TRAINING COURSE

ON

COMPUTER APPLICATIONS IN HYDROLOGY

(UNDER WORLD BANK AIDED HYDROLOGY PROJECT)

Module 18

Modelling

of

Coastal Aquifers

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MODELLING OF COASTAL AQUIFERS

INTRODUCTION

In coastal areas, groundwater is an important natural resource of freshwater, and is increasingly being used to meet the urban and agricultural water-supply needs. However, indiscriminate groundwater development in coastal regions can have serious consequences in the form of aggressive *seawater intrusion*, which ruins the water quality for human consumption. Protection of water quality in these aquifers requires an understanding of the dynamic relation between freshwater and adjacent seawater.

Coastal Hydrogeologic Conditions

In some areas, coastal hydrogeologic conditions can simply be represented by an individual confined, unconfined or island aquifer. More commonly,the hydrogeologic setting is that of a multi-layer aquifer system (Fig. 1). Under natural, undisturbed conditions, a seaward hydraulic gradient exists with freshwater discharging to the sea. The heavier saltwater flows in from the sea and a wedge-shaped body of saltwater develops beneath the lighter freshwater. Under steady state conditions a stationary interface exists, its shape and position being determined by the freshwater head and gradient.

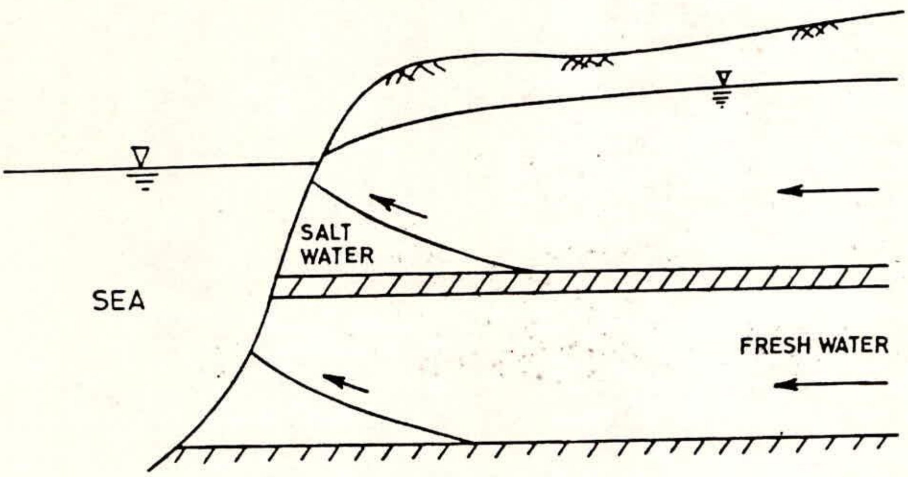

FIG.1: IDEALISED CROSS-SECTION OF A TWO LAYER COASTAL AQUIFER SYSTEM

The seawater intrusion occurs mainly on account of saltwater transport by advection and hydrodynamic dispersion. In reality, due to hydrodynamic dispersion, the zone of contact between freshwater and saltwater takes the form of a *transition zone* (or disperse interface) across which the salt concentration and hence density of water varies from that of freshwater to that of saltwater. In this zone, the diluted saltwater being lighter than original saltwater, rises and moves seaward, causing saltwater from the sea to flow towards the transition zone. This induces a cyclic flow of saltwater from the floor of the sea to the transition zone and finally back to the sea. As a result of this circulation of seawater, the toe of disperse interface is displaced towards the seaward side (Cooper et al., 1964).

Inland changes in recharge or discharge modify the flow within the freshwater region, inducing a corresponding movement of the interface. A reduction in freshwater flow towards the sea causes the interface to move inland and results in the intrusion of saltwater into the aquifer. Conversely, the interface retreats following an increase in freshwater flow. The rate of interface movement is governed by the boundary conditions and aquifer properties.

Thus, in order to plan a sustainable groundwater development of coastal regions, devising an optimal pumping policy is essential. Other alternative methods to control seawater intrusion include construction of physical barriers, injection barriers and extraction hydraulic barriers (Custodio, 1987). For the feasibility and success of the above measures, a prior knowledge of the immediate and long term transient behaviour of a coastal aquifer in response to a given measure is essential. In such cases, mathematical modelling of the coastal aquifer system is an indispensable tool that can simulate the physical complexities, as well as the spatial and temporal variations inherent in a coastal system.

MODELLING APPROACHES

The mathematical analysis of the seawater intrusion problem may involve several simplifying assumptions. Depending upon whether the freshwater and saltwater are taken as miscible or immiscible fluids, there are two distinct approaches to model a coastal aquifer system:

- ***Sharp interface approach***
- ***Miscible transport approach***

The sharp interface approach, which assumes that freshwater and saltwater being immiscible fluids are separated by an abrupt interface, is suitable when the width of transition zone is small relative to the thickness of the aquifer. Otherwise, it is vital to account for the effects of hydrodynamic dispersion using the miscible transport approach which explicitly represents the transition zone.

Sharp Interface Models

For sharp interface models two flow domains, freshwater and saltwater, are considered. These two domains are coupled by the interfacial boundary condition of continuity of flux and pressure. The equation of flow in the freshwater region is (Bear, 1979)

$$\frac{\partial}{\partial x}\left(K_{xf}\frac{\partial h_f}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_{yf}\frac{\partial h_f}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_{zf}\frac{\partial h_f}{\partial z}\right) + Q_f = S_{sf}\frac{\partial h_f}{\partial t} \tag{1}$$

and for saltwater region it is

$$\frac{\partial}{\partial x}\left(K_{xs}\frac{\partial h_s}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{ys}\frac{\partial h_s}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_{zs}\frac{\partial h_s}{\partial z}\right) + Q_s = S_{ss}\frac{\partial h_s}{\partial t} \tag{2}$$

where x and y are the coordinates in the horizontal plane; z is the coordinate along the vertical direction; t is time; h is the hydraulic head; K_x, K_y, and K_z are hydraulic conductivities in x-, y- and z-directions, respectively; S_s is the specific storage; Q is the source/sink term (-ve for sink); and subscripts f and s refer to freshwater and saltwater, respectively.

Due to the computational burden involved in performing a full three-dimensional analysis, several types of simplifying assumptions are employed to obtain the required solution. To reduce the dimensionality of the problem, seawater intrusion is often simulated in two-dimensional horizontal plane by invoking Dupuit assumptions (also known as the hydraulic approach). In this case, the representative equations are obtained by integrating the flow equations (Eqs. 1 and 2) for each fluid over the vertical dimension. The solution of these equations, subject to appropriate boundary conditions, provides the values of freshwater (h_f) and saltwater (h_s) hydraulic heads. The interface elevation (Z_i) is then obtained as follows:

$$Z_i = \frac{\gamma_s}{\gamma_s-\gamma_f}h_s - \frac{\gamma_f}{\gamma_s-\gamma_f}h_f \tag{3}$$

where γ_s is the specific weight of saltwater and γ_f is the specific weight of freshwater, respectively.

Instead of the *two-fluid* approach mentioned above, some of the sharp interface models simulate seawater intrusion based on the *one-fluid* approach which models flow in the freshwater region only. Here it is assumed that the saltwater flow region instantaneuosly adjusts to changes in the freshwater flow region. It is to be observed that the one-fluid approach neglects the influence of saltwater flow on the freshwater head distribution. Therefore, this approach is suitable for reproducing the long term responses only. The two-fluid approach is more appropriate for investigating short term responses (Essaid, 1986).

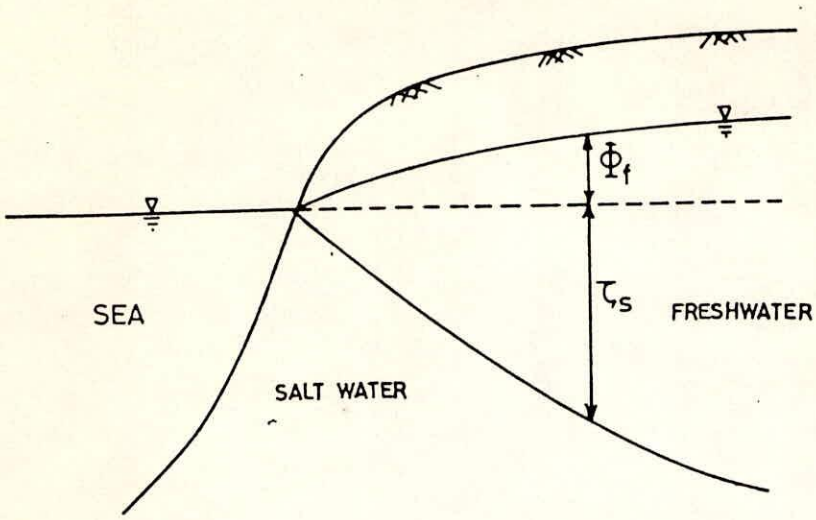

FIG.2 : THE GHYBEN-HERZBERG INTERFACE MODEL

The *Ghyben-Herzberg principle* proposed by Badon-Ghyben (1889) and Herzberg (1901) was the first quantitative analysis of the interface position and is still widely used in field to arrive at a quick estimate of the interface position. It relates the freshwater head above sea level (Φ_f) to the depth of the interface below sea level (ζ_s) for a system in static equilibrium; i.e., steady horizontal freshwater flow and stationary saltwater (Fig. 2). At the interface, the pressure due to the overlying column of freshwater must be equivalent to that due to the column of saltwater, therefore the following relation holds:

$$\zeta_s \gamma_s = (\zeta_s + \Phi_f)\gamma_f$$

$$\text{or} \quad \zeta_s = \delta \Phi_f \tag{4}$$

where $\delta = \gamma_f/(\gamma_s-\gamma_f)$. For common values of freshwater and salwater densities (1.0 gm/cm^3 and 1.025 gm/cm^3, respectively) the value of δ is 40. This implies, the depth to the interface below sea level is forty times the freshwater head. In groundwater literature, the assumption of stationary saltwater is commonly known as the *Ghyben-Herzberg approximation*, and is the basis of the one-fluid approach discussed earlier.

The erroneous result inherent in Eq. (4) is that the thickness of freshwater zone is represented as zero at the shore where the elevation of water table is zero. This is because the Ghyben-Herzberg principle relates the head at the water table to the position of interface.

An improvement upon the Ghyben-Herzberg principle is the relation (Eq. 3) given by Hubbert (1940) which relates the freshwater and saltwater heads at a point on the interface to its elevation.

Solution Strategies

The solution of the governing differential equations subject to appropriate boundary conditions gives the distribution of heads in the freshwater and saltwater zones, and simulates the general position, shape and behaviour of the interface. The analytical solutions to the sharp interface problem are generally derived for steady state flow conditions using the Dupuit assumptions and (or) the Ghyben-Herzberg approximation. The numerical solutions, a majority of them incorporating the Dupuit assumptions, are mostly based on the finite differences or finite elements.

Miscible Transport Models

In miscible transport models, the problem of seawater intrusion is posed as that of a *variable density fluid flow* accounting for the effects of dispersion. These models require the simultaneous solution of the coupled groundwater flow and advective-dispersive equations. For a variable density fluid, the flow equation is (Bear, 1979)

$$\frac{\partial}{\partial x}\left(\frac{\kappa_x \gamma}{\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\kappa_y \gamma}{\mu}\frac{\partial p}{\partial y}\right) + \frac{\partial}{\partial z}\left[\frac{\kappa_z \gamma}{\mu}\left(\frac{\partial p}{\partial z} + \gamma\right)\right] + W\gamma^* = S_s\frac{\partial p}{\partial t} + \phi\frac{\partial \gamma}{\partial c}\frac{\partial c}{\partial t} \tag{5}$$

where p is the fluid pressure; κ_x, κ_y, and κ_z are the intrinsic permeabilities in the x-, y- and z-directions, respectively; γ is the specific weight of fluid; S_s is the specific storage of porous medium; μ is the dynamic viscosity of fluid; γ^* is the specific weight of source or sink fluid; ϕ is porosity; c is solute concentration (defined as the mass of solute per unit volume of solvent); and W is the source/sink volume flux per unit volume of porous medium (+ve for inflow).

The last term on the right side of Eq. (5), represents the rate of change in specific weight due to a change in concentration over time. Since the contribution of this term compared to other terms is small, it is mostly neglected.

The Darcy velocities in x, y and z directions are given by

$$q_x = -\frac{\kappa_x}{\mu}\frac{\partial p}{\partial x} \tag{6a}$$

$$q_y = -\frac{\kappa_y}{\mu}\frac{\partial p}{\partial y} \tag{6b}$$

$$q_z = -\frac{\kappa_z}{\mu}\left(\frac{\partial p}{\partial z} + \gamma\right) \tag{6c}$$

The advective-dispersive equation describing the transport of dissolved salt (assuming no chemical reactions and no interaction with the solid matrix) is (Bear, 1979)

$$\frac{\partial}{\partial x}\left[\phi\left(D_{xx}\frac{\partial c}{\partial x} + D_{xy}\frac{\partial c}{\partial y} + D_{xz}\frac{\partial c}{\partial z}\right)\right] + \frac{\partial}{\partial y}\left[\phi\left(D_{yx}\frac{\partial c}{\partial x} + D_{yy}\frac{\partial c}{\partial y} + D_{yz}\frac{\partial c}{\partial z}\right)\right]$$
$$+ \frac{\partial}{\partial z}\left[\phi\left(D_{zx}\frac{\partial c}{\partial x} + D_{zy}\frac{\partial c}{\partial y} + D_{zz}\frac{\partial c}{\partial z}\right)\right] - q_x\frac{\partial c}{\partial x} - q_y\frac{\partial c}{\partial y} - q_z\frac{\partial c}{\partial z} \tag{7}$$
$$+ W(c^* - c) = \phi\frac{\partial c}{\partial t}$$

where c^* is the solute concentration in the source or sink fluid; and D_{xx}, D_{xy}, D_{xz} etc., are coefficients of hydrodynamic dispersion.

The coefficient of hydrodynamic dispersion is defined as the sum of the coefficient of mechanical dispersion and molecular diffusion in a porous medium. For an isotropic aquifer, the coefficients of hydrodynamic dispersion can be stated explicitly as (Bear,1979)

$$D_{xx} = \alpha_L\frac{v_x^2}{|v|} + \alpha_T\frac{v_y^2 + v_z^2}{|v|} + D_o \tag{8a}$$

$$D_{yy} = \alpha_L\frac{v_y^2}{|v|} + \alpha_T\frac{v_x^2 + v_z^2}{|v|} + D_o \tag{8b}$$

$$D_{zz} = \alpha_L\frac{v_z^2}{|v|} + \alpha_T\frac{v_x^2 + v_y^2}{|v|} + D_o \tag{8c}$$

$$D_{yx} = D_{xy} = (\alpha_L - \alpha_T)\frac{v_x v_y}{|v|} \tag{8d}$$

$$D_{xz} = D_{zx} = (\alpha_L - \alpha_T)\frac{v_x v_z}{|v|} \tag{8e}$$

$$D_{yz} = D_{zy} = (\alpha_L - \alpha_T)\frac{v_y v_z}{|v|} \tag{8f}$$

where α_L is the longitudinal dispersivity; α_T is the transverse dispersivity of the porous medium; D_o is the molecular diffusion coefficient; v_x, v_y and v_z are the components of seepage velocity; and $|v|$ is the magnitude of velocity vector.

Referring again to governing equations, the flow equation can also be formulated in terms of freshwater head by defining a freshwater head ψ as follows:

$$\psi = \frac{p}{\gamma_f} + z \tag{9}$$

The governing equations are coupled via the constitutive equation which relates the specific weight of the fluid to its concentration as follows (Frind, 1982):

$$\gamma(c) = \gamma_f[1 + c(\frac{\gamma_s}{\gamma_f} - 1)] \tag{10}$$

Solution Strategies

The solution of the governing differential equations subject to appropriate boundary conditions provides the spatial and temporal distribution of the salt concentration in the given domain for known values of pumpage and recharge. Due to the mathematical complexity of the miscible transport model the analytical solutions to this problem are few. The semi-analytical solution in vertical cross-section given by Henry (1960) for a hypothetical case of seawater intrusion in a confined coastal aquifer has now become a benchmark against which the numerical solutions are usually tested.

Most of the numerical solutions for miscible transport models are based on the finite element or finite differences method. However, the simultaneous solution of the coupled flow and transport equations is numerically difficult. The difficulty lies in the solution of transport equation which comprises both the advective and dispersive components of transport. The advective component of transport dominates for most of the field problems. Solution of such an equation by conventional techniques viz., finite differences or finite elements is susceptible to *numerical dispersion* (Huyakorn and Pinder, 1983). As is well known, numerical dispersion is a truncation error which crops up while solving the transport equation if the term proportional to the second order is neglected while approximating the first order derivatives. Procedures designed to curb numerical dispersion may require impractical small grid spacings and time steps, or may result in artificial oscillations. Oscillations manifest themselves either as concentration values higher than the maximum value, or negative concentration values.

An alternative technique, amongst others, to minimize the problem of numerical dispersion is the Method of Characteristics (Garder et al., 1964). This technique minimizes numerical dispersion by delinking the advective and dispersive components of transport. The advective transport is simulated through a set of moving particles and the dispersive transport by finite differences or finite elements.

Another important aspect while implementing the miscible transport approach is the formulation of boundary condition on the seaward side of a coastal aquifer. As mentioned previously, due to a natural gradient, freshwater discharges to the sea along with the diluted saltwater. The extent of this outlet portion or *window* on the seaward boundary of an aquifer is unknown a priori. Conventionally, the length of window is determined by trial and error before taking a simulation run (Huyakorn et al., 1987). However, since the window length is subject to an increase/decrease in the freshwater inflow and generally varies with time, it needs to be updated during the simulation itself in order to follow realistic changes in the flow field.

DISCUSSION OF MODELLING APPROACHES

Each of the modelling approaches has advantages and limitations, and can be employed succesfully under appropriate conditions. The sharp interface approach, in conjunction with the hydraulic approach, allows the problem to be reduced by one dimension. Thus, it can be applied areally to large physical systems. This approach does not give information concerning the nature of the transition zone; however it does represent the overall flow dynamics of the system and reproduces the general response of the interface to applied stresses.

The miscible transport approach should be adopted in areas where the transition zone is wide. When concentration gradients are low, the governing equations can be solved areally on a basin-wide scale. However, when the flow is density-dependent, the vertical dimension must be included. Because of computational constraints, studies based on this approach have been generally limited to two-dimensional vertical cross-sections. While simulating the movement of a narrow concentration front, some numerical instabilities and errors may occur, especially in areas where the transition zone approaches a sharp interface.

Volker and Rushton (1987) compared steady-state solutions for both sharp interface and miscible transport approaches and demonstrated that as the coefficient of hydrodynamic dispersion decreases, the two solutions approach each other. The choice of the approach used to model a particular sysytem ultimately depends on the nature of the coastal aquifer system as well as the goals of modelling effort.

REFERENCES

1 Badon-Ghyben, W., 1889, Nota in verband met de voorgenomen putboring nabj Amsterdam (Notes on the probable results of well drilling near Amsterdam), Tijdschr. Kon. Inst. Ing., The Hague, pp. 8-22.

2 Bear, J., 1979, Hydraulics of groundwater, McGraw-Hill, New York, 569 pp.

3 Cooper, H.H., Kohout, F.A., Henry, H.R. and Glover, R.E., 1964, Sea water in coastal aquifers, U.S. Geol. Surv. Water-Supply Pap. 1613-C, 84 pp.

4 Custodio, E., 1987, Methods to control and combat saltwater intrusion, In: Groundwater Problems in Coastal Areas, Studies and Reports in Hydrology, UNESCO, Paris, pp. 396-433.

5 Essaid, H.I., 1986, A comparison of the coupled freshwater-saltwater flow and the Ghyben-Herzberg sharp interface approaches to modelling of transient behaviour in coastal aquifer systems', J. Hydrol., Vol. 86, pp 169-193.

6 Frind, E.O., 1982, Simulation of long-term transient density-dependent transport in groundwater, Advances in Water Resources, Vol. 5, pp 73-88.

7 Garder, A.O., Jr., Peaceman, D.W. and Pozzi, A.L., Jr., 1964, Numerical calculation of multidimensional miscible displacement by the method of characteristics, Soc. Petrol. Eng. J., Vol. 4, No. 1, pp. 26-36.

8 Henry, H.R., 1960, Salt intrusion into coastal aquifers, Int. Assoc. Sci. Hydrol., Publ. No. 52, pp. 478-487.

9 Herzberg, A., 1901, Die wasserversorgung einiger Nordseebaden (The water supply on parts of the North Sea Coast), J. Gasbeleucht. Wasserversorg., Vol. 44, pp. 815-819, 842-844.

10 Hubbert, M.K., 1940, The theory of ground water motion, J. Geol., Vol. 48, No. 8, pp. 785-944.