Role of Simulation Models in Planning of Groundwater Development

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INTRODUCTION

For the survival of human race it is necessary that the rapid population growth, being experienced these days, is matched by a corresponding expansion of agricultural production. This understanding has led to the development of high yielding varieties of crops, increased dependence on chemical fertilizers and more intensive irrigation. All these measures have greatly increased the water requirement for irrigation. To meet these requirements, several canal irrigation projects have been constructed over the years. However, the water availability from such projects is rarely sufficient to meet the complete irrigation water requirement. The water deficit in these areas is usually met through groundwater development.

However, an indiscriminate groundwater development may lower the water table excessively causing several technical/socio-economic problems. These problems may include among others, reduction of base flow contribution to hydraulically connected streams, deterioration of groundwater quality, reduction of static storage, increase of pumping cost, drying up of wells etc. On the other hand, inadequate groundwater development in canal command areas may lead to problems associated with water table rise viz. water logging/salinization of the root zone. Therefore, it is necessary to plan the groundwater development judiciously.

Planning of groundwater development, like any other planning addresses the twin issues of feasibility and optimality. This usually involves arriving at such a development plan that maximizes the benefit from the use of groundwater and/or minimizes the cost, subject to the technical and other socio-economical constraints. Conventionally linked simulation-optimization models are invoked to arrive at the optimal groundwater development plans. Typically these models comprise a physically based groundwater flow model linked to an optimizer. The planning problem is posed as an optimization problem with the simulation model computing the state variables of the groundwater system appearing in the objective function and constraints.

Broadly speaking planning of groundwater development like all planning processes comprises twin objectives viz. determining *feasibility* and ensuring *optimality.* The feasibility check is primarily aimed at evolving such pumping patterns that satisfy the local technical/socio-economic constraints. The constraints on the groundwater development may include among others, the requirements of: maintaining the water table in a specified range, allowing adequate outflow to hydraulically connected water bodies (say streams, lakes and sea), preserving adequate static storage etc. Since the constraints represent the local concerns, there can not be any universal set of constraints. For example in coastal aquifers, certain outflow to sea is necessary for restricting the sea water intrusion to an *acceptable* level. Thus, in this case the minimum permissible outflow to sea may be derived from the maximum acceptable extent of the sea water intrusion. It is quite apparent that there could be a large number of pumping patterns that satisfy the imposed constraints. Thus, it is necessary to pick up the *most appropriate* or optimal pumping pattern from the array of the *feasible* patterns. This requires specifying the criteria for optimality that may broadly be stated as maximizing the net benefits (or minimizing the penalty) from the pumping activity.

Feasibility Checks

Traditionally, the *feasibility checks* in the groundwater planning process are conducted by performing numerical experiments (i.e., *simulation*) on groundwater flow models termed henceforth as *simulation models.* A simulation model typically comprises the following components.

i) An equation (usually differential) governing the flow
ii) An algorithm to solve the chosen equation numerically to compute the time and space distribution of the head
iii) A set of algorithms to compute the problem- specific state variables from the computed head distributions
iv) Computer codes to implement the selected algorithms

Feasibility of the *trial* pumping patterns can be checked through simulation that may broadly involve the following steps.

i) Identify the aquifer system (spatial extent, boundary/ initial conditions, parameters etc.).
ii) Quantify the proposed pumping pattern.
iii) Identify the constraints and the corresponding state variables of the groundwater system.
iv) Formulate the nodal forcing functions by adding algebraically the proposed pumping/ recharge and other "natural" source/ sink terms.
v) Project the nodal heads and hence the state variables relevant to the constraints.
vi) Check feasibility.

Optimality

It is apparent that an array of feasible pumping patterns may be arrived at by simulation as described in the preceding section. The next step towards the planning is to pick up the *most rewarding* (or *least penalizing*) optimal pattern from the array of the evolved feasible patterns. This would require specifying quantitatively objective function(s) that relate the *reward /penalty* to the pumping pattern. Apparently the objective function would be derived from the *intended* objective(s) of the pumping activity. Typically the functions may comprise among others one or more of the following expectations.

i) Maximizing the water production under specified constraints like: limiting the drawdowns/ water table depths/ sea water intrusion/ stream-aquifer interflows etc.
ii) Maximizing the net benefit from water production i.e., benefit from the pumping minus the cost of pumpage or water production per unit cost, under specified constraints discussed above. The cost of pumpage may be expressed in terms of the pumping pattern and unit pumping cost. The latter may be assumed to be constant or lift-dependent.
iii) Minimizing the maximum drawdown/ maximum water table depth/ pumping cost for a specified water production.
iv) Minimizing the pumpage from a prevalent well network for a specified level of aquifer remediation (i.e., attenuation of concentration of stipulated species in the groundwater)
v) Minimizing cost of pumping from a prevalent well network for a specified level of aquifer remediation
vi) Maximizing the aquifer remediation by pumping from a prevalent well network subject to the constraints described in (i).
vii) Maximizing the aquifer remediation for a specified financial allocation
viii) Maximizing the net benefits/calorific value of the cropping pattern that can be irrigated by the pumpage or conjunctively by pumpage and the available canal supplies

It is apparent that except for a few rather simplistic objective functions [like (i), ii) with constant pumping cost, (iv)], computation of all other functions described above for a given pumping pattern would require operation of a simulation model.

LINKED SIMULATION OPTIMIZATION MODELS

It follows from the preceding discussion that the planning process requires to check the feasibility and ensure the optimality. The two functions though described in *stand alone* mode in the preceding paragraphs are in fact performed simultaneously through a composite simulation-optimization modeling wherein the planning problem is posed as an optimization problem with the feasibility requirements appearing as constraints (Maddock 1972a, Kashyap and Chandra 1982, Gorelick 1983, Emch and Yeh 1998, Theodossiou 2004, Kashyap 2005, Pulido-Velazquez et al 2006). This apparently requires assimilation of the simulator into the optimizer as described in the following general formulation.

Decision Variables: $\mathbf{Q}$ (pumping rate matrix, well-wise in discrete mode or cell-wise in continuous mode)

$$\text{Optimize (maximize or minimize): } \Omega[\mathbf{F}\{\mathbf{Q}, \mathbf{f}_1(\mathbf{h})\}] \text{ with respect to } \mathbf{Q} \tag{1}$$

Where $\mathbf{F}$ = the chosen objective function vector, Ω = representative scalar objective function, $\mathbf{Q}$ = pumping rate matrix forming the decision variable vector, $\mathbf{f}_1$ = vector of state variables relevant to the objective function, $\mathbf{h}$ = nodal heads (steady state or at advancing times)

$$\text{Subject to the constraints: } \mathbf{g}[(\mathbf{f}_2(\mathbf{h})] \leq 0 \tag{2}$$

Where $\mathbf{g}$ = the constraint vector, $\mathbf{f}_2$ = vector of state variables relevant to the constraints. Heads ($\mathbf{h}$) appearing in (1) and (2) are computed from $\mathbf{Q}$ through a simulation model as follows.

$$\mathbf{h} = \Phi(\mathbf{Q}) \tag{3}$$

Where Φ = function imbibed in the simulation model.

Simulation Models

A simulation (or groundwater flow) model is essentially a tool to project the state variables of the groundwater system for an assigned pattern of forcing function, and known initial and boundary conditions and parameters. A brief description of various terms appearing in this definition is included in the following paragraphs.

State Variables: The state variables are essentially the variables that describe the "state" of a system. These variables may be divided in two categories viz. Mandatory and Problem-specific. The mandatory state variable is: Piezometric head or Water table elevation. This variable is henceforth termed as "head". The Problem specific state variables are essentially derived from the head distribution in space and time. These could include, depending upon the problem at hand, depth to water table, static storage, influent/ effluent seepage, outflow to sea, sea water intrusion etc.

Forcing Function: The forcing function may comprise among others the following constituents: Withdrawals (i.e., pumpage), Recharge (derived from- rainfall, applied irrigation, seepage from surface water bodies etc.), Evapotranspiration from the saturated zone

Initial conditions: Initial conditions, as the name implies comprise of the spatial distribution of the head at the instance when the assigned excitation commences to act. There are two possible interpretations of the Initial conditions. Mathematically, they are necessary for arriving at a unique solution of a differential equation. Conceptually, they can be visualized as the influence of the hydraulic conditions occurring prior to the activation of the assigned forcing function.

Boundary conditions: Here too there are two possible interpretations. Mathematically, they are necessary for arriving at a unique solution of a differential equation. Conceptually, they can be visualized as the influence of the hydraulic conditions occurring across the boundary of the domain, of the solution. Thus, to obtain a unique solution of the differential equation, it is necessary to define boundary conditions all along domain boundary. The boundary condition may either be a known head (head assigned) or a known flow rate (flow assigned) across the boundary. It can be thus concluded that for obtaining a unique solution it is necessary to know either the head or normal flows all along the boundary.

Boundary heads are assigned wherever an aquifer is terminating into a water body. At the interface between the two, the head may be assumed to be equal to the water elevation in the water body.
Normal flows need to be known for the part (s) of the domain boundary not interfacing with water bodies. These flows are more difficult to estimate (unless they are known to be zero i.e., an impervious boundary) and would usually require water balance of the adjoining areas.

Out of the two types of boundary conditions, the head assigned boundaries are more suitable for forecasting since the water elevations in the hydraulically connected water bodies may generally not be significantly influenced by the pumping/recharge pattern in the aquifer. Thus, the known prevalent water elevations may be assumed to hold good under the projected conditions (i.e., the pumping/recharge rates different from the prevalent ones). On the other hand, the lateral inflows across the boundary are very sensitive to any change in pumping/recharge. Thus, the inflow rates under the projected conditions may vary significantly from the prevailing ones. In other words the known prevalent inflow rates may not provide the necessary boundary conditions.

Model Parameters: The spatial distribution of the appropriate (that is, depending upon the type of aquifer) aquifer parameters need to be assigned for computing the head distributions corresponding to the assigned forcing function. The data from pumping tests shall rarely be adequate to meet this input requirement. The spatial distribution is usually obtained from a solution of *Inverse problem* (or Model calibration). The solution requires the historical data of forcing function, heads, initial and boundary conditions. It aims at evolving such distribution of the aquifer parameters, which lead to a closest match between the observed, and the model-computed heads. Typically this requires repeated *direct* modeling corresponding to a selected historical period, with varying values of aquifer parameters, and finally arriving at the *best possible* match. This approach Model calibration is usually followed up by a validation of the calibration. This is accomplished by using only a part of the available data base for the calibration. The unused part is subsequently employed to determine how well the calibrated model reproduces the observed state variables. The calibration is considered as *validated* provided the reproduction of the state variables in the validation stage is *almost* as good as in the calibration stage.

Recharge Parameters: A groundwater flow model invoked for the planning comprises the usual aquifer parameters viz. transmissivity and storage coefficient/specific yield. However, depending upon the local conditions and the expected level of rigor, additional parameters may be introduced to estimate recharge components of the forcing function. As such, there can not be a universal set of such parameters. Typically for Indian scenario, following set of parameters may be relevant (CBIP, 1987).

i) Rainfall recharge coefficient (α): This parameter relates the unknown rainfall recharge (R_r) to the known rainfall depth (P) as follows:

$$R_r = \alpha P$$

ii) Canal seepage factor (β): This parameter relates the unknown canal seepage (R_S) to the known canal water supply(Q) as follows:

$$R_S = \beta \times Q$$

Further, a major part of this canal seepage occurs along the main canals. The remaining part occurring through branches, distributaries, water courses etc may be well distributed over the entire area. As such, the recharge from canal seepage (R_S) is considered as comprising two components viz. R_{S1}: occurring along the main canal and R_{S2}: uniformly distributed over the entire area. The break-up is parameterized (in terms of a parameter ξ) as follows:

$$R_{S1} = \xi \times R_S$$
$$R_{S2} = (1-\xi) \times R_S$$

iii) Applied irrigation recharge parameters (F): Recharge from the irrigation (emanating from canal water and groundwater) applied on the field is related to the corresponding application depths. Recalling that a fraction β of the canal water is conceptualized as the seepage loss, the canal water available on the field is $(1-\beta)Q$. As such the recharge from the applied irrigation (R_i) is related to the available canal water $[(1-\beta)Q]$ and the groundwater (GW) as per the following parameterization:

$$\begin{aligned} R_i &= [Q(1-\beta) + GW] \times F \\ &= I_r \times F \end{aligned}$$

Where, I_r = Total applied irrigation and F = applied irrigation recharge parameters termed as irrigation application efficiency. In the present study the parameter F is considered as crop dependent.

Incorporation of the cropping pattern: The groundwater flow models provide the head field at the advancing times for an assigned forcing function (W) that is derived parametrically from data of rainfall, canal supplies and groundwater withdrawal (GW). This parameterization may be enhanced to link the GW component to the cropping pattern by deriving the necessary groundwater pumpage for given cropping and canal availability patterns. This lead to the following expression for GW_{ik}:

$$GW_{ik} = \left[\sum_j a_{jl} \delta c_{jk} - (1-\beta) Q_{ik} \right] + GW_{ik}^*$$

Where, GW_{ik} = groundwater withdrawal at i^{th} node in k^{th} time period, GW_{ik}^* = groundwater withdrawal for non-agricultural usage at i^{th} node in k^{th} time period, a_{jl} = fractional area under j^{th} crop in l^{th} zone, δc_{jk} = unit irrigation water requirement of j^{th} crop in k^{th} period, Q_{ik} = canal water released toward at i^{th} node during k^{th} period.

The corresponding sink term W_{ik} (representing the algebraic sum of all abstractions and fluxes) can be written as follows:

$$W_{ik} = GW_{ik} - [\alpha P_{ik} + (1-\xi)\beta Q_{ik} + (I_r \times F] + \mathrm{E}$$

OTHER STRATEGIES

Recalling that all optimizers essentially involve sequential computation of the objective functions and the constraints for a large array of the decision variable vector, the above formulation reveals that the optimization process would comprise repeated invoking of the simulator (Φ) for sequentially modified decision variable vector **Q**. Since a single simulation (say solution of the governing differential equation over a finite difference/ element grid) may require significant computation effort, the entire procedure of optimizing **Q** may turn out to be computationally expensive. This problem may be resolved by designing specific strategies aimed at reducing either the number of simulations (required for reaching the optimal solution) or the computational effort per simulation. The following strategies are usually invoked

Embedded Simulation.

A simulation model essentially solves the chosen (flow) differential equation for heads at pre-designated nodes/cells of a finite difference/element grid. The solution involves formulating a determinate system of algebraic equations ($G_j(\mathbf{h}) = 0$, j = 1,.....m; m being the number of nodes) in terms of known initial/ boundary conditions, parameters, stresses (comprising the *current* values of the decision variables **Q**, and other known natural components), and unknown heads (**h**) at the advancing time. The equations are solved for **h**, and subsequently other state variables ($\mathbf{f_1}$ and $\mathbf{f_2}$) relevant to the objective functions and the constraints respectively are computed employing appropriate auxiliary equations and the parameters.

In the embedding method (Aguado and Remson 1974, Gorelick and Remson 1982, Gorelick 1983), the nodal heads at the advancing times along with the pumpages (**Q**) are treated as the decision variables. And the algebraic equations [$\mathbf{G}(\mathbf{h}) = 0$] are treated as *equality* constraints, thus *embedding* the simulator (Φ) in the optimizer.

Response Matrix Approach

This is yet another strategy aimed at avoiding computationally expensive simulations in the course of optimization. In this approach the simulation part of the *LSO* models is de-linked through some pre-modeling home work (Deininger 1970, Maddock 1972a, Maddock III 1974, Gorelick 1982, Gorelick 1983, McPhee and Yeh 2004, Psilovikos 2006). The home work essentially comprises multiple simulation runs to generate a matrix of response functions $\boldsymbol{\delta}$. Assigning a unit pulse abstraction (a unit depth spread over a unit period) at j^{th} node, the response [say the drawdown, $\delta(ijk)$] at node i after an elapse of k time periods is computed through a pre-calibrated simulation model. Conducting multiple simulations runs with the abstraction assigned at the nodes in turn, the elements of three dimensional (or two dimensional in case of steady state planning) response matrix $\boldsymbol{\delta}$ are computed and stored. The head (h_{ik}) at node i at time k in response to a known pattern of simultaneous abstraction at all the nodes is subsequently computed by invoking the principle of superposition as follows.

$$h_{ik} = h_{i0} - \sum_{p=1}^{k-1} \sum_{j=1}^{m} Q(j,p)\delta(i,j,k-p)$$

Where $h_{i,0}$ = initial head at node i, Q(j,p) = abstraction rate at node j during the period from p to p+1 discrete times, and δ(i,j,k-p) is the response (say drawdown) at node i to unit abstraction rate at node j during the period from p to p+1 discrete times.

The planning problem is thus posed as follows:

Decision Variables: **Q** (pumping rate matrix, well-wise in discrete mode or cell-wise in continuous mode)

Optimize (maximize or minimize): $\Omega[\mathbf{F}\{\mathbf{Q}, \mathbf{f_1(h)}\}]$ with respect to **Q**
Subject to the constraints: $\mathbf{g}[(\mathbf{f_2(h)}] \leq 0$

The array h is computed through the kernel approach.

It may be pointed out that the above invoked principle of superposition shall be valid only if the response (say drawdown) is linearly related to the forcing function (say abstraction). The linearity holds for the confined aquifers or nearly holds for thick unconfined aquifers.

ANN Methodology

Artificial neural network (ANN) methodology is being increasingly employed to simulate the aquifer response to a variety of inputs including pumping pattern and weather (Ranjithan et al 1993, Coppolla et al. 2003, 2005, Feng et al 2008), and for addressing complex groundwater management problems (Rogers and Dowla 1994, Johnson and Rogers 1995, Coppolla E. Jr. et al. 2003, Bhattacharya and Datta 2005, Singh and Datta 2007). The methodology although inspired by the working of human brain and bearing a somewhat exotic name, is essentially a specialized regression strategy. However, unlike the general regression the function relating inputs to the outputs is rather regimented. The function comprises an input layer, hidden layers and an output layer. The input layer contains the input variables (termed as input nodes) that comprise the physical inputs and a bias term assigned a constant value of 1.0. Similarly the output layer has the output variables (again termed as output nodes). There may be several hidden layers containing several nodes, their number not being known a priori. Nodes are connected in the forward direction (i.e., commencing from the input layer and terminating at the output layer) across the layers by transfer functions.

INDIAN SCENE

In India the groundwater development is planned by conducting lumped water balance studies on historical data. Government of India set up a committee in 1996 to standardize the procedure for implementing this approach. The committee finalized its recommendations in 1997. The recommendations, usually termed as GEC-97 norms (Government of India 1997, Kashyap 2003) are widely invoked to estimate the ground water resource in the country. The norms essentially comprise two steps towards the resource estimation. The first step involves an estimation of the recharge from rainfall in monsoon season by conducting a lumped water balance study invoking the historical data of water table elevations, draft, recharge etc. Subsequently in the second step the annual utilizable recharge is estimated rather empirically as a fraction of the estimated recharge allowing for the *losses* comprising evapotranspiration and lateral outflows to drains. Where as the first step involving estimation of recharge is quite rational (being based upon the well known continuity equation), the second step aimed at estimating the utilizable recharge is rather arbitrary. As such not surprisingly application of the norms in many studies is known to have led to a variety of anomalous results.

Although studies on the estimation of ground water resource in India generally continue to be based upon the GEC-97 norms, there has been over the years a larger assimilation of the simulation approach in the ground water practice. As a consequence a number of simulation studies, usually restricted to calibration and limited projections are being taken up. However, simulation has yet not been assimilated in the main stream resource estimation being conducted by the state and the central groundwater departments. Main reason for this paradox is that generally the simulation studies in India terminate with the computation of the piezometric head distributions. Such distributions though important have no implications in respect of the feasibility. No attempts are made to process the computed head distributions to derive additional state variables like stream-aquifer interflows, static storage, number of wells going dry, water-logged area, sea water intrusion etc. which may determine the feasibility of any pumpage proposal. As a consequence the simulation studies do not get into the "feasibility checking" mode, and remain not only isolated from the main stream GEC-97 based resource estimation, but also devoid of any objectives. Consequently such studies remain by and large *ceremonial* rather than functional, and the potential of modeling in respect of resource estimation is not realized. Reasons for this are not hard to understand. The most widely used simulation package (MODFLOW) stops at the projection of the water elevations, and does not process them for deriving additional variables of interest. Further, simulation is not considered as a part of GEC-97 driven resource estimation. Nevertheless functional simulation studies with well defined objectives are usually being taken up by academic/ research organizations (like Kashyap 1992a,b; 1994, 1997) in consultancy mode with the resulting reports largely remaining unpublished.

CONCLUSION

A rigorous planning of groundwater development requires application of a simulation model and an optimizer in a linked mode. An external linkage may be computationally too expensive to address real life problems. The embedded approach wherein the simulation model is embedded into the optimizer as a set of equality constraints may be a viable approach for tackling the steady state problems over not too large areas. The response matrix approach involving generation of response coefficients is yet another viable approach for linear groundwater systems. Replacement of physically based simulation models by approximate but computationally inexpensive regression/ ANN models is another effective option. The practicing groundwater fraternity in India has yet to adopt groundwater simulation modeling as a tool for checking the feasibility and planning the groundwater development.

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