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COMPARISON OF SOME VARIABLE PARAMETER SIMPLIFIED HYDRAULIC FLOOD ROUTING MODELS

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LIST OF SYMBOLS

A	Flow area
B	Channel width
C	Wave celerity corresponding to discharge Q at a section
C_1, C_2, C_3	Coefficients of the conventional Muskingum routing scheme.
C_1', C_2', C_3'	Coefficients of the Muskingum-Koussis routing scheme
EVOL	relative error in flow volume
F	Froude number
F_o	Froude number corresponding to reference discharge level
I_1 & I_2	Inflow discharges at the beginning and end of the routing time interval
K	Muskingum travel time
K_o	Muskingum travel time corresponding to initial steady flow
N	the total number of discharge ordinates
Q	discharge at any section of the channel reach during unsteady flow; also discharge at section(2) specifically.
Q_o	reference discharge
Q_2	discharge at section(2); also discharge at the end of the routing time
Q_3	discharge at section (3) of the routing reach during unsteady flow
Q_m	discharge at mid-section of the reach
Q_n	normal discharge
Q_{oi}	the ith observed discharge

Q_{ci}	the ith computed discharge using the method under consideration
Q_{PE}	relative error in peak discharge
Q_{pc}	the computed peak outflow discharge
Q_{po}	the observed peak outflow discharge
$\bar{Q}_{oi}$	mean of the observed discharges
q	discharge/unit width
S_o	bed slope
T_{PQE}	error in time to peak discharge
t	notation for time
$t(Q_{pc})$	time corresponding to computed peak discharge
$t(Q_{po})$	time corresponding to observed peak discharge
v_o	velocity corresponding to reference discharge
v_m	velocity at the middle of the reach
y	flow depth
y_m	flow depth at the middle of the reach
y_3	flow depth at section(3) of the routing reach during unsteady flow
Δt	routing time interval
θ	Muskingum weighting parameter
θ_o	θ corresponding to initial steady flow

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ABSTRACT

The three point variable parameter Muskingum-Cunge method envisaged by Ponce and Yevjevich is based on the concept of relating the physical diffusion with the numerical diffusion at every routing time level which is achieved by averaging the discharges and the wave celerities at both the inflow and outflow sections of the previous time level, and at the inflow section corresponding to present time level. It has been found that this method is able to reproduce the St.Venant's solution for a given inflow hydrograph and for a given channel reach closely than the constant parameter Muskingum-Cunge Method. Another variable parameter Muskingum flood routing method proposed by Perumal is based on the concept of linear variation of flow depth along the reach. Like variable parameter Muskingum-Cunge method, in this method also, both the parameters can be varied at every routing time level, but still adopting the linear form of solution. In this report the solutions of both these methods have been compared with the St.Vanant's routing solution for a given inflow hydrograph and the given channel reach. The results show that the variable parameter Muskingum method developed by Perumal performs better than the variable parameter Muskingum-Cunge method developed by Ponce and Yevjevich.

1.0 INTRODUCTION

Numerous one dimensional flood routing models with varying complexities are available in literature. These models range from conventional Muskingum method to the model based on complete solution of St. Venant's equations which fully describe the one dimensional flow in channels and river reaches. Simplified hydraulic models belong to the class of models derived by solving the continuity equation and the simplified momentum equation which is obtained by curtailing or approximating some of the terms present in the the momentum equation of the St. Venant's equations, using the criterion of order of magnitude of these terms with reference to the bed slope. The convection-diffusion equation, kinematic wave equation, and the variable parameter Muskingum method developed by Perumal (1987) are the examples of such simplified models. These sim plified models are non-linear in nature as the parameters involved in these models which characterise the flow phenomena vary with time and magnitude of flow. These simplified models are reduced to linear form by fixing the parameters about a reference discharge. Thus the parameters involved in the non-linear simplified models are kept constant in the case of linear models while simulating the flood wave movement.The linearised St. Venant's equation porposed by Harley (1967), the linearized convection-diffusion equation of Hyami (1951), the Kalinin-Milyukov method (Kalinin-Milyukov,1958), the linear Kinematic wave equation, and its approximations proposed by Cunge (1969) and Koussis (1976) are examples of such models.

It has been found that these linear and non-linear simplified models are sufficient for planning purposes, flood forecasting, preliminary design of hydraulic structures etc. Therefore attempting the solution of full St.Venant's equations for all purposes is a overkill of available computational facilities in the sense that information at numerous locations along the routing reach are produced while they are required only at limited locations. Simplified models give the required information only at desired locations without involving much computations when compared with the solutions of full St. Venant's equations.

Although the highly developed mathematical tools for the analysis of linear systems have rendered linear models attractive choices in flood hydrology (Koussis and Osborne, 1986), the adoption of complete linear models, wherein the parameters are kept constant for the entire event of flood wave movement, does not represent the realistic system. Ofcourse, the wide use of constant parameters simplified hydraulic models such as Kalinin-Milyukov, and Muskingum-Cunge methods in practice demonstrate that the accuracy of routing results is not severely affected.

However, this aspect has not been conclusively proved for all circumstances of flood wave movement. The constant parameters of these models are estimated based on the assumption that the flow variations take place around a reference discharge. This limitation produces distortion, in the predicted outflow hydrograph when wide variations in the flow variable are considered. Keefer and McQuivey (1974) state that if the model is linearized about a high discharge, the low flows arrived too soon and are over damped and if it is linearized around a low discharge the peaks arrived late and are under damped.

The most desirable way the non linearity in the flood routing process may be taken into account is to use such a model that remains linear at one time level, but the linear characteristics may change from one time level to another time level. Thus the parameters involved in the modelling vary from one time step to another time step just as the flow variable involved in the phenomena. This concept has been adopted by Ponce , Yevjevich (1978) & Koussis (1978) while they applied the Muskingum method based on the diffusion analogy principle. Whereas Ponce and Yevjevich (1978) considered the variation of both K and Θ, the travel time and weighting parameter respectively of the Muskingum method, one time step to another, Koussis (1978) considered the variation of K only keeping Θ constant. Recently Perumal (1987) has developed a variable parameter Muskingum method with parameter K and Θ varying from one time level to another time level with the solution procedure remaining linear.

This report is concerned with the evaluation of the performance of some variable parameter simplified hydraulic models in reproducing the St. Venant's solutions. The models considered are the variable parameter Muskingum-Cunge method (Ponce and Yevjevich, 1978) which is considered to be one of the best simplified model available to-date (Koussis and Osborne, 1986), and the variable parameter Muskingum method proposed by Perumal (1987).

2.0 REVIEW

In this section, only those flood routing models which take into account the nonlinearity of the routing process by remaining in the linear solution domain at any time level, but varying the flow characteristics from one time level to another time level have been reviewed. It is well known that the routing process is nonlinear in nature and therefore flood routing models with variable coefficients can be expected to perform better. It has been shown by Keefer and McQuivey (1974) that if the inflow hydrograph into a channel reach is considered in several blocks with each block having its own reference or linearizing discharge then the convolution of these inflow blocks with the corresponding unit hydrographs of the channel reach developed based on the reference discharge of each block yield routed hydrographs comparable well with the observed hydrograph, than that routed hydrograph obtained based on the convolution of the inflow hydrograph with the unit hydrograph corresponding to a single reference discharge for the entire inflow hydrograph. This envisages the need for adopting variable parameter routing models.

Koussis (1978) developed a variable parameter Muskingum method based on the diffusion analogy principle, using the same concept as adopted by Cunge (1969), with constant weighting parameter θ and varying travel time K. Koussis (1978) has found from his experience that θ is not varying considerably with discharge, but K varies with discharge. Koussis varied the value of K at each time step by averaging the travel speed of the flood wave estimated at the upstream and downstream sections of the reach by introducing the correction in the rating curve at the respective sections using "Jones formula" (Henderson, 1966) as given below:

$$Q = Q_n \left(1 + \frac{1}{CS_o} \frac{\partial y}{\partial t}\right)^{1/2} \qquad .....(1)$$

in which,

Q = the discharge at a section during unsteady flow

Q_n = the normal discharge at the same section corresponding to the flow depth y observed during unsteady flow

C = the travel speed corresponding to discharge Q at a section

t = notation denoting time

By iteratively solving equation (1), the travel speed at the upstream and downstream sections may be obtained corresponding to each time level of the Muskingum method solution. Koussis (1978) estimated the outflow discharge Q, using the following expression obtained by assuming linear variation of inflow over the routing time interval Δt:

$$Q_2 = C_1' I_2 + C_2' I_1 + C_3' Q_1 \qquad .....(2)$$

Wherein the coefficients C_1, C_2 and C_3 are given as:

$$C_1' = 1 - \frac{K}{\Delta t}(1 - \beta)$$

$$C_2' = \frac{K}{\Delta t}(1-\beta) - \beta \text{ and} \qquad \ldots\ldots (3)$$

$$C_3' = \beta$$

Where $\beta = e^{-\Delta t/K(1-\theta)}$

Following the same approach of Cunge (1969), Koussis estimated the parameters θ and K in terms of Channel and flow characteristics by relating the numerical diffusion with the physical diffusion. The form of the parameters so estimated are given as:

$$\theta = 1 - \frac{\Delta t/K}{\ln\left(\frac{\lambda+1+\Delta t/K}{\lambda+1-\Delta t/K}\right)} \qquad \ldots\ldots (4)$$

where

$$\lambda = \frac{Q_o}{BS_o C \Delta X}$$

Q_o = Reference discharge.

and $K = \Delta x/C$ ..... (5)

The symbols B and Δx represents respectively, the channel width and reach length. The estimation of discharge at the outflow section requires one more iteration procedure using equation (2) besides the iteration required for the correction of rating curve at downstream section for the estimation of travel speed based on the loop rating curve. Therefore it can be realized that although the Koussis procedure is physically based, it involves tedious iterative computations.

Ponce and Yevjevich (1978) suggested a simple variable parameter method based on the Muskingum-Cunge procedure. Usually the routing time interval being fixed, and Δx and S_o are specified for each computational cell constituting of four grid points, as shown in figure (1), their method involves the determination of flood wave celerity and the unit width discharge, q for each computational cell. The values of c and q at grid point (j,n) are defined by:

$$C = \left.\frac{dQ}{dA}\right|_{j,n} \qquad \ldots\ldots (6)$$

$$q = \left.\frac{Q}{B}\right|_{j,n} \qquad \ldots\ldots (7)$$

in which Q = discharge; A = flow area;

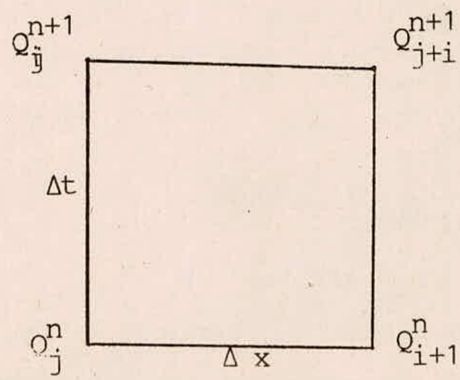

Fig. 1- SPACE TIME DISCRETIZATION OF MUSKINGUM METHOD

The following ways of determining c and q were investigated by Ponce and Yevjevich for the computation of variables θ and K of Muskingum-Cunge method for each time level:

1) directly by using a two point average of the values at grid points (j,n) and (j+1,n);

2) directly by using a three point average of the values at grid points (j,n), (j+1,n) and (j, n+1) and

3) by iteration, using a four point average calculation.

They concluded that three point and four point iterative schemes of varying C and q yield better results and both are comparable. In view of iterations involved in four point scheme, it may be considered that three point average procedure is desirable for use in practice. Besides, this method is also much simpler than the method suggested by Koussis (1978). Recently Koussis and Osborne (1986) have concluded that the four point iterative routing scheme does not offer any advantage over the three point scheme. This conclusion is in agreement with the findings of Ponce and Yevjevich (1978). However both Ponce and Yevjevich's (1978) and Koussis (1978) approaches for varying the parameters of the Muskingum method at each routing time level are arbitrary and not based on the methematics of the Muskingum method solution.

Recently Perumal (1987) has presented a variable parameter simplified hydraulic method based on the approximation of the St. Venant's equations for routing floods, without considering lateral inflow, in channels having uniform rectangular cross-section and constant bed slope. The method was developed by assuming that the friction slope S_f is constant at any instant of time over the channel routing reach, and by adopting the concept that during unsteady flow there exists a one to one relationship, at any instant of time, between the stage at the middle of the routing reach & the discharge downstream of it. The form of the governing equation for obtaining the solution is same as that of Muskingum method which is given as:

$$I - Q = \frac{d}{dt} K \left[Q + \theta \; (I-Q) \right] \qquad ..... (8)$$

in which,

$$K = \frac{\Delta x}{\left[\frac{5}{3} - \frac{4}{3} \left(\frac{Y_3}{B + 2Y_3} \right) \right] v_3} \qquad ..... (9)$$

and

$$\theta = \frac{1}{2} - Q_3 \left[\frac{1}{2} - \frac{\frac{1}{2}\left(\frac{1}{2} - 1\right)}{\underline{|2}} G_m + \frac{\frac{1}{2}\left(\frac{1}{2} - 1\right)\left(\frac{1}{2} - 2\right)}{\underline{|3}} G_m^2 \right.$$

$$\left. + \; .... \right] \frac{\left[1 - \frac{4}{9} F^2 \left(1 - \frac{2y_m}{B + 2y_m}\right)^2 \right]}{S_o B \left[\frac{5}{3} - \frac{4}{3} \left(\frac{y_m}{B + 2y_m} \right) \right] v_m \Delta x} \qquad ..... (10)$$

where

$$G_m = \frac{1 - \frac{4}{9} F^2 \left(1 - \frac{2y_m}{B + 2y_m} \right)^2}{S_o B \left[\frac{5}{3} - \frac{4}{3} \frac{y_m}{(B + 2y_m)} \right] v_m} \frac{\partial Q}{\partial x} \qquad ..... (11)$$

The symbols y_3, v_3 and Q_3 respectively denote the flow depth, velocity and discharge at section (3) which is located downstream of the mid-section of the reach where the discharge during unsteady flow is uniquely related with the flow depth at mid-section of the reach, and Y_m and V_m represent the flow depth and velocity at mid-section of the reach during unsteady flow. F is the Froude number corresponding to flow at the mid-section of the reach. The definition sketch of the reach considered in this study is shown in fig.2. For wide rectangular channels equations (9) and (10) reduce to:

SECTION ①-① : CORRESPONDS TO THE INFLOW POINT

SECTION ②-② : CORRESPONDS TO THE OUTFLOW POINT

SECTION ③-③ : CORRESPONDS TO THE POINT WHERE THE DISCHARGE Q_e IS UNIQUELY RELATED WITH THE STAGE AT THE MIDSECTION OF THE REACH

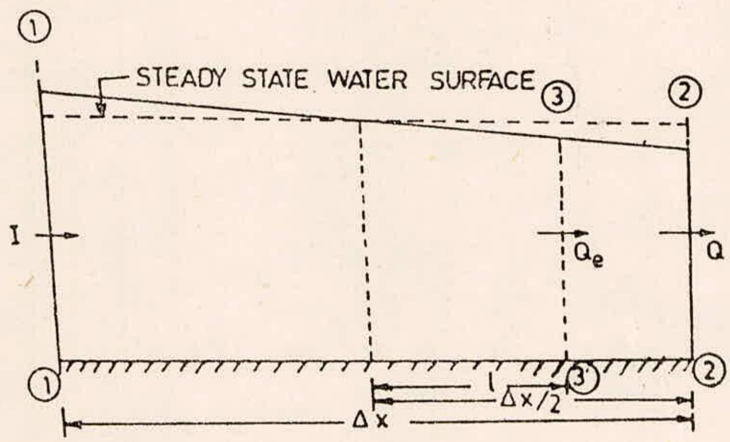

FIG.2- DEFINITION SKETCH OF THE REACH UNDER CONSIDERATION

$$K = \frac{\Delta x}{\frac{5}{3} v_3} \qquad \ldots(12)$$

and

$$\theta = \frac{1}{2} - Q_3 \frac{[\frac{1}{2} - \frac{1/2(1/2-1)}{\lfloor 2}G_m + \frac{1/2(1/2-1)(1/2-2)}{\lfloor 3}G_m^2 + \ldots](1-\frac{4}{9}F^2)}{S_o B (\frac{5}{3} v_m) \Delta x} \qquad \ldots(13)$$

when neglecting the terms G_m, G_m^2, ... etc., θ reduces to

$$\theta = \frac{1}{2} - \frac{Q_3 \ (1 - \frac{4}{9} F^2)}{2S_o B (\frac{5}{3} v_m) \Delta x} \qquad \ldots(14)$$

It was shown that when the variables are fixed corresponding to a reference discharge value Q_o,

$$K = \frac{\Delta x}{\frac{5}{3} v_o} \qquad \ldots(15)$$

and

$$\theta = \frac{1}{2} - \frac{Q_o(1 - \frac{4}{9} F_o^2)}{2S_o B(\frac{5}{3} v_o)\Delta x} \qquad \ldots(16)$$

The above expression for K and θ were obtained by Dooge et al. (1982) based on linear St. Venant's solution approach, for the case of constant parameters Muskingum flood routing method. When the rectangular channel is not wide and after eliminating G_m, G_m^2, ...etc. K and Θ reduce to:

$$K = \frac{\Delta x}{[\frac{5}{3} - \frac{4}{3}(\frac{y_3}{B + 2y_3})] V_3} \qquad ...(17)$$

$$\Theta = \frac{1}{2} - \frac{y_m Q_3 [1 - \frac{4}{9} F^2 (1 - \frac{2y_m}{B + 2y_m})^2]}{2S_o [\frac{5}{3} - \frac{4}{3}(\frac{y_m}{B + 2y_m})] Q_m \Delta x} \qquad ...(18)$$

The developed method employed equations (17) and (18) for routing floods in four different channel having prismatic rectangular cross-section with different constant bed slopes and Manning's roughness coefficients, and the results were compared with the corresponding St. Venant's solutions. three different solution approaches were used for routing floods in each channel corresponding to a reach length 40 km. These approaches consist of considering the entire 40 km. length as a single reach and obtaining the solution by varying Θ and K; considering the entire 40 km. length as a single reach but obtaining the solution by varying K and keeping Θ constant; and considering the 40 km reach consists of 8 equal sub-reaches and obtaining the solution by successively routing through these reaches by varying both Θ and K. It was found that the last solution approach was able to reproduce more closely the St. Venant's solution of both stage and discharge hydrographs when compared with the other two approaches.

The study also brought out the theoretical reason for the reduced outflow in the beginning of the Muskingum solution and suggested the needed remedial measure to avoid it. Also it was shown using the developed theory that for Muskingum method the maximum value of θ is 0.5 and its negative value is admissible.

Comparision of variable parameter Muskingum-Cunge method of Ponce and Yevjevich (1978), and the variable parameter Muskingum method proposed by Perumal (1987) indicate, in general, that the latter is able to reproduce the stage hydrograph information also, which the former cannot do as part of the developed procedure.

3.0 PROBLEM DEFINITION

It is required to compare the solutions of the variable parameter Muskingum-Cunge method developed by Ponce and Yevjevich (1978) and the variable parameter Muskingum method developed by Perumal (1987) with the St.Venant's solutions obtained for a given inflow hydrograph and the given rectangular channel reach with different bed slope and Manning's roughness coefficients. It is assumed that there exists no lateral inflow or outflow within the reach under study.

4.0 METHODOLOGY

The variable parameter Muskingum-Cunge method introduced by Ponce and Yevjevich (1978) and adopted herein for the comparison of its solution with the variable parameter Muskingum method developed by Perumal (1987) is described below:

4.1 Variable Parameter Muskingum-Cunge Method:

The Muskingum method solution is given as:

$$Q_{m+1} = C_1 I_{m+1} + C_2 I_m + C_3 Q_m \qquad \text{..... (19)}$$

where

Q_{m+1} = outflow at time $(m+1)\Delta t$

I_{m+1} = inflow at time $(m+1)\Delta t$

I_m = inflow at time $m\Delta t$

Q_m = outflow at time $m\Delta t$

The coefficients c_1, C_2 and C_3 are given as:

$$C_1 = \frac{\Delta t - 2K\theta}{\Delta t + 2K(1-\theta)} \qquad \text{..... (20)}$$

$$C_2 = \frac{\Delta t + 2K\theta}{\Delta t + 2K(1-\theta)} \qquad \text{..... (21)}$$

$$C_3 = \frac{2K(1-\theta) - \Delta t}{\Delta t + 2K(1-\theta)} \qquad \text{..... (22)}$$

Where, Δt is the routing period. In the Muskingum-Cunge version, the parameters K and θ are calculated as (Ponce and Yevjevich, 1978):

$$K = \frac{\Delta x}{C} \qquad \text{..... (23)}$$

$$\theta = \frac{1}{2}\left(1 - \frac{Q_o}{S_o B C \Delta x}\right) \qquad \text{..... (24)}$$

in which, Δx = reach length; C = flood wave celerity; Q_o = reference discharge; S_o = channel bed slope; B = channel width. The above expression for θ has been derived for wide rectangular channel cross section.

4.1.1. Variable Parameters

Usually, Δt is fixed, and Δx and S_o are specified for each computational cell consisting of four grid points as shown in figure 3.

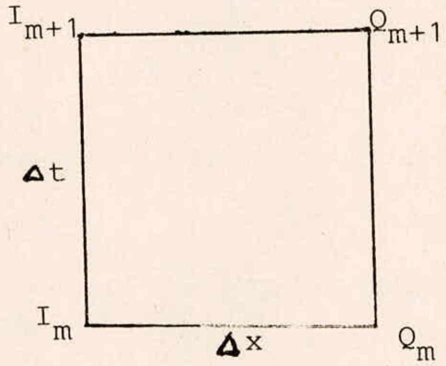

FIG. 3 SPACE-TIME DISCRETIZATION OF MUSKINGUM METHOD

The reference discharge Q_o and the wave celerity C required at time $(m+1)\Delta t$ are computed respectively by taking the three point average of I_m, I_{m+1} and Q_m and

$\left.\frac{dI}{dA}\right|_{m}$, $\left.\frac{dI}{dA}\right|_{m+1}$ and $\left.\frac{dQ}{dA}\right|_{m}$ as follows (Ponce and Yevjevich, 1978):

$$\left. Q_o \right|_{m+1} = \frac{I_m + I_{m+1} + Q_m}{3} \quad \text{..... (25)}$$

$$\left.\frac{dQ}{dA}\right|_{m+1} = \frac{\left.\frac{dI}{dA}\right|_{m} + \left.\frac{dI}{dA}\right|_{m+1} + \left.\frac{dQ}{dA}\right|_{m}}{3} \quad \text{.... (26)}$$

4.2 Variable Parameter Muskingum Method Proposed by Perumal:

The theoretical basis behind this method is described elsewhere (Perumal, 1987). In this section only the procedure for the application of the method is described.

4.2.1. Solution procedure:

The initial parameter values for K and θ viz., K_o and θ_o were evaluated using equations (17) and (18) respectively. Using these parameter values, the coefficients of the conventional Muskingum method were evaluated as:

$$C_1 = \frac{-K_o\theta_o + \Delta t/2}{K_o(1-\theta_o) + \Delta t/2} \quad \text{.... (27)}$$

$$C_2 = \frac{K\ \theta_o + \Delta t/2}{K_o\ (1-\theta_o)+\Delta t/2} \quad \text{.... (28)}$$

$$C_3 = \frac{K_o(1-\theta_o) - \Delta t/2}{K_o(1-\theta_o) + \Delta t/2} \quad \text{.... (29)}$$

Then the discharge Q_2 at the outflow section denoted as section 2, corresponding to inflow I_2 is determined using Muskingum equation as:

$$Q_2 = C_1 I_2 + C_2 I_1 + C_3 Q_1 \quad \text{.... (30)}$$

where,

I_2 = inflow ordinate at the beginning of the current routing time level

I_1 = inflow ordinate at Δt time units ahead of the current routing time level

Q_1 = outflow ordinate at Δt time units ahead of the current routing time level

Knowing I_2 and Q_2 , the discharge at section (3) as depicted in figure (3) was evaluated as:

$$Q_3 = Q_2 + \theta_o \quad (I_2 - Q_2) \quad \text{.... (31)}$$

Corresponding to this discharge, the normal depth at the middle of the reach was evaluated using Newton-Raphson method based on the normal depth discharge relationship as:

$$Q_3 = \frac{B^{5/3} S_o^{1/2}}{n} \quad \frac{y_m^{5/3}}{(B+2y_m)^{2/3}} \quad \text{.... (32)}$$

Then the discharge at the middle of the reach was evaluated as:

$$Q_m = (I_2 + Q_2)/2 \quad \text{.... (33)}$$

Knowing Q_m, y_m, Q_3 and F^2, the new θ was computed using equation (18) corresponding to Q_2. The flow depth at section (3) was evaluated as:

$$y_3 = y_m + (Q_3 - Q_m) \Big/ \left[\left(\frac{5}{3} - \frac{4}{3}\left(\frac{y_m}{B+2y_m}\right)\right)\frac{Q_m}{y_m}\right] \quad \ldots\ldots (34)$$

The velocity v_3 at section (3) was computed as:

$$v_3 = \frac{Q_3}{By_3} \quad \ldots\ldots (35)$$

Knowing v_3 and y_3 and the distance of routing reach Δx, the new travel time K was computed using equation (17).

These revised K and θ values were used for the next step of solution corresponding to the new input ordinate. These steps were repeated for the entire solution procedure, thus varying the values of K and θ at every time step, but at the same time adopting the linear solution procedure. The flow depth at the outflow section corresponding to the solution Q_2 was computed as:

$$y_2 = y_m + (Q_2 - Q_m) \Big/ \left[\left(\frac{5}{3} - \frac{4}{3}\frac{y_m}{(B+2y_m)}\right)\frac{Q_m}{y_m}\right] \quad \ldots\ldots (36)$$

The procedure described above correspond to the variable parameter Muskingum method developed by Perumal (1987). However, the stage hydrograph which can be developed using equation (36) has not been attempted herein as only the discharge can be computed by the variable parameter Muskingum-Cunge method for the purpose of compariso

5.0 APPLICATION

The performance of the variable Parameter Muskingum-Cunge method (VPMC) proposed by Ponce and Yevjevich (1978), and the variable parameters Muskingum method proposed by Perumal were compared by reproducing the St.Venant's solutions obtained by routing floods in a given rectangular channel for a given inflow hydrograph assuming the flow follows Manning's friction law, and there is no presence of lateral inflow or outlow in the channel. This section describes the test series adopted including inflow hydrographs, channel geometry and flow resistance properties, and criteria adopted for comparing the performance of these two methods.

5.1 Test Series:

The test approach for comparing both methods under consideration is to use hypothetical inflow-outflow hydrographs. Accordingly a hydrograph defined by a mathematical function is routed through the given channel for a specified distance using St.Venant's equation, which govern the one dimensional flow in open channels, and thus the 'observed' outflow hydrograph at the end of the specified distance is established. Now the same inflow hydrograph is routed in the same channel using each of the method for the same specified distance, and the resulting respective routed hydrographs are compared with the corresponding St. Venant's solution. The criteria for comparision based on the reproduction of various characteristics of outflow hydrograph are defined at section 5.2. The logic behind the use of hypothetical inflow-outflow hydrographs for verifying such methodologies has been already established (Kundzewicz, 1986).

5.1.1 Inflow hydrographs:

In order to get a better understanding on the performance of both the routing methods considered herein, it was decided to use the same inflow hydrograph for all the test runs. The hypothetical inflow hydrographs defined by a four parameter Pearson type-III distribution which is expressed by the following equation was adopted in this study:

$$Q(t) = Q_b + (Q_p - Q_b) \left(\frac{t}{t_p} \right)^{\frac{1}{(\gamma-1)}} e^{\frac{(1-t/t_p)}{(\gamma-1)}} \quad .... \quad (37)$$

where,

Q_b = base flow = 100 m^3/sec

Q_p = peak flow = 1000 m^3/sec

t_p = time to peak = 10 hours

γ = Skewness factor = 1.15

This hydrograph was adopted by Weinmann (1977) based on the consideration of steepness of hydrograph and magnitude of initial flow. The same hydrograph was also used by Perumal (1987) for testing his variable parameter Muskingum method both for rectangular, and trapezoidal channel cross section reaches. The hydrograph based on equation (37) is shown in all the discharge hydrograph plots presented in this report.

5.1.2 Channel geometry and flow resistance properties:

The rectangular channel with the width of 50 m was used for all the test runs, and the routing computations were tested for their performance on four different channel configurations which are characterised by the following bed slope and friction values as given in Table-1.

TABLE -1 CHANNEL CONFIGURATIONS

Channel Type	Bed Slope	Manning's n-value
1.	0.0002	0.04
2.	0.0002	0.02
3.	0.002	0.04
4.	0.002	0.02

These configurations were earlier adopted by Weinmann (1977) possibly due to the reason that the first two configurations represent a worst case for which the approximate routing procedures are expected to perform poorly, and the last two configurations represent the best case for which they are expected to perform well. The same configurations were used also by Perumal (1987) in testing his variable parameter Muskingum method for rectangular and trapezoidal channel cross section reaches.

Four test runs as indicate in Table-2 were made for comparing the performance of both VPMC method, and the variable Parameter Muskingum Method proposed by Perumal in reproducing the corresponding test run solutions, of St. Venant's equations. In all the runs, the routing time interval Δt was considered as 15 minutes in order to avoid any numerical error in the solutions using equations (19) and (30). The solutions were obtained by considering the single reach for all runs.

TABLE - 2 TEST RUN DETAILS

Test Run No.	Channel type	Reach length in km.	Bed slope	Manning's n-value
1	1	40	0.0002	0.04
2	2	40	0.0002	0.02
3	3	40	0.002	0.04
4	4	40	0.002	0.02

5.2 Comparison Criteria

The following comparison criteria were adopted for checking the effeciency of both methods viz, the VPMC, and the Variable parameter Muskingum method proposed by Perumal in reproducing the St.Venant's solution for the given rectangular channel and the given inflow hydrograph:

5.2.1 The hydrograph fitting consideration

The closeness with which the solutions of both routing methods, follows the true solution i.e. the St.Venant's solution, including the closeness of shape and size of hydrograph, can be measured using the criterion of variance explained by each of the method. The expression for variance explained in % is given as:

$$\text{Variance explained in \%} = \frac{\text{Total variancae} - \text{Remaining variance}}{\text{Total variance}} \quad ..(38)$$

where,

$$\text{the total variance} = \frac{1}{N}\sum_{i=1}^{N}(Q_{0i} - \bar{Q}_{oi})^2 \quad ....(39)$$

$$\text{the remaining variance} = \frac{1}{N}\sum_{i=1}^{N}(Q_{oi} - Q_{ci})^2 \quad ....(40)$$

Q_{oi} = the ith discharge observation

$\bar{Q}_{oi}$ = mean of the observed discharges

Q_{ci} = the ith discharge computed using the method under consideration

N = the total number of discharge ordinates.

5.2.2 Magnitude of flood peak consideration

Relative error in peak discharge in % is given as:

$$Q_{PE} = \frac{(Q_{pc} - Q_{po})}{Q_{po}} \times 100 \quad ....(41)$$

Where,

Q_{pc} = the computaed peak outflow discharge by the method under study

Q_{po} = the observed peak outflow discharge

5.2.3 Time of peak consideration:

Error in time of peak discharge (hours) is given as:

$$T_{PQE} = t\,(Q_{pc}) - t\,(Q_{po}) \qquad \ldots(42)$$

where,

$t\,(Q_{pc})$ = time corresponding to computed peak discharge

$t\,(Q_{po})$ = time corresponding to observed peak discharge

5.2.4 Conservation of mass consideration:

The relative error in the flow volume in percentage of the total inflow volume is expressed as:

$$EVOL = \frac{\sum_{i=1}^{N} Q_{ci} - \sum_{i=1}^{N} I_i}{\sum_{i=1}^{N} I_i} \times 100 \qquad \ldots(43)$$

where, 13.

I_i = the ith inflow discharge

6.0 RESULTS AND DISCUSSION

Table 3 presents the results of variance explained, relative errors in peak discharge, errors in time to peak discharges and the relative error in flow volume for all 4 test runs made in this study for the purpose of comparing the performance of VPMC method proposed by Ponce and Yevjevich (1978) and variable parameter Muskingum method proposed by Perumal (1987). Figures 4 to 7 show the inflow hydrograph, the outflow hydrographs computed by these two routing methods and the respective St.Venant's solutions for test run Nos. 1 to 4 respectively. It is seen from these plots that the performance of the variable parameter Muskingum method proposed by Perumal is better able to reproduce the St.Venant's solutions when compared with the VPMC method proposed by Ponce and Yevjevich (1978).

It is seen from Table 3 that the variance explained by Perumal's method is always greater than that by VPMC method. The variance explained by the Perumal's method for run Nos. 2-4 was more than 99%, and for run No. 1, it was 97.28%. The corresponding value for run no. 1 of VPMC method was 87.21% only.

The conservation of mass is maintained well by the Perumal's method as seen from Table 3. In all the runs the volume difference created was $< |0.5|$ %. But comparatively in the case of VMPC method the conservation of mass was not satisfied. The volume computed by this method was always more than the total inflow volume supplied. The

TABLE - 3 COMPARISON OF RESULTS

Test Run No.	Channel Type	Variance explained in %		Q_{PE} in %		T_{PQE} (hr)		EVOL	
		method-1*	method-2**	method-1	method-2	method-1	method-2	method-1	method-2
1	1	87.21	97.28	3.01	-6.54	--1.00	-0.25	15.91	-0.44
2	2	96.75	99.37	2.15	-1.81	-0.50	0.0	7.12	-0.49
3	3	97.89	99.26	-0.50	-1.21	-0.25	0.0	3.34	-0.34
4	4	99.60	99.99	-0.2	0.0	0.0	0.0	1.47	-0.29

Note:

* Method-1 Variable Parameter Muskingum - Cunge method

**Method-2 Variable Parameter Muskingum Method proposed by Perumal

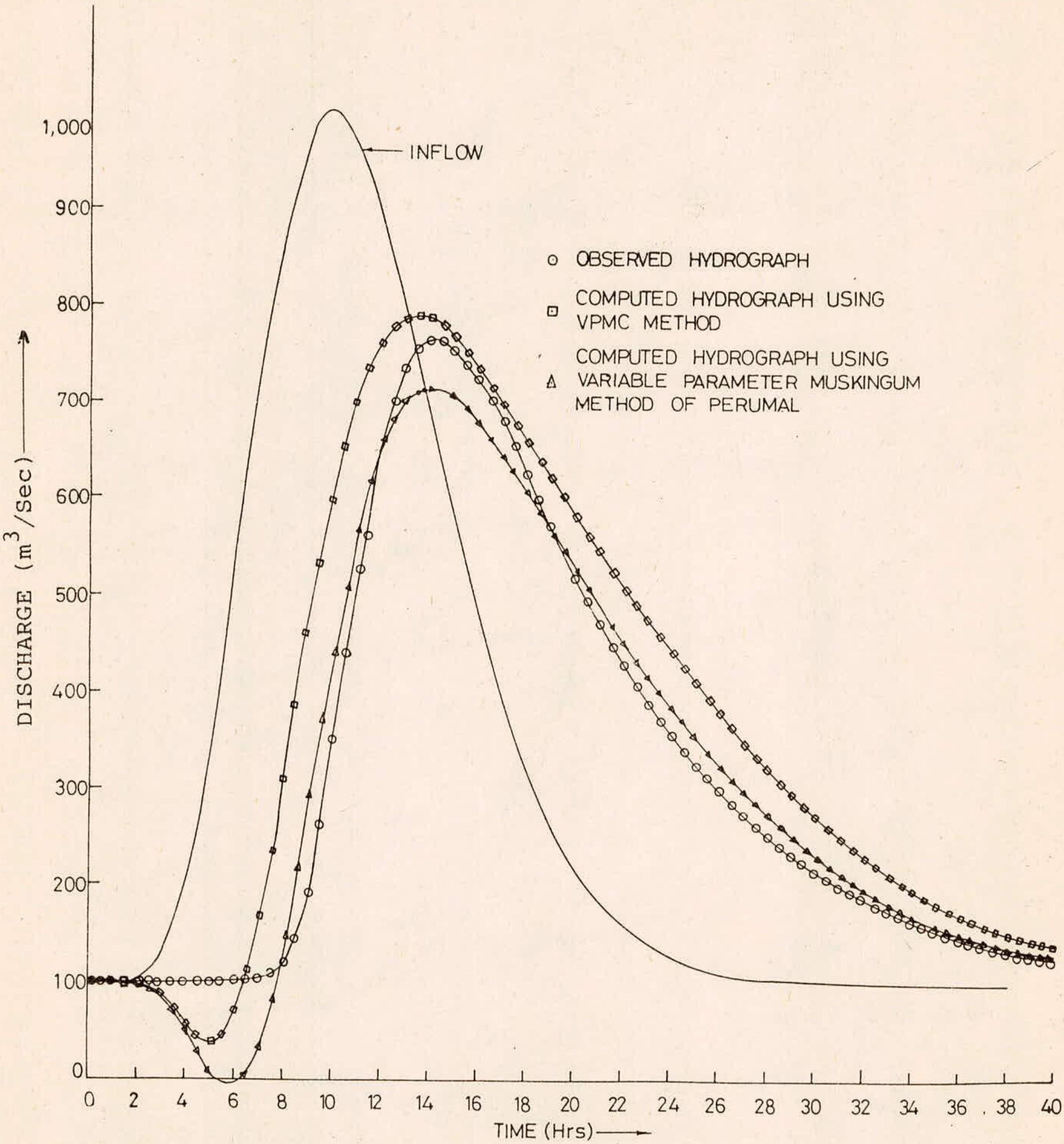

FIG. 4. COMPARISON OF VARIABLE PARAMETER MUSKINGUM METHODS' SOLUTIONS WITH St. VENANT'S SOLUTION (FOR CHANNEL TYPE-1)

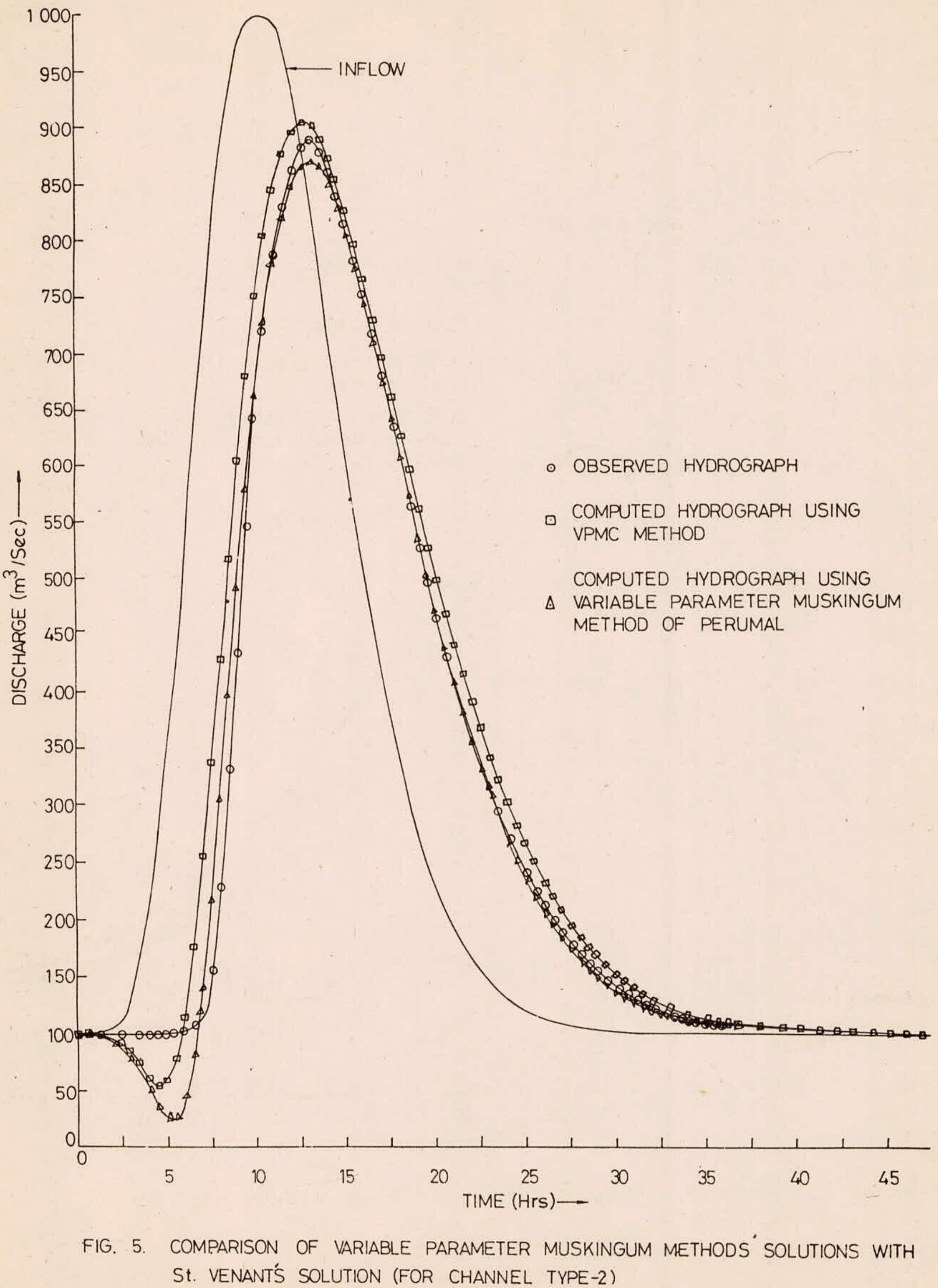

FIG. 5. COMPARISON OF VARIABLE PARAMETER MUSKINGUM METHODS' SOLUTIONS WITH St. VENANT'S SOLUTION (FOR CHANNEL TYPE-2)

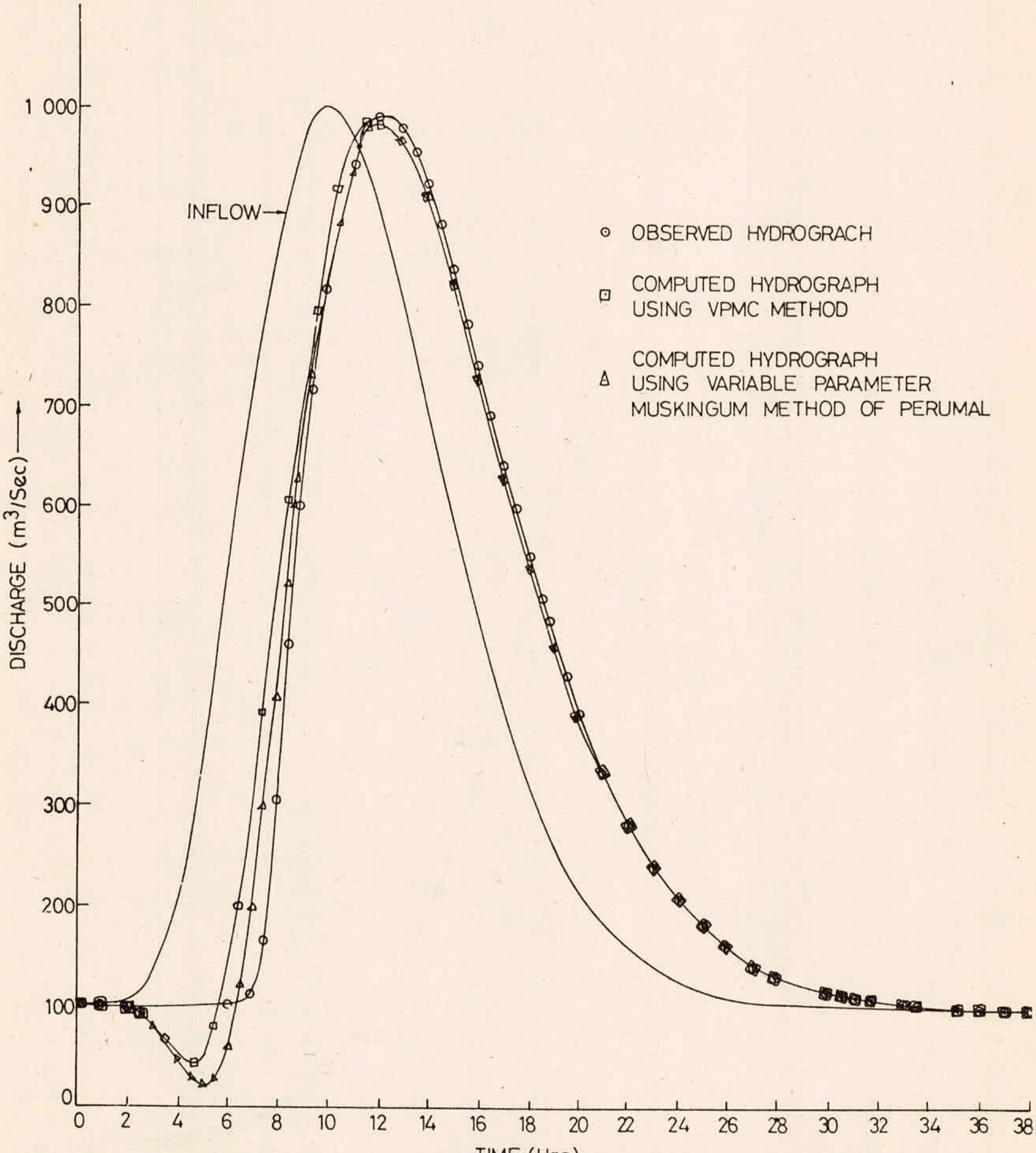

FIG. 6. COMPARISON OF VARIABLE PARAMETER MUSKINGUM METHODS' SOLUTIONS WITH St. VENANT'S SOLUTION (FOR CHANNEL TYPE-3)

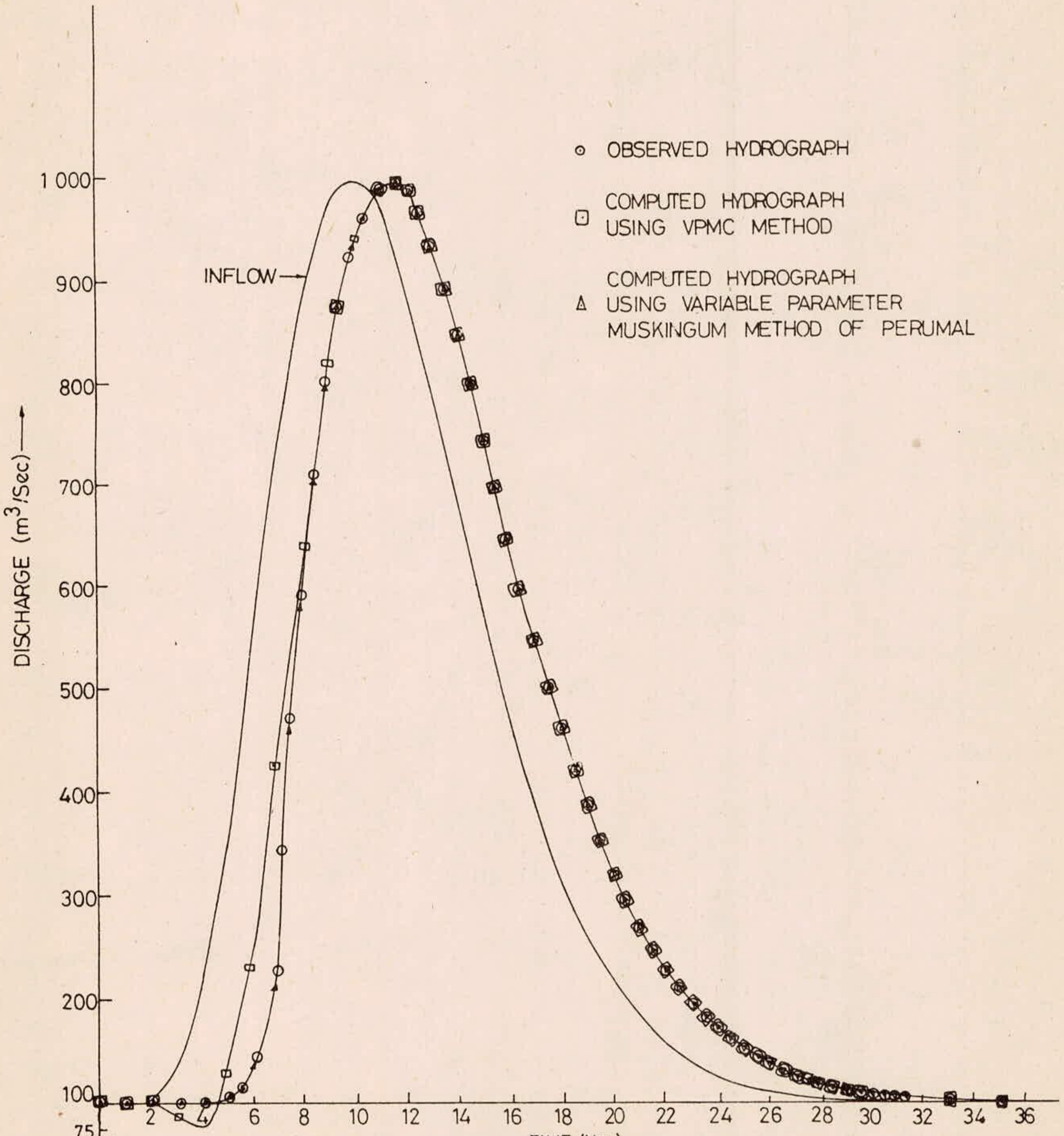

FIG. 7. COMPARISON OF VARIABLE PARAMETER MUSKINGUM METHODS' SOLUTIONS WITH St. VENANT'S SOLUTION (FOR CHANNEL TYPE-4)

conservation of mass by this method was poor in the case of test run No.1, with the computed volume exceeding the total inflow volume by about 16%.

It can be seen from table-3 that the reproduction of peak flows and time to peak by both models are comparable with the observed values.

The overall poor performance of the VPMC method over the Perumal's method may be attributed to the reason that the variation in the parameters from one time level to another time level is achieved in a rational manner in the latter case while it is not so in the former case. The approach of averaging the wave celerities and the reference discharges by the former method as given by equations (25) and (26) is arbitrary.

An added advantage of Perumal's method is that it can also produce stage hydrograph as part of the developed procedure. This aspect is important for flood forecasting purposes.

However both these method do not perform well for run No. 1 i.e. when the bed slope is very small and channel roughness is high. But the performance of Perumal's method is better than the VPMC method in this case.

But when the bed slope increases and bed roughness decreases, both methods reproduce the St. Venant's solutions closely, with the performance of variable parameter Muskingum method proposed by Perumal (1987) being better than that of Ponce and Yevjevich (1978) always.

7.0 CONCLUSIONS

It is concluded from this comparative study of variable parameter Muskingum-Cunge method proposed by Ponce and Yevjevich (1978) and the variable parameter Muskingum method proposed by Perumal (1987) that the latter is able to reporduce the St. Venant's solution hydrograph more closely than the former, especially with regard to the characteristics of overall reproduction of the hydrograph and the conservation of mass.

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