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SEEPAGE FROM PARALLEL CANALS

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LIST OF SYMBOLS

a	Dimension of rectangular strip
b	Dimension of rectangular strip
B	Width of the canal at the water surface
B_1	Width of the first canal at the water surface
B_2	Width of the second canal at the water surface
D	Centre to centre distance between the parallel canals
E	Saturated thickness before the onset of recharge
F_w	Special functions tabulated for calculating water table rise
H	Maximum depth of water in the canal
H_1	Maximum depth of water in the first canal
H_2	Maximum depth of water in the second canal
h_o	Initial depth of saturation
$\bar{h}$	Weighted mean of the depth of saturation
K	Coefficient of permeability
t	Time reckoned from the onset of recharge
q	Recharge from unit length of a line source
s	Rise in water table height
T	Transmissivity of the aquifer
x,y	Cartesian coordinate
w	Constant rate of percolation
Φ	Storage coefficient
α	T/Φ

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ABSTRACT

The evolution of water table due to recharge from a strip source has been analysed. The analysis has been extended to find the evolution of water table due to seepage from two parallel canals. It has been found that for two identical canals the points of maximum height of water table start moving towards each other from the centres of the canals and stop at a distance less than (B/2 + H) from the centres of the canals, B being the width of the canals at the water level and H the maximum depth of water in the canals. It is observed that for two identical parallel canals the water table in the region between the canals takes the shape of a plateau with lapse of time. The maximum rise of water table in the region between the canals has been predicted for various sizes and spacings of canals. It is found that for two identical parallel canal system, the interference of the canals is not significant upto a time factor 0.045 if their centre to centre spacing is more than sixteen times the width of the canal.

1.0 INTRODUCTION

The process of seepage from a canal starts as soon as water is filled in it. As the time elapses, in the first stage, the soil layers around the canal get saturated. The saturated water front, in the next stage, moves slowly downwards and, after a certain period of time, it reaches the water table below the bed of the canal. During this downward propagation of seepage from the canal into the flow domain, the seepage water is used for the saturation of wetted zone, where the pores were previously filled with air. After reaching the water table, only a part of the infiltrating water is stored within the extending saturated zone, whereas the remaining part recharges the groundwater. The time delay for the seeping water to reach the water table, after the onset of seepage from the canal, which is hydraulically unconnected from aquifer, can be obtained by using the Green and Ampt equation. It may be noted that seepage rate from the canal is not the recharge rate at the water table at all time. With wetting front position some where between the canal bed and initial water table position at very large depth and for initially dry soil, the seepage rate varies in time, but the recharge rate is constant and zero. If the water content behind the wetting front is close to saturation, recharge rate rises abruptly from zero to the prevailing seepage rate at the time the saturation front encounters the water table (Abdulrazzak&Morel-Seytoux, 1983). Ultimately the recharge due to seepage from the canal results in rise of water table in the form of water mounds. The study of seepage prior to the initiation of recharge to groundwater is not in the scope of the study presented.

Theoretically the steady state of seepage from a canal can develop only if the wetting front can propagate large depth without encountering the water table. If the water table is lying at a depth more than 1.5 times the width of canal, for all practical purposes it can be assumed to be at large depth. In practical situation, however, the water seeping down from canal will reach the normal groundwater at a finite depth and, in the process, the seepage flow gets influenced by the near horizontal flow of groundwater, thus forcing the stream lines to follow a horizontal rather than a vertical trend.

Many research workers have investigated the steady free seepage from a canal to deep water table. A number of theoretically well established results are known after Kozeny (1931), Vedernikov (1934), Riesenkampf (1938), Muscat (1946) etc. The investigations of above authors are equally based on the application of both hodograph and conformal mapping. The difference in their detailed analysis are, in general, due to difference in the profiles of the canal considered in their analysis. If a flow domain comprises homogeneous and isotropic material, for the shape of a canal given by Kozeny, the maximum or asymptotic width of the downwards seeping sheet of water is approximately given by (B + 2H), where, B is width of canal at the top water surface and H is the maximum depth of water in it. The discharge flowing down as seepage is equal to K(B + 2H), K being coefficient of permeability of the medium. Although slight deviations in the shape of canal from that given by Kozeny, will cause no appreciable error in the seepage discharge, the assumption of existence of water table at very large depth limits its applicability for the cases where ground table lies at a shallow depth below the bed of the canal. The seepage from canal travelling downwards and meeting the deep groundwater table will raise a groundwater ridge from which groundwater can flow both ways. As the time passes, the water table will continue to rise in the absence of drainage in the vicinity of flow domain.

In many canal systems the canal seldom carries a fixed discharge or have constant water depth in it. The discharge in a canal depends on availability of water and its demand in the command area. On account of fluctuating water level in canal, largely for the reason mentioned above, the steady seepage condition is seldom reached. Also, in some command areas, there may be two parallel canals running simultaneously or intermittently depending on requirements at various times. For instance, both the canals may run simultaneously in Khariff crop season when adequate supplies for both the canals are available, whereas when supplies in the river get diminished, only one canal may run in those periods. In all such cases the seepage from canals remains in unsteady state.

The computations of the unsteady free seepage has been dealt with by few authors. Averjanov (1950) has expressed the time dependent flow rate of seepage as a product of the water loss under a free steady state ($t = \infty$) and a factor greater than unity. The multiplying factor has been determined theoretically and by experiments. Hantush (1967) has derived an expression for rise in water table height due to recharge from a basin of finite length and width. If the dimension of length is increased to a very large value, the solution will correspond to rise in water table due to recharge from a canal. However, the solution involves numerical integration. Glover (1974) has analysed the evolution of water table due to recharge from a line source, but has not taken the width of recharge body into consideration. Shestakov (1965) has tabulated special functions by adopting numerical integration to determine rise in water table due to recharge from a strip source of infinite length.

In the present report, starting from Glover's solution, the evolution of water table due to recharge from a canal of given width has been analysed and a closed form solution has been obtained. The analysis has been extended to study the evolution of water table due to recharge from two parallel canals which are at a certain distance apart. The analysis has been carried out to quantify the rise in water table at different time periods and at various locations along a transverse section through the canals.

2.0 REVIEW

Evolution of water table due to recharge from a rectangular water body has been analysed by Marino (1975) who has used numerical method consisting of alternating direction implicit scheme. The numerical method can be extended for analysing the evolution of water due to recharge from a canal

Hantush (1967) has developed approximate analytical expression for the rise and fall of the water table in an infinite unconfined aquifer in response to uniform recharge from a rectangular spreading basin. Hantush described the water table rise by the following expression:

$$h^2 = h_o^2 + \frac{(w\bar{h}t)}{2\phi} . F[(a+x)/2\sqrt{(K\bar{h}t/\emptyset)}, (b+y)/2\sqrt{(K\bar{h}t/\emptyset)}]$$

$$+F[(a+x)/2\sqrt{(K\bar{h}t/\emptyset)}, (b-y)/2\sqrt{(K\bar{h}t/\emptyset)}]$$

$$+F[(a-x)/2\sqrt{(K\bar{h}t/\emptyset)}, (b+y)/2\sqrt{(K\bar{h}t/\emptyset)}]$$

$$+F[(a-x)/2\sqrt{(K\bar{h}t/\emptyset)}, (b-y)/2\sqrt{(K\bar{h}t/\emptyset)}]$$

in which

$\bar{h}$ = weighted mean of the depth of saturation during the period of flow (a constant of linearization), 2a, 2b=dimension of the rectangular strip, w = constant rate of percolation and

$$F(p,q) = \int_0^1 Erf(p/\sqrt{v}).Erf(q/\sqrt{v})dv.$$

Hantush's expression involves numerical integration.

The following expression for rise of water table due to recharge from a canal can be derived from the above expression of Hantush:

$$s(x,t) = \frac{wt}{2\phi} \left\{ \int_o^1 Erf\left[\frac{a + x}{2\sqrt{(Khtv/\phi)}}\right] dv + \int_o^1 Erf\left[\frac{a - x}{2\sqrt{(Khtv/\phi)}}\right] dv \right\}$$

Assuming the canal to be a line source, Glover has derived the following expression for rise in water table due to uniform vertical recharge:

$$s(x,t) = \frac{q\sqrt{(4\pi\alpha t)}}{2\pi KD} \frac{x}{\sqrt{(4\alpha t)}} \int_{\frac{x}{\sqrt{(4\alpha t)}}}^{\infty} \frac{e^{-u^2}}{u^2} du$$

$$= \frac{q\sqrt{(4\pi\alpha t)}}{2\pi KD} \frac{x}{\sqrt{(4\alpha t)}} \left[\frac{\sqrt{(4\alpha t)}e^{-\frac{x^2}{4\alpha t}}}{x} - \sqrt{\pi} + \sqrt{\pi} Erf\left(\frac{x}{\sqrt{4\alpha t}}\right) \right]$$

The prediction of water table rise by Glover's method will not be accurate if it is applied to find the rise in water table near the canal having large width.

Shestakov (1965) has tabulated special functions numerically for determination of water table rise due to recharge from a strip source of width 2b and of infinite length. The expression for rise of water table given by Shestakov is as follows:

$$s(x,t) = \frac{wt}{\phi} F_W(\tau)$$

in which $s(x,t)$ = rise in water table at a distance x from the centre of the strip,

w = uniform percolation rate,

ϕ = storage coefficient, and

t = time reckoned from the on set of recharge,

For the centre of the strip $(x=0)$, $F_W = F_W(\tau_o)$;
for $x \leq b$,

$$F_W = [F_W(\tau_x) + F_W(\tau''_x)]/2 \ ;$$

and for $x > b$,

$$F_W = [F_W(\tau'_x) - F_W(\tau''_x)]/2$$

$$\tau'_x = \frac{Tt}{\phi(b+x)^2}, \qquad \tau''_x = \frac{Tt}{\phi(b-x)^2}$$

$$\tau_o = \frac{Tt}{\phi b^2}$$

It can be seen that Shestakov's and Hantush's approaches are similar. The special functions tabulated by Shestakov are given in Table 1.

Table 1. Shestakov's special functions for evaluation of water table rise due to recharge from a strip source

τ	$F_w(\tau)$	τ	$F_w(\tau)$
0.00	1.000	14	0.268
0.05	1.000	16	0.253
0.10	0.994	18	0.240
0.20	0.963	20	0.0229
0.30	0.924	25	0.207
0.40	0.884	30	0.190
0.50	0.849	35	0.178
0.60	0.818	40	0.167
0.70	0.790	50	0.151
0.80	0.764	60	0.138
0.90	0.741	70	0.128
1.00	0.720	80	0.120
1.50	0.639	90	0.113
2.0	0.583	100	0.108
2.5	0.537	156.3	0.089
3.0	0.503	200	0.078
4.0	0.451	278	0.062
5.0	0.413	400	0.054
6.0	0.384		
7.0	0.361		
8.0	0.342		
9.0	0.325		
10.0	0.311		
12.0	0.287		

3.0 PROBLEM DEFINITION

Figure (1) shows a canal constructed in a homogeneous and isotropic pervious medium of infinite aerial extent. The dimensions of the canal are as shown in the figure. 'B' is the width of top water surface and 'H' is the maximum depth of water in the canal. The water table is at large depth below the canal bed such that the canal is unconnected with aquifer. The coefficient of permeability of the flow domain is K. The thickness of the saturated depth of aquifer is E. There is no drainage channel in the vicinity of the canals. It is required to determine the rise of water table at different locations across the canal. The rise of water table is to be evaluated in time, reckoned from the instant the water front reaches the groundwater table. It is also required to determine the evolution of water table due to recharges from two parallel canals located at a distance D apart. The two parallel canals have been depicted in Figure 2.

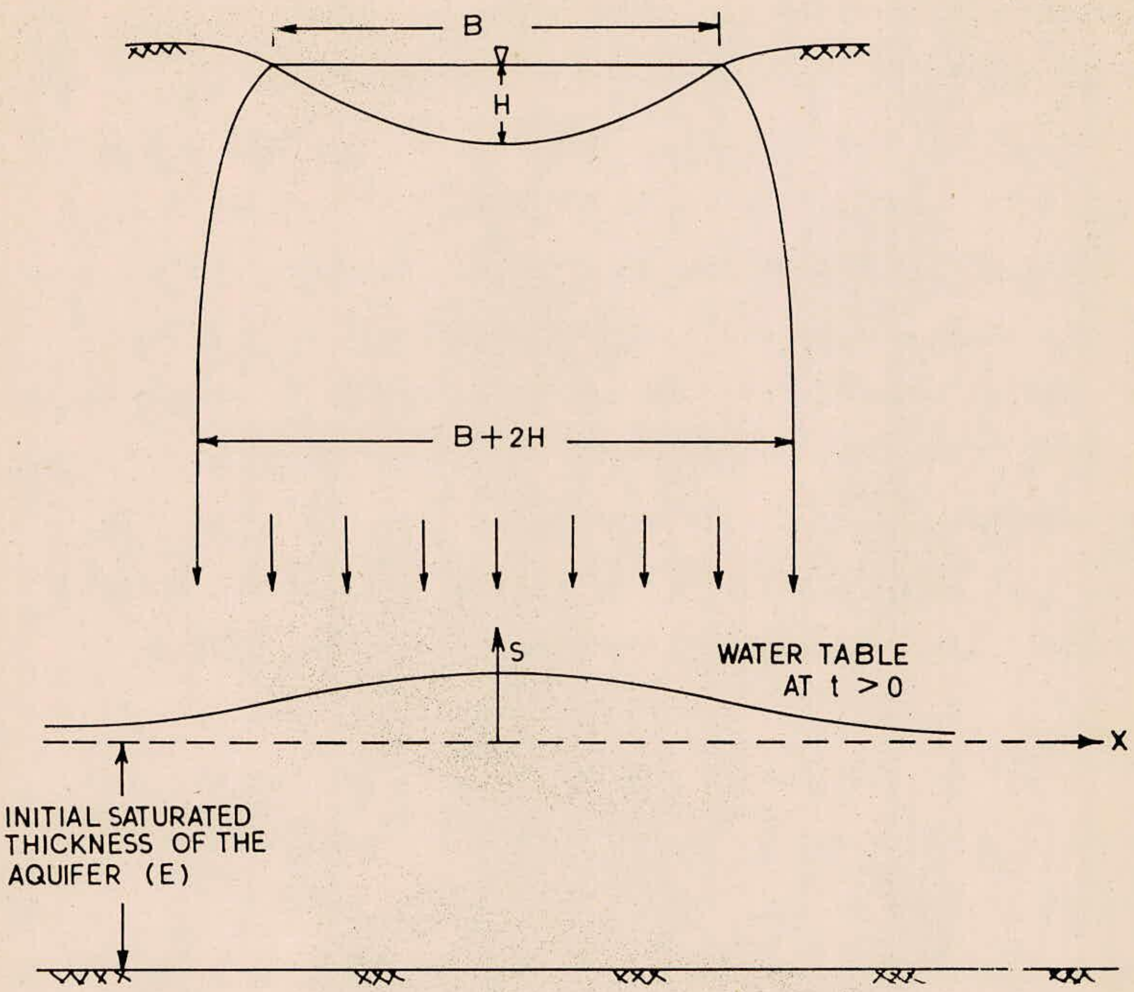

FIG. 1 Schematic section of a canal

4.0 METHODOLOGY

The assumptions made to carry out the analysis for evolution of water table by Glover's basic equations are as follows:

i) The aquifer is homogeneous, isotropic, aerially infinite, and resting on a horizontal impermeable base.

ii) The hydraulic properties of the aquifer remain constant with both time and space.

iii) The rate of seepage is constant with respect to time.

iv) The flow due to seepage is vertically downward until it reaches the water table.

v) The water table remains below the bottom of the recharging body.

vi) The average head over the depth of saturation is approximately equal to the height of the water table above the base of the aquifer.

vii) The rise of water table relative to the initial depth of saturation is small.

viii) Dupuit's assumptions are valid.

The evolution of water table due to recharge from a line source can be obtained from the solution of the following linearised one dimensional Boussinesq's equation:

$$\frac{\partial^2 s}{\partial x^2} = \frac{1}{\alpha} \cdot \frac{\partial s}{\partial t} \qquad (1)$$

in which 's' is the rise in water table, $\alpha = \frac{T}{\phi}$, and T is the transmissivity of the aquifer and ϕ is the storage coefficient.

The solution of the above equation is required to satisfy the following boundary conditions for the line source:

$$\text{At} \quad x=0, \quad T\frac{\partial s}{\partial x} = -\frac{q}{2}$$

$$\text{and at} \quad x=\infty , \quad s(\infty,t)=0$$

in which q is the seepage loss per unit length of the line source.

The initial condition required to be satisfied is:

$$s(x,0)=0$$

Solution to the differential equation (1), which satisfies the above initial and boundary conditions has been given by Glover (1974) as below:

$$s(x,t) = \frac{q\sqrt{4\pi\alpha t}}{2\pi T} \cdot \frac{x}{\sqrt{4\alpha t}} \int_{\frac{x}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u^2} du$$

$$= \frac{q\sqrt{4\pi\alpha t}}{2\pi T} \frac{x}{\sqrt{4\alpha t}} \left[\frac{\sqrt{4\alpha t} \cdot e^{-\frac{x^2}{4\alpha t}}}{x} - \sqrt{\pi} + \sqrt{\pi}\,\mathrm{Erf}\left(\frac{x}{\sqrt{4\alpha t}}\right) \right]$$

$$= \frac{q\sqrt{\alpha t}}{\sqrt{\pi}T} e^{-\frac{x^2}{4\alpha t}} - \frac{qx}{2T} + \frac{qx}{2T} \mathrm{Erf}\left(\frac{x}{\sqrt{4\alpha t}}\right) \qquad \ldots (2)$$

In actual case a canal has a certain finite width. Therefore, it would be appropriate to treat it as a strip source instead of a line source. If the water table is at very large depth below the bed of canal, the width of the strip can be taken approximately to be (B+2H). Also, if the water table lies at large depth, according to Kozeny the seepage rate per unit width of strip is K. A strip source can be regarded to be consisting of a number of line sources. As the governing differential equation of flow is linear, the rise of water table due to seepage from a strip source can be obtained by integrating the rise of water table due to each line source.

Assuming that the strip source is of width (B+2H) and seepage rate per unit width = K, the expression for rise of water table, s, at a distance x from the centre of the strip for $x \leq -(B/2+H)$ and $x \geq (B/2+H)$ is derived as follows:

$$s(x,t) = \int_{-(\frac{B}{2}+H)}^{(\frac{B}{2}+H)} \left[\frac{K\sqrt{4\pi\alpha t}}{2\pi T} \sqrt{\frac{(x-v)^2}{4\alpha t}} \int_{\frac{x-v}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u^2} du\right] dv \qquad \ldots (3)$$

According to Glover,

$$\int_{\frac{x-v}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u^2} du = \frac{\sqrt{4\alpha t}}{x-v} e^{-\frac{(x-v)^2}{4\alpha t}} - \sqrt{\pi} + \sqrt{\pi}\, \frac{2}{\sqrt{\pi}} \int_0^{\frac{x-v}{\sqrt{4\alpha t}}} e^{-u^2} du$$

$$= \frac{\sqrt{4\alpha t}}{x-v} e^{-\frac{(x-v)^2}{4\alpha t}} - \sqrt{\pi} + \sqrt{\pi}\, \mathrm{Erf}\left(\frac{x-v}{\sqrt{4\alpha t}}\right)$$

Making use of this relationship in equation (3) and simplifying

$$s(x,t) = \int_{-(\frac{B}{2}+H)}^{(\frac{B}{2}+H)} \frac{K\sqrt{4\pi\alpha t}}{2\pi T}\left[e^{-(x-v)^2/4\alpha t} - \sqrt{\pi}\sqrt{\frac{(x-v)^2}{4\alpha t}} + \frac{\sqrt{\pi}(x-v)}{\sqrt{4\alpha t}}\,\mathrm{Erf}\left(\frac{x-v}{\sqrt{4\alpha t}}\right)\right]dv \qquad \ldots (4)$$

Splitting the limits of integration and integrating

$$s(x,t) = \frac{K\sqrt{4\pi\alpha t}}{2\pi T}\left[\int_{-(B/2+H)}^{0} e^{-(x-v)^2/4\alpha t}\,dv + \int_{0}^{(B/2+H)} e^{-(x-v)^2/4\alpha t}\,dv\right.$$

$$-\frac{K\sqrt{4\pi\alpha t}.\sqrt{\pi}}{2\pi T}\left[\int_{-(B/2+H)}^{0} \frac{(x-v)}{\sqrt{4\alpha t}}\,dv + \int_{0}^{(B/2+H)} \frac{(x-v)}{\sqrt{4\alpha t}}\,dv\right]$$

$$+\frac{K\sqrt{4\pi\alpha t}.\sqrt{\pi}}{2\pi T}\left[\int_{-(\frac{B}{2}+H)}^{0} \frac{(x-v)}{\sqrt{4\alpha t}}\,\mathrm{Erf}\frac{(x-v)}{\sqrt{4\alpha t}}\,dv + \int_{0}^{(B/2+H)} \frac{(x-v)}{\sqrt{4\alpha t}}\,\mathrm{Erf}\frac{(x-v)}{\sqrt{4\alpha t}}\,dv\right]$$

$$= \frac{K\sqrt{4\pi\alpha t}}{2\pi T}\left[\sqrt{\pi\alpha t}\,\mathrm{Erf}\left(\frac{x+B/2+H}{\sqrt{4\alpha t}}\right) - \sqrt{\pi\alpha t}\,\mathrm{Erf}\left(\frac{x-B/2-H}{\sqrt{4\alpha t}}\right)\right]$$

$$-\frac{K\sqrt{4\pi\alpha t}.\sqrt{\pi}}{2\pi T}\cdot\frac{x(B+2H)}{\sqrt{4\alpha t}}$$

$$+\frac{K\sqrt{4\pi\alpha t}.\sqrt{\pi}}{2\pi\ T}\int_{-(B/2+H)}^{0} \frac{x-v}{\sqrt{4\alpha t}}\,\mathrm{Erf}\left(\frac{x-v}{\sqrt{4\alpha t}}\right)dv + \int_{0}^{(B/2+H)} \frac{x-v}{\sqrt{4\alpha t}}\,\mathrm{Erf}\left(\frac{x-v}{\sqrt{4\alpha t}}\right)dv \qquad \ldots (5)$$

An integral of the form $\int Y.\mathrm{Erf}(Y)dY$ is given by

$$\int Y.\mathrm{Erf}(Y)dy = \frac{Y^2}{2}\mathrm{Erf}(Y) + \frac{Y}{2\sqrt{\pi}}e^{-Y^2} - \frac{1}{2\sqrt{\pi}}\int e^{-Y^2}dy + \text{constant}$$

Using this relationship in equation (5) and integrating

$$s(x,t) = \frac{K\sqrt{4\pi\alpha t}}{2\pi t}\Big[\sqrt{\pi\alpha t}\ \mathrm{Erf}\Big(\frac{x+B/2+H}{\sqrt{4\alpha t}}\Big) - \sqrt{\pi\alpha t}\ \mathrm{Erf}\Big(\frac{x-B/2-H}{\sqrt{4\alpha t}}\Big)$$

$$- \frac{x.\sqrt{\pi}\ (B+2H)}{\sqrt{4\alpha t}} + \frac{\sqrt{\pi}(x+B/2+H)^2}{2.\ \sqrt{4\alpha t}}\mathrm{Erf}\Big(\frac{x+B/2+H}{\sqrt{4\alpha t}}\Big)$$

$$+ \frac{x+B/2+H}{2}\ e^{\frac{-(x+B/2+H)^2}{4\alpha t}} - \frac{\sqrt{\pi\alpha t}}{2}\ \mathrm{Erf}\Big(\frac{x+B/2+H}{\sqrt{4\alpha t}}\Big)$$

$$- \frac{\sqrt{\pi}(x-B/2-H)^2}{2.\sqrt{4\alpha t}}\ \mathrm{Erf}\ \Big(\frac{x-B/2-H}{\sqrt{4\alpha t}}\Big) - \frac{(x-B/2-H)}{2}.\ e^{-\frac{(x-B/2-H)^2}{4\alpha t}}$$

$$+ \frac{\sqrt{\pi\alpha t}}{2}\ \mathrm{Erf}\ \Big(\frac{x-B/2-H}{\sqrt{4\alpha t}}\Big)\Big]$$

Simplifying and rearranging,

$$s(x,t) = \frac{K\alpha t}{2T}\mathrm{Erf}\Big(\frac{x+B/2+H}{\sqrt{4\alpha t}}\Big) - \frac{K\alpha t}{2T}\ \mathrm{Erf}\ \Big(\frac{x-B/2-H}{\sqrt{4\alpha t}}\Big)$$

$$+ \frac{K}{4T}\ (x+B/2+H)^2\ \mathrm{Erf}\ \Big(\frac{x+B/2+H}{\sqrt{4\alpha t}}\Big) + \frac{K\sqrt{\alpha t}}{2T\sqrt{\pi}}(x+B/2+H)\ e^{-\frac{(x+B/2+H)^2}{4\alpha t}}$$

$$- \frac{K}{2T}\ \frac{\sqrt{\alpha t}}{\sqrt{\pi}}(x-B/2-H)\ e^{-\frac{(x-B/2-H)^2}{4\alpha t}} - \frac{K(x-B/2-H)^2}{4T}\ \mathrm{Erf}\Big(\frac{x-B/2-H}{\sqrt{4\alpha\ t}}\Big)$$

$$- \frac{K\sqrt{x^2}(B+2H)}{2T} \qquad \ldots (6)$$

The above expression can be written as

$$s(x,t) = F(x,B,H,t) - \frac{K\sqrt{x^2}(B+2H)}{2T} \qquad \ldots (7)$$

The expression for rise of water table, $s(x,t)$ under the strip source at location x, for $-(\frac{B}{2}+H) \le x \le (\frac{B}{2}+H)$ is derived as follows:

$$s(x,t) = \int_0^{(B/2+H+x)} \left[\frac{K\sqrt{\alpha t}}{\sqrt{\pi}\ T} e^{-v^2/4\alpha t} - \frac{K\ v}{2\ T} + \frac{K\ v}{2\ T} \mathrm{Erf}\left(\frac{v}{\sqrt{4\alpha t}}\right)\right] dv$$

$$+ \int_0^{(B/2+H-x)} \left[\frac{K\sqrt{\alpha t}}{\sqrt{\pi}\ T} e^{-v^2/4\alpha t} - \frac{K\ v}{2\ T} + \frac{K\ v}{2\ T} \mathrm{Erf}\left(\frac{v}{\sqrt{4\alpha t}}\right)\right] dv$$

Integrating and simplifying,

$$s(x,t) = \frac{K\alpha t}{2\ T} \mathrm{Erf}\left(\frac{x+B/2+H}{\sqrt{4\alpha t}}\right) - \frac{K\alpha t}{2\ T} \mathrm{Erf}\left(\frac{x-B/2-H}{\sqrt{4\alpha t}}\right)$$

$$+ \frac{K}{4T}(x+B/2+H)^2 \mathrm{Erf}\left(\frac{x+B/2+H}{\sqrt{4\alpha t}}\right) - \frac{K}{4T}(x-B/2-H)^2 \mathrm{Erf}\left(\frac{x-B/2-H}{\sqrt{4\alpha t}}\right)$$

$$+ \frac{K\sqrt{\alpha t}}{2\sqrt{\pi}T}(x+B/2+H) e^{-(x+B/2+H)^2/4\alpha t} - \frac{K\sqrt{\alpha t}}{2\sqrt{\pi}T}(x-B/2-H) e^{-(x-B/2-H)^2/4\alpha t}$$

$$- \frac{K}{2T}\left[x^2 + (B/2+H)^2\right] \qquad \ldots (8)$$

$$= F(x,B,H,t) - \frac{K}{2T}\left[x^2 + (B/2+H)^2\right] \qquad \ldots (9)$$

The evolution of rise in water table due to seepage from two parallel canals is next considered. The width of water surface and depth of water for the left and right canals, as shown in Fig.(2), are B_1, H_1, and B_2, H_2, respectively. The distance between centre to centre of the canals is D. The rise in water table at any point along a transverse section across the two parallel canals at time say, t, can be computed by summing up the values of rise in water table for

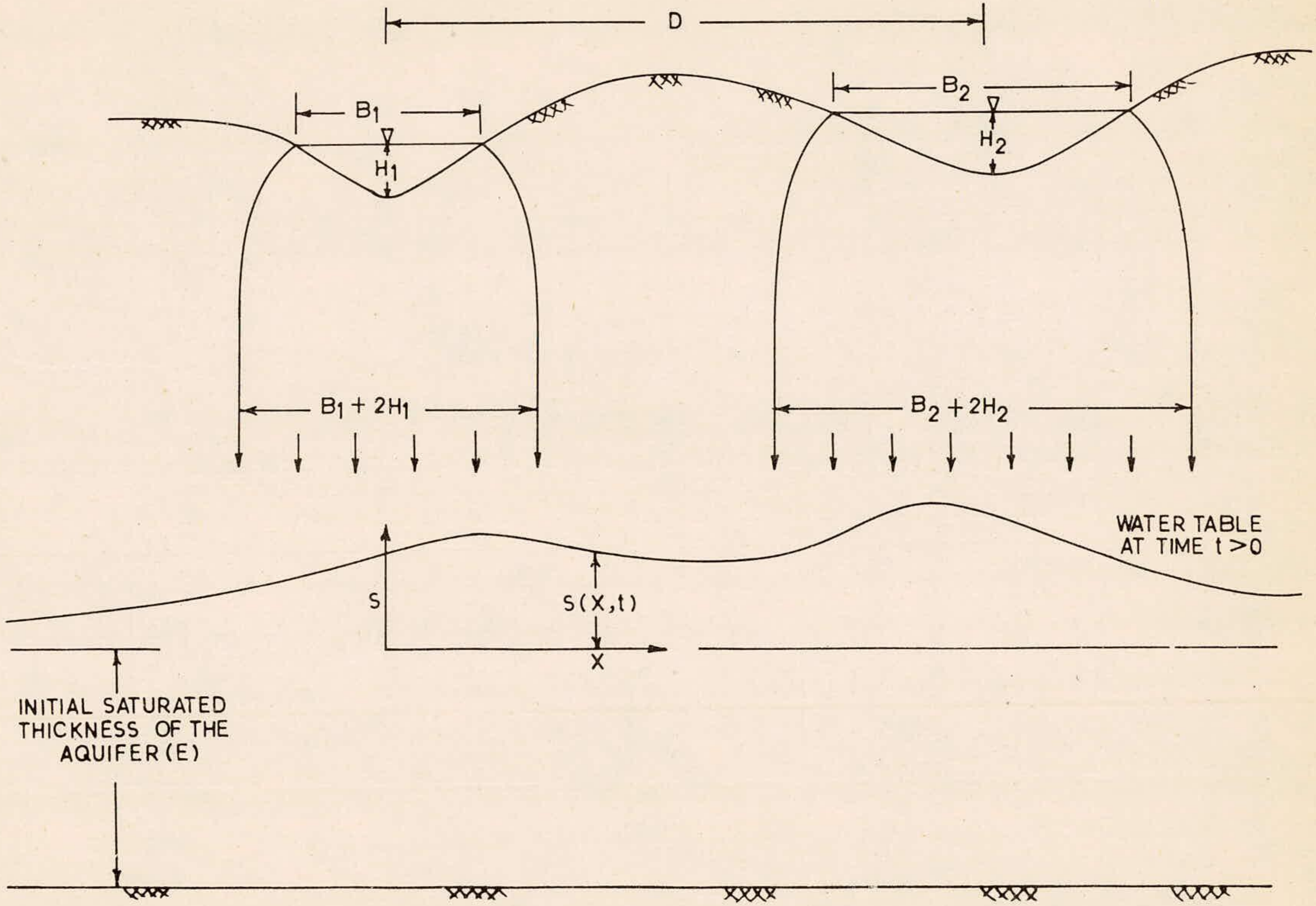

FIG. 2 Schematic section of two parallel canals

each canal with the help of equations (7) and (9). For different locations of point x, as shown in Fig.(2), the expressions for rise in water table are as follows:

For $x \leq -(B_1/2+H_1)$

$$s(x,t) = F(x,B_1,H_1,t)+F(D-x,B_2,H_2,t) - \frac{K\sqrt{x^2}(B_1+2H_1)}{2T} - \frac{K\sqrt{(D-x)^2}(B_2+2H_2)}{2T} \qquad (10)$$

for, $-(B_1/2+H_1) \leq x \leq (B_1/2+H_1)$

$$s(x,t) = F(x,B_1,H_1,t)+F(D-x,B_2,H_2,t) - \frac{K}{2T}[x^2 + (B_1/2+H_1)^2] - \frac{K\sqrt{(D-x)^2}\,(B_2+2H_2)}{2T} \qquad \ldots (11)$$

for, $(B_1/2+H_1) \leq x \leq D-(B_2/2+H_2)$

$$s(x,t) = F(x,B_1,H_1,t)+F(D-x,B_2,H_2,t) - \frac{K\sqrt{x^2}(B_1+2H_1)}{2T} - \frac{K\sqrt{(D-x)^2}(B_2+2H_2)}{2T} \qquad \ldots (12)$$

for, $D-(B_2/2+H_2) \leq x \leq D+(B_2/2+H_2)$

$$s(x,t) = F(x,B_1,H_1,t)+F(D-x,B_2,H_2,t) - \frac{K\sqrt{x^2}(B_1+2H_1)}{2T} - \frac{K}{2T}[(D-x)^2+(B_2/2+H_2)^2] \qquad \ldots (13)$$

and for, $x \geq D+(B_2/2+H_2)$

$$s(x,t) = F(x,B_1,H_1,t)+F(D-x,B_2,H_2,t) - \frac{K\sqrt{x^2}(B_1+2H_1)}{2T} - \frac{K\sqrt{(D-x)^2}(B_2+2H_2)}{2T} \qquad \ldots (14)$$

5.0 RESULTS

The results of rise of water table at different time and locations, on account of recharge from a single canal and from two parallel canals spaced at a known distance apart are presented In the computations the Error Functions have been evaluated by using rational approximations of Hastings (vide Abramowitz and Stegun, 1970). The evolution of water table from a recharging strip (canal) has been compared with results calculated by Shestakov's special functions (1965) which have been evaluated numerically. The results of the present study for a single canal have been found to compare well with those of Shestakov.

The results of percentage rise of water table at different locations across a section at the end of various nondimensional time, after the onset of recharge from a single canal, having H/E=0.003, are presented in Fig. 3 and 4 for values of B/E=0.03 and 0.06 respectively. It is seen that for B/E=0.03, the maximum percentage rise of water table at the centre of the strip after nondimensional time, $Kt/2\phi E=0.05$, is 0.626. The percent rise at this time at a distance, x/E=0.5, from the centre of the canal is 0.107. The corresponding percentage rise for a canal of larger width having B/E=0.06 and H/E=0.003 is 0.190. The gradient of the water table at a distance, x/E=0.5 from the centre of the recharge strip of width, B/E=0.03 is found to be 0.46 percent. The corresponding gradient for a canal of width, B/E=0.06 is

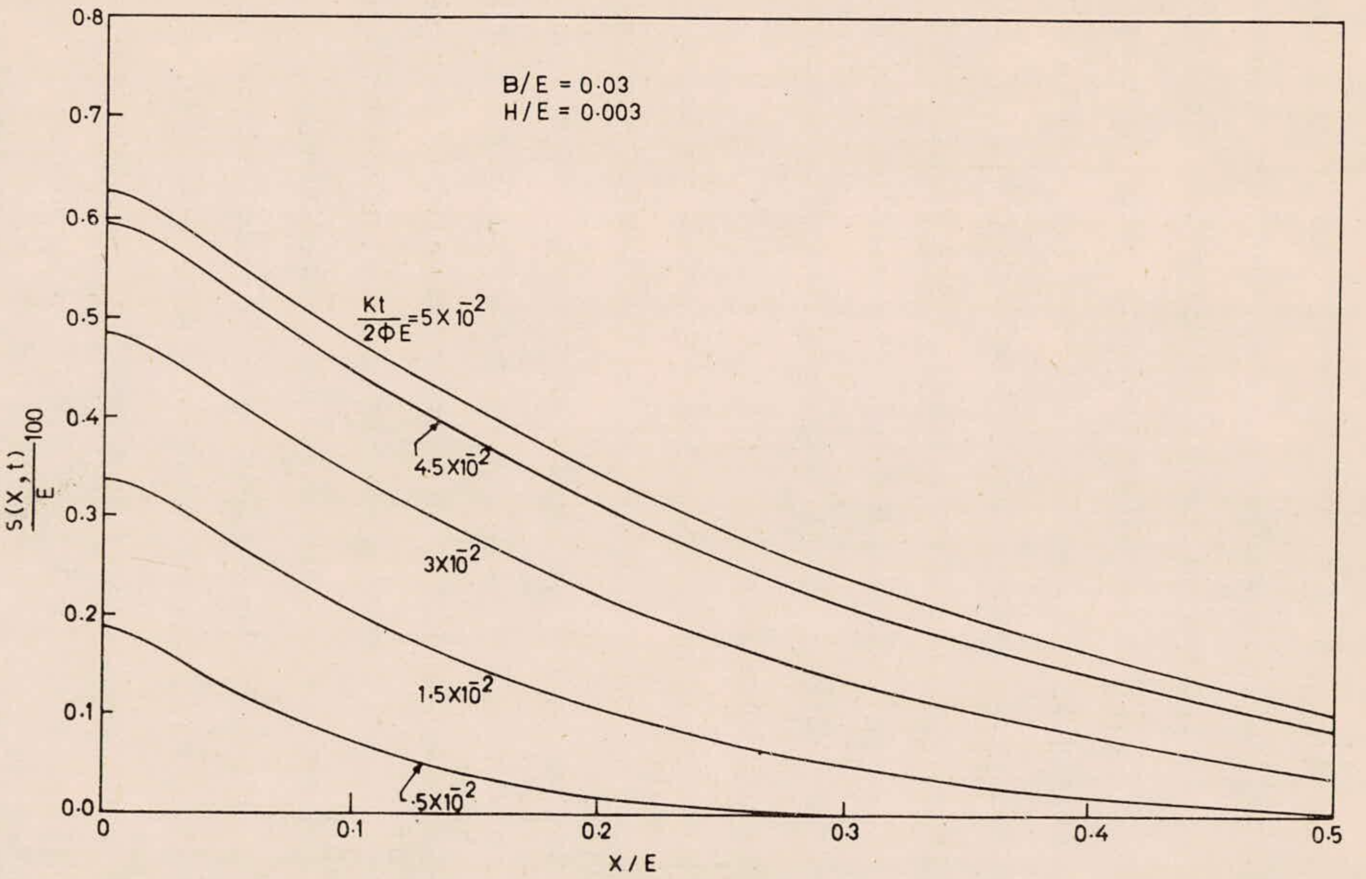

FIG.3 Variation of percentage rise of water table due to recharge from a canal of width B/E = 0.03

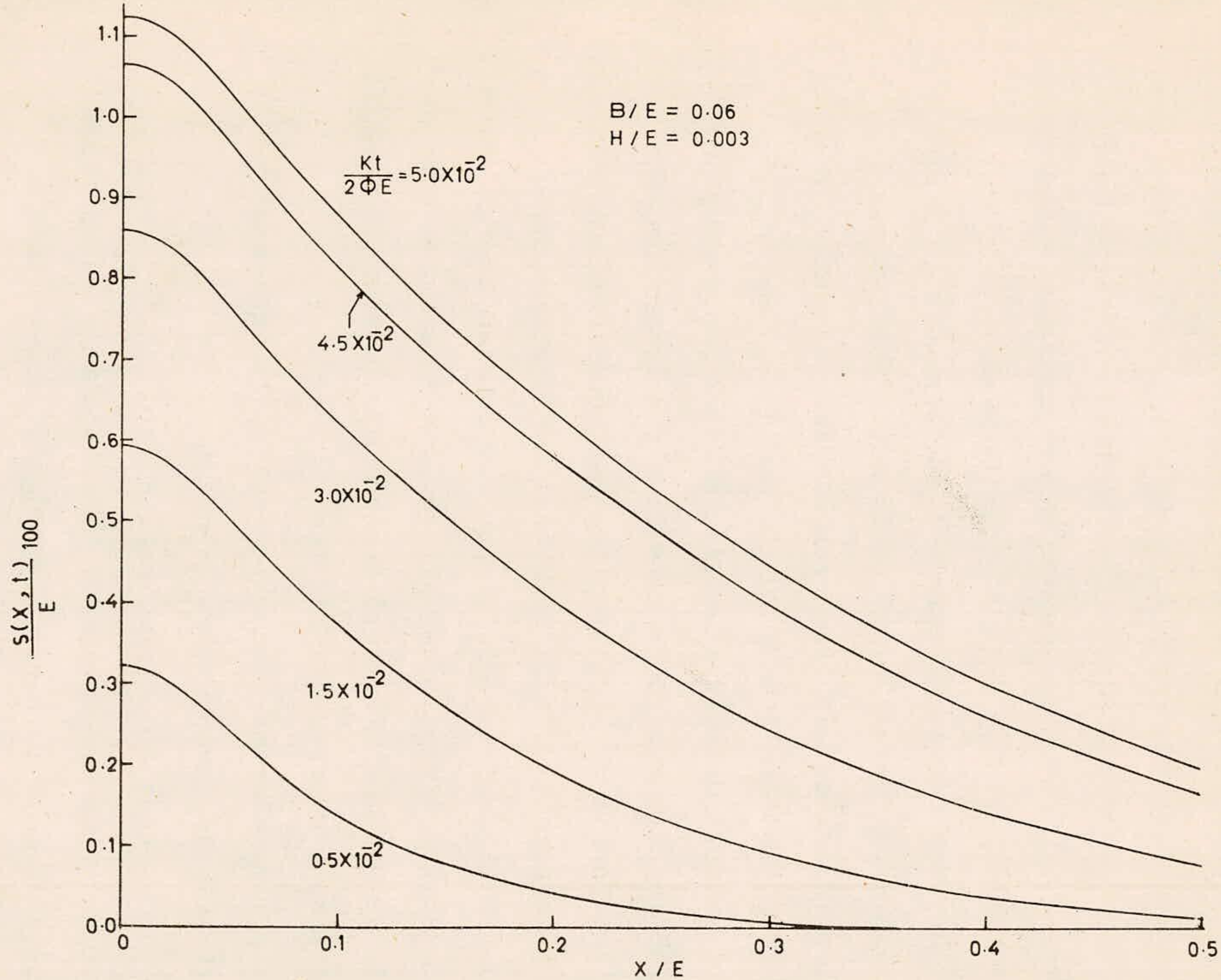

FIG. 4 Variation of percentage rise o. water table due to recharge from a canal of width B/E=0.06

0.856 percent. Since the gradient of groundwater generally varies between 0.1 to 1 percent (Mandel and Shiftan, 1981), it can, therefore, be assumed that for intermittantly running irrigation canal, the effect of rise of water table is insignificant at $x/B > 10$. It can also be seen that as time passes, the water mound has a tendency to dissipate more and more on both sides and the flow tends to become horizontal at large distances from the canal.

The rise of water table and locations of points of maximum rise at different time due to recharge from two identical parallel canals have been studied for spacings of 120 m and 180m between the canals and for B/E=0.03 and 0.06. The aquifer parameters assumed are: $T=100m^2$/day, K=0.1m/day and ϕ=0.1. The resulting rise of water table across a section of the parallel canals at various times and locations are plotted in Figs. 5 through 8. It is seen that, in the beginning of recharge, the two water table ridges are located at the centre of the recharging strips (canals). As the recharge continues, the points of maximum rise move towards each other but do not cross a distance of (B/2+H) from the centres of the two strips. As time elapses, the water table, in the zone in between the canals, tends to become flat.

The percent rise in water table (S/E) at different nondimensional time for various locations across the parallel canals are presented in Figs. 9 through 12 for various spacings and sizes of canals. The results indicate that at locations, x/E=0.6, the percentage rise due to recharge from a canal of width B/E=0.03 is, about 0.1 percent after a nondimensional time factor of $5x10^{-2}$.

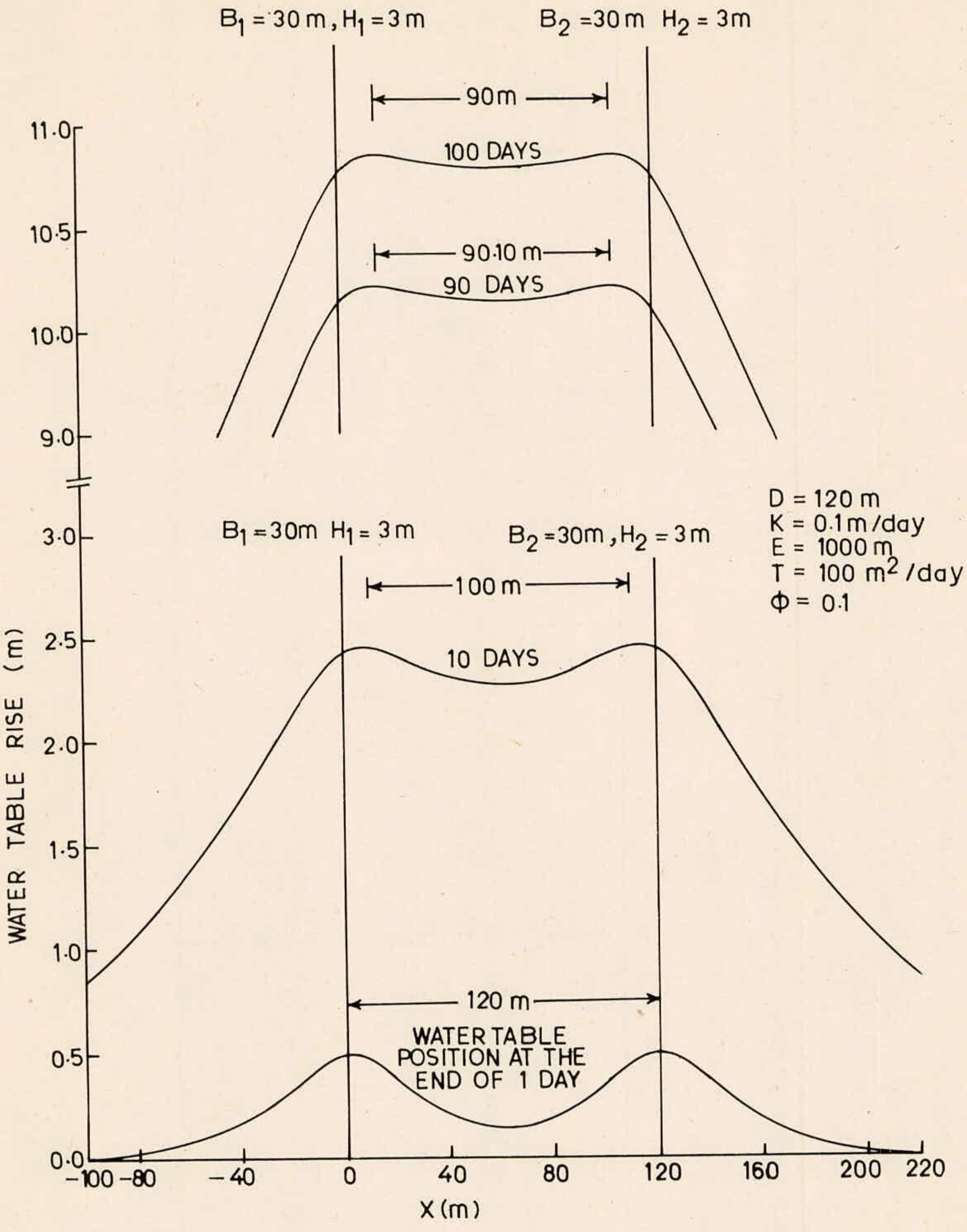

Fig.5 Water table rise due to recharge from two identical parallel canals of 30m width spaced at a distance of 120m apart.

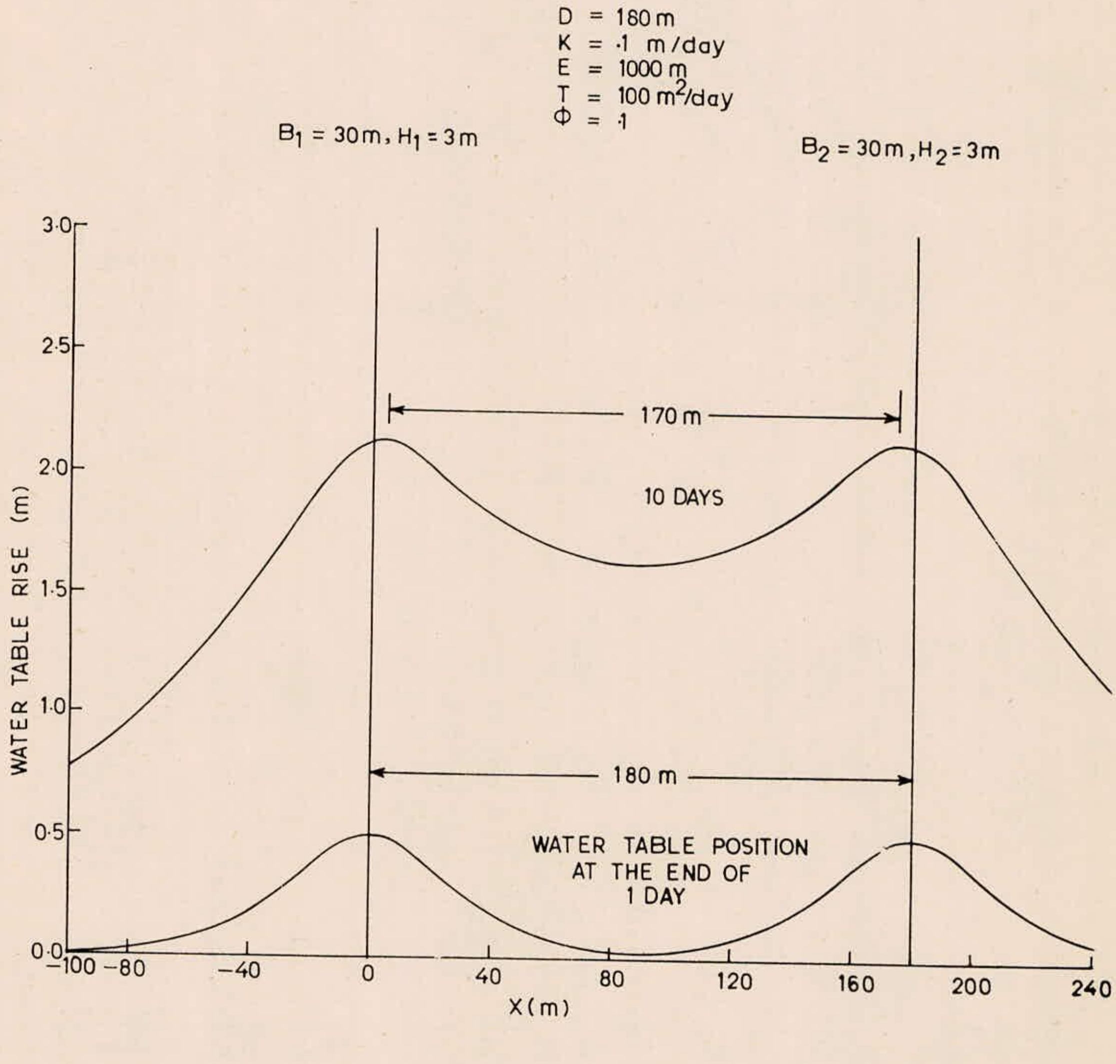

FIG.6 a. Water table rise due to recharge from two identical parallel canals of 30m width spaced at a distance of 180m apart

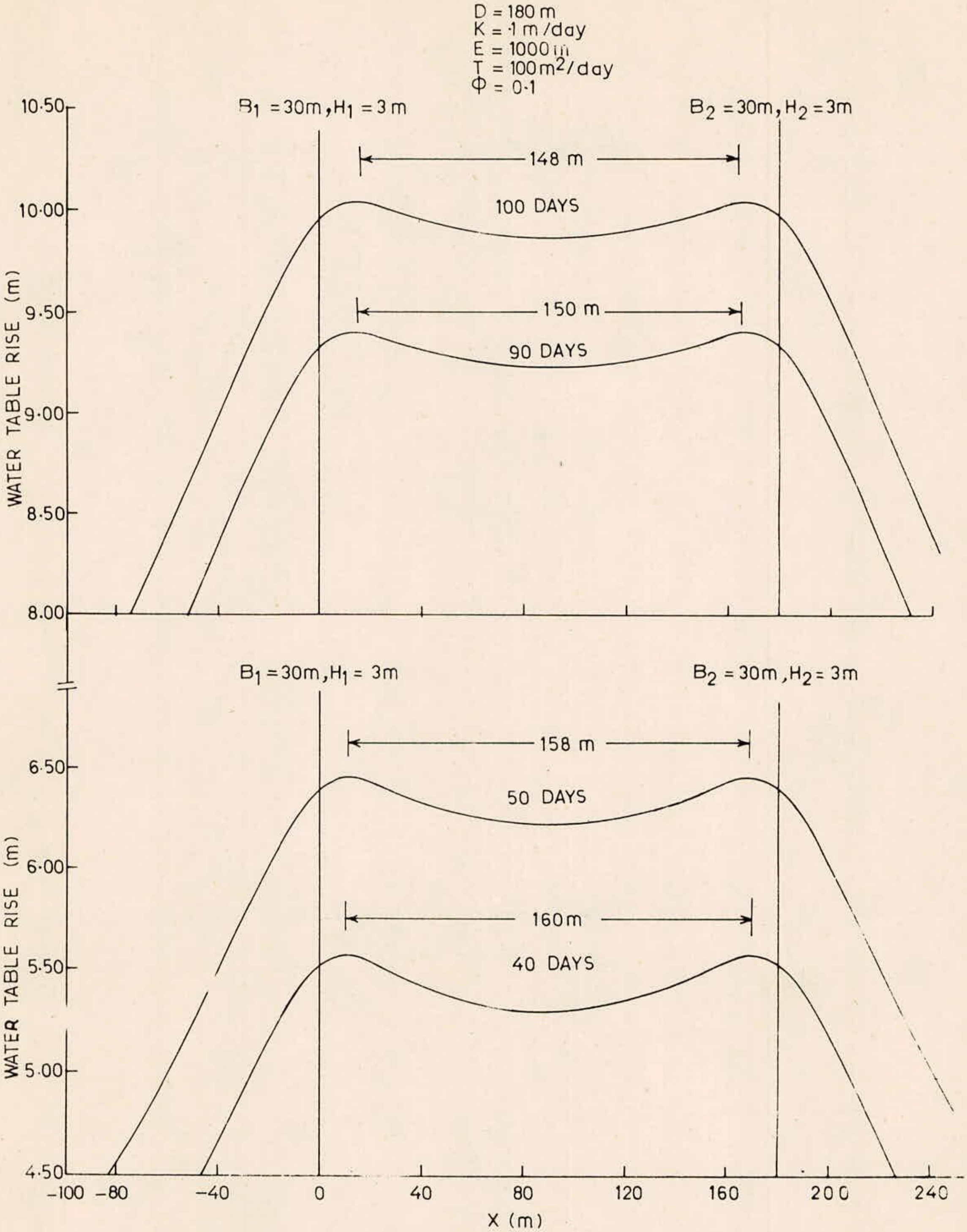

Fig.6b. Water table rise due to recharge from two identical parallel canals of 30 m width spaced at a distance of 180m apart.

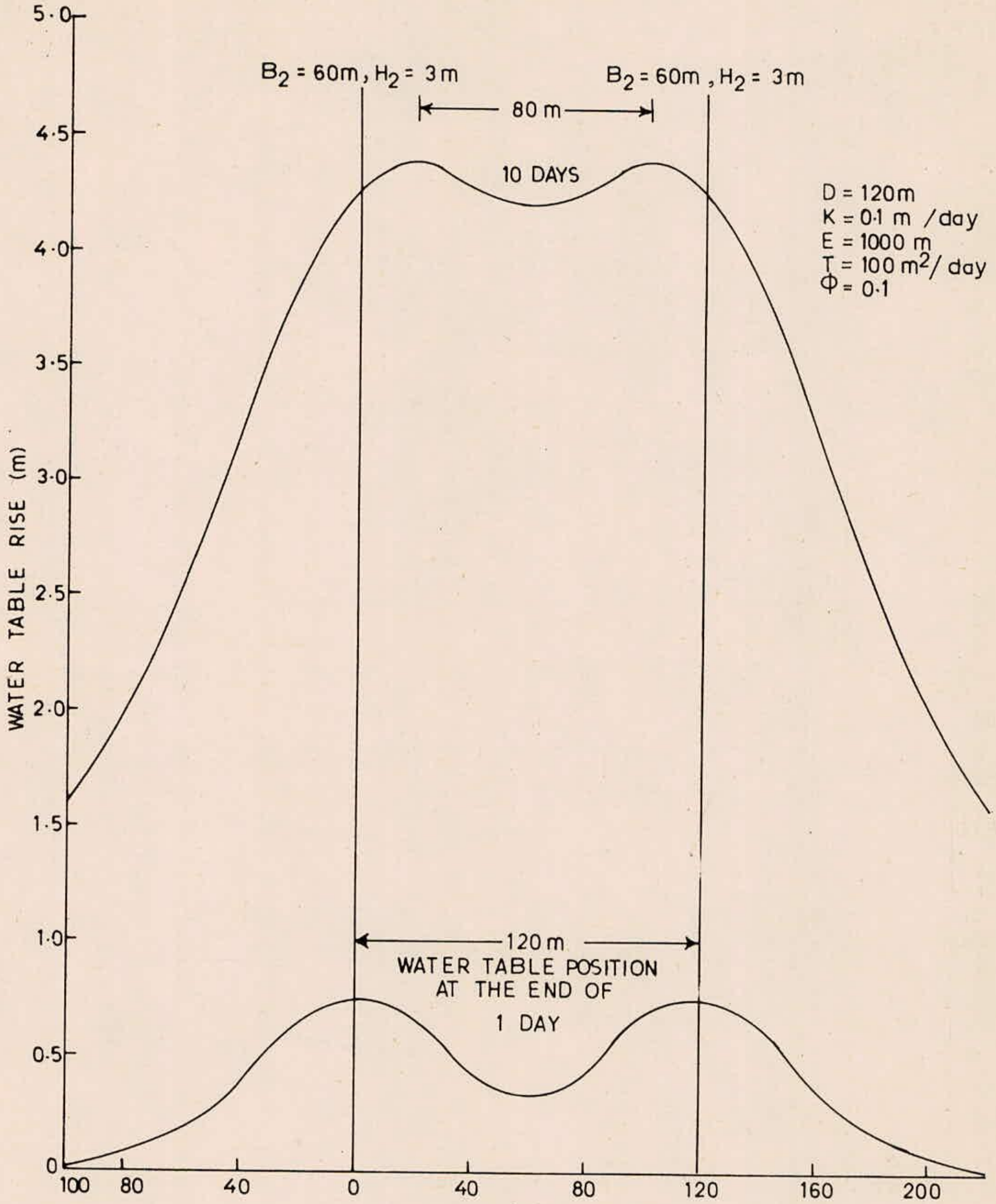

Fig.7(a) Water table rise due to recharge from two identical parallel canals of 60m width spaced at a distance of 120m apart

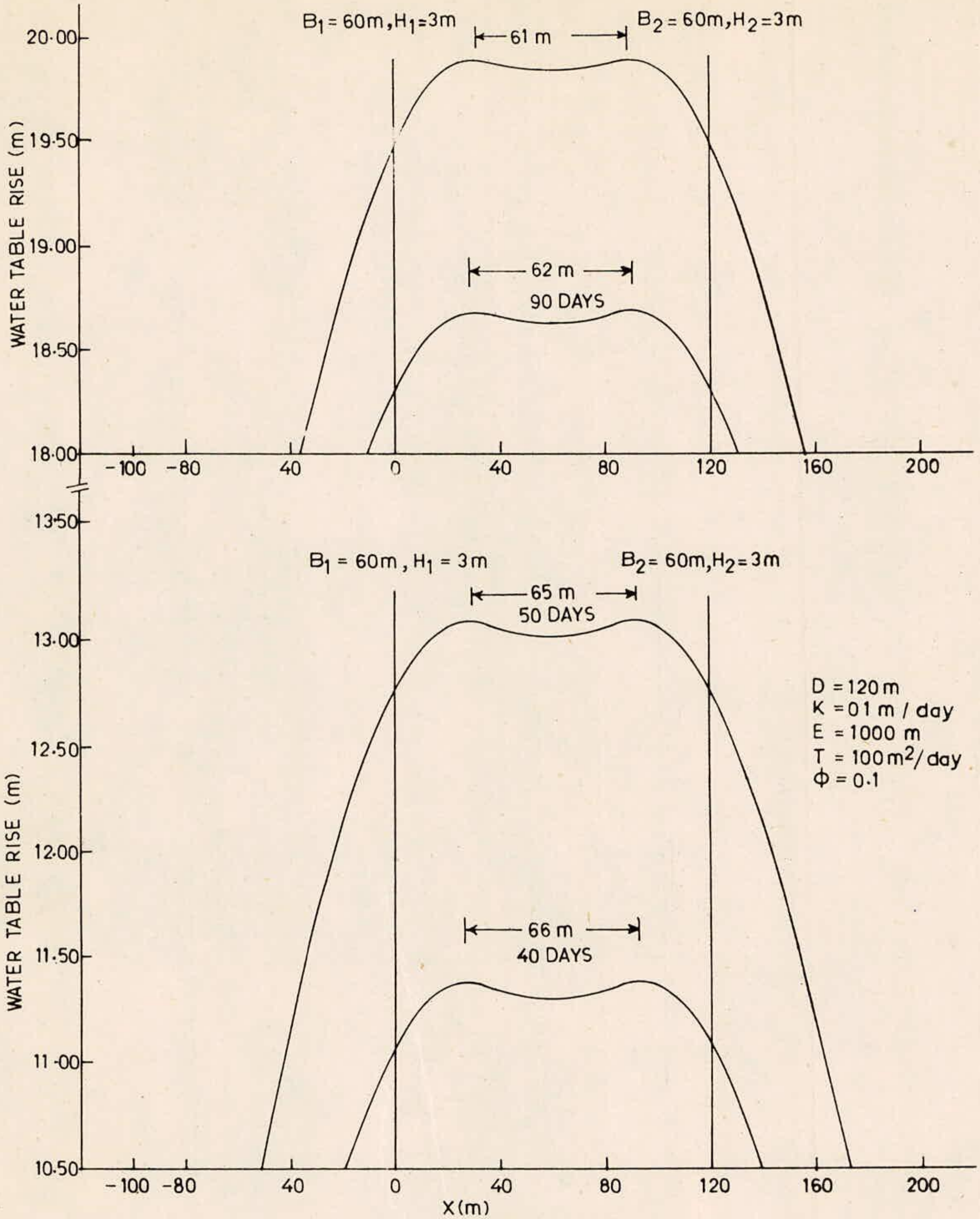

Fig.7(b) Water table rise due to recharge from two identical parallel canals of 60m width spaced at a distance of 120 m apart.

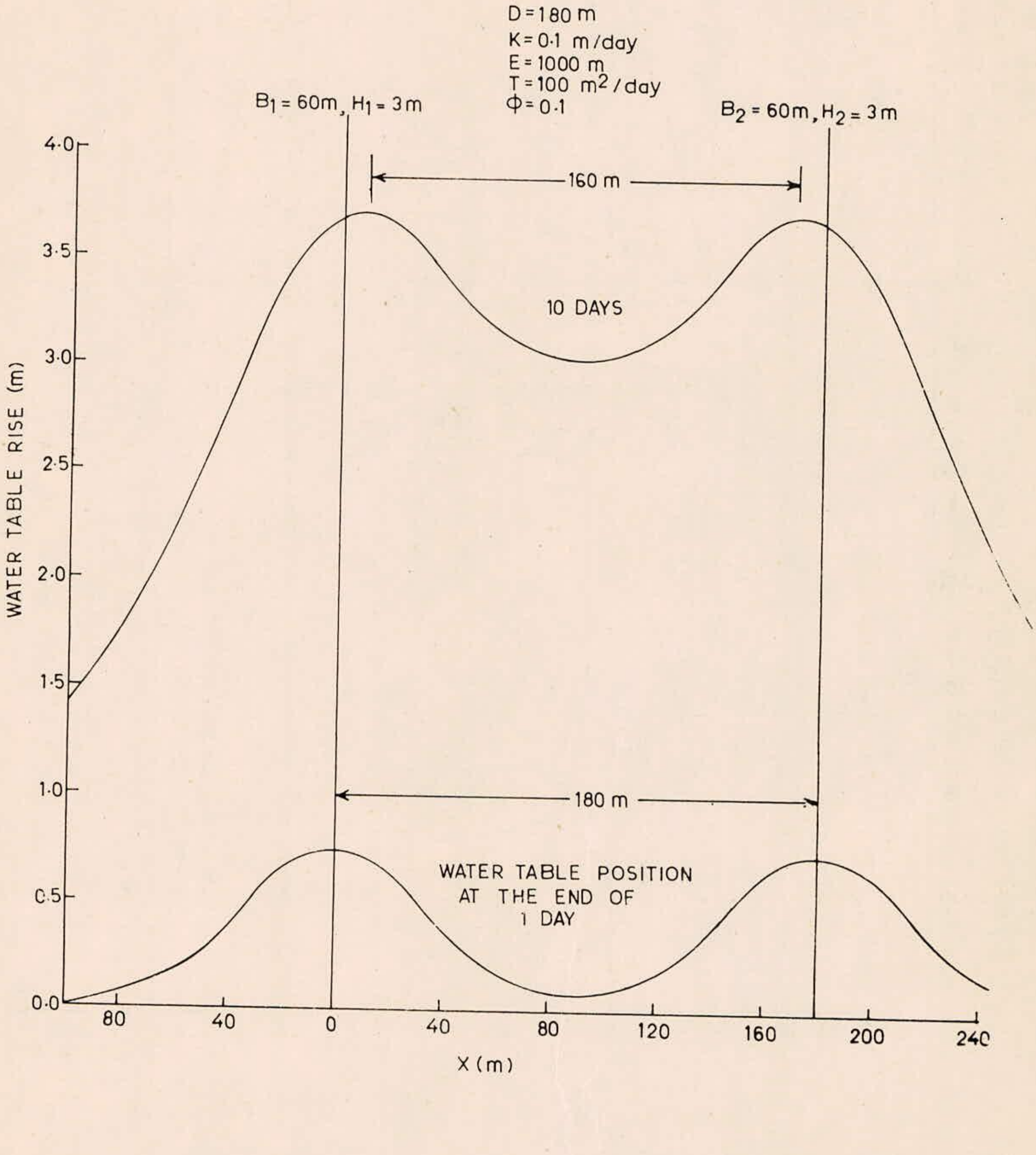

Fig.8 (a) Water table rise due to recharge from two identical parallel canals of 60m width spaced at a distance of 180m apart

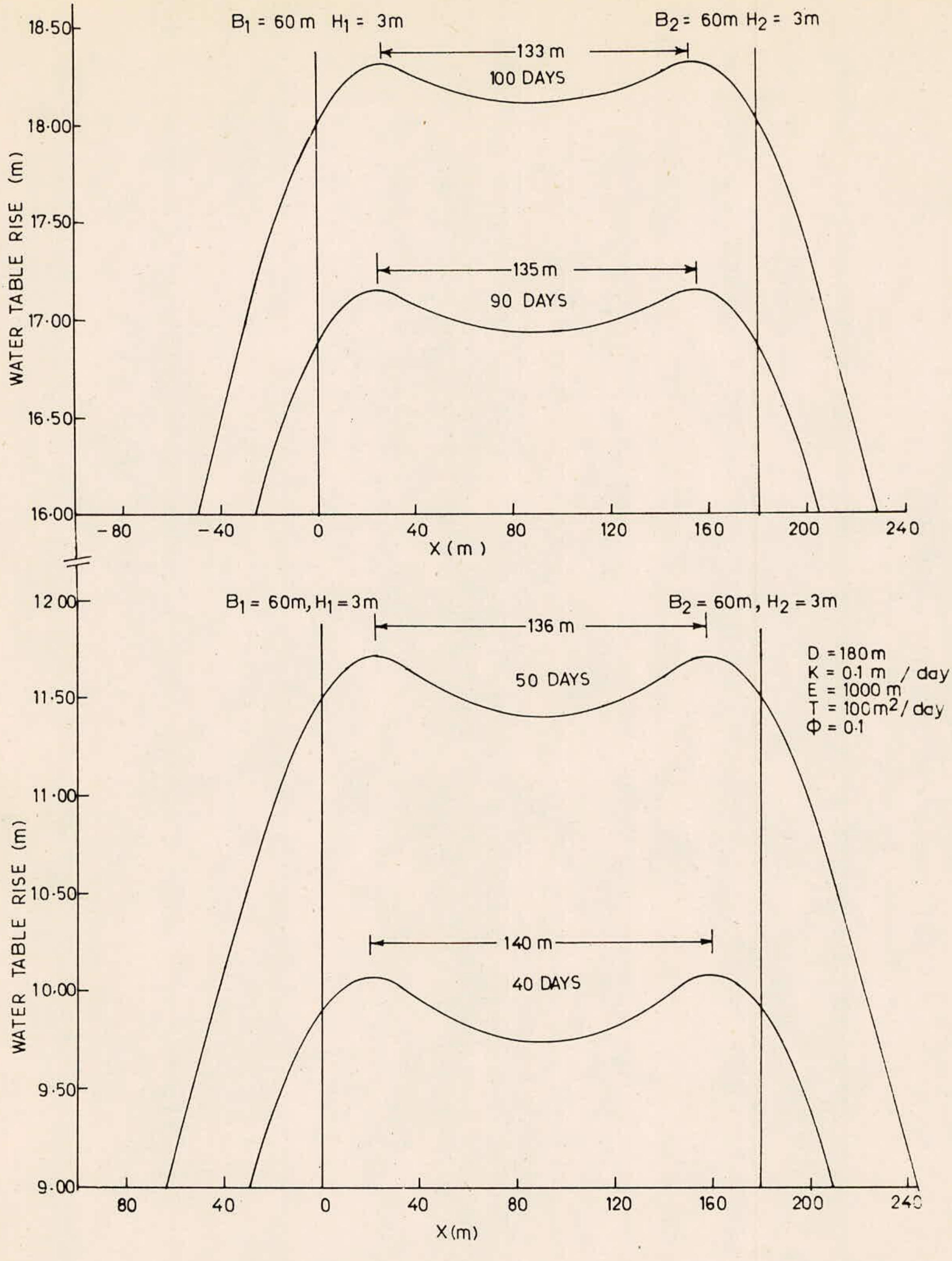

Fig.8 (b) Water table rise due to recharge from two identical parallel canals of 60m width spaced at a distance of 180m apart.

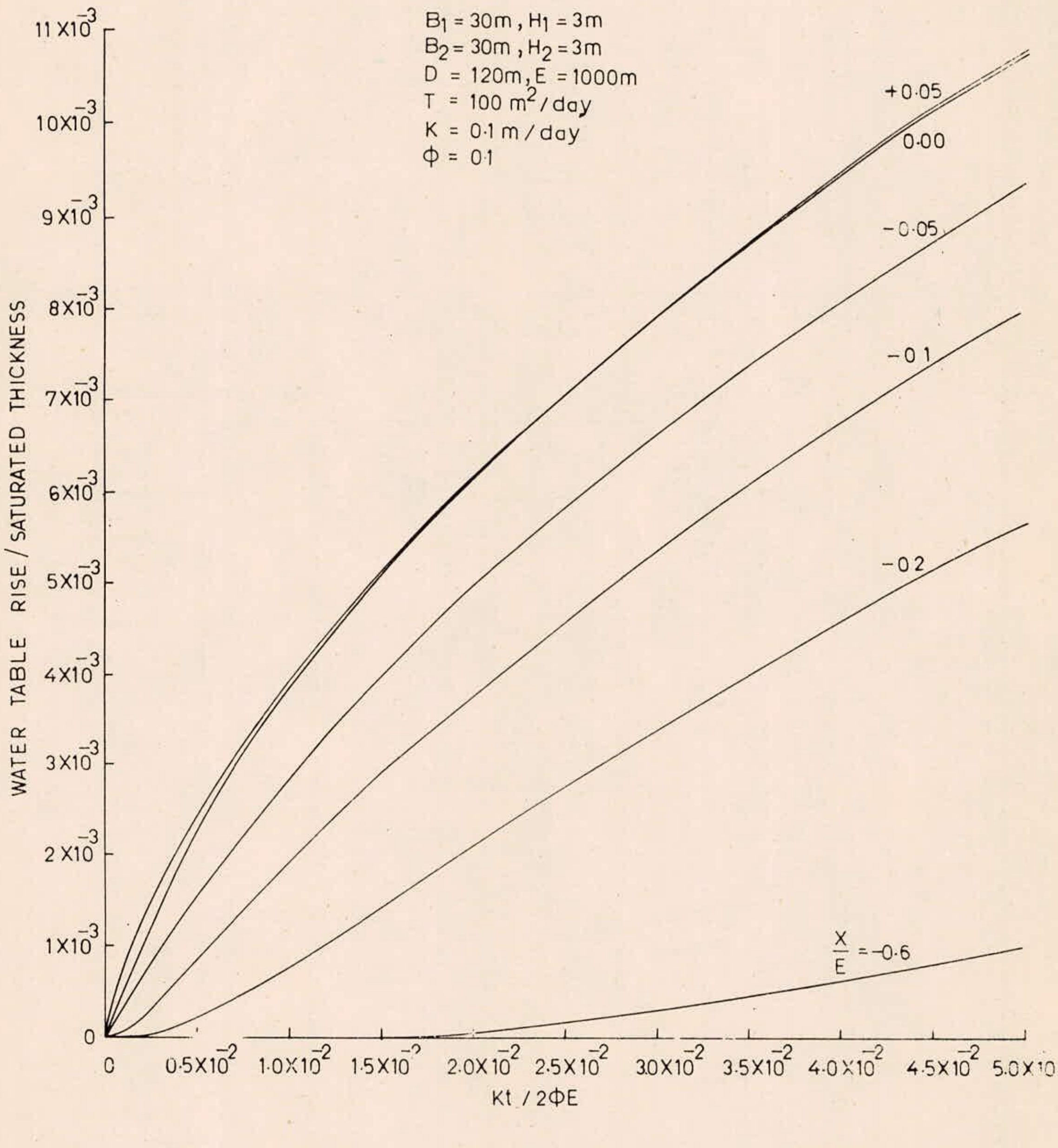

Fig.9. Variation of water table rise due to recharge from two identical parallel canals of width B/E=0.03 spaced at a distance of D/E=0.120

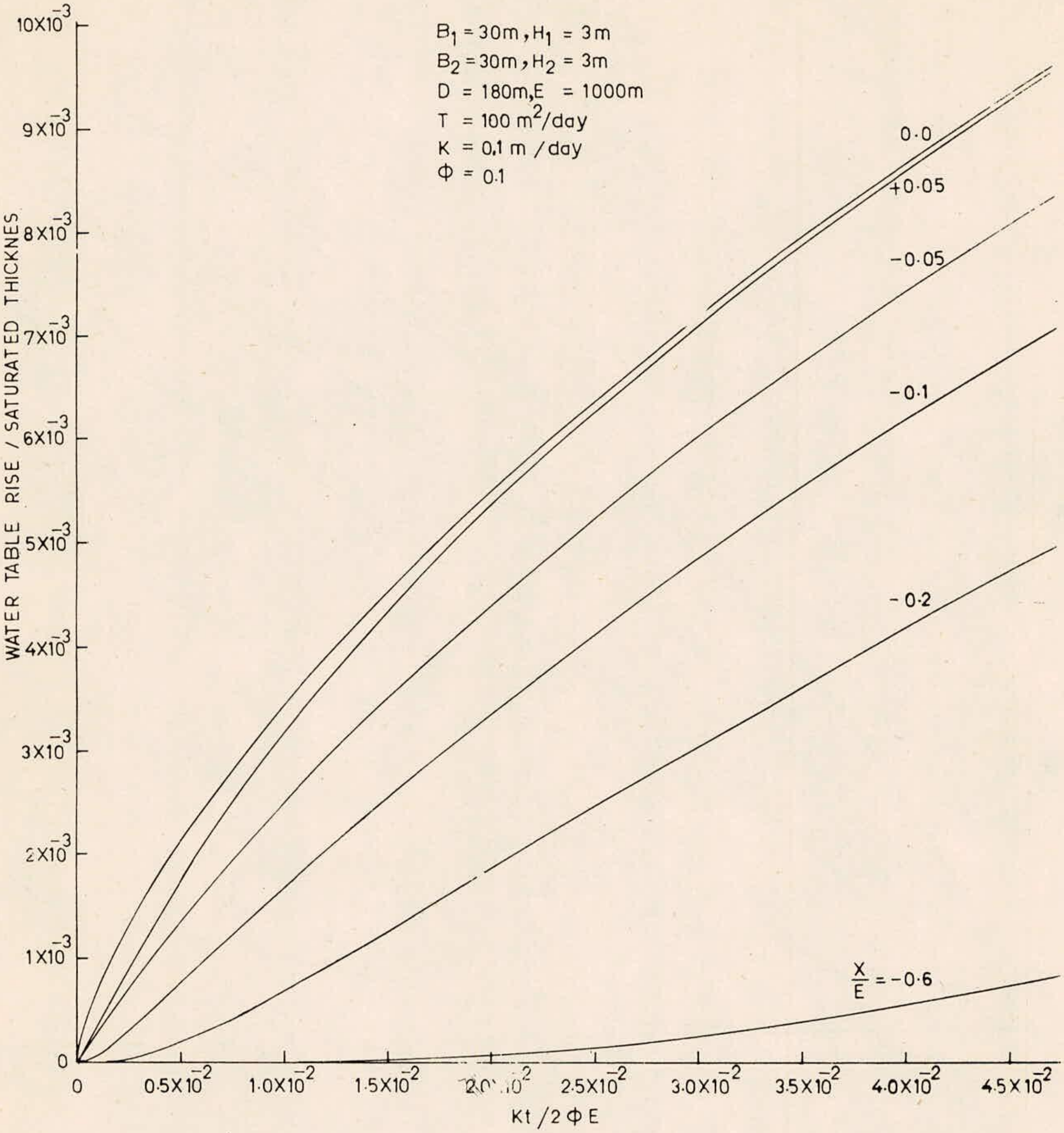

FIG. 10 Variation of water table rise due to recharge from two identical parallel canals of width B/E=0.03 spaced at a distance of D/E=0.180

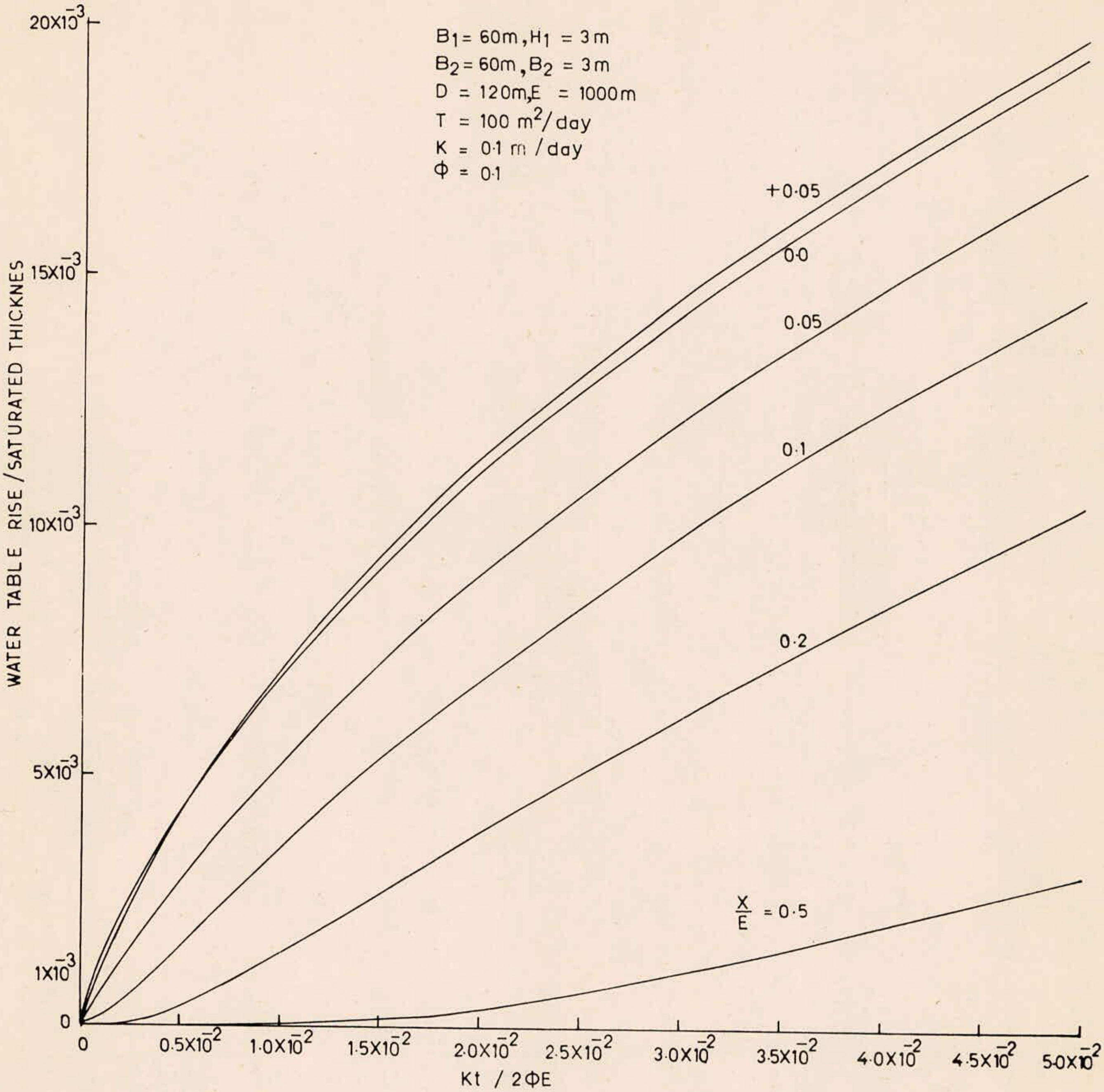

FIG.11 Variation of water table rise due to recharge from two identical parallel canals of width B/E=0.06 spaced at a distance of D/E=0.120

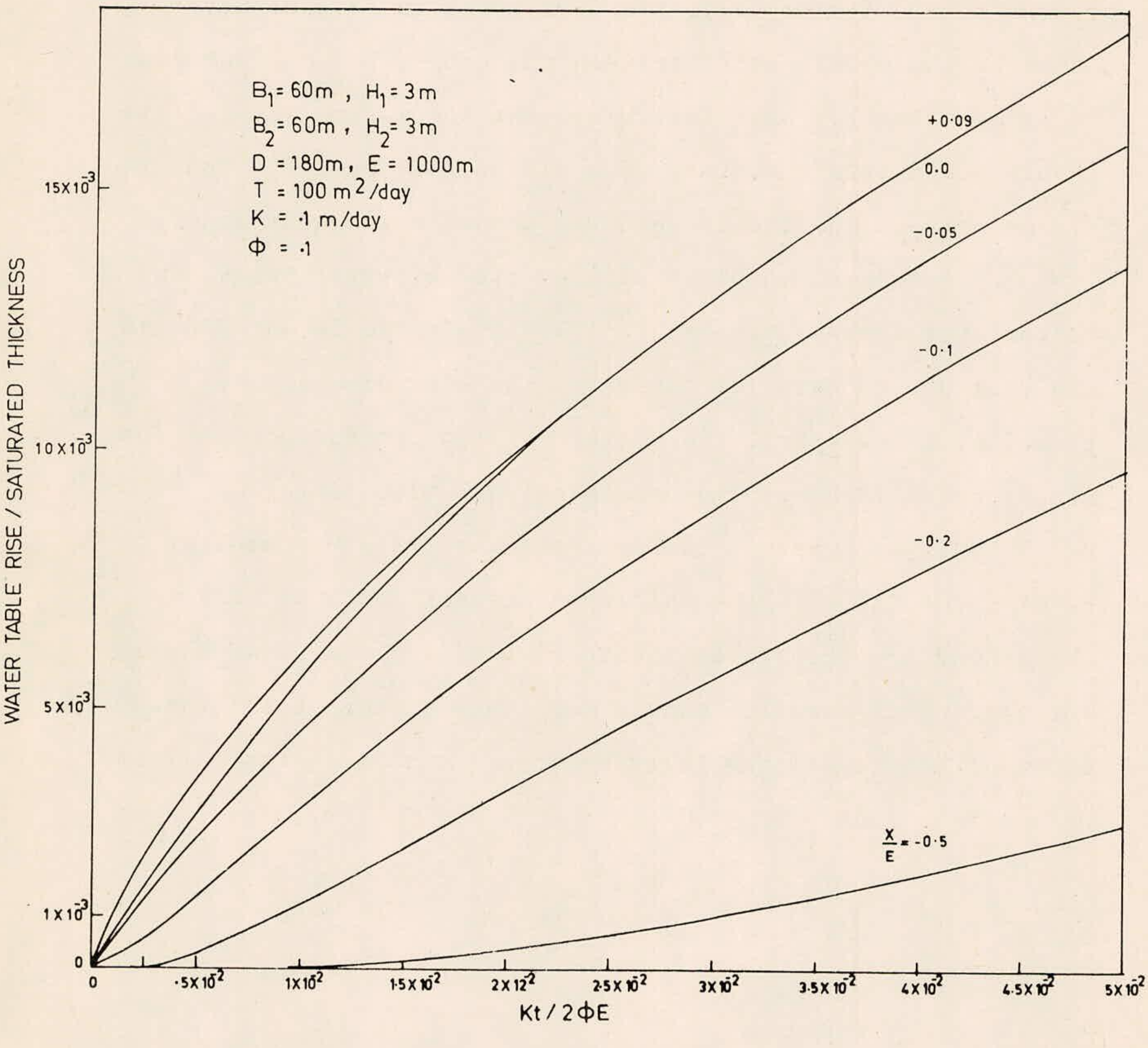

FIG.12 Variation of water table rise due to recharge from two identical parallel canals of width B/E=0.06 spaced at a distance of D/E=0.180

The maximum rise of water table for different widths of parallel canals and spacings between them are presented in Table 2. The results show the reduction in maximum rise as spacing between the parallel canals is varied from 80m to 480m. It is seen from the table that for identical parallel canals, beyond a time factor 0.15, the difference in rise of water table at the middle point between the canals and maximum rise is insignificant. This indicates that the region between the canals would take the shape of a plateau ultimately. Thus the region between the canals tends to become a stagnant zone.

A comparison of results of rise of water table, due to recharge from single canal of width 30m and 3m water depth and that due to parallel canals of the same dimensions, spaced at 120m. apart, shows that maximum percentage rise for single canal (S/E) at the nondimensional time factor of 0.045 is 0.593, whereas the percentage of rise of parallel canals is 1.022. If the spacing increases to 480m. the corresponding percentage of rise is 0.693. This shows that for spacing of parallel canals more than sixteen times the width of the canal, the interference of canals is insignificant upto a time factor 0.045.

Table .2 Effect of spacing of parallel canals on maximum rise of water table

Width of first canal B_1 (m)	Depth of water in First canal (m)	Width of second canal B_2 (m)	Depth of water in second canal H_2	Spacing between canals, D (m)	Non-dimensional time $Kt/2\Phi E$	Maximum rise as percentage of saturated thickness	Percentage rise at middle point between the canals
30	3	30	3	80	0.015	0.572	0.570
					0.045	1.082	1.080
					0.150	2.085	2.084
30	3	30	3	120	0.015	0.520	0.509
					0.045	1.022	1.015
					0.150	2.020	2.016
30	3	30	3	180	0.015	0.459	0.427
					0.045	0.941	0.922
					0.150	1.926	1.916
30	3	30	3	240	0.015	0.414	0.355
					0.045	0.871	0.835
					0.150	1.840	1.820
30	3	30	3	480	0.015	0.344	0.153
					0.045	0.693	0.545
					0.150	1.557	1.467
30	3	30	3	∞	0.015	0.336	-
					0.045	0.593	-
					0.150	1.096	-
60	3	60	3	80	0.015	1.048	1.047
					0.045	1.983	1.982
					0.150	3.822	3.822

Contd..

60	3	60	3	120	0.015	0.946	0.936
					0.045	1.868	1.863
					0.150	3.700	3.697
60	3	60	3	180	0.015	0.826	0.786
					0.045	1.715	1.692
					0.150	3.525	3.514
60	3	60	3	240	0.015	0.741	0.653
					0.045	1.581	1.533
					0.150	3.364	3.337
60	3	60	3	480	0.015	0.608	0.282
					0.045	1.249	1.000
					0.150	2.868	2.690
60	3	60	3	∞	0.015	0.592	-
					0.045	1.064	-
					0.150	1.986	-

6.0 CONCLUSIONS

Based on the study presented, the following conclusions are drawn:

a) Closed form solution for evolution of water table in time due to recharge from a single strip and from two parallel strips, identified as canals, have been obtained.

b) The increase in saturated thickness of aquifer due to recharge from the canal is insignificant at a distance of ten times the width of the canal.

c) For two identical parallel canal system, the interference of the canals is not insignificant upto a time factor 0.045 if their centre to centre spacing is more than sixteen times the width of the canal.

d) The region between the two parallel canals under continuous recharge takes the shape of a plateau in course of time.

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