	63.21447	1.69897	2.69742	1.80082	1.37650
54.5	498.16	54.36131	1.73640	2.69737	1.73529	1.28050
50.25	498.13	50.17053	1.70114	2.69734	1.70045	1.22980
49.23	498.13	50.17053	1.69223	2.69734	1.70045	1.22980
20	498.07	42.26444	1.30103	2.69729	1.62598	1.12270

12.26	497.74	10.43739	1.08849	2.69700	1.01859	0.35890
7	497.68	6.85356	0.84510	2.69695	0.83592	0.17950
7.5	497.7	7.96972	0.87506	2.69697	0.90144	0.24100
11.15	497.72	9.16455	1.04727	2.69699	0.96211	0.30080
40.63	498	33.84953	1.60885	2.69723	1.52955	0.98710
56.9	498.2	60.19380	1.75511	2.69740	1.77955	1.34520
30.48	498.03	37.34859	1.48401	2.69726	1.57227	1.04670
10	497.97	30.51253	1.00000	2.69720	1.48448	0.92520
27.9	497.98	31.60679	1.44560	2.69721	1.49978	0.94610
40	498.13	50.17053	1.60206	2.69734	1.70045	1.22980
41.64	498.23	64.75081	1.61951	2.69743	1.81125	1.39180
32.48	498.07	42.26444	1.51162	2.69729	1.62598	1.12270
27.77	497.97	30.51253	1.44358	2.69720	1.48448	0.92520
10	497.93	26.31708	1.00000	2.69717	1.42024	0.83870
9	497.88	21.48456	0.95424	2.69712	1.33213	0.72410
25.91	497.95	28.37840	1.41347	2.69719	1.45299	0.88250
10	497.94	27.33862	1.00000	2.69718	1.43678	0.86080
14.1	**497.85**	**18.80684**	1.14922	2.69710	1.27432	0.65160
8.68	497.73	9.79126	0.93852	2.69699	0.99084	**0.33010**
16	497.9	23.36236	1.20412	2.69714	1.36852	0.77090
13.33	497.81	15.49810	1.12483	2.69706	1.19028	0.55020
7.5	497.7	7.96972	0.87506	2.69697	0.90144	0.24100
6.7	497.68	6.85356	0.82607	2.69695	0.83592	0.17950
9.21	497.75	11.10286	0.96426	2.69701	1.04543	0.38740
13.5	497.83	17.11492	1.13033	2.69708	1.23337	0.60160
11	497.78	13.21466	1.04139	2.69704	1.12106	0.47050
8.5	497.72	9.16455	0.92942	2.69699	0.96211	0.30080
6.17	497.66	5.81681	0.79029	2.69693	0.76468	0.11630

4.22	497.56	1.85556	0.62531	2.69685	0.26847	-0.22810
4.43	497.55	1.57487	0.64640	2.69684	0.19724	-0.26530
722	499.96	576.57815	2.85854	2.69894	2.76086	2.71510
590.3	499.7	469.58322	2.77107	2.69871	2.67171	2.59830
645.5	499.79	505.45335	2.80990	2.69879	2.70368	2.64000
430	499.44	372.95781	2.63347	2.69848	2.57166	2.46760
993.9	500.35	756.23443	2.99734	2.69927	2.87866	2.87360
481	499.35	341.95272	2.68215	2.69841	2.53397	2.41810
272	499.05	247.79066	2.43457	2.69814	2.39408	2.23110
320.9	499.26	312.21426	2.50637	2.69833	2.49445	2.36580
330.7	499.29	321.98591	2.51943	2.69835	2.50784	2.38360
266	499.04	244.89749	2.42488	2.69814	2.38898	2.22420
210	498.95	219.57845	2.32222	2.69806	2.34159	2.15930
136.9	498.62	137.94811	2.13640	2.69777	2.13972	1.87430
126.3	498.58	129.26585	2.10140	2.69773	2.11148	1.83340
140	498.63	140.16003	2.14613	2.69778	2.14662	1.88430
128	498.6	133.57386	2.10721	2.69775	2.12572	1.85410
242.1	498.94	216.84539	2.38399	2.69805	2.33615	2.15180
268.4	499.05	247.79066	2.42878	2.69814	2.39408	2.23110
180	498.8	180.27645	2.25527	2.69793	2.25594	2.04010
140	498.77	172.85363	2.14613	2.69790	2.23768	2.01430
127	498.6	133.57386	2.10380	2.69775	2.12572	1.85410
136.6	498.65	144.63340	2.13545	2.69780	2.16027	1.90390
128	498.59	131.41157	2.10721	2.69774	2.11863	1.84380
110	498.51	114.71140	2.04139	2.69767	2.05961	1.75790
84.29	498.45	102.88782	1.92578	2.69762	2.01236	1.68860
80.29	498.42	97.20292	1.90466	2.69760	1.98768	1.65230

77.28	498.38	89.85953	1.88807	2.69756	1.95356	1.60190
75	498.36	86.28955	1.87506	2.69754	1.93596	1.57590
74.18	498.36	86.28955	1.87029	2.69754	1.93596	1.57590
70.04	498.32	79.35385	1.84535	2.69751	1.89957	1.52220
63.68	498.25	67.87540	1.80400	2.69745	1.83171	1.42200
43.49	498.1	46.13797	1.63839	2.69732	1.66406	1.17720
45	498.12	48.80873	1.65321	2.69733	1.68850	1.21250
34.46	497.99	32.71914	1.53732	2.69722	1.51480	0.96670
25	497.94	27.33862	1.39794	2.69718	1.43678	0.86080
13.57	497.86	19.68081	1.13258	2.69711	1.29404	0.67610
12.87	497.84	17.95152	1.10958	2.69709	1.25410	0.62680
11.2	497.78	13.21466	1.04922	2.69704	1.12106	0.47050
9.3	497.71	8.55734	0.96848	2.69698	0.93234	0.27110
5	497.6	3.19142	0.69897	2.69688	0.50398	-0.08440
4.08	497.56	1.85556	0.61066	2.69685	0.26847	-0.22810
3.5	497.5	0.50238	0.54407	2.69679	-0.29897	-0.45970
3.23	497.49	0.35627	0.50920	2.69678	-0.44822	-0.50030
3.62	497.52	0.86406	0.55871	2.69681	-0.06346	-0.38030
55	498.18	57.24268	1.74036	2.69739	1.75772	1.31320
44.92	498.12	48.80873	1.65244	2.69733	1.68850	1.21250
60	498.19	58.70954	1.77815	2.69740	1.76871	1.32930
1749	501.37	1332.44141	3.24279	2.70016	3.12465	3.22850
460.1	499.52	401.57581	2.66285	2.69855	2.60377	2.50960
467.5	499.52	401.57581	2.66978	2.69855	2.60377	2.50960
326.3	499.29	321.98591	2.51362	2.69835	2.50784	2.38360

289.8	499.15	277.59768	2.46210	2.69823	2.44342	2.29770
247.9	498.95	219.57845	2.39428	2.69806	2.34159	2.15930
210	498.9	206.07405	2.32222	2.69801	2.31402	2.12120
184	498.79	177.78590	2.26482	2.69792	2.24990	2.03160
210	498.88	200.78509	2.32222	2.69800	2.30273	2.10550
323.5	499.27	315.45576	2.50987	2.69834	2.49894	2.37180
405.4	499.39	355.57679	2.60788	2.69844	2.55093	2.44040
485.3	499.59	427.42966	2.68601	2.69861	2.63086	2.54500
309.9	499.24	305.77842	2.49122	2.69831	2.48541	2.35380
280	499.12	268.48879	2.44716	2.69820	2.42893	2.27820
211.6	498.9	206.07405	2.32552	2.69801	2.31402	2.12120
176.8	498.73	163.18482	2.24748	2.69787	2.21268	1.97890
146.1	498.65	144.63340	2.16465	2.69780	2.16027	1.90390
127.6	498.57	127.13673	2.10585	2.69773	2.10427	1.82300
100	498.5	112.69891	2.00000	2.69767	2.05192	1.74660
97.66	498.49	110.70315	1.98972	2.69766	2.04416	1.73530
85	498.41	95.34170	1.92942	2.69759	1.97928	1.63990
77.82	498.58	129.26585	1.89109	2.69773	2.11148	1.83340
68.37	498.32	79.35385	1.83487	2.69751	1.89957	1.52220
85.01	498.41	95.34170	1.92947	2.69759	1.97928	1.63990
75.76	498.35	84.53006	1.87944	2.69753	1.92701	1.56270
91.16	498.46	104.81646	1.95980	2.69763	2.02043	1.70040
98.4	498.5	112.69891	1.99300	2.69767	2.05192	1.74660
90	498.44	100.97600	1.95424	2.69761	2.00422	1.67660
120	498.54	120.84909	2.07918	2.69770	2.08224	1.79090
90	498.44	100.97600	1.95424	2.69761	2.00422	1.67660
82.86	498.4	93.49738	1.91834	2.69758	1.97080	1.62740
120.8	498.55	122.92833	2.08207	2.69771	2.08965	1.80170

119.7	498.56	125.02421	2.07809	2.69772	2.09699	1.81240
120.6	498.58	129.26585	2.08135	2.69773	2.11148	1.83340
95.93	498.47	106.76191	1.98195	2.69764	2.02842	1.71220
77	498.35	84.53006	1.88649	2.69753	1.92701	1.56270
70	498.28	72.69167	1.84510	2.69747	1.86148	1.46590
51.5	498.22	63.21447	1.71181	2.69742	1.80082	1.37650
51.96	498.22	63.21447	1.71567	2.69742	1.80082	1.37650
90	498.43	99.08103	1.95424	2.69760	1.99599	1.66450
65	498.26	69.46360	1.81291	2.69746	1.84176	1.43680
60	498.25	67.87540	1.77815	2.69745	1.83171	1.42200
43.95	498.13	50.17053	1.64296	2.69734	1.70045	1.22980
45	498.13	50.17053	1.65321	2.69734	1.70045	1.22980
37.77	498.04	38.55079	1.57715	2.69726	1.58603	1.06600
34.05	497.97	30.51253	1.53212	2.69720	1.48448	0.92520
35.1	497.99	32.71914	1.54531	2.69722	1.51480	0.96670
41.74	498.08	43.53789	1.62055	2.69730	1.63887	1.14110
38	498.06	41.00875	1.57978	2.69728	1.61288	1.10400
35	497.98	31.60679	1.54407	2.69721	1.49978	0.94610
22	497.92	25.31383	1.34242	2.69716	1.40336	0.81640
12.94	497.86	19.68081	1.11193	2.69711	1.29404	0.67610
12	497.83	17.11492	1.07918	2.69708	1.23337	0.60160
10.2	497.75	11.10286	1.00860	2.69701	1.04543	0.38740
9.63	497.72	9.16455	0.98363	2.69699	0.96211	0.30080
9	497.7	7.96972	0.95424	2.69697	0.90144	0.24100
9.29	497.7	7.96972	0.96802	2.69697	0.90144	0.24100
5	497.62	3.98480	0.69897	2.69690	0.60041	-0.01560
8210	508.28	9020.86866	3.91434	3.95525		-0.01560

2.70610	3.91434