REVIEW OF DIFFERENT SOFTWARES AVAILABLE FOR GROUNDWATER FLOW MODELS

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PREFACE

Application of large numerical simulation models to groundwater problems involves both art and science. Successful modellers understand the science behind the models they use and have considerable experience in applying models to practical problems. The art of modelling is learned by practicing how to apply models. In the process, one learns how to describe the problem domain, select boundary conditions, assign model parameters, and calibrate the models.

In other words, one becomes a successful modeller by knowing the science and practicing the art. A detailed review of the widely used groundwater flow models is a prerequisite to the practicing art and science of groundwater modelling. The present is an effort in this direction wherein the groundwater flow models that are widely used for the modelling of groundwater systems, have been reviewed in terms of modelling philosophies for different components of groundwater system, different type of internal and external boundary conditions, temporal variation of external stresses, etc.. Four such models namely PLASM (developed by Prickett and Lonnquist, 1971), MODFLOW (developed by McDonald and Harbaugh, 1988), GWFL3D (developed by Walton) and the model developed by Boonstra and deRidder, have been reviewed.

The present study has been carried out by Shri S.K. Singh, Scientist 'C' under the guidance of Dr. P.V. Seethapathi, Scientist 'F' as per the work programme of the Ground Water Modelling and Conjunctive Use Division.

(S.M. Seth)

Director

Abstract

Groundwater modelling is a powerful tool that can aid in studying groundwater problems and help increase our understanding of groundwater system facilitating the prediction and management of groundwater.

The groundwater flow models that are widely used for the modelling of groundwater systems, have been reviewed in terms of modelling philosophies for different components of groundwater system, different type of internal and external boundary conditions, temporal variation of external stresses, etc.. The models that have been reviewed includes PLASM (developed by Prickett and Lonnquist, 1971), MODFLOW (developed by McDonald and Harbaugh, 1988), GWFL3D (developed by Walton) and groundwater flow Model developed by Boonstra and deRidder. For each model detailed development of flow equations and codes, modelling techniques and capabilities along with the limitations for modelling specific components have been discussed.

Contents

1.0 INTRODUCTION

The prediction of the behavior of groundwater system has been one of the main concerns of groundwater hydrologists as most of the management decisions are based on the probable future behaviour of the system. This involves formulating a correct conceptual model, selecting parameter values to describe spatial variability within the groundwater flow system, as well as spatial and temporal trends in hydrologic stresses and past and future trends in water levels. Although some decisions can be made using best engineering and best geological judgments, in many instances especially for complex groundwater problems, human reasoning alone is inadequate. The best tool available to help groundwater hydrologists meet the challenge of prediction is usually a groundwater model.

A model is a device that represents the physical processes involved in a physical system to some approximation. 'Laboratory sand tank model' is a 'physical model' that simulates the groundwater flow directly. a 'mathematical model' simulates groundwater flow indirectly by means of governing equations that represent the physical processes in the system, together with the equations that describe the conditions at the boundary. For unsteady problems, the initial conditions are also needed. Mathematical models can be solved analytically or numerically.

The analytical methods give accurate solutions and have the advantage of immediate availability and give a good insight into the dependence of solution on various physical parameters. But this advantage may partly or completely be lost when the form of solution is very complicated. The main limitation of the analytical methods is that they are available only for relatively simple problems and require the boundary of the system to be of regular geometric shape. For most of the problems of practical interest with irregular shape of the boundaries, spatial

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variability in parameters and boundary conditions, non uniformity of initial conditions, the analytical solution is not feasible and virtually impossible except for very simple cases.

Most groundwater modelling efforts are aimed at predicting the consequences of a proposed action. However, there are two other important type of applications, viz, i) model can be used as an interpretive sense to gain insight into the controlling parameters in a site-specific setting as a framework for studying system dynamics and/or organizing field data. This type of modelling effort does not necessarily require calibration; ii) models can also be used to study processes in generic geologic settings. Generic modelling studies are used to analyze flow in hypothetical hydrogeologic systems and may be useful to help frame regulatory guidelines for a specific regions. This type of modelling effort too does not necessarily require calibration.

The groundwater models can be divided broadly into two categories: flow models and solute transport models. Groundwater flow models solves the distribution of head, where as solute transport models solve for concentration of solute as affected by advection, dispersion, etc..

The present report deals with the review of different softwares available for groundwater flow models. Four such models based on finite difference code (that are widely used) namely PLASM (developed by Prickett and Lonnquist, 1971), MODFLOW (developed by McDonald and Harbaugh, 1988), GWFL3D (developed by Walton) and the model developed by Boonstra and deRidder, have been reviewed in detail w.r.t. modelling philosophies for different components of groundwater system, different type of internal and external boundary conditions, temporal variation of external stresses, etc.. Chapter 2 deals with the PLASM, chapter 3 Boonstra and deRidder model, chapter 4 deals with MODFLOW and CHapter 5 deals with GWFL3D.

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2.0 PLASM

PLASM (Prickett-Lonnquist Aquifer Simulation Model) was developed by Prickett and Lonnquist(1971) and was one of the first readily available, well documented groundwater flow models.

PLASM is a finite difference model that can simulate two-dimensional groundwater flow problems in both areal and profile orientation. It is a block centred model but can accommodate mess-centred flux boundary conditions. PLASM requires the user to assign a storage parameter (either storage coefficient or specific yield) to the grid area and to specify transmissivities for the paths between the nodes according to the scheme illustrated in Fig.2.1.

In the original PLASM, uniform grid spacing was assumed and the storage factor was specified equal to storage coefficient (or specific yield) multiplied by $\Delta x \Delta y$. In the 1981 version for IBM compatible microcomputers, the input was changed so that the user now specifies transmissivity (or hydraulic conductivity), storage coefficient (or specific yield), and arrays of Δx and Δy. The model internally performs the corrections required for irregular nodal spacing. The user may select inconsistent units of gallons, feet, and days, or consistent units of cubic meters, meters, and days.

A specified head boundary is simulated by specifying a high value (usually 1E21) for the storage coefficient at specified head nodes. In this way a large volume of water is made available, preventing the assigned heads at specified head nodes from changing during the simulation. For simulating specified flow conditions, PLASM allows fluxes across the water table to be input directly. Leakage to or from a river, lake, or reservoir can be simulated using head-dependent conditions. The model handles all

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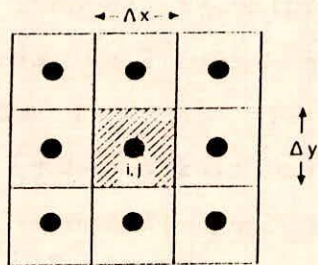
Area of influence for
storage coefficient $(S_{i,j})$,
recharge $(R_{i,j})$ & pumping $(Q_{i,j})$

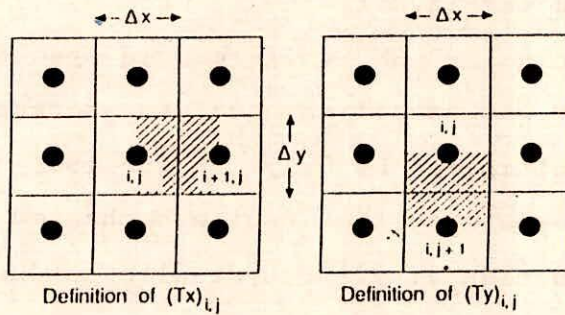Definition of $(Tx)_{i,j}$ Definition of $(Ty)_{i,j}$

Fig.2.1 Definition of Areas of Influence in PLASM

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head-dependent boundaries by input of leakance (K_z'/b'), source bed head, and a reference elevation. The flux or leakage rate is calculated using following equation.

$$Q_L/A = (K_z'/b')(h_{source} - h) \qquad \ldots(2.1)$$

where,

Q_L = volumetric flux;

A = area of the cell through which leakage occurs;

h_{source} = head in the source reservoir (e.g., a lake or river);

h = head in the aquifer immediately below or adjacent to the source;

K_z' = vertical hydraulic conductivity of the interface(e.g., river bed sediments) separating the aquifer from the source;

b' = thickness of the interface.

The head dependent boundary option in PLASM can be used to represent seeps and springs. In the PC version of the PLASM, all head dependent conditions may be simulated using the following three variables:

1. leakance of the source bed sediments (R);

2. head in the source reservoir (RH); and

3. a reference elevation (RD).

Leakance is calculated using the following equation.

$$\text{Leakance} = K_z'/b' \qquad \ldots(2.2)$$

R,RH, and RD may not vary with time but the code could be modified to allow for time dependence. All head dependent flows are assumed to follow a linear relation where the flux is calculated from

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$$Q = R(RH-h)\Delta x\Delta y \qquad \text{for } h > RD \qquad\qquad \ldots(2.3)$$

$$Q = R(RH-RD)\Delta x\Delta y \qquad \text{for } h \leq RD \qquad\qquad \ldots(2.4)$$

where, h = head in the aquifer as calculated by the model.

The head-dependent option in P1ASM can be used to simulate leakage through a confined bed or leakage through river or lakebed sediments. When simulating leakage through a leaky confining bed, R is the leakance of the confining bed as defined in (2.2), RH is the head in the over laying aquifer, and Rd is the elevation of the confining bed. When the head in the aquifer falls below RD, the aquifer converts to unconfined conditions. In case of leakage to or from a river or lake, R is the leakance of the riverbed or lakebed sediments, RH is the head in the river or lake, and RD is the elevation of the riverbed or lakebed sediments.

Flow to drains can be simulated using equations (2.2) through (2.4) by setting RH equal to the elevation of the drain. Where RD is the elevation at which flow to the drain stops and is generally equal to the elevation of the drain (RH). In this case, R represents the conductance of the drain pipe and backfill.

Springs can be simulated as drains where RH represents the seepage elevation, usually the ground surface. In this case, RD is assumed equal to RH. The value of R controls the rate of flow and is estimated from field measurements of head and spring discharge or is estimated during model calibration.

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3.0 BOONSTRA AND deRIDDER MODEL

Boonstra and deRidder (1981) developed a finite difference groundwater flow model based on the modelling ideas of Tyson and Weber (1963) which includes some of the ideas developed by Dietrich and Goodwill (1972). The model essentially simulates the two dimensional steady or unsteady flow in a groundwater basin. Dupuits assumptions for the groundwater flow in the aquifer has been adopted. Only one aquifer system, i.e., single layer of aquifer, bounded at the bottom by an impermeable layer can be modelled. A confined, or semi-confined or unconfined layer as the upper boundary of the aquifer, can be simulated. Effect of storage in a semi-confining layer at the top of the aquifer has been neglected and hence, cannot be simulated. Horizontal flow in the semi-permeable layer forming upper boundary of the aquifer and flow through the semi-permeable layer can be modelled.

The model deals with the saturated flow in the aquifer. The flow in unsaturated zone which governs the processes of evapotranspiration, percolation and infiltration, has not been modelled. Hence, all the external stresses on the aquifer need to be calculated separately which form an input to the model in the form of net recharge to the aquifer. An additional feature of the model is that a range between which the water table in the aquifer is allowed to vary, can be prescribed. If the water table exceeds a certain limit, the model introduces an artificial flow rate that keeps the calculated water table within that limit.

3.1 Physical Background:

The model is based on the two main equations which govern the flow of groundwater through the aquifer. These are momentum equation (in the form of Darcy's law) and the continuity equation (in the form of mass-conservation equation). Both the above equations can be represented by the following differential

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equation for the unsteady flow in an aquifer, overlain by a semi permeable layer.

$$\frac{\partial}{\partial x}(KD\frac{\partial h}{\partial x})+\frac{\partial}{\partial y}(KD\frac{\partial h}{\partial y})=R-P-\phi\frac{\partial h}{\partial t}+\frac{K'}{D'}(h'-h) \qquad \ldots(3.1)$$

where,

K = hydraulic conductivity of the aquifer for horizontal flow, directional anisotropy has not been taken into account, (LT^{-1});

D = saturated thickness of the aquifer, (L);

h = hydraulic head (piezometric head) in the aquifer, (L);

R = net rate of recharge, zero for confined and semi confined aquifers, (LT^{-1});

P = net rate of abstraction, (LT^{-1});

ϕ = storage coefficient for confined and semi confined aquifer or specific yield for unconfined aquifers, (dimensionless);

t = time, (T);

h'= hydraulic head in the covering layer, (L);

K'= vertical hydraulic conductivity of the semi-permeable layer which forms the upper boundary of the aquifer, zero for the confined aquifers, (LT^{-1});

D'= saturated thickness of the semi-permeable layer, (L).

In practice, the specific yield may be considered to equal the drainable pore space (effective porosity) because in unconfined aquifers, the effect of elasticity of both the fluid and the aquifer material are generally so small that they can be neglected. It has been assumed that the values of storage coefficient are equal for both upward and downward movement of water table.

It has been assumed that the saturated thickness D' in the (semi-permeable) covering layer is less than the thickness D of the aquifer and its vertical hydraulic conductivity K' is low as compared to that of the aquifer material. The horizontal flow in the covering layer has been neglected (in exact formulation the

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covering layer has been assumed to consists of anisotropic material with a hydraulic conductivity K' in vertical direction and zero hydraulic conductivity in all horizontal directions.

The differential equation governing one-dimensional flow of groundwater in the covering layer is:

$$R - \phi \frac{\partial h}{\partial t} = \frac{K'}{D'} (h' - h) \qquad \qquad ...(3.2)$$

where, ϕ' is the specific yield of the covering layer.

In the formulation of above equation, it has been assumed that the covering layer has a free water table, so that its saturated thickness D' is not constant but may vary with time. Because of its low permeability pumping has not been considered from this layer.

3.2 Discretization:

The solution of the above differential equations governing the flow through confined/semi-confined/unconfined aquifer using finite difference method requires the discretization of space and time. In this model, the space has been discretized into a number of small but finite areas. Each sub-area thus formed, is assigned a node to which all the hydrogeologic parameters of the sub-area (also called nodal area) are referred. It is assumed that all the inflow and outflow pertaining to a nodal area occur at the node of the area. The values of storage coefficient/specific yield and piezometric head assigned to a node are representative for that nodal area. Thus, each node is considered to be representative of its nodal area. It has been assumed that the flow occurs from a node to another node, hence, a certain value of hydraulic conductivity is assigned to the path which connects two neighbouring nodal area through their nodes.

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The accuracy of the results depends upon the size of the nodal area (mesh size). The finer the mess size, the more accurate the result is. In order to discretize a groundwater basin into a number of nodal areas, a network of rectangles, squares, or polygons is superimposed upon it. Guidelines for the design of nodal network has been given by Boonstra and Ridder, Page 84-90.

Since, the recharge and the abstraction at nodes changes with time and thus piezometric head changes with time, the model also requires the discretization of time. Thus, the total time of simulation is divided into a number of successive time steps. For more accurate results the time step should be finer. At the end of each time step the piezometric head distribution over the entire space of aquifer are computed and the calculation is repeated for successive time steps.

3.3 Finite Difference Formulation:

Replacing the continuous derivatives appearing in (3.1) and (3.2) by their finite difference approximation (fig.3.1), we get:

$$\sum_i (h_i - h_b) \frac{W_{i,b} K_{i,b} D_{i,b}}{L_{i,b}} = A_b P_b + A_b S_b \frac{\Delta h_b}{\Delta t} - \frac{A_b K'_b}{D'_b}(h'_b - h_b) \qquad \ldots (3.3)$$

where,

$W_{i,b}$ = length of the side between nodes i and b, (L);

$L_{i,b}$ = distance between nodes i and b, (L);

A_b = area associated with node b, (L);

S_b = storage coefficient/specific yield associated with node b, (dimensionless)

P_b = net rate of abstraction associated with node b, (LT^{-1});

h_i = piezometric head in the aquifer at node i, (L);

h = piezometric head in the aquifer at node b, (L);

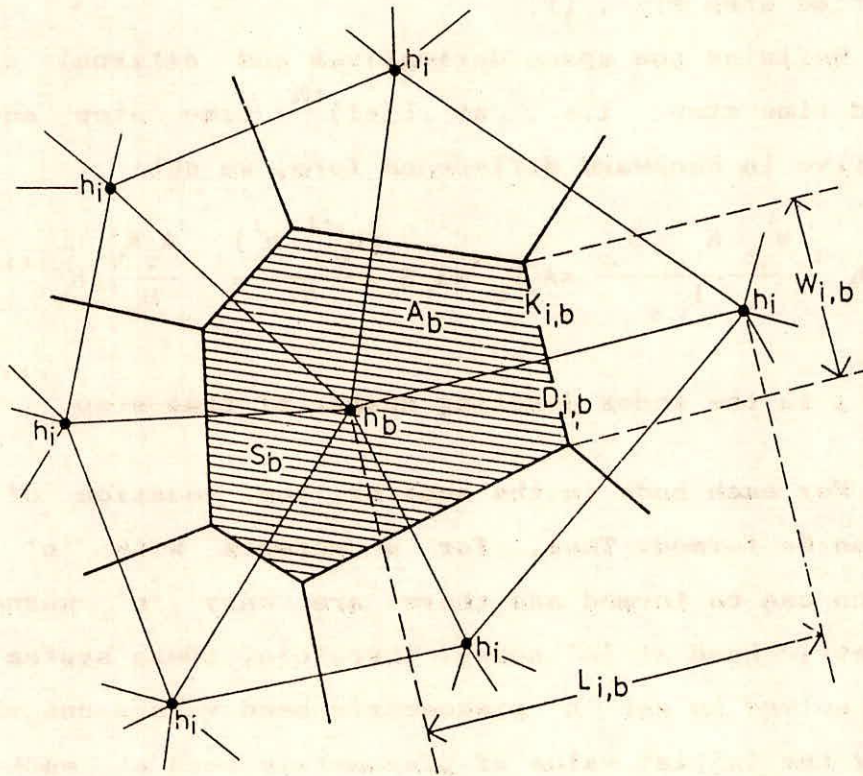K'_b = vertical hydraulic conductivity of the covering layer associated with node b, (LT^{-1});

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Fig. 3.1 Nodal geometry

D_b' = saturated thickness of covering layer associated with node b, (L);

h_b' = piezometric head in the covering layer at node b, (L);

Δh_b = change in the piezometric head in the aquifer at node b over a time step, (L);

Δt = time step size, (T).

Defining the space derivatives and external stress at a forward time step, i.e., at $(j+1)^{th}$ time step and the time derivative in backward difference form, we get:

$$\sum_i (h_i^{j+1} - h_b^{j+1}) \frac{W_{i,b} \; K_{i,b} \; D_{i,b}}{L_{i,b}} = A_b P_b^{j+1} + A_b S_b \frac{(h_b^{j+1} - h_b^j)}{\Delta t} - \frac{A_b K'}{D_b'} (h_b'^{j+1} - h_b^j)$$

$$...(3.4)$$

where, j is the index denoting number of time-step.

For each node in the aquifer, one equation of the above type can be formed. Thus, for a network with 'n' nodes, 'n' equation can be formed and there are only 'n' unknowns, i.e., peizometric head at 'n' nodes. Therefore, these system of equation can be solved to get 'n' piezometric head values one at each node, knowing the initial value of piezometric head at each node. For each successive time step, similar type of system of simultaneous equations can be formulated and piezometric head at all nodes at the end of the time steps can be obtained.

3.4 Solution Techniques:

For solving the set of simultaneous equations represented by (3.4), an implicit numerical integration technique (Rietmeyer and Monton 1967) has been used. This method does not suffer from instability criterion. The initial piezometric head at each node having been specified, the piezometric head at each nodal point at the end of first time step is obtained with given set of parameters $K_{i,b}$, $D_{i,b}$, S_b, $L_{i,b}$, $W_{i,b}$ and the recharge and the

abstraction rate during the first time step. This completes the first iteration in the first time-step, the components of water balance for each nodal area are calculated and all flows are balanced at each node by setting their sum equal to a residual term. Then, the piezometric head at each node is adjusted using following equation.

$$h_b^{j+1} = h_b^j + \frac{\text{residual for nodal area } b}{\text{SUM}}$$

$$\text{SUM} = \sum_i [(W_{i,b} K_{i,b} D_{i,b} / L_{i,b}) + (A_b S_b / \Delta t) + (A_b K_b / D_b')] \qquad \ldots (3.5)$$

For a true solution, the sum of all residual values of the water balances over all the nodal areas should be zero. Therefore, iterative calculations are required to find the proper water balances for each nodal area and corresponding piezometric head at each node. After each iteration, the sum of all residual values of the water balances over all nodal areas is calculated and is compared with a specified minimum value (or closure criteria). If the sum is more than this specified minimum value, the iteration is repeated till it satisfies the closure criteria. This completes the solution for first time step. For the second time-step, the piezometric head obtained at the end of first time step forms the starting condition, and the whole process stated above is repeated. Similarly, the peizometric heads at the end of third, forth, ... are obtained in succession. The method of solving equations as given above is essentially that of Gauss-Seidal (Raltson 1961).

The model makes use of two other solution technique for solving set of simultaneous equation given by (3.4), viz., Gauss elimination method, and Gauss-Jordan method. In these methods the equations are solved directly instead of iteratively. In the model, a modification to the Gaussian eliminating technique known as Gauss Jordan approach, has been used.

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4.0 MODFLOW

4.1 Model Description:

The governing partial differential equation for the three dimensional unsteady (transient) movement of incompressible groundwater through heterogeneous and anisotropic medium may be described as

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right) - W = S_s \frac{\partial h}{\partial t} \qquad \ldots(4.1)$$

where,

    $x, y,$ and z are the cartesian coordinates aligned along the major axes of conductivity K_{xx}, K_{yy}, and K_{zz};

    h is the piezometric head (L);

    W is the volumetric flux per unit volume and represents source and/or sinks (T^{-1});

    S_s is the specific storage of the porous material of the aquifer (L^{-1}); and

    t is the time (T)

here,

$$S_s = S_s(x,y,z)$$
$$K_{xx} = K_{xx}(x,y,z)$$
$$K_{yy} = K_{yy}(x,y,z)$$
$$K_{zz} = K_{zz}(x,y,z)$$
$$h = h(x,y,z,t)$$
$$W = W(x,y,z,t) \qquad \ldots(4.2)$$

Thus, in general the specific storage and the conductivities may the functions of space and time. Therefore, the flow under non-equilibrium conditions in a heterogeneous and

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anisotropic medium is described by (4.1). Equation (4.1) when combined with boundary conditions (flow and/or head conditions at the boundaries of the aquifer system) and initial condition (in case of transient flow, specification of head conditions at t=0), constitute a mathematical model of transient groundwater flow.

The analytical solution of (4.1) is not feasible for complex systems, so numerical methods must be employed to obtain approximate solutions. Finite difference approach is one of such numerical methods, wherein the continuous system described by (4.1) is replaced by a set of discrete points in space and time, and the partial derivatives are replaced by finite differences between the functional values at these points. Thus, the process leads to systems of simultaneous linear algebraic difference equation and their solution yields values of head at specific points and time. These values are an approximation to the time varying head distribution that would be given by an analytical solution of the partial differential equation of flow.

4.2 Discretization Convention:

For the formulation of finite difference equations, the aquifer system needs to be discretized into a mesh of points termed nodes, forming rows, columns, and layers. Such spatial discretization of an aquifer system is shown in fig. 4.1. To conform with computer array convention, an i,j,k coordinate system is used. If an aquifer system consists of 'nrow' rows, 'ncol' columns and 'nlay' layers, then

i	is the row index,	i = 1,2,... nrow;
j	is the column index,	j = 1,2,...ncol;
k	is the layer index,	k = 1,2,...nlay.

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For example, fig. 4.1 shows a system with nrow=5, ncol=9 and nlay=5. With respect to cartesian coordinate system, points along a row are parallel to x axis, points along a column are parallel to the y axis, and points along vertical are parallel to z axis. In spatial discretization, nodes represents prisms of porous material termed cells in conceptual sense. Within each cell the hydraulic properties are constant so that any value associated with a node applies to or is distributed over the extent of a cell.

The width of cells along rows is designated as Δr_j for the jth column; the width of cells along columns are designated as Δc_i for i^{th} row; and the thickness of layers in vertical are designated as Δv_k for the kth layer (Fig.4.1). Thus the cell with the coordinates of $(i,j,k) = (5,3,2)$ has a volume of $\Delta r_3 . \Delta c_5 . \Delta v_2$.

There exist two conventions for defining the configuration of cells with respect to the location of nodes, viz., the block centered formulation and the point centered formulation. In both systems the aquifer is divided with two sets of parallel lines which are perpendicular to each other.

In a block-centered formulation, the blocks formed by the sets of parallel lines are the cells and the nodes are at the centre of the cells. In a point-centered formulation the nodes are assumed at the intersection points of the sets of parallel lines and the cells are drawn around the nodes with faces half way between nodes. In either case of configuration, the spacing of nodes should be such that the hydraulic properties of the system are uniform over the extent of a cell. Both type of grid configurations are shown in fig. 4.2.

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Columns (J)

1 2 3 4 5 6 7 8 9

Rows (I)

Layers (K)

Δv_K

Δc_I

Δr_j
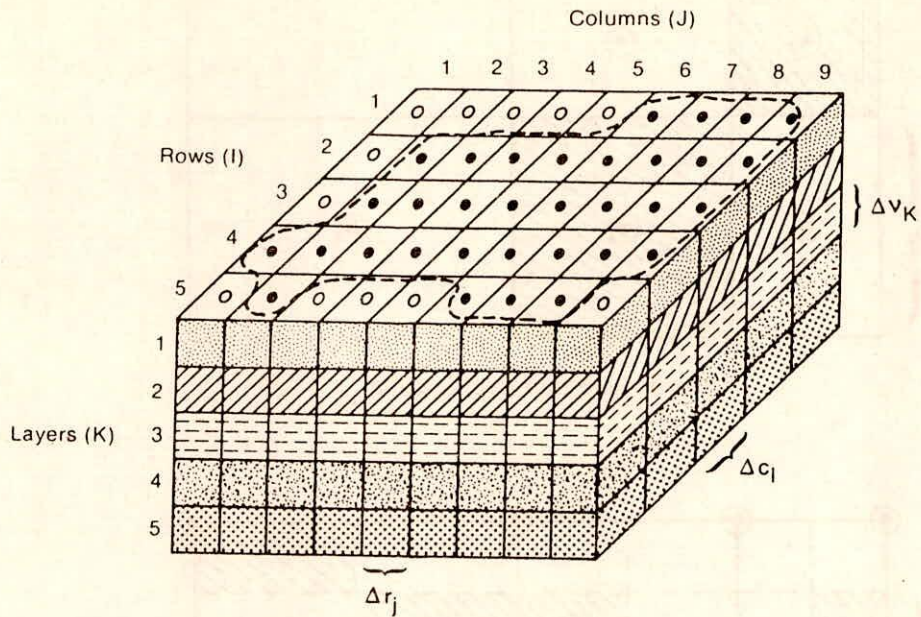
Explanation

– – – – Aquifer Boundary

●　　　Active Cell

0　　　Inactive Cell

Δr_J　　Dimension of Cell Along the Row Direction. Subscript (J) Indicates the Number of the Column
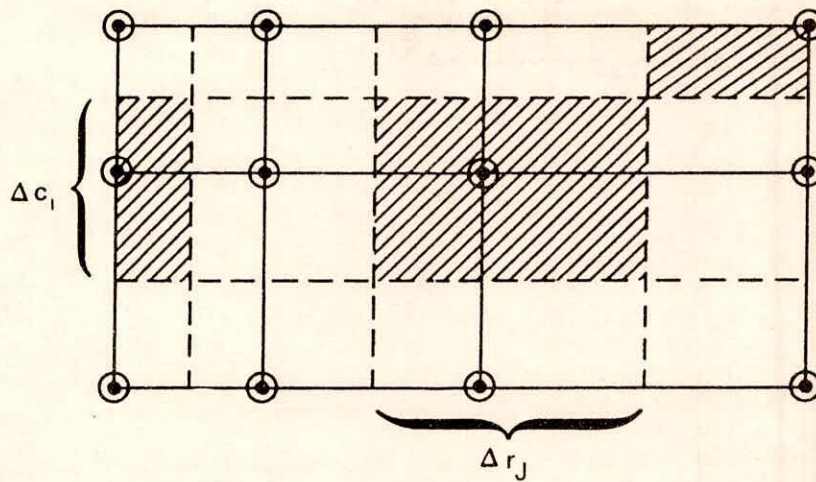
Δc_I　　Dimension of Cell Along the Column Direction. Subscript (I) Indicates the Number of the Row

Δv_K　　Dimension of the Cell Along the Vertical Direction. Subscript (K) Indicates the Number of the Layer

Fig.4.1 A Discretized Hypothetical Aquifer System.

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Δr_J
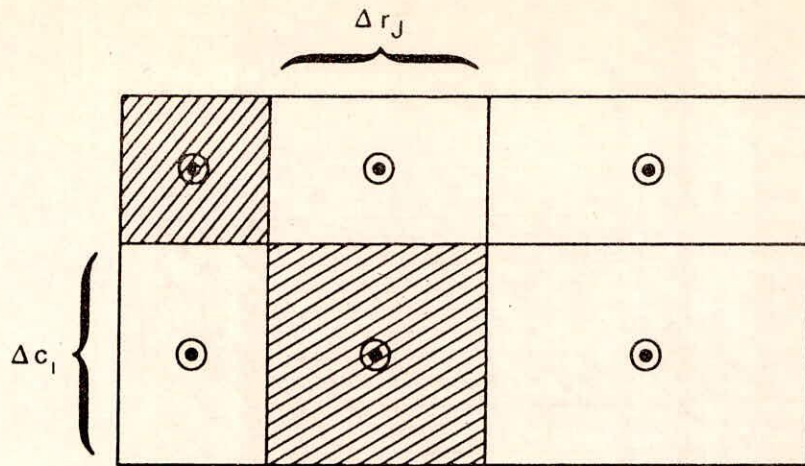
Block-Centered Grid System

Δc_I

Δc_I

Δr_J

Point-Centered Grid System

Explanation

⊙ Nodes

—— Grid Lines

— — — Cell Boundaries for Point
Centered Formulation

///// Cells Associated With
Selected Nodes

Fig. 4.2 Block Centred and Point Centred Grids.

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4.3 Finite Difference Equation:

The following development of finite difference equation holds good for both type of grid configuration described earlier. The groundwater flow equation may be written in finite difference form applying continuity equation. Thus, the sum of all flows into and out of cell must be equal to the rate of change in storage within the cell. Under the assumption that the groundwater is incompressible, the continuity equation (expressing the balance of flow) for a cell can be written as

$$\Sigma Q_i = S_s \frac{\Delta h}{\Delta t} . \Delta V \qquad \qquad ...(4.3)$$

where,

Q_i is the flow rate into the cell ($L^3 t^{-1}$);

S_s is the specific storage defined as the ratio of volume of water which can be injected per unit volume of aquifer material per unit change in head (L^{-1});

ΔV is the volume of the cell (L^3); and

Δh is the change in head over a time interval Δt (L)

The right hand side of (4.3) represents the volume of water taken into the storage over a time interval Δt, given a change in head of Δh. Thus, (4.3) is stated in terms of inflow and storage gain. Outflow and loss in storage are represented by defining outflow as negative inflow and loss as negative gain.

For a three dimensional problem each cell is surrounded by six adjacent cells. Fig.4.3 shows a cell i,j,k and six adjacent cells, i.e.,i-1,j,k ; i+1,j,k ; i,j-1,k ; i,j+1,k ; i,j,k-1 ; and i,j,k+1 . Thus net flow to the cell i,j,k is the algebraic summation of the flows into the cell from six adjacent cells.
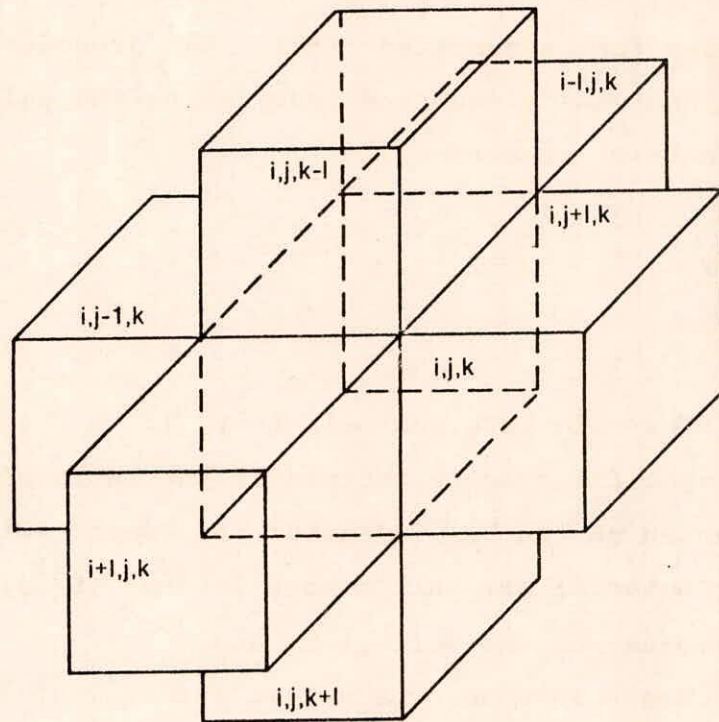
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Fig.4.3 Cell i,j,k and Indices for the Six Adjacent Cells.

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Using Darcy's law, flow from each adjacent cell into the cell i,j,k can be obtained. Flow into the cell i,j,k in row direction from cell i,j-1,k (fig.4.4) is given by

$$q_{i,j-1/2,k} = KR_{i,j-1/2,k} \, \Delta c_i \Delta V_k \frac{(h_{i,j-1,k} - h_{i,j,k})}{\Delta r_{j-1/2}} \qquad \ldots (4.4)$$

where,

$q_{i,j-1/2,k}$ is the volumetric flow discharge through the face between the cells i,j,k and i,j-1,k $(L^3 t^{-1})$;

$KR_{i,j-1,k}$ is the hydraulic conductivity along the row between nodes i,j,k and i,j-1,k ; and

$\Delta r_{j-1/2}$ is the distance between nodes i,j,k and i,j-1,k (L)

The index j-1/2 indicates the space between nodes, (fig.4.4). It does not indicate a point exactly half way between nodes. For example, $KR_{i,j-1/2,k}$ represents average hydraulic conductivity in the entire region between nodes i,j,k and i,j-1,k.

Since the grid dimensions and hydraulic conductivity remain constant throughout the solution process, the above equation may be rewritten by combining the constants into single constant called hydraulic conductance or simply 'conductance' of the cell.

$$q_{i,j-1/2,k} = CR_{i,j-1/2,k}(h_{i,j-1,k} - h_{i,j,k}) \qquad \ldots (4.5)$$

where, $CR_{i,j-1/2,k} = KR_{i,j-1/2,k} \Delta c_i \Delta V_k / \Delta r_{j-1/2}$

$CR_{i,j-1/2,k}$ is the conductance in i^{th} row and k^{th} layer between nodes i,j-1,k and i,j,k $[L^2 t^{-1}]$. Thus conductance is the product of hydraulic conductivity and cross-sectional area of flow divided by length of flow path; in this case, the distance between
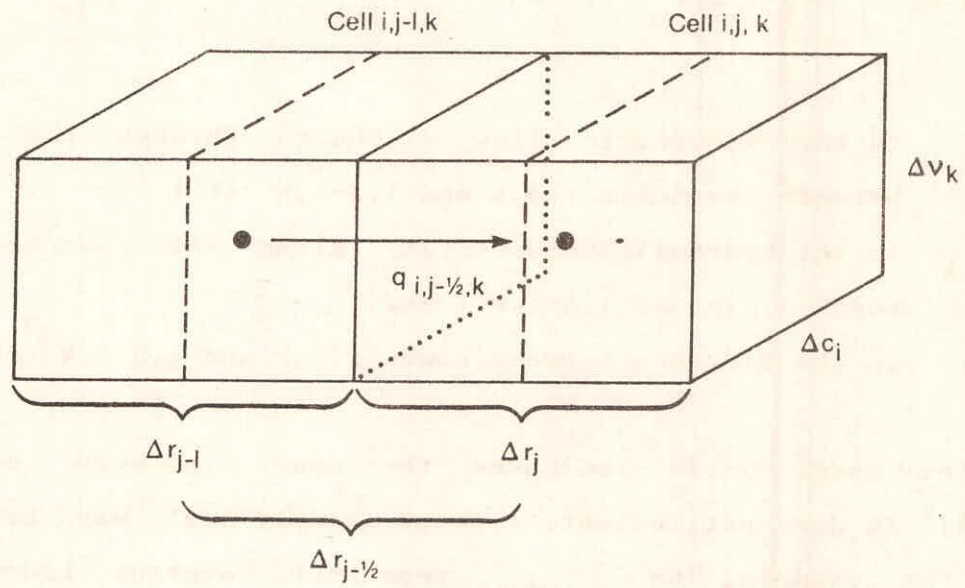
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Cell i,j-l,k   Cell i,j, k

Δv_k

$q_{i,j-\frac{1}{2},k}$

Δc_i

Δr_{j-l} Δr_j

$\Delta r_{j-\frac{1}{2}}$

Fig.4.4 Flow into Cell i,j,k from Cell i,j-1,k.

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the nodes. Here, C represents the conductance and R represents in row direction.

Similar expressions can be written approximating the flows into or out of the cell i,j,k through the remaining five faces. Such expressions are as written below.

$$q_{i,j+1/2,k} = CR_{i,j+1/2,k} (h_{i,j+1,k} - h_{i,j,k}) \qquad \ldots(4.6)$$

$$q_{i-1/2,j,k} = CC_{i-1/2,j,k} (h_{i-1,j,k} - h_{i,j,k}) \qquad \ldots(4.7)$$

$$q_{i+1/2,j,k} = CC_{i+1/2,j,k} (h_{i+1,j,k} - h_{i,j,k}) \qquad \ldots(4.8)$$

$$q_{i,j,k-1/2} = CV_{i,j,k-1/2} (h_{i,j,k-1} - h_{i,j,k}) \qquad \ldots(4.9)$$

$$q_{i,j,k+1/2} = CV_{i,j,k+1/2} (h_{i,j,k+1} - h_{i,j,k}) \qquad \ldots(4.10)$$

Equations (4.5) through (4.10) represent the flow into the cell i,j,k from six adjacent cells. Seepage from the stream beds, drains, areal recharge, evapotranspiration and flow from wells are taken care of by additional terms which accounts for flow into the cell from outside the aquifer. These flows may depend on the head in the receiving cell but are independent of the heads in all other cells of the aquifer or they may be entirely independent of head in receiving cell. Flow from outside the aquifer which is represented by W in (4.1), may be expressed in general as

$$a_{i,j,k,n} = P_{i,j,k,n} h_{i,j,k} + q_{i,j,k,n} \qquad \ldots(4.11)$$

where,

$a_{i,j,k,n}$ is the flow from the n-th external source into cell i,j,k $[L^3 T^{-1}]$

$P_{i,j,k,n}$ is a constant $[L^2 T^{-1}]$

$q_{i,j,k,n}$ is a constant $[L^3 T^{-1}]$

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For example, let cell i,j,k represents a well and $q_{i,j,k}$ represents discharge. $q_{i,j,k,1}$ is the discharge being pumped. In this case, the discharge from the well is assumed to be independent of head. Hence,

$$P_{i,j,k,1} = 0 \; ; \text{ and}$$

$$a_{i,j,k,1} = -q_{i,j,k,1} \qquad \qquad \ldots (4.12)$$

If the second external source $(n=2)$ is taken to be seepage from river bed. Seepage is proportional to the head difference between the river stage $(R_{i,j,k})$ and head in the receiving cell i,j,k $(h_{i,j,k})$. Hence,

$$a_{i,j,k,2} = CRIV_{i,j,k,2} \; (R_{i,j,k} - h_{i,j,k})$$

or, $a_{i,j,k,2} = -CRIV_{i,j,k,2} \; h_{i,j,k} + CRIV_{i,j,k,2} \; R_{i,j,k} \qquad \ldots (4.13)$

where,

$CRIV_{i,j,k,2}$ is the conductance of the river bed in cell i,j,k $[L^2 T^{-1}]$

The conductance $CRIV_{i,j,k,2}$ corresponds to $P_{i,j,k,2}$ and the term $CRIV_{i,j,k,2} \; R_{i,j,k}$ corresponds to $q_{i,j,k,2}$. Similarly, all other external sources or stresses can be represented by an expression of the form of (4.11). If there are N external sources or stresses affecting a single cell, the combined flow is expressed by:

$$QS_{i,j,k} = \sum_{n=1}^{N} a_{i,j,k,n}$$

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$$= \sum_{n=1}^{N} P_{i,j,k,n} \; h_{i,j,k} + \sum_{n=1}^{N} q_{i,j,k,n}$$

$$= P_{i,j,k} \; h_{i,j,k} + Q_{i,j,k} \qquad \ldots (4.14)$$

where, $P_{i,j,k} = \sum_{n=1}^{N} P_{i,j,k,n}$

and $Q_{i,j,k} = \sum_{n=1}^{N} q_{i,j,k,n}$

While writing the continuity equation of the form given by (4.8), for cell i,j,k, the term ΣQ_i consists of flow to the cell from six adjacent cells, and all other external flow rate to the cell. The flow from six adjacent cells into cell i,j,k is given by (4.5) through (4.10) and the flow from the external sources into cell i,j,k is represented by (4.14). Substituting these equations in (4.3), we get

$$CR_{i,j-1/2,k} \; (h_{i,j-1,k} - h_{i,j,k}) +$$
$$CR_{i,j+1/2,k} \; (h_{i,j+1,k} - h_{i,j,k}) +$$
$$CC_{i-1/2,j,k} \; (h_{i-1,j,k} - h_{i,j,k}) +$$
$$CC_{i+1/2,j,k} \; (h_{i+1,j,k} - h_{i,j,k}) +$$
$$CV_{i,j,k-1/2} \; (h_{i,j,k-1} - h_{i,j,k}) +$$
$$CV_{i,j,k+1/2} \; (h_{i,j,k+1} - h_{i,j,k}) +$$
$$P_{i,j,k} \; h_{i,j,k} + Q_{i,j,k}$$
$$= SS_{i,j,k} \; (\Delta r_j \Delta c_i \Delta v_k)(\Delta h_{i,j,k} / \Delta t) \qquad \ldots (4.15)$$

where,

$\Delta h_{i,j,k}/\Delta t$ is a finite difference approximation for head change with respect to time $[LT^{-1}]$

$SS_{i,j,k}$ is the specific storage of cell i,j,k $[L^{-1}]$; and

$\Delta r_j \Delta c_i \Delta k_k$ is the volume of cell i,j,k $[L^3]$

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The above equation can be written in backward difference form by specifying flow term at t_m, the end of the time interval, and approximating the time derivative of head over the interval t_{m-1} to t_m, i.e.,

$$CR_{i,j-1/2,k}(h^m_{i,j-1,k} - h^m_{i,j,k}) + CR_{i,j+1/2,k}(h^m_{i,j+1,k} - h^m_{i,j,k})$$

$$+CC_{i-1/2,j,k}(h^m_{i-1,j,k} - h^m_{i,j,k}) + CC_{i+1/2,j,k}(h^m_{i+1,j,k} - h^m_{i,j,k})$$

$$+CV_{i,j,k-1/2}(h^m_{i,j,k-1} - h^m_{i,j,k}) + CV_{i,j,k+1/2}(h^m_{i,j,k+1} - h^m_{i,j,k})$$

$$+P_{i,j,k}h^m_{i,j,k} + Q_{i,j,k} = SS_{i,j,k}(\Delta r_j \Delta c_i \Delta v_k)\frac{(h^m_{i,j,k} - h^{m-1}_{i,j,k})}{t_m - t_{m-1}}$$

$$...(4.16)$$

An equation of the above type can be written for each of the 'n' cells in the system; and, since there is only one unknown head for each cell, we are left with a system of 'n' equations with 'n' unknowns. Such a system of equations can be solved simultaneously.

4.4 Provision for Boundary Conditions and Initial Condition:

The type of boundaries that may be imposed in the model include constant head, no-flow, constant flow, and head dependent flow boundaries. These various types of boundaries are represented by the difference cell types. Cells can be designated by three types viz, inactive cell; constant head cell; and, variable head cell. Variable head cells are those in which head vary with time. Therefore, an equation of the type of (4.16) is required for each variable head cell. Head remains constant with time in constant

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head cells and these cells do not require an equation, however, the adjacent variable head cells will contain non-zero conductance terms representing flow from the constant head cell. 'No flow cells' are those to which there is no flow from adjacent cells. Neither an equation for a no flow cell nor the equations for the adjacent cells containing a term representing flow from the no flow cell, are formulated. The use of no-flow cells and constant head cells to simulate boundary conditions is given in fig. 4.5. Constant-flow and head-dependent flow boundaries can be represented by a combination of no-flow cells and external sources.

In most cases, the actual number of equations of the form of equation 4.16 will be less than the total number of model cells. This is because the number of equations is only equal to the number of 'variable head cells'. The objective of transient simulation is to predict the head distribution at successive times with the given initial head distribution and the boundary conditions. The initial head distribution consists of a value of $h_{i,j,k}^1$ at each point in the mesh at time t_1, the beginning of the first of the discrete time steps into which the time axis is divided in the finite difference process. The first step in the solution process is to calculate values of $h_{i,j,k}^2$, i.e., head at time t_2 which mark the end of the first time step. Therefore, in (4.16), the subscript m is taken as 2 and thus the subscript m-1, which appear in only one head term, is taken as 1. Once such equations are formed for each variable head cells, an iterative method is used to obtain the values of $h_{i,j,k}^2$. An iterative method starts with an initial trial solution. This trial solution is used to calculate through a procedure of calculation, an interim solution which more nearly satisfies the system of
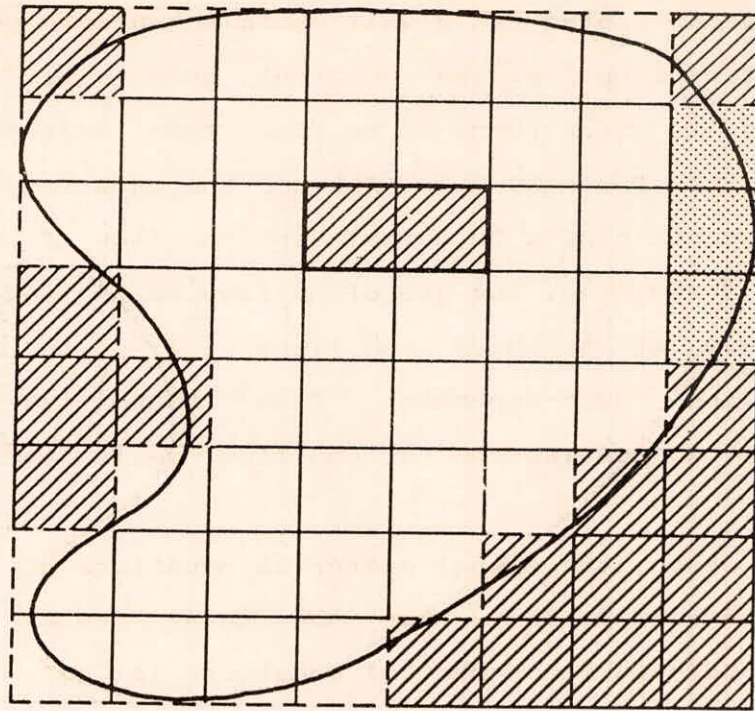
Explanation

——— Aquifer Boundary

– – – Model Impermeable Boundary

Inactive Cell

Constant-Head Cell

Variable-Head Cell

Fig.4.5 Discretized Aquifer Showing Boundaries and Cell Designations.

equations. The interim solution then becomes a new trial solution and the procedure is repeated. Each repetition is called an 'iteration'. The process is repeated until an iteration occurs in which the trial solution and the interim solution are nearly equal, i.e., for each node, the difference between the trial head value and the interim head value is smaller than some arbitrary established value, usually termed as 'closure criterion'. The interim solution is then regarded as a good approximation to the solution of system of equation under given initial and boundary conditions. For the solution of the present problem a strongly implicit programme has been used. The 'closure criterion' has been taken as 0.001 m.

4.5 Model Structure and Simulation Packages:

The model has a modular structure having a main program and a large number of highly independent subroutines called 'modules'. For a typical simulation, the functions which must be performed are shown in fig. 4.6, which shows a flow chart for the overall structure of the program. The works within the rectangles are performed by individual modules which are called by the MAIN (main program). Thus, MAIN is an organized collection of FORTRAN CALL STATEMENTS which invoke modules to read data, perform calculations, and print results; MAIN does not do work but it merely calls modules which do work.

A 'package' consists of all modules associated with a particular hydrologic feature, a solution method or the overall control of the simulation. For example, all the modules concerned with the simulation of wells, combined together form the 'well package'. Similarly, there are other packages to simulate the

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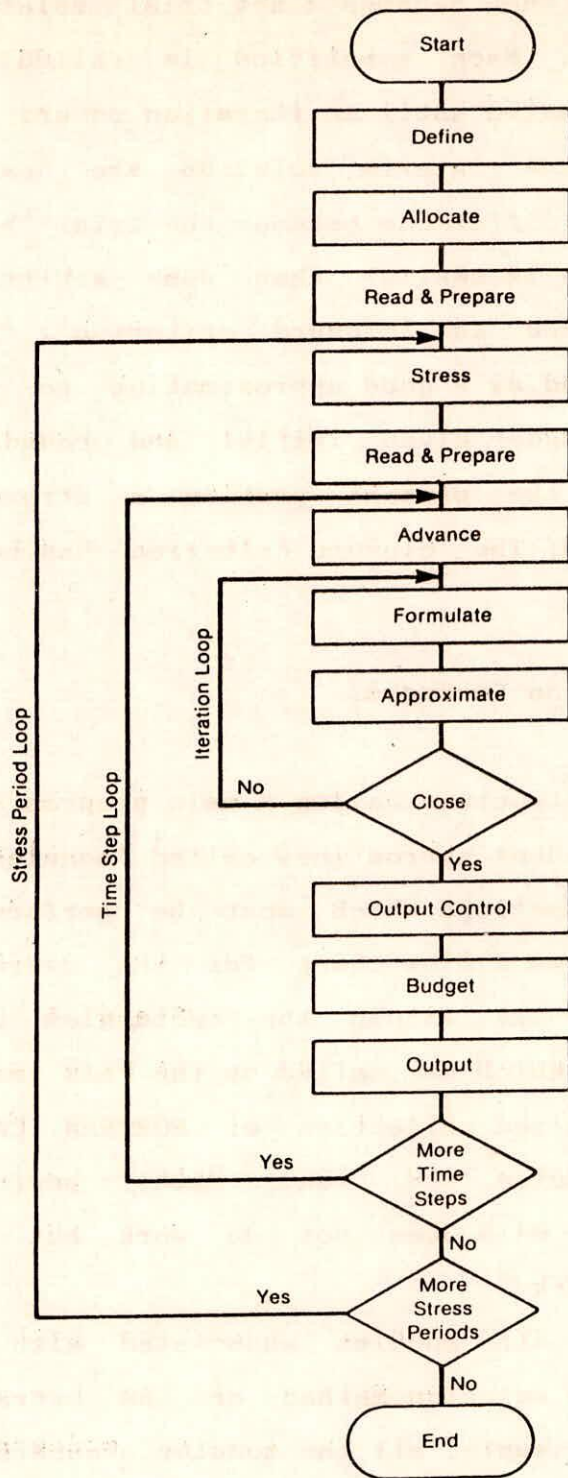
```
        ┌─────────┐
        │  Start  │
        └────┬────┘
             │
        ┌────┴────┐
        │ Define  │        DEFINE — Read data specifying number of rows,
        └────┬────┘        columns, layers, stress periods, and major program
             │             options.
        ┌────┴────┐
        │ Allocate│        ALLOCATE — Allocate space in the computer to
        └────┬────┘        store data.
             │
     ┌───┴─────────┐       READ AND PREPARE — Read data which is constant
     │Read & Prepare│      throughout the simulation. Prepare the data by
     └───┬─────────┘       performing whatever calculations can be made at
             │             this stage.
        ┌────┴────┐
        │ Stress  │        STRESS — Determine the length of a stress period
        └────┬────┘        and calculate terms to divide stress periods into time
             │             steps.
     ┌───┴─────────┐
     │Read & Prepare│      READ AND PREPARE — Read data which changes
     └───┬─────────┘       from one stress period to the next. Prepare the data
             │             by performing whatever calculations can be made at
        ┌────┴────┐        this stage.
        │ Advance │
        └────┬────┘        ADVANCE — Calculate length of time step and set
             │             heads at beginning of a new time step equal to heads
        ┌────┴────┐        calculated for the end of the previous time step.
        │Formulate│
        └────┬────┘        FORMULATE — Calculate the coefficients of the
             │             finite difference equations for each cell.
        ┌────┴────┐
        │Approximate│      APROXIMATE — Make one cut at approximating a
        └────┬────┘        solution to the system of finite difference equations.
             │
         ◇ Close ◇  No
             │ Yes
     ┌───┴─────────┐       OUTPUT CONTROL — Determine whether results
     │Output Control│      should be written or saved on disk for this time step.
     └───┬─────────┘       Send signals to the BUDGET and OUTPUT pro-
             │             cedures to indicate exactly what information should
        ┌────┴────┐        be put out.
        │ Budget  │
        └────┬────┘        BUDGET — Calculate terms for the overall volu-
             │             metric budget and calculate and save cell-by-cell
        ┌────┴────┐        flow terms for each component of flow.
        │ Output  │
        └────┬────┘        OUTPUT — Print and save heads, drawdown and
             │             overall volumetric budgets in accordance with
         ◇ More  ◇  Yes    signals from OUTPUT CONTROL procedure.
         ◇ Time  ◇
         ◇ Steps ◇
             │ No
         ◇ More   ◇  Yes
         ◇ Stress ◇
         ◇ Periods◇
             │ No
        ┌────┴────┐
        │   End   │
        └─────────┘
```

Stress Period Loop   Time Step Loop   Iteration Loop

Fig. 4.6 Overall Program Structure.

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effects of rivers, areal recharge, drains, evapotranspiration and general head boundaries.

All the modules related to the internal flow between model cells and flow into storage for a block-centered formulation are members of 'block-centred flow ' package. The packages related to external internal flow are termed 'flow component' packages. An other set of package termed 'solver' package includes modules needed to implement a particular solution algorithm, e.g., Strongly Implicit Procedure (SIP) and Slice-Successive Over-relaxation (SSOR). The 'solver' and the 'flow component' packages are, in effect, the options available to the users. User specifies which of the 'flow component' packages are required for a simulation and which 'solver' package is desired.

The basic package is used irrespective of the options selected. It handles initial conditions, boundary conditions and discretization. The list of all the packages of the model is given in Table 4.1 along with their abbreviation and description. The primary modules (Subroutines which are called from the Main program) are arranged in a matrix format to illustrate the classification by package and by procedure in Table 4.2. All row in the matrix correspond to procedures and the columns correspond to packages. A 'X' entered in the block of the matrix indicate that the module exists for that particular procedure. Absence of 'X' means a module representing the procedure is not required. Entries marked with subscript 'S' indicate the primary modules which utilize sub-modules in accomplishing their function. Sub-modules are the secondary modules contained in a particular package. Entries marked with the super-script 'U' show the modules which utilize utility modules. Utility modules are secondary modules available to many packages. The modelling philosophy

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Package Name	Abbreviation	Package Description
Basic	BAS	Handles those tasks that are part of the model as a whole. Among those tasks are specification of boundaries, determination of time-step length, establishment of initial conditions, and printing of results.
Block-Centered Flow	BCF	Calculates terms of finite-difference equations which represent flow within porous medium; specifically, flow from cell to cell and flow into storage.
Well	WEL	Adds terms representing flow to wells to the finite-difference equations.
Recharge	RCH	Adds terms representing areally distributed recharge to the finite-difference equations.
River	RIV	Adds terms representing flow to rivers to the finite-difference equations.
Drain	DRN	Adds terms representing flow to drains to the finite-difference equations.
Evapotranspiration	EVT	Adds terms representing ET to the finite-difference equations.
General-Head Boundaries	GHB	Adds terms representing general-head boundaries to the finite-difference equations.
Strongly Implicit Procedure	SIP	Iteratively solves the system of finite-difference equations using the Strongly Implicit Procedure.
Slice-Successive Overrelaxation	SOR	Iteratively solves the system of finite-difference equations using Slice-Successive Overrelaxation.

Flow Component Packages

Stress Packages

Solver Packages

Table 4.1 List of Packages.

| Procedures | Flow Component Packages | | | | | | | | Solver Packages | |
| | | Stress Packages | | | | | | | | |
	B A S	B C F	W E L	R C H	R I V	D R N	E V T	G H B	S I P	S O R
Define (DF)	X									
Allocate (AL)	X	X	X	X	X	X	X	X	X	X
Read & Prepare (RP)	X_U	X_{US}							X	X
Stress (ST)	X									
Read & Prepare (RP)			X	X_U	X	X	X_U	X		
Advance (AD)	X									
Formulate (FM)	X	X_S	X	X	X	X	X	X		
Approximate (AP)									X_S	X_S
Output Control (OC)	X									
Budget (BD)		X_{US}	X_U	X_U	X_U	X_U	X_U	X_U		
Output (OT)	X_U									

Table 4.2 Organization of Modules by Procedures and Packages.

contained in different packages have been discussed in subsequent paragraph.

4.5.1 River package:

The river package in the model simulates the effects of exchange of flow between surface water features and ground water system. This is accomplished by adding the terms representing seepage to or from the surface features (e.g. rivers) to the finite difference form of groundwater flow equation (4.16) for each cell affected by the seepage.

The stream to be modelled is divided into reaches so that each reach is completely contained in a single cell. Fig.4.7 shows the discretization of a stream into reaches, few small reaches are ignored. Stream-aquifer seepage is simulated between each reach and the model cell that contains the reach.

For the situation in which the open water of a stream is separated from the ground water system by a layer of low permeability stream-bed material, the stream-aquifer inter-connection is represented as a conductance through which one dimensional flow occurs. Figs.4.8 and 4.9 show the cross-section of an aquifer containing a stream and the conceptual representation of stream-aquifer inter-connection in simulation.

The following assumptions have been made:

1) measurable headlosses between the stream and aquifer are limited to those across the stream bed layer itself, i.e., there is no significant head loss between the bottom of the stream bed layer and the point represented by the underlying model node.

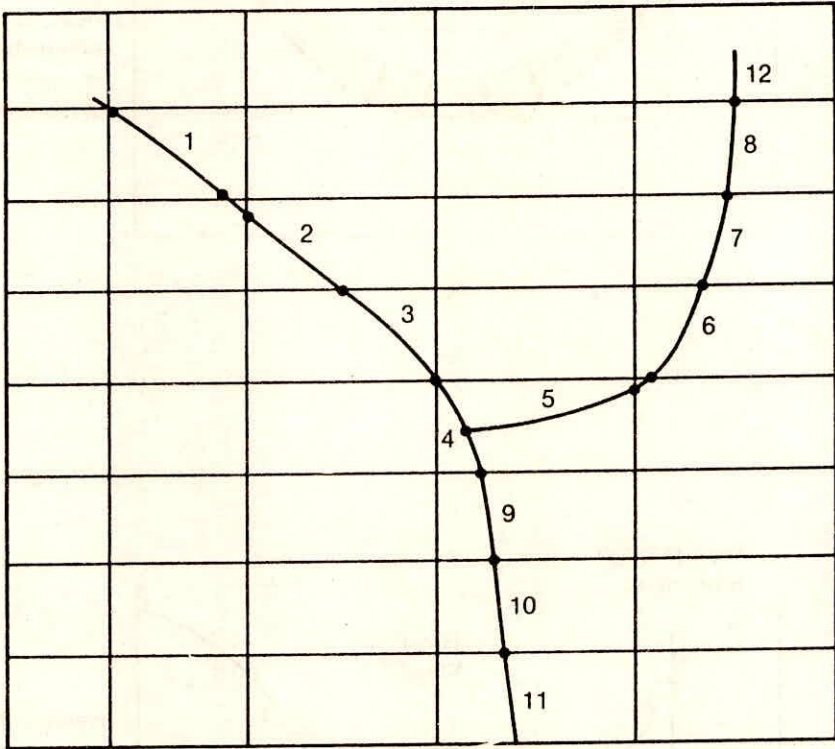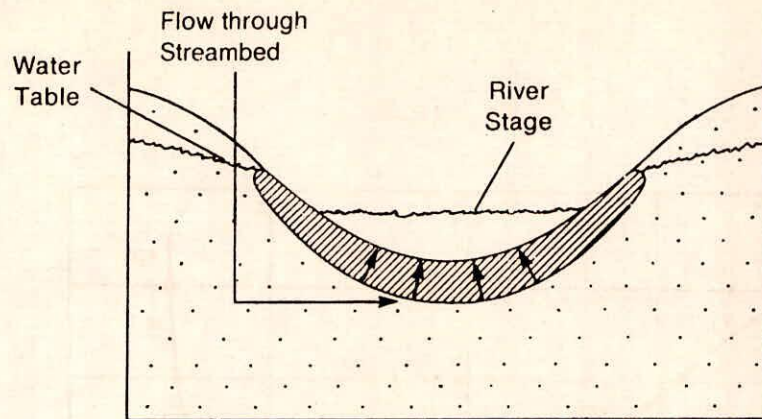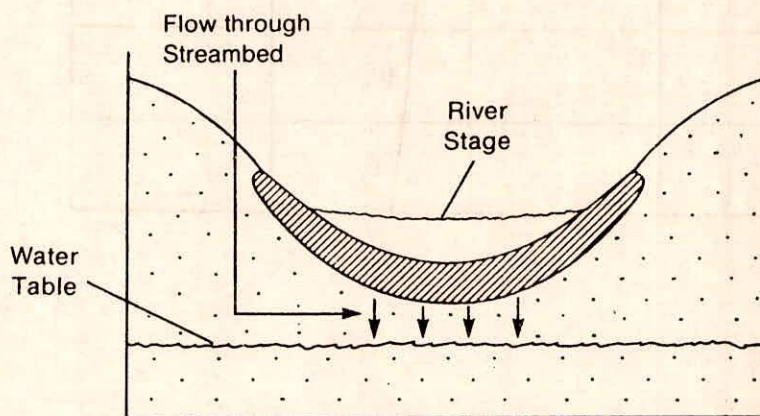2) underlying model cell remains fully saturated, i.e., its

Fig.4.7 Discretization of a Stream into Reaches.
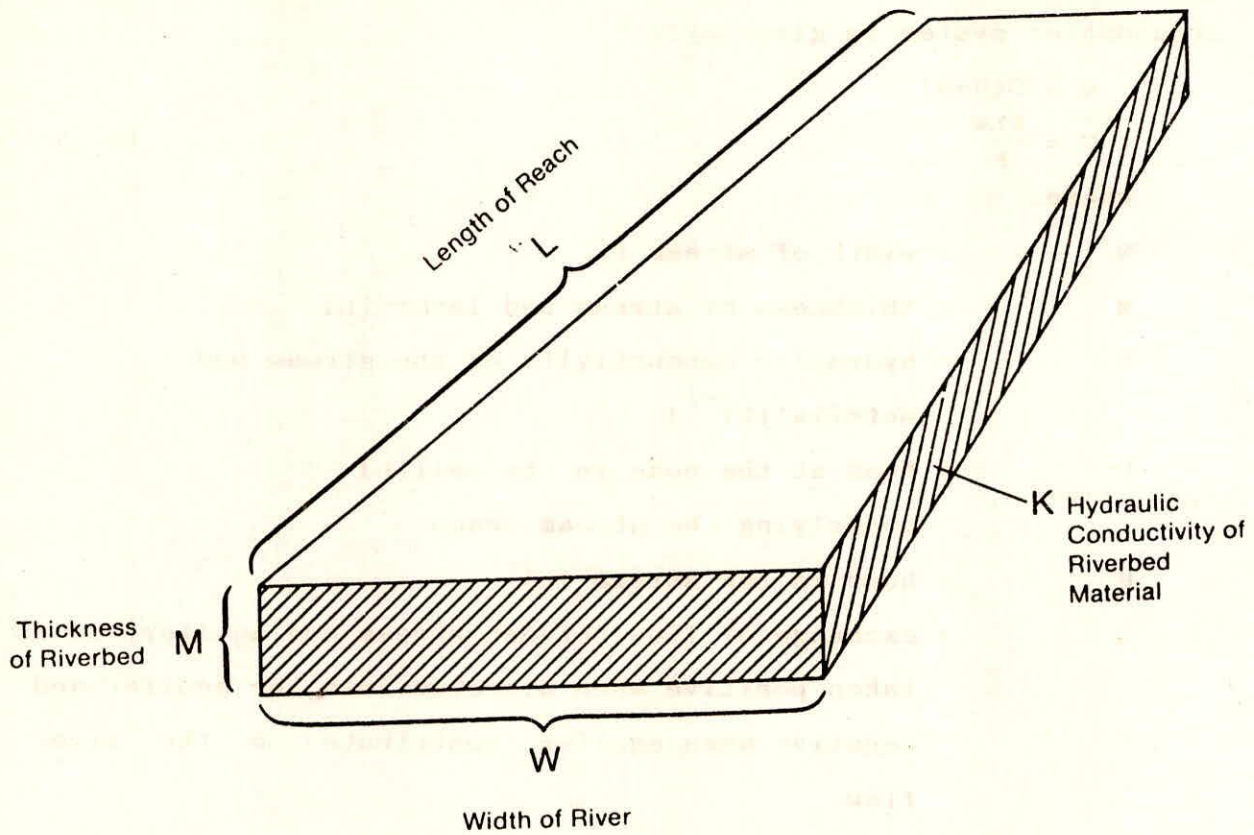
Fig.4.8 Cross Section of an Aquifer Containing a Stream.

Fig.4.9 Idealization of Streambed Conductance in an Individual Cell.

water level does not drop below the bottom of the stream bed layer.

Under the above assumptions, flow between the stream and the groundwater system is given by:

$$Q = C(H-h)$$
$$C = \frac{KLW}{M} \qquad \qquad ...(4.17)$$

where,

W   = width of stream (L)

M   = thickness of stream bed layer (L)

K   = hydraulic conductivity of the stream bed material (LT^{-1})

$h_{i,j,k}$ = head at the node in the cell(L) underlying the stream reach

H   = head in the stream (L)

Q   = exchange of flow between stream and aquifer; it is taken positive when directed into the aquifer and negative when aquifer contribute to the river flow.

In many instances, no discrete low permeability stream bed layer is present. In these cases, the single conductance term, C, needs to be formulated, which can be used in (4.17) to relate flow between the stream and the aquifer represented by node i,j,k to the corresponding head difference. The flow in this case, in general is three-dimensional, hence, its representation through a single conductance term is inappropriate one. The effective conductance in this case can be calculated

  a) using reliable field measurements of stream seepage and associated head difference.

  b) adjustment during model calibration.

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The initial choice of conductance may be taken as given by (4.17), where L is replaced by the vertical interval between the stream bed and node i,j,k. If distinct layers can be recognized within this interval, these should normally be treated as conductances in series in formulating an equivalent conductance. However, it may be pointed out that formulation of a single conductance term to account for a three-dimensional flow process is inherently an empirical exercise, and hence, adjustment of conductance during calibration is almost always required.

In the model, the flow between a stream and a node i,j,k is simulated according to the following equations.

$$Q = C(H-h_{i,j,k}), \quad \text{when} \quad h_{i,j,k} > H_B$$

$$Q = C(H-H_B) \quad \text{when} \quad h_{i,j,k} < H_B \quad \quad \text{...(4.18)}$$

where,

H_B = elevation of the bottom of river bed, (L).

Equation (4.18) shows that declines in head below R_B produce no increase in flow through stream bed layer. The flow simply retains the constant value given by (4.17) as long as aquifer head in the cell i,j,k remains below R_B. Fig.4.10 shows a graph of flow between the stream and cell i,j,k as a function of head, $h_{i,j,k}$ as calculated using (4.18). Equation (4.18) and fig.4.10 generally prevails in stream-aquifer interaction whether or not a discrete stream bed layer is present.

This simplified model of stream-aquifer is independent of the location of stream reach within the cell, and the level of water in the stream is uniform over the reach.
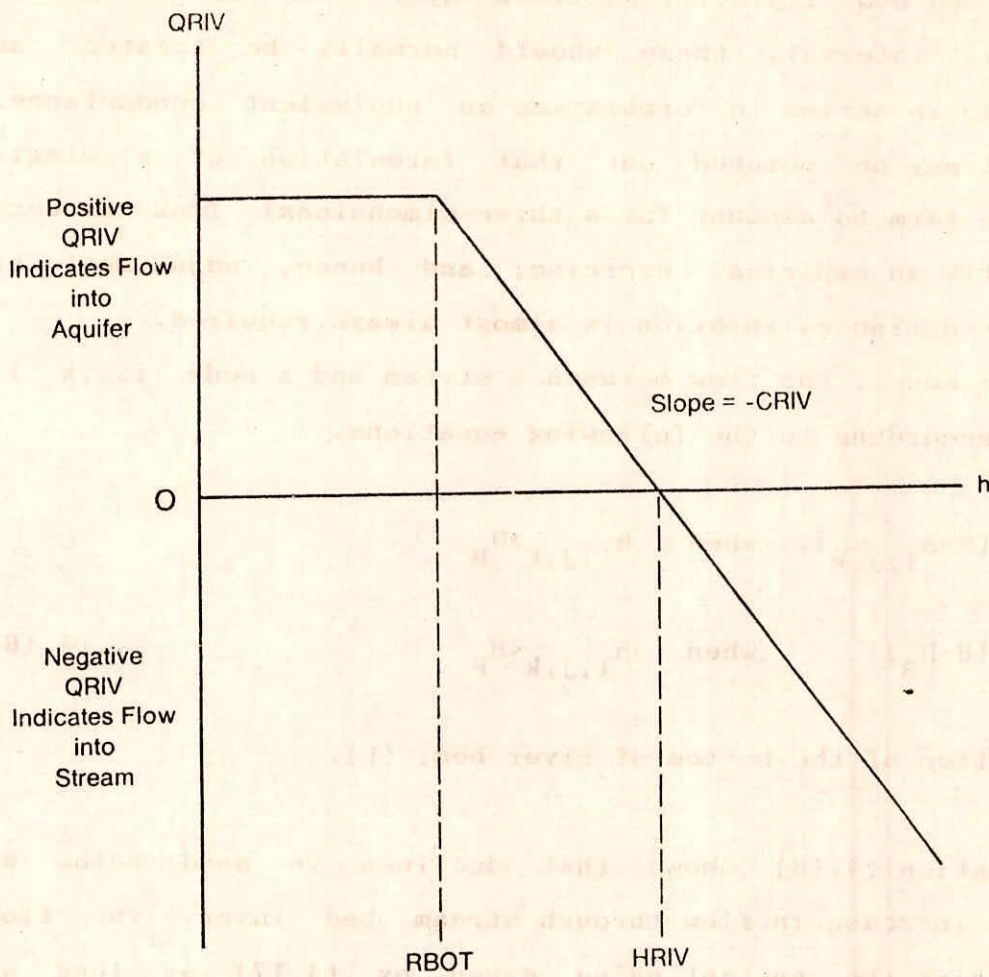
Fig.4.10 Plot of Flow, QRIV, from a Stream into a Cell as a
Function of Head, h, in the Cell.

4.5.2 Recharge package:

The recharge package simulates the effect of areally distributed recharge to ground water system. Generally, areal recharge occurs as a result of precipitation that percolates to the ground water system. As an external stress, recharge applied to the model is defined as

$$QR_{i,j} = I_{i,j}\, \Delta r_j \Delta c_i \qquad \qquad \ldots(4.19)$$

where,

$QR_{i,j}$ = rate of recharge applied to the model at horizontal cell location (i,j) expressed as a fluid volume per unit time, $(L^3 T^{-1})$

$I_{i,j}$ = recharge flux applicable to the map area $(\Delta r_j \Delta c_i)$, (LT^{-1})

The recharge $QR_{i,j}$ is applied to a single cell within the vertical column of cells located as (i,j). There is no need to allow for recharge to occur simultaneously at middle depths in the same vertical column because natural recharge enters the ground water system at its top.

The recharge package can potentially be used to simulate recharge from sources other than precipitation, e.g., artificial recharge, return flow from irrigation, etc. If the ability to apply recharge to more than one cell in a vertical column of cells is required, then the well package which allows recharge or discharge to be specified at any model cell, can be used.

4.5.3 Well package:

The well-package simulates the withdrawal/recharge of water from/to the aquifer through wells at a specified rate during a given stress period, where the rate is independent of both the cell area and the head in the cell.

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Well discharged is handled in the well-package by specifying the rate, Q at which each individual well adds water to the aquifer or removes water from it. Negative values of Q are used to indicate well discharge, while positive values of Q indicate a recharging well.

At each iteration, the value of Q for each well is substracted from the RHS value of (4.16) for the cell containing the well.

4.5.4 Evapotranspiration package:

The evapotranspiration package simulates the effect of plant transpiration and direct evaporation in removing water from the saturated ground water regime. The following assumption has been made.

i) when the head in the cell is above certain specified elevation termed as 'ET surface', evapotranspiration loss from the water table occurs at a maximum rate specified by the user;

ii) when the difference in $h_{i,j,k}$ and the 'ET surface' is more than a specified interval (extinction depth or cutoff depth) evapotranspiration from the water table ceases;

iii) when the $h_{i,j,k}$ is between the limits of extinction depth, evapotranspiration from the water table varies linearly with water table elevation.

The volumetric rate of evapotranspiration loss from a given cell is required for the implementation of finite difference approach. The above points can be expressed in the form of following equations which gives the volumetric rate of evapotranspiration from a given cell.

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$$Q_{ET_{i,j}} = Q_{ET_{i,j}} \quad \text{when} \quad h_{i,j,k} > h_{s_{i,j}}$$

$$Q_{ET} = 0 \quad \text{when} \quad h_{i,j,k} < h_{s_{i,j}} - d_{i,j}$$

$$Q_{ET_{i,j}} = Q_{ETM_{i,j}} \left[\frac{h_{i,j,k} - (h_{s_{i,j}} - d_{i,j})}{d_{i,j}} \right]$$

when $(h_{s_{i,j}} - d_{i,j}) < h_{i,j,k} > h_{s_{i,j}}$

...(4.20)

where,

$$Q_{ET_{i,j}} = R_{ET_{i,j}} \Delta r_j \Delta c_i$$

$Q_{ET_{i,j}}$ = volumetric loss rate due to evapotranspiration from cell i,j, $(L^3 T^{-1})$;

$R_{ET_{i,j}}$ = rate of loss per unit surface area of water table within the map area $\Delta r_j \Delta c_i$, (LT^{-1});

$Q_{ET_{i,j}}$ = maximum volumetric rate of evapotranspiration from cell i,j, $(L^3 T^{-1})$;

$h_{i,j,k}$ = head or water table elevation in the cell from which the evapotranspiration occurs, (L);

$d_{i,j}$ = extinction or cutoff depth for cell i,j, (L);

$h_{s_{i,j}}$ = water table elevation at which evapotranspiration occurs at maximum rate, (L);

Fig.4.11 shows a graph of evapotranspiration loss, $Q_{ET_{i,j}}$ vs head in cell i,j,k based on above equations.

4.5.5 General head boundary package:

The general head boundary package determines the transfer of flow across the boundaries. A linear relationship between the
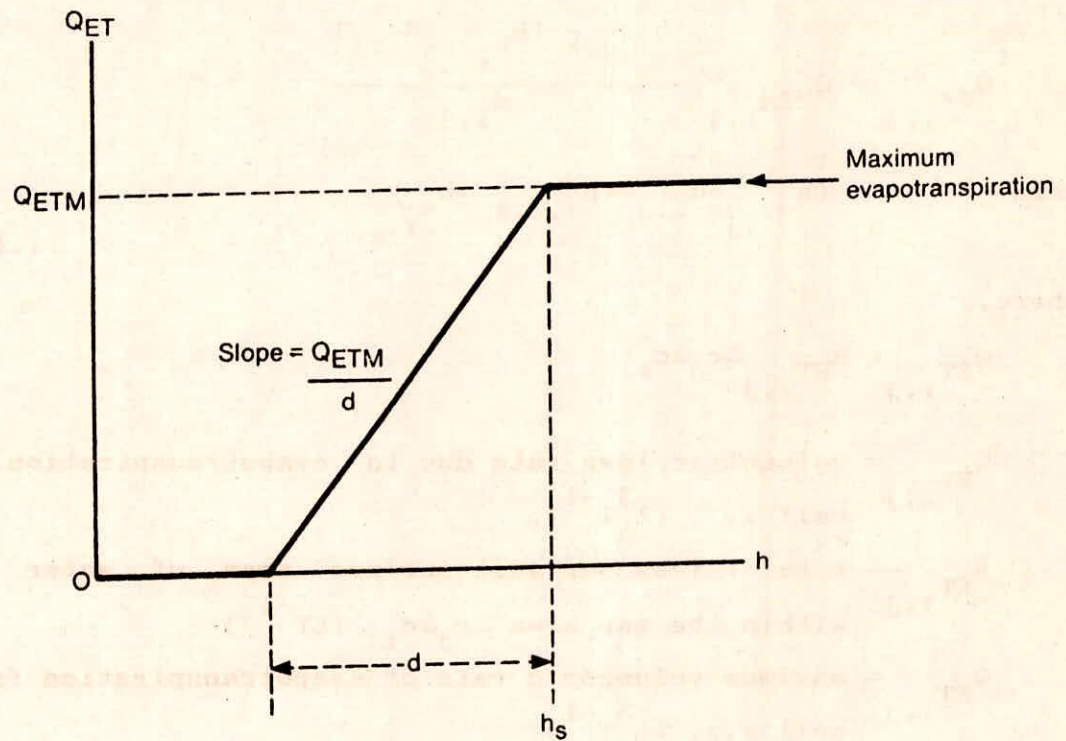
Fig.4.11 Plot of Q_{ET}, as a Function of Head, h, in a Cell.
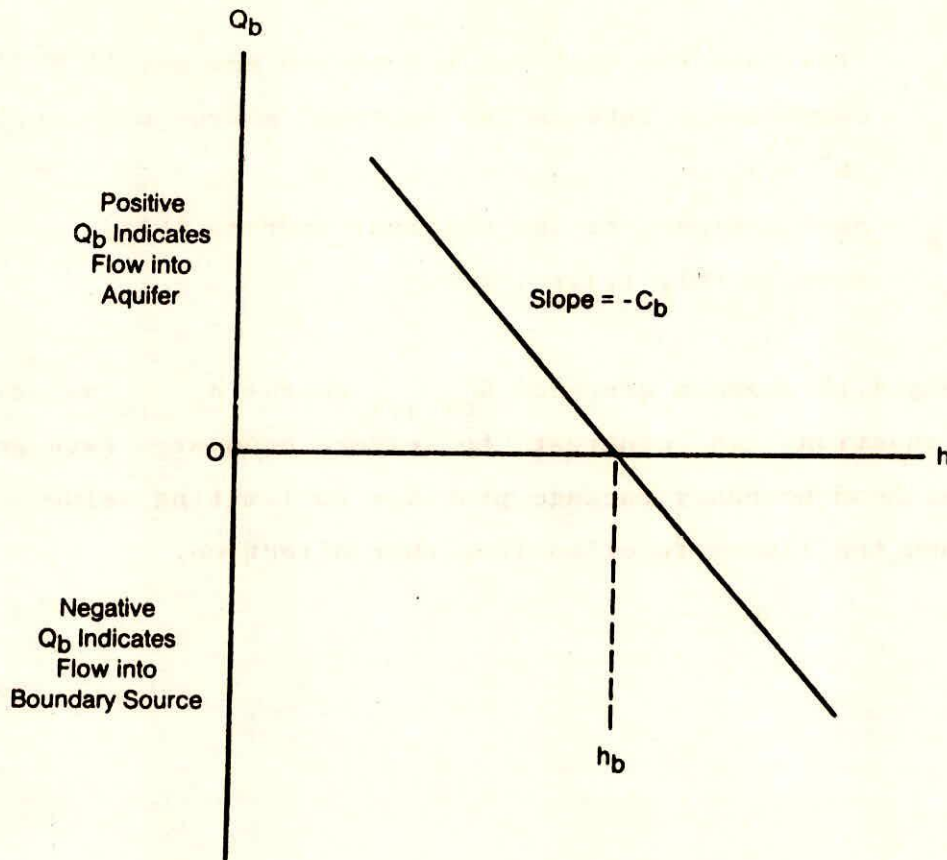
44

Fig.4.12 Plot of Flow, Q_b, from a General Head Boundary Source into a Cell as a Function of Head, h, in the Cell.

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flow into the cell and head in the cell is assumed, which is

$$Q_{bi,j,k} = C_{bi,j,k}(h_{bi,j,k} - h_{i,j,k})$$ ...(4.21)

where,

$Q_{bi,j,k}$ = flow into the cell i,j,k from the source, $(L^3 T^{-1})$;

$C_{bi,j,k}$ = conductance between the external source and cell i,j,k $(L^2 T^{-1})$;

$h_{bi,j,k}$ = head assigned to the external source, (L);

$h_{i,j,k}$ = head in cell i,j,k , (L).

Fig.4.12 shows a graph of $Q_{bi,j,k}$ versus $h_{i,j,k}$ as given by above equation. In contrast to river boundary packages, the general head boundary package provides no limiting value of flow to bound the linear function in either direction.

5.0 WALTON (GWFL3D) MODEL:

Walton(1989) developed a micro computer program(GWFL3D) for simulating quasi-three dimensional groundwater flow. This model incorporates both numerical and analytical technique and is a extensive modification to the popular mainframe computer program PLASM developed by Prickett and Lonnquist(1971). The modelhas the provisions of simulating production wells, flowing wells, multi-aquifer production wells, large diameter wells, partially penetrating production wells, aquifer layering, aquitard storativity, delayed gravity response, drains and mines, in addition to the aquifer system and simulation capabilities described in PLASM.

Similar to PLASM, this model solves finite difference equations in one-, two- and quasi-three-dimension for unsteady flow in heterogeneous aquifer system using a modified ADI algorithms. The program calculates the spatial and temporal distribution of piezometric head. It can simulate the following aquifer system and features.

1. Unconfined, semi-confined and confined aquifers;
2. Constant head, constant flux and no flow aquifer boundaries;
3. Conversion of aquifer storativity;
4. Multi-aquifer condition;
5. Evapotranspiration from groundwater;
6. Production well with constant or variable discharge or injection well recharge;
7. Induced infiltration from surface water (e.g., from streams, etc);

In the model, aquifer system conditions are user defined in a interactive mode. it is a modular model. This model was verified (Walton,1989,pp 2) with public domain program presented by McDonald and Harbaugh(1984) and with the data and equations

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presented by Prickett and Lonnquist(1971), Trescott et al.(1976), Bennett et al.(1982), Rushton and Redshaw(1979). The techniques for simulating i)leakage through aquitards ii) induced infiltration from stream bed, conversion from confined to unconfined case, water table conditions and multiple aquifer systems, are the same as in PLASM (Prickett and Lonnquist, 1971, pp 30-52).

5.1 Discretization:

The time discretization and space discretization are interdependent and should be specified in a manner which can represent the appreciable head changes adjacent to source/sink nodes during the initial time increment. There is a provision for non-uniform time increments which are user-defined as a part of time variable data base. The proper choice of time-increment is a must for accurate head/drawdown calculations. The total simulation time is divided into a number of small time increments.

The spatial discretization has been obtained by finite difference grid lines forming 'rows' and 'columns' superimposed over the map (plan) of the aquifer system. 'I' denotes the column index and 'J' denotes the row index. The origin of the coordinate axes is assumed at the upper left corner of the plan map. Therefore, 'I' increases from left to right and 'J' increases from top to bottom. Nodes are assumed at the intersections of the grid lines. The grid spacing should be small in comparison to the simulation area. Smallest grid spacings should be specified where the head gradient varies significantly such as around production or injection wells, rivers, mines and drains. There is a provision for variable grid spacings. Grid spacings should be gradually increased outward from these features. Grid spacings are uniform along individual columns and rows. The aquifer system is discretized in vertical into a number of layers which are numbered

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from the land surface downward using the 'K' notation. Grid spacings is the same in all aquifer system units or layers. The origin of the system is taken at the upper left corner of the top most unit or layer. Water table condition is assumed to prevail in the uppermost layer at the top of the aquifer system, when the aquifer layering is simulated.

5.2 Solution Technique:

The large set of finite difference equations are solved using the algorithms used in PLASM, where the equations are based on square grid. The finite difference equation for a square grid in a single layer aquifer system (Fig.5.1) as given in PLASM (Prickett and Lonnquist, 1971, p4) is given below:

$$Q_n + T_{i-1,j,2}(h_{i-1,j} - h_{i,j}) + T_{i,j,1}(h_{i,j+1} - h_{i,j})$$

$$= T_{i,j,2}(h_{i,j} - h_{i+1,j}) + T_{i,j-1,1}(h_{i,j} - h_{i,j-1})$$

$$+ S\Delta x_i \Delta Y_j (h_{i,j} - ho_{i,j})/\Delta t + Q_{i,j} \qquad \ldots(5.1)$$

where,

$T_{i,j,1}$ = aquifer layer transmissivity between nodes (i,j) and (i,j+1), $(L^2 T^{-1})$;

$T_{i,j,2}$ = aquifer layer transmissivity between nodes (i,j) and (i+1,j), $(L^2 T^{-1})$;

$h_{i,j}$ = calculated head at the end of simulation time increment Δt at node (i,j), (L);

$ho_{i,j}$ = calculated head at the end of previous simulation time increment at node i,j , (L);

Δt = simulation time increment, (T);

S = aquifer layer storativity, (dimensionless);

$Q_{i,j}$ = rate of withdrawal at node (i,j), $(L^3 T^{-1})$;

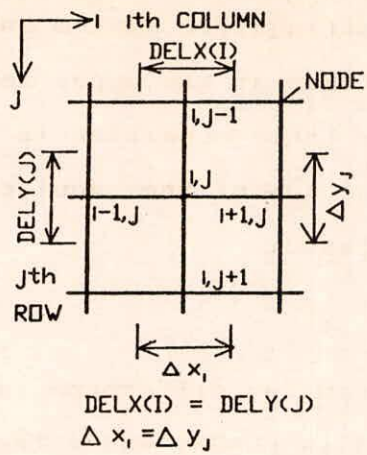Q_n = source/sink flow rate at node (i,j), (positive if source), $(L^3 T^{-1})$;

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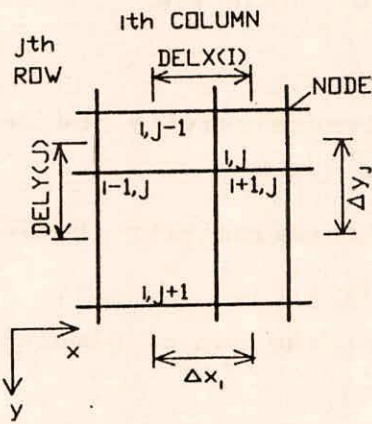
Fig.5.1 Uniform Finite Difference Grid.

Fig.5.2 Variable Finite Difference Grid.

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Δx_i = grid spacing in X-direction, (L);

ΔY_j = grid spacing in Y-direction, (L).

The left hand side of the above equation represents inflow to the node i,j and the right hand side represents outflow from the node i,j including change in storage in the grid. For a variable grid spacing (Fig.5.2), the finite difference equation can be written as (Walton, 1989, pp. 17)

$$Q_n + T_{i-1,j,2}(h_{i-1,j} - h_{i,j})[2\Delta Y_j/(\Delta X_{i+1} + \Delta X_i)]$$

$$+ T_{i,j,1}(h_{i,j+1} - h_{i,j})[2\Delta X_i/(\Delta Y_i + Y_{j+1})]$$

$$= T_{i,j,2}(h_{i,j} - h_{i+1,j})[2\Delta Y_j/(\Delta X_i + \Delta X_{i+1})]$$

$$+ T_{i,j-1,1}(h_{i,j} - h_{i,j-1})[2\Delta X_i/(\Delta Y_j + \Delta Y_{j+1})]$$

$$+ S\Delta X_j \Delta Y_j(h_{i,j} - ho_{i,j})/\Delta t + Q_{i,j} \qquad \ldots(5.2)$$

Comparing the above equation with (5.1), it is obvious that the algorithm for solving large sets of finite difference equations based on a square grid can be utilized with valuable grids, provided $T_{i,j,2}$ and $T_{i-1,j,2}$ are multiplied by the arithmetic mean expression $2\Delta Y_j/(\Delta X_{i+1} + \Delta X_i)$ and $T_{i,j,1}$ and $T_{i,j-1,1}$ are multiplied by $2\Delta X_i/(\Delta Y_{j+1} + \Delta Y_j)$. GWFL3D simulates variable grid spacing by adjusting transmissivities accordingly similar to that in PLASM.

5.3 Closure Criteria:

The model uses iterative alternating direction implicit method (Prickett and Lonnquist, 1971, pp 3-7,46-48) to solve the finite difference simultaneous equations given by (5.2). These equations are solved in subroutine SIMEQ. Water balance components are calculated in the subroutine WATBL called from the subroutine

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SIMEQ upon the request of the user. The iterations are repeated till the user specified closure criteria for head convergence is met. The model does not use the water balance summed for all nodes as the measure of convergence error as done in PLASM, because precision of water balances at all nodes must be high for accurate results and this precision tends to be low at nodes where simulation factors or storativities are multiplied by very small head-losses or head-differences which can cause severe program problems.

5.4 Computer Memory Limitations:

The model has the following computer memory limitations.

1. the aquifer simulation layers must be less than 5 because of the limited micro-computer core storage (640K);

2. the numbers of columns or rows should be limited to 20 because of limited micro computer core storage;

3. if delayed gravity yield is simulated, the number of layers must be less than 1;

4. the number of time increment must be less than 51, if no delayed gravity yield is simulated;

5. if delayed gravity yield is simulated, the number of time increments must be less than 21, because of limited micro-computer core storage.

5.5 Simulations:

The following simulations can be made by assigning the typical values for the parameters.

1. Water table conditions may be simulated in the upper most layer by setting the top aquitard hydraulic conductivity and thickness equal to zero, thus creating an aquiclude;

2. Leaky artesian conditions with a constant head with source bed may be simulated in the upper most layer by specifying

the source bed head and the aquitard hydraulic conductivity and its thickness.

5.6 Additional Modelling Facilities:

The model has the following additional modelling facilities as compared to PLASM. These facilities have been discussed below:

5.6.1 Modelling irregular and recharge boundaries:

No-flow boundaries may be simulated by assigning zero transmissivities to the boundary nodes. Constant head boundaries may be simulated by assigning very large values for storage coefficients at boundary nodes (value: 1×10^{20}). For irregular boundaries, the nodes lie on both side of the boundaries. In other words, aquifer boundaries are located midway between adjacent nodes lying outside and inside the boundaries. The irregular boundaries may be simulated by adjusting transmissivities, storage factors with the help of following equations (Walton, 1989, pp 18-19).

$$T_{xa} = T_{xu} [(\Delta Y/2 + Y_b)/(\Delta X/2 + X_b)]$$

$$T_{ya} = T_{yu} [(\Delta X/2 + X_b)/(\Delta Y/2 + Y_b)]$$

$$SF1a = SF1u[(\Delta X/2 + X_b)(\Delta Y/2 + Y_b)/(\Delta X \Delta Y)]$$

$$R_a = R_a [(\Delta X/2 + X_b)(\Delta Y/2 + Y_b)/(\Delta X \Delta Y)] \qquad \qquad ...(5.3)$$

where,

T_{xa} = aquifer layer transmissivity in x-direction adjusted for irregular boundary location in gpd/ft, $(L^2 T^{-1})$;

T_{ya} = aquifer layer transmissivity in Y-direction adjusted for irregular boundary location in gpd/ft, $(L^2 T^{-1})$;

T_{xu} = aquifer layer transmissivity in X-direction without boundary in gpd/ft, $(L^2 T^{-1})$;

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T_{yu} = aquifer layer transmissivity in X-direction without boundary in gpd/ft, (L^2T^{-1});

SFla=aquifer layer storage factor adjusted for irregular boundary adjacent to node in gal/ft;

SFlu=aquifer layer storage factor without boundary in gal/ft;

Ra =simulation factor adjusted for irregular boundary adjacent to node in gpd/ft;

Ru =simulation factor without boundary in gal/ft;

ΔX =grid spacing in X-direction in ft;

ΔY =grid spacing in Y-direction in ft;

X_b =distance between grid line and boundary in x-direction in ft;

Y_b =distance between grid line and boundary in y-direction in ft;

If there is no boundary in x-direction, $X_b=\Delta X/2$ and if there is no boundary in y-direction, $Y_b=\Delta Y/2$. For a square grid $\Delta X=\Delta Y$.

5.6.2 Simulation of Well-fields:

The model simulates the drawdown caused by the following type of wells in the simulation of regional aquifer system.

    1. production tube-well (fully penetrating the aquifer);

    2. partially penetrating tube-well;

    3. fully penetrating finite diameter well (considering well storage capacity);

    4. multi-aquifer production tube-well (considering the well to be open to a number of aquifers);

    5. Flowing well.

First four types of wells as given above are simulated through the subroutine SIMEQ, while the flowing well is simulated through the subroutine FWEL. In all the above cases the simulated drawdowns/heads are obtained through the subroutine RESLT.

A <u>fully penetrating tube-well</u> is simulated by imposing a discharge rate on a grid block centred at a node (similar

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technique has been adopted in PLASM for simulating a production well). In real situation, the diameter of the production well is much smaller as compared to the dimensions of the grid block in which the well is located. In this case, the head calculated at production well node is the average head for the grid block and not the head in the production well. In order to calculate the head in the production well, the following equations have been used.(Walton,1989,pp 21).

$$r_e = 0.28[(T_{yy}/T_{xx})^{1/2}\Delta X^2 + (T_{xx}/T_{yy})^{1/2}\Delta Y^2]^{1/2} /$$

$$[(T_{yy}/T_{xx})^{1/4} + (T_{xx}/T_{yy})^{1/4}]$$

$$h_w = h_{gb} - Q \ln(r_e/r_w)/[2\pi(T_{xx}\,T_{yy})^{1/2}] \qquad \qquad \ldots(5.4)$$

where,

r_e =equivalent well block radius at which the head in the aquifer isequal to the numerically calculated head for the production-well grid block (in ft);

T_{xx} & T_{yy} =aquifer layer transmissivities at well node in x and y-directions respectively in gpd/ft;

ΔX & ΔY =well block grid spacings in x and y-directions respectively

h_w =head in the production well in ft;

h_{gb} =average head in well block in ft;

Q =production well discharge rate in gpd;

r_w =effective radius of production well in ft.

The production well head may be adjusted for the well loss occurring due to turbulent flow near well, using the following equation proposed by Jacob, 1947, pp 1047-1070)

$$s_{wl} = C_{wl} Q^2 \qquad \qquad \ldots(5.5)$$

where,

s_{w1} = additional drawdown due to well loss in ft;

C_{w1} = well loss constant in $\sec^2/ft^5$;

Q = production well discharge rate in ft^3/s;

All the variables/constants appearing in above equations are input to the model and are user-defined in subroutine RESLT.

A <u>partially penetrating well</u> may be simulated in GWFL3D through subroutine PARPEN using the following equation given by Hantush(1964).

$$s_t = s + s_p$$

$$s_p = 114.6Q[2m^2/[\pi^2(1-d)(l_o-d_o)]\sum_{n=1}^{\infty}(1/n^2)\{\sin(n\pi l/m)-\sin(n\pi d/m)\}$$

$$\{\sin(n\pi l_o/m)-\sin(n\pi d_o/m)\}W[u,n\pi r(P_V/P_H)^{1/2}/m]$$

$$u=1.87r^2 S/(Tt)$$

$$...(5.6)$$

where,

s_t = total drawdown due to partially penetrating wells in ft;

s = drawdown with fully penetrating wells in ft;

s_p = drawdown due to effects of partial penetration of wells in ft;

Q = discharge rate in gpm;

r = distance between production and observation wells in ft;

S = aquifer storativity as a decimal;

T = aquifer transmissivity in gpd/ft;

t = pumping period in days; m = aquifer thickness in ft;

P_V & P_H = aquifer hydraulic conductivity in vertical and horizontal direction respectively;

l = distance from aquifer top to production well screen base in ft;

d = distance from aquifer top to production well screen top in ft;

l_o = distance from aquifer top to observation well screen base in ft;

d_o = distance from aquifer top to observation well screen top in ft;

Approximate expressions are utilized for estimating values of $W\{u, n\pi r(P_V/P_H)^{1/2}/m\}$. All the variables/constants appearing in above equations are user-defined in subroutine PARPEN. Production well discharge is assumed constant.

A fully penetrating large diameter well with storage within it may be simulated through subroutine WELSTOR. The model uses the following equations to calculate the drawdown with well storage effect (Walton, 1989, pp 25-27).

$$Q = Q_{at} + \pi(r_w^2 - r_c^2)s_{ws}7.48/\Delta t$$

$$s_{ws} = sQ_{at}/Q \qquad \qquad \ldots(5.7)$$

where,

Q = constant discharge from the production well in gpm;

Q_{at} = average discharged derived from aquifer or aquifer layer during simulation period in gpm;

r_w = production well effective radius in ft;

r_c = pump column pipe radius in ft;

s_{ws} = drawdown with well storage impacts at the end of simulation period in ft;

Δt = time at the end of present simulation period in min.

Using the above two equations and an iterative procedure the value of Q_{at} and S_{ws} are obtained. The first trial value of s_{ws} is based on the numerically calculated drawdown without well storage impacts. The first and the second trial values of s_{ws} are compared and if the difference exceeds on user-specified tolerance error, the iteration is repeated. Thus, when the tolerance criteria is met, the calculated value of Q_{at} and s_{ws} are declared as true values. All the variables are specified by the user in the subroutine WELSTOR. The variable discharge from the well can not be simulated.

A multi-aquifer production well can be simulated through the subroutine SIMEQ in respect of the following variables.

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a) Calculation of water level in abstraction well taping multiple aquifers;

b) Calculation of the contribution of individual aquifer to well discharge;

c) Calculation of redistribution of flow/head between aquifers through the well under non pumping conditions;

Fig.5.3 shows a multi-aquifer production well. Recharging well's negative discharges can also be simulated. The discharges into the production well from different aquifers to which the well is open, are calculated with the following equations (Walton, 1989, pp 27-29).

$$Q_{i,j,b} = 2\pi T_{i,j,b} h_{i,j,b} / \ln(r_e/r_w) - \{2\pi T_{i,j,b} / \ln(r_e/r_w)\}$$

$$\sum_{b=m}^{n} [T_{i,j,b} h_{i,j,b} / \ln(r_e/r_w)] / \sum_{b=m}^{n} [T_{i,j,b} / \ln(r_e/r_w)]$$

$$+ T_{i,j,b} QT_{i,j} / [\ln(r_e/r_w) \sum_{b=m}^{n} \{T_{i,j,b} / \ln(r_e/r_w)\}] \quad \ldots(5.8)$$

where,

r_e = equivalent production well block radius in ft is given by eqn.(5.4);

$T_{i,j,b}$ = aquifer layer transmissivity at production well node i,j in aquifer layer b in gpd/ft;

$h_{i,j,b}$ = head in the aquifer layer b at node i,j at the end of the time increment in ft;

r_w = effective radius of production well in ft;

$QT_{i,j}$ = discharge from production well in gpd;

m,n = indices denoting aquifer layers open to the well;

$Q_{i,j,b}$ = discharge from aquifer layer b into well in gpd.

Using the above equation values of $h_{i,j,b}$ are obtained by iterative technique. Knowing the values of $h_{i,j,b}$, the composite water level in the well is calculated from the following equation.
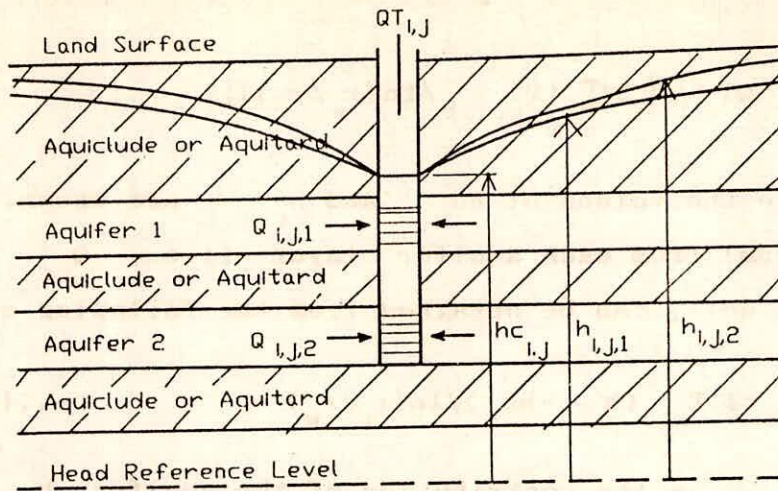
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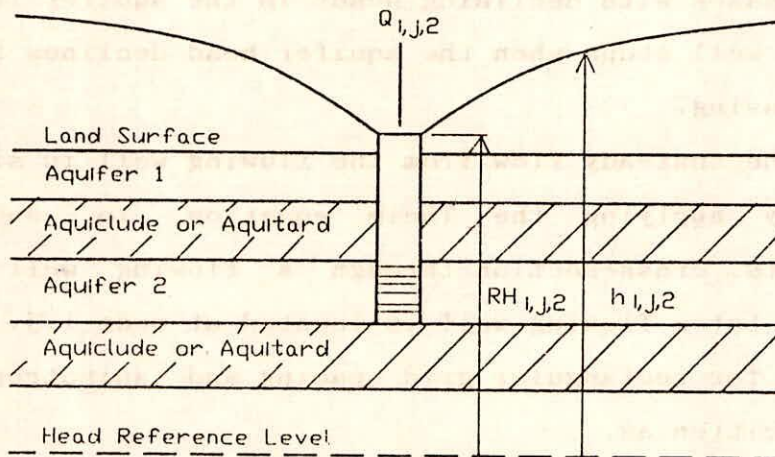**Fig.5.3 Cross Section Through a Multi-Aquifer Well.**

Fig.5.4 Cross Section Through a Flowing Well.

$$hc_{i,j} = \sum_{b=m}^{n} [T_{i,j,b} h_{i,j,b}/\ln(r_e/r_w)] / \sum_{b=m}^{n} [T_{i,j,b}/\ln(r_e/r_w)] -$$

$$QT_{i,j}/[2\pi \sum_{b=m}^{n} \{T_{i,j,b}/\ln(r_e/r_w)\}] \qquad \qquad ...(5.9)$$

Once the values of $hc_{i,j}$ and $h_{i,j,b}$ are known, the rate of flow (in gpm) from each aquifer layer (i.e., $Q_{i,j,b}$) into the production well, can be obtained from the following equation.

$$Q_{i,j,b} = 2\pi T_{i,j,b}(h_{i,j,b} - hc_{i,j})/\ln(r_e/r_w) \qquad \qquad ...(5.10)$$

$Q_{i,j,b}$ is the contribution of the aquifer b to the well discharge. The variables appearing in (5.8) through (5.10) are input to the model and are read through the subroutine SIMEQ.

Head variation in a <u>flowing well</u> can be simulated in in the model through the subroutine FWEL. The model uses the Thiem equation (Todd,1980, pp 117) which defines the steady state flow from a well. The discharge from a flowing well varies with time and decreases with declining heads in the aquifer layer. The flow from the well stops when the aquifer head declines below the top of the casing.

The unsteady flow from the flowing well is simulated in the model by applying the Theim equation to successive time increments. cross-section through a flowing well is shown in fig.5.4. Let a flowing well is located at node i,j. Then the Thiem equation for rectangular grid spacing and anisotropic conditions may be written as.

$$Q_{i,j,k} = 2\pi (T_{xx} T_{yy})^{1/2} (h_{i,j,k} - RH_{i,j,k})/\ln(r_e/r_w)$$

$$= RF_{i,j,k} h_{i,j,k} - RF_{i,j,k} RH_{i,j,k} \qquad \qquad ...(5.11)$$

where,

$Q_{i,j,k}$ = discharge rate from the flowing well at node i,j and open to k^{th} layer of the aquifer in gpd;

T_{xx} & T_{yy} = aquifer layer transmissivity in x and y-directions respectively at well node in ft;

r_e = equivalent radius of flowing-well block expressed by eqn. 5.4;

r_w = effective radius of flowing well;

$h_{i,j,k}$ = head at flowing well node in k^{th} aquifer layer;

$RH_{i,j,k}$ = head in the flowing well;

$RF_{i,j,k}$ = flowing well simulation factor assigned to the flowing well node

Delayed gravity yield under water table conditions for a single aquifer can be simulated in GWFL3D by assigning to each node an effective storativity and an effective recharge defined (Rushton and Redshaw, 1979, pp. 259-262) as:

$$SE_{i,j,k} = S_{i,j,1} + SY_{i,j,1} [1-\exp\{-alpha_{i,j,1} \Delta t_t \}]$$

$$QR_{i,j,1} = alpha_{i,j,1} SY_{i,j,1} \sum_{i=1}^{n-1} \Delta s_m \exp(1-alpha_{i,j,1})(t_n - t_m) 7.48 A_{i,j,1}$$

$$...(5.12)$$

where,

$alpha_{i,j,1} = 3PV_{i,j,1} /SY_{i,j,1}$ m7.48)= reciprocal of delayed index in day^{-1}

$SE_{i,j,1}$ = effective storativity of the aquifer layer with delayed gravity yield during current time increment as a decimal;

$S_{i,j,1}$ = storativity of the aquifer layer as a decimal;

$SY_{i,j,1}$ = specific yield of the aquifer layer as a decimal;

Δt_t = time since start of the pumping in days;

t_m = time at the end of the mth time increment in days;

t_n = time at the end of the nth time increment in days;

n = current time increment number

$QR_{i,j,1}$ = effective recharge arising from delayed gravity yield due to previous drawdown in gpd (during m^{th} time increment, a

drawdown s_m occurred which produces a delayed gravity yield contribution at a later time t_n);

$A_{i,j,1}$ = plan area of the vector volume of the aquifer layer in which delayed gravity yield occurs (in sq ft);

$PV_{i,j,1}$ = aquifer layer vertical hydraulic conductivity in gpd/sq ft;

$h_{i,j,1}$ = head in the aquifer layer at the end of the current time increment in ft.

The time t_d (in days) after which the effects of the delayed gravity yield are negligible may be determined with the following equation (Walton, 1989, p. 41):

$$t_d = 37.5 m S_y / P_v \qquad \qquad ...(5.13)$$

where,

m = thickness of the aquifer layer in ft;

S_y = specific yield of the aquifer layer as a decimal;

P_v = vertical hydraulic conductivity of the aquifer layer in gpd/sq ft.

For precise simulation of delayed gravity yield, small time increments and finer grid spacings near the production well and small convergence criteria for nodal heads should be used. This simulation assumes single production well inaffected by other factors (output/inputs) or wells.

5.6.3 Simulation of multi-aquifer system:

The layered aquifer or multi-aquifer system as shown in fig.5.5 can be simulated by assigning an aquifer layering simulation factor $RA_{i,j}$ (in gpd/ft) to each node where more than one aquifer layers occurs. $RA_{i,j}$ is given by the following equation (Walton, 1989, pp. 38-39).
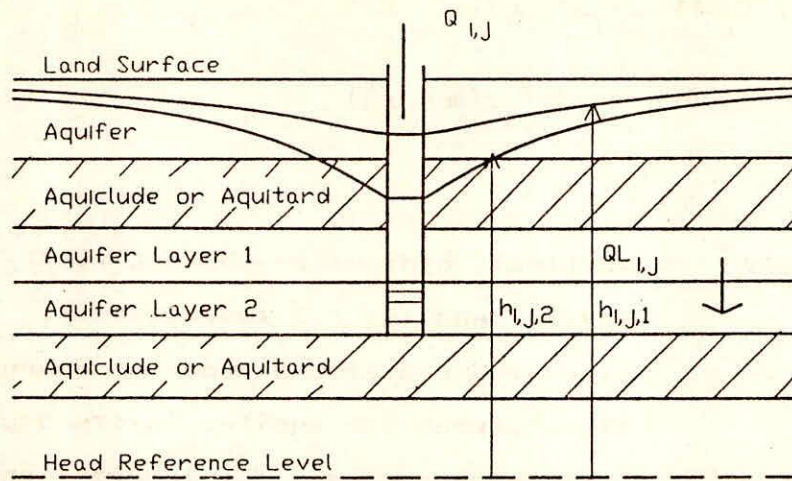
Fig.5.5 Cross Section Through a Multi-Aquifer System.

$$RA_{i,j} = C_1 C_2 / (C_1 + C_2)$$

$$C_1 = PV_{i,j,1} \; AI_{i,j} / (m_{i,j,1}/2)$$

$$C_2 = PV_{i,j,2} \; AI_{i,j} / (m_{i,j,2}/2) \qquad \qquad \ldots (5.14)$$

where,

PV_{ij1} & PV_{ij2} = vertical hydraulic conductivity in gpd/sq.ft of layer 1 and layer 2 respectively;

AI_{ij} = plan area centered at node i,j through which cross flow between the aquifer layers occur in sq ft;

$m_{i,j,1}$ & $m_{i,j,2}$ = thicknesses of aquifer layers 1 and 2 respectively in ft.

The rate of exchange of flow between the aquifer layers (QL_{ij}) in gpd can be calculated using following equation.

$$QL_{i,j} = RA_{i,j} (h_{i,j,1} - h_{i,j,2}) \qquad \qquad \ldots (5.15)$$

where,

$h_{i,j,1}$ & $h_{i,j,2}$ = heads in the aquifer layer 1 and aquifer layer 2 respectively in ft.

When the head in a layer becomes less than the elevation of the top of the layer, the storativity of the layer is changed from for artesian to water table condition.

The effect of aquitard storativity can be simulated in GWFL3D for the periods when time is less than t_{as}. t_{as} is defined as the time since last pumping rate change during which release from the aquitard is appreciable. t_{as} in days can be obtained from the following equation:

$$t_{as} = S'm'/P' \qquad \qquad \ldots (5.16)$$

where,

S' = aquitard storativity as a decimal;

m' = aquitard thickness in ft;

p' = aquitard hydraulic conductivity in gpd/sq ft.

In order to simulate aquitard release, an aquitard storativity simulation factor is assigned to each node where aquitard exists and is given by the following equation. (Walton, 1989, pp. 40-41).

$$RT_{i,j,k} = P'_{i,j,k} A_{i,j,k} / [\{\pi P'_{i,j,k} t_{ap_{i,j,k}} m'_{i,j,k} / (3 m'^2_{i,j,k} S'_{i,j,k} 7.48)\}^{1/2}$$

$$m'_{i,j,k}][1 + 2\sum_{n=1}^{200} \exp\{-n^2 / [P'_{i,j,k} t_{ap_{i,j,k}} m'_{i,j,k} / (3 m'^2_{i,j,k} S'_{i,j,k} 7.48)]\}]$$

$$...(5.17)$$

where,

$RT_{i,j,k}$ = aquitard storativity simulation factor in gpd/ft;

$A_{i,j,k}$ = horizontal area of aquitard vector volume within which water is released from storage in sq ft;

$S_{i,j,k}$ = aquitard storativity as a decimal;

T_{ap} = time after last pumping rate change in days.

The aquitard storage release rate $QS_{i,j,k}$ in gpd is calculated as:

$$QS_{i,j,k} = RT_{i,j,k} ho_{i,j,k} - RT_{i,j,k} h_{i,j,k} \qquad ...(5.18)$$

where,

$h_{i,j,k}$ = head in aquifer layer at time t in ft;

$ho_{i,j,k}$ = head in aquifer layer at the end of previous time increment in ft.

6.0 CONCLUDING REMARKS

The following groundwater flow models that are widely used for the modelling of groundwater systems, have been reviewed in terms of modelling philosophies for different components of groundwater system, different type of internal and external boundary conditions, temporal variation of external stresses, etc..

1. PLASM (developed by Prickett and Lonnquist, 1971);

2. MODFLOW (developed by McDonald and Harbaugh, 1988);

3. GWFL3D (developed by Walton) and;

4. Groundwater flow Model developed by Boonstra and deRidder.

For each of the above models, detailed development of flow equations and codes, modelling techniques and capabilities along with the limitations for modelling specific components have been discussed.

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