Groundwater Modeling

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INTRODUCTION

Groundwater flow modeling implies performing numerical experiments on a groundwater flow model. The objective of such experimentation for practicing engineers is usually to check the feasibility of any human intervention into the groundwater system, e.g., pumpage, recharge etc. For groundwater academics, the objective could be to understand various processes involved in the groundwater system.

GROUNDWATER FLOW MODEL

A groundwater flow model is essentially a tool to project the State variables of the groundwater system for an assigned pattern of forcing function, and known initial and boundary conditions and parameters.

A brief description of various terms appearing in this definition is included in the following paragraphs.

State Variables

The state variables are essentially the variables that describe the "state" of a system. These variables may be divided in two categories viz. Mandatory and Problem-specific. The mandatory state variable is: Piezometric head or Water table elevation. This variable is henceforth termed as "head". The Problem specific state variables are essentially derived from the head distribution in space and time. These could include, depending upon the problem at hand, depth to water table, static storage, influent/ effluent seepage, outflow to sea, sea water intrusion etc.

Forcing Function

The forcing function may comprise, among others, the following constituents:

- Withdrawals (i.e., pumpage)
- Recharge (derived from- rainfall, applied irrigation, seepage from surface water bodies etc.)
- Evapotranspiration from the saturated zone

Initial and Boundary Conditions

Initial conditions: Initial conditions, as the name implies comprise of the spatial distribution of the head at the instance when the assigned excitation commences to act. There are two possible interpretations of the Initial conditions. Mathematically, they are necessary for arriving at a unique solution of a differential equation. Conceptually, they can be visualized as the influence of the hydraulic conditions occurring prior to the activation of the assigned forcing function.

Boundary conditions: Here too there are two possible interpretations. Mathematically, they are necessary for arriving at a unique solution of a differential equation. Conceptually, they can be visualized as the influence of the hydraulic conditions occurring across the boundary of the domain, of the solution. Thus, to obtain a unique solution of the differential equation, it is necessary to define boundary conditions all along domain boundary. The boundary condition may either be a known head (head assigned) or a known flow rate (flow assigned) across the boundary. It can be thus concluded that for obtaining a unique solution it is necessary to know either the head or normal flows all along the boundary.

Boundary heads are assigned wherever an aquifer is terminating into a water body. At the interface between the two, the head may be assumed to be equal to the water elevation in the water body.

Normal flows need to be known for the part (s) of the domain boundary not interfacing with water bodies. These flows are more difficult to estimate (unless they are known to be zero i.e., an impervious boundary) and would usually require water balance of the adjoining areas.

Out of the two types of boundary conditions, the head assigned boundaries are more suitable for forecasting since the water elevations in the hydraulically connected water bodies may generally not be significantly influenced by the pumping/recharge pattern in the aquifer. Thus, the known prevalent water elevations may be assumed to hold good under the projected conditions (i.e., the pumping/recharge rates different from the prevalent ones). On the other hand, the lateral inflows across the boundary are very sensitive to any change in pumping/recharge. Thus, the inflow rates under the projected conditions may vary significantly from the prevailing ones. In other words the known prevalent inflow rates may not provide the necessary boundary conditions.

Model Parameters

The spatial distribution of the appropriate (that is, depending upon the type of aquifer) aquifer parameters need to be assigned for computing the head distributions corresponding to the assigned forcing function. The data from pumping tests shall rarely be adequate to meet this input requirement. The spatial distribution is usually obtained from a solution of *Inverse problem* (or Model calibration). The solution requires the historical data of forcing function, heads, initial and boundary conditions. It aims at evolving such distribution of the aquifer parameters, which lead to a closest match between the observed, and the model-computed heads. Typically this requires repeated *direct* modeling corresponding to a selected historical period, with varying values of aquifer parameters, and finally arriving at the *best possible* match. This approach Model calibration is usually followed up by a validation of the calibration. This is accomplished by using only a part of the available data base for the calibration. The unused part is subsequently employed to determine how well the calibrated model reproduces the observed state variables. The calibration is considered as *validated* provided the reproduction of the state variables in the validation stage is *almost* as good as in the calibration stage.

Hard rock formations: A hard rock formation may typically comprise the top weathered zone under-lain by a fractured rock mass. The weathered zone is essentially characterized by high-frequency fracturing in all directions. And as such it is usually treated as a porous medium indexed by the usual flow and storage parameters. However, the behaviour of the underlying fractured rock may be quite different. The fractured rock is usually viewed as a combination of two distinct elements – solid original blocks of rock and the adjoining fractures. Each of these elements has its own flow and storage parameters. The flow parameters of the composite fractured rock mass are almost entirely due to the fractures with the solid rock providing negligible permeability. On the other hand the contribution of fractures towards the storage parameters is usually quite negligible. As such these parameters are primarily governed by the inter-grannular porosity of the rock. It is apparent that Model calibration and the subsequent validation are quite important for models of groundwater flow in hard rocks

COMPONENTS OF A GROUNDWATER FLOW MODEL

Typically a groundwater flow model comprises of the following components:

- An equation (algebraic or differential) governing the flow
- An algorithm to solve the chosen equation numerically to compute the time and space distribution of the head
- A set of algorithms to compute the problem- specific state variables from the pre-computed head distributions
- Computer codes to implement the selected algorithms

A brief description of these components is incorporated in the following paragraphs.

Governing Equations

Any equation governing the groundwater flow is essentially an expression of the continuity equation which, in the context of groundwater flow, can be heuristically stated as follows:

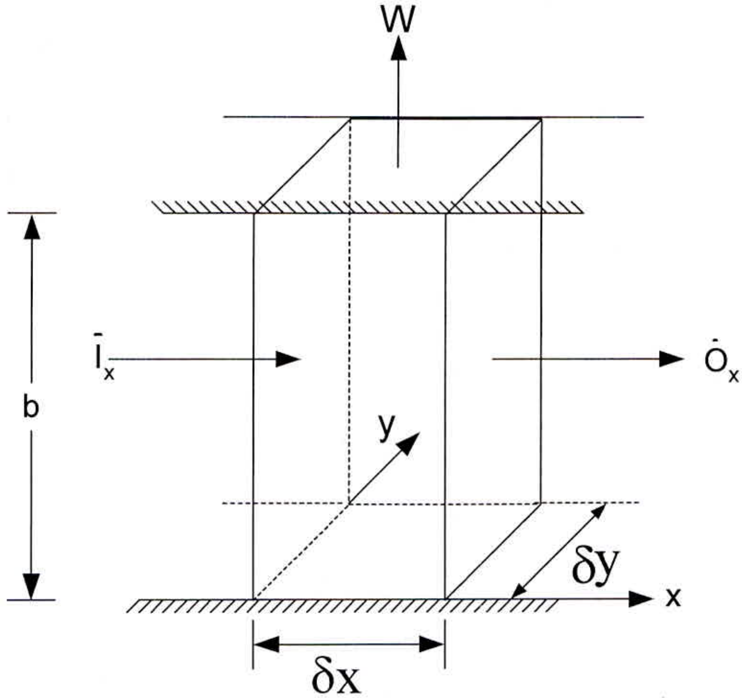
Across any selected domain of saturated flow, the difference between the inflow and the outflow rates equals the rate of change of the storage of water in that domain.

The selected domain is usually an infinitesimally small element. Thus, in case of a general three dimensional flow it is an infinitesimally small volume. However, if the flow occurs predominantly in two orthogonal directions with very little or no flow in the third orthogonal direction (e.g., two- dimensional horizontal flow), the domain could be infinitesimally small area with a unit/ physical dimension in the no- flow direction. Further, if the flow occurs predominantly only in one direction, the domain is an infinitesimally small length with unit/ physical dimensions in other two no- flow directions.

Two types of inflows/ outflows are considered while writing the continuity equation for groundwater flow. First type comprises the ones occurring on account of the prevalent hydraulic gradients. The second type comprises *external* inflow or outflow (also termed as Source or Sink tems), i.e., driven by the forcing function. The gradient driven inflow and the outflow rates are expressed in terms of the space derivatives of the head (viz., piezometreic head/ water table elevation) and a flow parameter (Hydraulic conductivity in general or Transmissivity in case of one/ two- dimensional horizontal flow) by invoking Darcy's law. The rate of change of the storage is expressed in terms of the time derivative of the head, this time invoking an appropriate storage parameter (Specific storage in general or Storage coefficient in case of one/ two-dimensional horizontal flow).

Plugging in the expressions for the gradient driven inflow and outflow rates and the rate of change of storage in the continuity equation, leads a differential equation comprising the spatial and temporal derivatives of the primary state variable, flow and storage parameters, and the forcing function.

The procedure described above is illustrated by deriving a differential equation governing two- dimensional horizontal flow in a confined aquifer. The derivation essentially involves writing down the continuity equation for an element having infinitesimally small dimensions (δx and δy) in lateral directions x and y, and extending over the entire thickness in the vertical direction (see the following figure).



Gradient driven flow rates in x- direction:

$$\dot{I}_x = -K_{xx}\frac{\partial h}{\partial x}b\delta y$$

$$= -T_{xx}\frac{\partial h}{\partial x}\delta y$$

$$\dot{O}_x = -\left[T_{xx}\frac{\partial h}{\partial x} + \frac{\partial}{\partial x}\left(T_{xx}\frac{\partial h}{\partial x}\right)\delta x\right]\delta y$$

$$\dot{I}_x - \dot{O}_x = \frac{\partial}{\partial x}\left(T_{xx}\frac{\partial h}{\partial x}\right)\delta x\ \delta y$$

Similarly $\dot{I}_y - \dot{O}_y = \frac{\partial}{\partial y}\left(T_{yy}\frac{\partial h}{\partial y}\right)\delta x\ \delta y$

Total gradient driven (Inflow - Outflow) rate

$$= \frac{\partial}{\partial x}\left(T_{xx}\frac{\partial h}{\partial x}\right)\delta x\ \delta y + \frac{\partial}{\partial y}\left(T_{yy}\frac{\partial h}{\partial y}\right)\delta x\ \delta y$$

Where x, y are the coordinates along two principal permeability directions, t is the time coordinate, h(x, y, t) is the head, I and O dots represent the inflow and outflow rates, K is hydraulic conductivity, T is Transmissivity, and suffixes denote the directions.

Forcing function driven out flow rate = W δx δy

Where W is the net abstraction per unit area per unit time (LT^{-1}).

$$\text{Total }(\dot{I} - \dot{O}) = \frac{\partial}{\partial x}\left(T_{xx}\frac{\partial h}{\partial x}\right)\delta x\delta y + \frac{\partial}{\partial y}\left(T_{yy}\frac{\partial h}{\partial y}\right)\delta x\delta y - W\,\delta x\delta y$$

$$\text{Rate of change of storage } = \frac{\partial h}{\partial t}S\delta x\delta y$$

Resulting governing differential equation:

$$\frac{\partial}{\partial x}\left(T_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_{yy}\frac{\partial h}{\partial y}\right) - W = S\frac{\partial h}{\partial t}$$

Where W is the forcing function (net external abstraction rate LT^{-1}) and S is storage coefficient.

Solution Algorithms

The differential equation governing the flow can be solved to obtain the spatial distribution of the head at pre- selected successively advancing discrete times. A realistic solution that accounts for the heterogeneity, anisotropy, and time and space variation of the forcing function, would have to be necessarily numerical in nature. A variety of numerical algorithms are available for solving the differential equations governing the groundwater flow. The easiest among them is the Finite difference method (FDM). A brief description of this method follows in the next paragraph.

FINITE DIFFERENCE METHOD

This method essentially involves the following steps:

Discretization of space and time

This is the first step of the modeling. Space, i.e., the area over which the system response is to be simulated, is discretized by a finite number of points- usually known as *nodes.* Typically the nodes may lie at the intersections of rows and columns superposed over the space. Similarly the time domain, i.e., the period over which the response is to be simulated, is discretized by a finite number *of discrete times.* Thus, a spatial distribution of any variable (say Storage coefficient) implies data comprising the values of the variable at each node. Similarly a spatial and temporal distribution of any variable (say, piezometric head) implies data comprising nodal values of the variable at the selected discrete times.

Marching in time domain

A strategy of "marching in time domain" is adopted for computing the nodal values of the head at successively advancing discrete times. This essentially involves: knowing the nodal heads at the beginning of a time step and computing the heads at the end of the time step. These computed heads form the "known heads" in the subsequent time step, and thus, the solution commences from the Initial condition and "marches" in the time domain.

Computation of the nodal heads at the end of a time step is accomplished as follows:

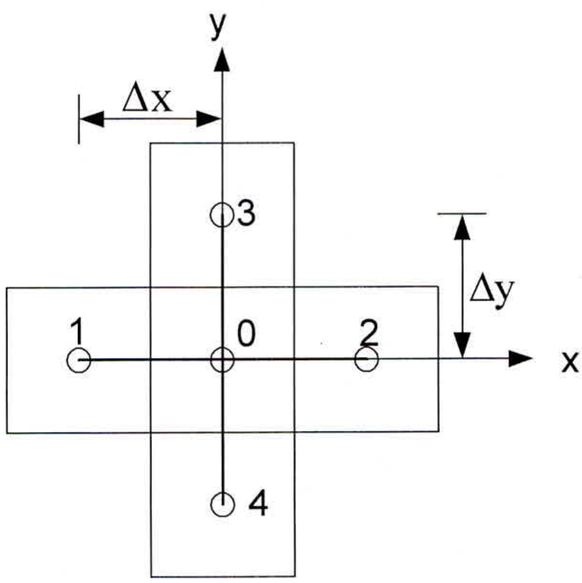
Formulation of linear algebraic equations

FDM essentially involves transforming the problem of solving the governing differential equation into a problem of solving a determinate system of linear equations. Formulation of the system of equations is discussed in the following paragraphs.

Interior Nodes: At each interior node, the space and time derivatives of the head appearing in the governing differential equations are approximated by corresponding finite differences. (This leads to inevitable truncation errors. However, these errors may be controlled by having not-so-large space and time steps.) This provides one linear equation for each node.

The procedure is illustrated below by writing down the finite difference form of the governing differential equation derived earlier.

Consider an interior node "0" surrounded by four nodes "1", "2", "3" and "4" as shown below.



Interior Node

The space and time derivatives of "h" at node "0" are expressed in terms of the respective finite differences as follows:

$$\frac{\partial}{\partial x}\left(T_{xx}\frac{\partial h}{\partial x}\right)=\frac{1}{\Delta x}\left[\frac{h_2-h_0}{\Delta x}\left(\frac{T_0+T_2}{2}\right)-\frac{h_0-h_1}{\Delta x}\left(\frac{T_0+T_1}{2}\right)\right]$$

$$\frac{\partial}{\partial y}\left(T_{yy}\frac{\partial h}{\partial y}\right)=\frac{1}{\Delta y}\left[\frac{h_3-h_0}{\Delta y}\left(\frac{T_0+T_3}{2}\right)-\frac{h_0-h_4}{\Delta y}\left(\frac{T_0+T_4}{2}\right)\right]$$

$$S\frac{\partial h}{\partial t}=S_o\frac{h_0-h_0^i}{\Delta t}$$

Where (h) are the unknown nodal heads at the end of the time- step, S is the Storage coefficient, h^i is the known head at the beginning of the time- step, Δt, Δx and Δt are the time and space steps, and the subscripts denote the node numbers.

Substitution of these finite- differences in the governing equations leads to the following form of a linear equation in terms of five unknown heads.

$$A(h_o)+B(h_1)+C(h_2)+D(h_3)+E(h_4)=F$$

Boundary nodes: An additional linear equation is obtained for each boundary node by invoking the respective known boundary condition.

Thus, as many equations are available as the number of the unknowns, viz., the nodal heads at the end of the time step.

Solution of linear algebraic equations

Theoretically the determinate system of equation can be solved by any standard numerical algorithm e.g., Gauss elimination, Gauss Seidel etc. However, the total memory requirement can be prohibitively large even for moderately sized domains. Consider this: if there are 1000 nodes (not an unusually large number), the memory required for storing the coefficient matrix alone would be one million words! However, the memory requirement can be significantly reduced by utilizing the "sparseness" of the coefficient matrix. Many specific algorithms like IADIE, LSOR, SIP etc. have been devised on these lines. These algorithms, apart from reducing the memory requirement, also reduce the round-off errors.

PROBLEM- SPECIFIC STATE VARIABLES

The end product from the solution of the governing differential equation comprises the mandatory state variable, i.e., nodal heads at successive discrete times. Other state variables which may be derived from these distributions may include among others, nodal depths to water table, influent/ effluent seepage, static storage, sea water intrusion etc.

FEASIBILITY CHECKS

The feasibility of a trial pumping/ recharge pattern can be checked through groundwater flow modeling by broadly implementing the following steps:

1. Identify the aquifer system (spatial extent, boundary/ initial conditions, parameters etc.)
2. Quantify the proposed pumping/ recharge pattern
3. Identify the constraints and the corresponding state variables of the groundwater system
4. Formulate the nodal forcing functions by adding algebraically the proposed pumping/ recharge and other "natural" source/ sink terms
5. Project the nodal heads and hence the relevant state variables
6. Check feasibility

The constraints may be derived from technical, social, socio-economic considerations. It is apparent that there can not be any universal constraints. The constraints essentially represent the local concerns. For example, in coastal aquifers, certain outflow to sea is necessary for restricting the sea water intrusion to an *acceptable* level. Thus, the minimum permissible outflow to sea may be derived from the maximum acceptable extent of the sea water intrusion. Then, the feasibility of any proposed pumping pattern may be checked by comparing the projected outflow with the pre-stipulated minimum permissible outflow. If the projected outflow is found to be smaller, the proposed pumping pattern may be moderated iteratively until the projected outflow gets equal to the minimum permissible limit.

UNCERAINITY IN PROJECTIONS

The governing differential equation imbibed in a model may be based upon a few assumptions e.g., horizontality of the flow, uniqueness of the parameters, linearity of flow. Further additional assumptions may have to be made while implementing the model e.g., principal permeability directions, boundary conditions, spatial distribution of the aquifer parameters, time and space distributions of the forcing function etc. These assumptions, necessitated by a gap between the data requirement (which is always huge) and the data availability (which alas is always limited), may not always hold. Further, there are inevitable numerical errors! All this introduces an uncertainty in the model projections.

The uncertainty level may be controlled to an extent, by choosing a model with an appropriate differential equation, and then subsequently using hydrologic/ geohydrologic "sixth sense" to bridge the data gap while formulating the data base for the model. The latter obviously would come with experience. This makes groundwater modeling as much an art as science. Finally it is good to remember that a model is at its best a simplistic version of the system and needs to be evolved as the understanding of the system improves and additional data become available. As such, the worst thing any modeler (and more so a groundwater modeler) can do is to forsake the common sense and have a blind faith in his model.

CONCLUSION

Groundwater modeling essentially involves projection of the problem- specific state variables of the groundwater system for a given forcing function, invoking the continuity equation at a micro level

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