DETERMINATION OF AQUIFER PARAMETERS USING MARQUARDT ALGORITHM FOR WELL RECOVERY DATA

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ABSTRACT

The problem of determination of aquifer parameters has been formulated as an optimization problem and has been solved using the non-linear programming technique. The Marquardt algorithm has been used to solve the problem. THe results are comparable with the other methods of determination of parameters.

INTRODUCTION

The graphical methods of determination of aquifer parameters are in use for a long time. However, with the wider availability of computers, the numerical methods are also becoming popular. The optimization techniques have been used to solve a number of problems related with water resources development. The techniques which have been most commonly used in hydrology are linear programming and dynamic programming. Due to several reasons, the use of non-linear programing technique in hydrology is somewhat limited although a large number of real life problems have non-linear objective function and/or constraints. In the present paper, the problem of determination of aquifer parameters has been formulated as an optimization problem and has been solved using the non-linear programming technique.

THEORY

The partial differential equation describing unsteady radially symmetric flow in a nonleaky homogeneous confined aquifer of constant thickness can be written as

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{S}{T}\frac{\partial h}{\partial t} \qquad (1)$$

where h is the piezometric head at a radius r from the pumped well at a time t since the start of pumping. T and S are the transmissivity and storage coefficient, respectively. Equation (1) assumes that the pumping well fully penetrates the aquifer layer during the test. A solution of the equation (1) which satisfies the continuity requirement and the conditions

$$h(r, 0) = h_o$$

$$h(\infty, t) = h_o$$

was given by Theis (1935). According to Theis

$$h_o - h = \frac{Q_w}{4\pi T} \int_{u=r^2S/(4Tt)}^{\infty} (e^{-u}/u)\, du \qquad (2)$$

where h_o is the initial piezometric head and Q_w is the constant rate of pumping.

Equation (2) can also be written as

$$s = \frac{Q_w}{4\pi T} W(u) \qquad (3)$$

Where s is the draw down and W(u) is the Theis well function which represents the exponential integral of equation (2).

The equation (2) is nonlinear in T and S. These parameters can be estimated by nonlinear regression analysis. In this paper, a non-linear programming algorithm has been used to determine the aquifer parameters T and S. This algorithm was developed by Marquardt (1963). The algorithm is briefly explained below.

Let s_i^* be the draw down in the aquifer at any instant. It can be computed by substituting the initial trial values of parameters T^* and S^* in the equation (3). Let ΔT and ΔS be the respective increments in $T^*$and S^* which will yield the improved estimates T and S at the end of each trial the corresponding draw down value given as,

$$s_i = f(T,S) \tag{4}$$

Expanding s_i by Taylor series about the trial values

$$s_i = f(T^* + \Delta T, S^* + \Delta S) \tag{5}$$

or

$$s_i = f(T^*,S^*) + \frac{\partial f(T^*, S^*)}{\partial T^*} \Delta T + \frac{\partial f(T^*,S^*)}{\partial S^*} \Delta S$$

or

$$s_i = s_i^* \frac{\partial s_i^*}{\partial T^*} \Delta T + \frac{\partial s_i^*}{\partial S^*} \Delta S \tag{6}$$

The increments ΔT and ΔS are determined such that the sum of squares of the difference between the observed and the calculated draw downs is minimum. Thus the objective function is written as :

$$\min_{\Delta T,\ \Delta S} \quad \text{sum} = \sum_{i=1}^{N} (s_i^o - s_i)^2 \tag{7}$$

where s_i^o is the observed draw down at any instant.

SOLUTION ALGORITHM

The linearized model given by equation (6) is substituted in equation (7). The normal equations can be obtained by taking the partial derivatives of the objective function, equation (7), with respect to T and S and setting them equal to zero

$$\frac{\partial \text{ sum}}{\partial T^*} = 0$$

$$\frac{\partial \text{ sum}}{\partial S^*} = 0$$

These equations will take the form

$$(A^T A)\ \Delta A = A^T (s^o - s^*) \tag{8}$$

where A is the matrix of partial derivatives of computed draw downs with respect to trial values of the parameters T and S, s^o and s^* are the vectors of observed and calculated draw downs, respectively, and

$$\Delta A = \begin{bmatrix} \Delta T \\ \Delta S \end{bmatrix}$$

The normal equations (8) are solved for ΔA and the new draw downs are calculated by substituting the improved estimates (T,S) of the parameters in equation (3). The error criterion is checked and if the same is not satisfied, the process is repeated with the updated estimates of the parameters.

In order to ensure convergence with relatively poor starting values, equation (8) is modified as

$$(A^T A + \lambda I)\ \Delta A = A^T (s^o - s^*) \tag{9}$$

where λ is the convergence factor and I is the identity matrix. Initial values of λ are large and decrease towards zero as convergence is reached

APPLICATION EXAMPLE

The algorithm explained above was used to determine the parameters T and S of a confined aquifer. A pumping test was carried out and the draw downs were observed during pumping as well as recovery.

The pumping well has negligible storage and the observations are taken at the well itself and at other points. The initial guess of T and S are given along with their lower and upper limit.

A well was pumped for 7000 min at the rate of 18.9236 m^3/10 min. The draw down observations were taken at a well at a distance 99.9 m. The initial guess for T and S were 0.4 and 0.0017. The lower and upper limits of T and S were 10.95 & 0.500. The values of T and S were obtained as 5.65 sq m/10 min and 0.00083 respectively. The results obtained using the conventional methods, CGWB(1982), are T = 5.76 sq m/10 min and S = 0.00074.

CONCLUSIONS

The Marquardt algorithm, which can be used to solve a non-linear programming problem, has been used to determine the aquifer parameters using the pumping test data. The results are obtained using field data are sufficiently close to the results obtained using the conventional methods.

REFERENCES

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