STREAM AQUIFER INTERACTION FOR A FULLY PENETRATING STREAM

Let the aquifer and the river be at initially rest condition. Let, there be a sudden drawdown of magnitude σ in the river stage and the river stage remains unchanged in the new position.

Solution to the one dimensional Boussinesq isequation that satisfies the above initial and boundary condition is given by

$$S(x,t) = \sigma\left[1-\text{erf}\left(\frac{x}{\sqrt{4\frac{T}{\phi}t}}\right)\right]$$

in which $S(x,t)$ is the drawdown at a distance x from the river bank, t is the time measured since the sudden change in the river stage, σ, took place, and $\text{erf}(X) = \frac{2}{\sqrt{\pi}}\int_0^X e^{-v^2}dv$. The return flow to the river from both sides of the river bank from areach of length, L_r, is given by

$$q_r(t) = 2L_r T \frac{\partial S}{\partial x}\Big|_{x=0} = 2L_r T\left[\frac{2}{\sqrt{\pi}} e^{-\frac{x^2}{4\frac{T}{\phi}t}} \frac{1}{\sqrt{4\frac{T}{\phi}t}}\right]_{x=0}$$

$$= \frac{2L_r T}{\sqrt{\pi\frac{T}{\phi}t}}$$

If the river stage changes with time the return flow can be computed as follows. Let the time span be discretised by uniform time step. The time step size may be a day or a week or a month. The return flow during n^{th} unit time step is given by

$$q_r(n) = 2L_r\sqrt{\frac{T\phi}{\pi}}\int_0^n \frac{\partial\sigma(\tau)}{\partial\tau}\frac{d\tau}{\sqrt{t-\tau}}$$

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$$= 2L_r\sqrt{\frac{T\phi}{\pi}} \int_0^1 \frac{\partial\sigma(\tau)}{\partial\tau} \frac{d\tau}{\sqrt{t-\tau}}$$

$$+ 2L_r\sqrt{\frac{T\phi}{\pi}} \int_1^2 \frac{\partial\sigma(\tau)}{\partial\tau} \frac{d\tau}{\sqrt{t-\tau}}$$

$$+ 2L_r\sqrt{\frac{T\phi}{\pi}} \int_{\gamma-1}^{\gamma} \frac{\partial\sigma(\tau)}{\partial\tau} \frac{d\tau}{\sqrt{t-\tau}}$$

$$+ 2L_r\sqrt{\frac{T\phi}{\pi}} \int_{n-1}^{n} \frac{\partial\sigma(\tau)}{\partial\tau} \frac{d\tau}{\sqrt{t-\tau}}$$

Let the change in river stage during a particular time step be uniform i.e. $\frac{\partial\sigma}{\partial\tau}$ changes from time step to step. Then

$$q_r(n) = 2L_r\sqrt{\frac{T\phi}{\pi}} \sum_{\gamma=1}^{\gamma} [\sigma(\gamma) - \sigma(\gamma-1)] \int_{\gamma-1}^{\gamma} \frac{d\tau}{\sqrt{n-\tau}}$$

Integrating

$$q_r(n) = 4L_r\sqrt{\frac{T\phi}{\pi}} \sum_{\gamma=1}^{n} [\sigma(\gamma) - \sigma(\gamma-1)](\sqrt{n-\gamma+1} - \sqrt{n-\gamma})$$

Let a coefficient $\delta(m)$ be defined as

$$\delta(m) = 2L_r\sqrt{\left(\frac{T\phi}{\pi}\right)} \int_0^1 \frac{d\tau}{\sqrt{m-\tau}} = 4L_r\sqrt{\left(\frac{T\phi}{\pi}\right)} (\sqrt{m} - \sqrt{m-1})$$

The return flow $q_r(n)$ can be expressed as

$$q_r(n) = \sum_{\gamma=1}^{n} [\sigma(\gamma) - \sigma(\gamma-1)]\delta(n-\gamma+1)$$

Using this relation the return flow during time period n can be estimated for varying river stage for a fully penetrating river.