FLOW TOWARDS PARTIALLY PENETRATING WELLS WITH STORAGE IN CONFINED AQUIFERS

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SYNOPSIS

The formulation of strategy for the optimal management of the groundwater demands a knowledge of the relationship between the pumpage from the well to the drawdowns in the aquifers in spatial as well as temporal coordinates. The existing literature does not provide a suitable relationship for partially screened wells with storage. Most of the wells in India are dug wells of large diameter with huge amounts of storage in them. Use of the existing relationships for the evaluation of aquifer parameters may yield erroneous values in such cases. Hence, a study of transient flow towards finite diameter wells with storage has been attempted with a view to develop type curves required for the proper estimation of aquifer parameters. In the present investigation, a digital model based on finite element technique is developed and it is used to obtain the solutions for partially penetrating/screened wells with storage in confined aquifers. An attempt has been made to generalise these solutions and type curves at discrete distances from the discharging wells have been presented. From the critical observation of these type curves ' storage affected zones' is demarcated in spatial and temporal coordinates. A simple method has been suggested to determine the transmissivity of the aquifer from the early time record of pumping.

1.0 INTRODUCTION

The formulation of strategy for the optimal management of the ground water demands a knowledge of the relationships between the pumpages in the well to the drawdowns in the aquifer in spatial and temporal coordinates. Though, tere are several such relationships available in well hydraulics, most of them consider the discharging well to be a line sink, and do not consider the storage in the well. Most of the wells in southern part of the country are dug wells of large diameter with considerable amount of storage. The existing methods of analyses to the wells of this type either for modelling the aquifer or for simulating the drawdown history are inadequate. Hence, a study of transient flow towards finite diameter wells with storage has been attempted, with a view to develop type curves required for the identification of aquifer parameters. This paper deals with the solutions for partially screened wells in confined aquifers.

In the present investigation, a digital model based on the finite element technique was prepared and analysed for the case of wells with storage. An attempt has been made to generalise the solutions and type curves have been presented at discrete distances from the discharging well for different storage parameters. Also, a simple method is suggested to determine the transmissibility of the aquifer from the early time pumping history.

2.0 MATHEMATICAL MODEL

A partially screened well with storage in a confined aquifer is shown in Fig.1. The aquifer system is considered to be of infinite aerial extent with the well at its centre such that all physical conditions are symmetrical with respect to the axis of the well. The aquifer of thickness m, is bounded by an aquiclude both on top and bottom. The aquifer is considered to be homogeneous and isotropic and has negligible dip. The well of radius r_w, has penetrated completely the aquifer but it is screened only for a length l in the commencing from the top of the aquifer. The well is considered to be pumped at constant discharge Q and the steady state conditions are yet to reach. The casing of the well, r_c, starts from the ground level and extends upto the junction of aquifer and aquiclude. The drawdown in the well at any time t (total time from the starting of the pump) is designated by s_w. The height of the non pumping piezometric surface is indicated by h_o. For the purpose of discretization, the infinite aquifer system is replaced by a finite system, with the inflow potential boundary for the aquifer located at $r = r_o$ from the axis. The height of the piezometric surface at any radial distance r is designated by h.

The governing differential equation for the case of axially symmetric flow is,

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial h}{\partial r} + \frac{\partial^2 h}{\partial z^2} = \frac{S}{T} \cdot \frac{\partial h}{\partial t} \qquad (1)$$

In equation 1, z is the elevation of a point at a radial distance r in the domain. The origin of the coordinate system is located at the point 'O' in the well, where the axis of symmetry intersects the base of the aquifer. The positive direction of z axis is taken to be vertically upward. The 'h' is the piezometric head at a point (r,z) measured above thebase of the aquifer. 'S' is the storativity and 'T' is transmissibility of the aquifer. The drawdown at any point (r,z) is assumed to be

$$s(r) = h_o - h\ (r,m) \qquad (2)$$

This value is used in the calculation of the well function $W(u^*)$ for preparing the type curves.

Boundary conditions:

The boundary conditions for this problem are as follows:-

DE	$z = m$	$r_w \leqslant r \leqslant r_o$	$\frac{\partial h}{\partial z} = 0$	(3)
EF	$r = r_o$	$0 \leqslant z \leqslant m$	$h = h_o$	(4)
FA	$z = o$	$r_w \leqslant r \leqslant r_o$	$\partial h/\partial z = o$	(5)
AC	$r = r_w$	$0 \leqslant z \leqslant (m-l)$	$\partial h/\partial z = o$	(6)
CD	$r = r_w$	$(m-l) \leqslant z \leqslant m$	the distance presecribed condition.	(7)

i.e. $$2 \pi K \int_{z=(m-l)}^{m} r_w \left[\frac{\partial h(t)}{\partial r} \right]_{r=r_w} dz + \pi r_c^2 \frac{\partial}{\partial t} h_w(t) = -Q \qquad (7a)$$

For the case of fully screened well, the boundary conditions for AC and CD are replaced by the following equation :-

AD $r = r_w$ $0 \leqslant z \leqslant m$ the discharge prescribed condtion (8)

i.e. $$2 \pi K \int_{z=o}^{m} r_w \left[\frac{\partial h(t)}{\partial r} \right]_{r=r_w} dz + \pi r_c^2 \frac{\partial}{\partial t} h_w(t) = -Q \qquad (8A)$$

It may be noted that $(\partial h/dr)$ varies with z, while it is a constant for a fully screened well in a confined aquifer.

Initial conditions:

The initial condition is

$h(r,o) = h_o$ (9)

where,

- r_w = radius of the discharging well
- r_c = radius of the casing well
- Q = discharge pumped from well
- m = thickness of the aquifer
- h_o = height of non-pumping piezometric surface
- r_o = radial distance from the origin O to the inflow boundary
- l = length of the screened portion
- K = permeability of the aquifer

The analytical solution for the mathematical model that describes the problem exactly would be highly complicated. So a numerical scheme based on finite element technique was attempted to solve the above set of equations.

3.0 FINITE ELEMENT MODEL

The variational forms of the field equation may be obtained by considering an equivalent variational problem and adopting Euler-Lagrange equation from the calculus of variation. By comparing the terms in these two equations and with the subsequent integration of terms, the funttional over the flow region can be obtained.

In minimising the functional, the requirement on the flow boundary must also be met. These requirements lead to the addition of some extra terms to the functional over the flow region. Considering the boundary conditions in the present problem the additional term that is to be added to the functional would be the one which corresponds to the prescribed discharge condition. For the impermeable boundaries and for the portion of the boundary where head is prescribed no new terms need be added. Thus, the functional which describes the entire flow field along with the boundaries is given by,

$$[\ I\ (h)\ ]_{\bar{R}} = \int_{t}^{t+\Delta t} \int_{R} \frac{1}{2}\ K.\ \frac{\partial h}{\partial x_i}.\ \frac{\partial h}{\partial x_i}.\ dR.dt + \int_{t}^{t+\Delta t} \int_{R} S_s.\ h.\ \frac{\partial h}{\partial t}\ .dR.dt + \int_{t}^{t+\Delta t} \int_{B} h.\bar{q}.\ dB.dt \qquad (10)$$

where,

Δt = increment in time domain

S_s = specific storage of the aquifer

i = a particular component along the coordinate axis

(summation is implied over the full range of a repeated subscript)

To solve the above problem the flow field is discretized into a net work consisting of inter-connected finite elements. The functional over the entire region of flow is assumed to be contributed by each element.

The flow domain is divided into a finite number of ring shaped elements, with rectangular cross-section. By drawing diagonals each rectiangle is sub-divided into four triangles. The variation of head in each triangle is taken to be linear.

The conductance and porosity matrices for the rectangular elements are obtained by combining the corresponding individual matrices of the triangles coupled with suitable algebraic manipulation for the elimination of the central node.

The final equation which is to be integrated with respect to time can be obtained as

$$\int_{t}^{t+\Delta t} [\, C_{JI} \,] \, h_J \, dt + \int_{t}^{t+\Delta t} [\, P_{JI} \,] \, \dot{h} \, dt + \int_{t}^{t+\Delta t} \{ Q_I \} = 0 \qquad (11)$$

where,

$[C]$ = global conductance matrix

$[P]$ = global porosity matrix

$\{Q\}$ = discharge vector

For the integration with respect to time, the Crank-Nicolson Method is adopted and the equation (11) reduces to

$$[D] \{ h^{t+1} \} = [E] \{ h^{t} \} - \frac{\Delta t}{2} \{ Q^{t+1} + Q^{t} \} \qquad (12)$$

where,

$[D] = [P] + \frac{\Delta t}{2} \; [C]$

$[E] = [P] - \frac{\Delta t}{2} \; [C]$

The methodology adopted for the solution of the equations 11,12 after implementing the boundary conditions of the discharging portion of the well was described in an earlier paper (Seethapathi, 1983).

4.0 ANALYSIS AND RESULTS

For this analysis, twelve cases namely, PDC-1 to PDC-12 are solved and analysed. The geometric and hydrogeological parameters adopted herein are shown in Table 1. Three different screening ratios, S_r, are chosen viz., 0.25, 0.50 and 0.75. The storage parameter, β is varied from 10^2 to 10^5 for each screening ratio $\left(\beta = \frac{r_c^2}{r_w^2} \cdot S\right)$ Plots are drawn on a log-log scale for different βs, between the non-dimensional time parameter, u*, and the non-dimensional drawdown, W(u*), for the well boundary and also for the interior points, with r/r_w as a parameter. Two sets of such curves are presented herein for different r/r_w ratios (1,16) for each screened ratio (Figs. 2 through 7).

4.1 Shape of the type curves

General pattern and shape of the type curves for partially screened wells do not differ from those for fully screened wells, except that the drawdowns obtained in these cases are more than in the case of fully screened wells (refer SEETHAPATHI,1983). They increase with descrease in the screening ratio which is equal to the ratio of the depth of the screened portion of the well to the depth of aquifer. In these cases also (similar to the type curves of fully screened wells) three distinct regions could be identified on the type curves.

4.2 Deviation time

The product of the nondimensional time parameter u_{wd} on the well face corresponding to the deviation time, t_d , and the storage parameter, β remains practically constant and is independent of the screening ratio , S_r , with its value, being equal to 2.0 i.e., $u_{wd} \cdot \beta = 2$ for any screening ratio. This is also apparent from the fact that in this region, the aquifer storage contributes negligibly small to the discharge from the well. That means, neither the aquifer properties nor the screening ratio will play any role in this region. In region II, the type curves for different screening ratios remain distinct , while they merge into one straight line in Region I. Also, the curvature of the type curves increases with increase in the screening ratio. In this region, the well storage and the aquifer storage contribute to the discharge of the well. In the earlier portion of region II the discharge from the aquifer storage decreases with reduction in the screening ratio. Hence, to maintain the constant discharge from the well, more water will be drawn from well storage, for wells with smaller screening ratio. Thus, in the early time of region II, the drawdowns from the aquifer will be less in comparison to those for a fully screened well.
From $u_{wd} \cdot \beta = 2.0$, $T = 0.125\ r_c^2 / t_d$

4.3 Effect of Partial Screening on Time Domain

The merging time, t_m, now defined as the timeat which the type curves merged with the Hantush type curve for ' wells with no storage' is different for different screening ratios, but these values remain practically constant for different storage parameters, and it changed with change in r/r_w ratio. These values are presented in a tabular form (Table 2). From this table, it is seen that as r/r_w , increases, the merging time decreases as in the case of fully screened wells in confined aquifers. However, it is also noticed that for a particular r/r_w ratio the value of $u^*_m . \beta$ decreases with decrease in screening ratio. For example, for $r/r_w = 1$ and for the fully screened well, $u^*_m \beta$ is found to be equal to 1×10^{-3} and for the screening ratio of 0.25, the value of $u^*_m \beta$ for the corresponding value of r/r_w is 0.62×10^{-3} , the corresponding values of t_m being 16 hrs and 26 hrs respectively for $r_c = 1.525m$ and $T = 0.6194\ cm^2/sec$.

4.4 The effect of Partial screening on space Domain

It is observed from the type curves of different screening ratios, and for $r/m = 1.0$ (i.e., in this case for the ratio $r/r_w = 160$) that the effect ofthe partial screening is comparatively small. From the data available (graphs are not presented herein) that for $r/m = 2$ (i.e., in this case $r/r_w = 320$) the partial screening effects have completely vanished. The merging time has become almost same for all the screening ratios.

CONCLUSION

[1] The type curves for different storage parameters and for different screening ratio have been drawn between the modified non-dimensional time parameters and the non-dimensional drawdown at discrete radial distances from the discharging well.

[2] A simple method has been suggested to determine the transmissibility of the aquifer from early time record of the observation.

[3] The storage affected zone has been demarcated both in spatial and temporal coordinates which facilitates to earmark the usage of the curves presented herein.

In conclusion, it can be said that the type curves for partially screened wells are, in general, similar to the ones for fully screened wells. The merging time, t_m, increases with a decrease in screening ratio. The deviation time, however, remains constant. Lastly, the effect of the partial screening on the drawdowns is felt up to a radial distance equal to twice the thickness of the aquifer.

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Table - 1

Solved Cases for Partially Screened wells in Confined Aquifer for Darcy Flow.

Discharge from the well (Q) = 2.8 m^3/min

Radius of the pumping well r_w = 7.625 cm

Thickness of the aquifer(m) = 1220 cm

Transmissibility (T) = 0.6194 cm^2/sec

Case No.	r (cm)	Beeta	Screening ratio	Diffusivity (m2/sec)
PDC-1	15.25	1.0 E+02	0.25	0.1548
PDC-2	48.23	1.0 E+03	0.25	0.1548
PDC-3	76.25	1.0 E+04	0.25	0.6193
PDC-4	114.38	1.0 E+05	0.25	2.7526
PDC- 5	15.25	1.0 E+02	0.50	0.1548
PDC-6	48.23	1.0 E+03	0.50	0.1548
PDC-7	76.25	1.0 E+04	0.50	0.6193
PDC-8	114.38	1.0 E+05	0.50	2.7526
PDC-9	15.25	1.0 E+02	0.75	0.1548
PDC-10	48.23	1.0 E+03	0.75	0.1548
PDC-11	76.25	1.0 E+04	0.75	0.6193
PDC-12	114.38	1.0 E+05	0.75	2.7526

Table - 2

Nondimensional Deviation time (at the well) and Merging time for confined Aquifer-Darcy Flow

S.No.	Screening Ratio S_r	r/r_w	$u^*_m \beta^{-3} \times 10^{-3}$	c_m	$u_{wd}\,\beta$	c_d
1	1.00	1.0	1.00	250.00	2.0	0.125
		8.0	300.00			
		16.0	1200.00			
		64.0	10000.00			
		160.0	200000.00			
2	0.75	1.0	0.95	263.16	2.0	0.125
		8.0	280.00			
		16.0	1000.00			
		64.0	9600.00			
		164.0	19600.00			
3	0.50	1.0	0.80	312.50	2.0	0.125
		8.0	200.00			
		16.0	900.00			
		64.0	8400.00			
		160.0	18800.00			
4	0.25	1.0	0.62	403.23	2.0	0.125
		8.0	120.00			
		16.0	700.00			
		64.0	7000.00			
		160.0	17700.00			

u^*_m - modified nondimensional merging time parameter

β - storage parameter

t_m - merging time

u_{wd} - nondimensional deviation time parameter on the well face

t_d - deviation time

c_m - coefficient in the expression for t_m

c_d - coefficient in the expression for t_d

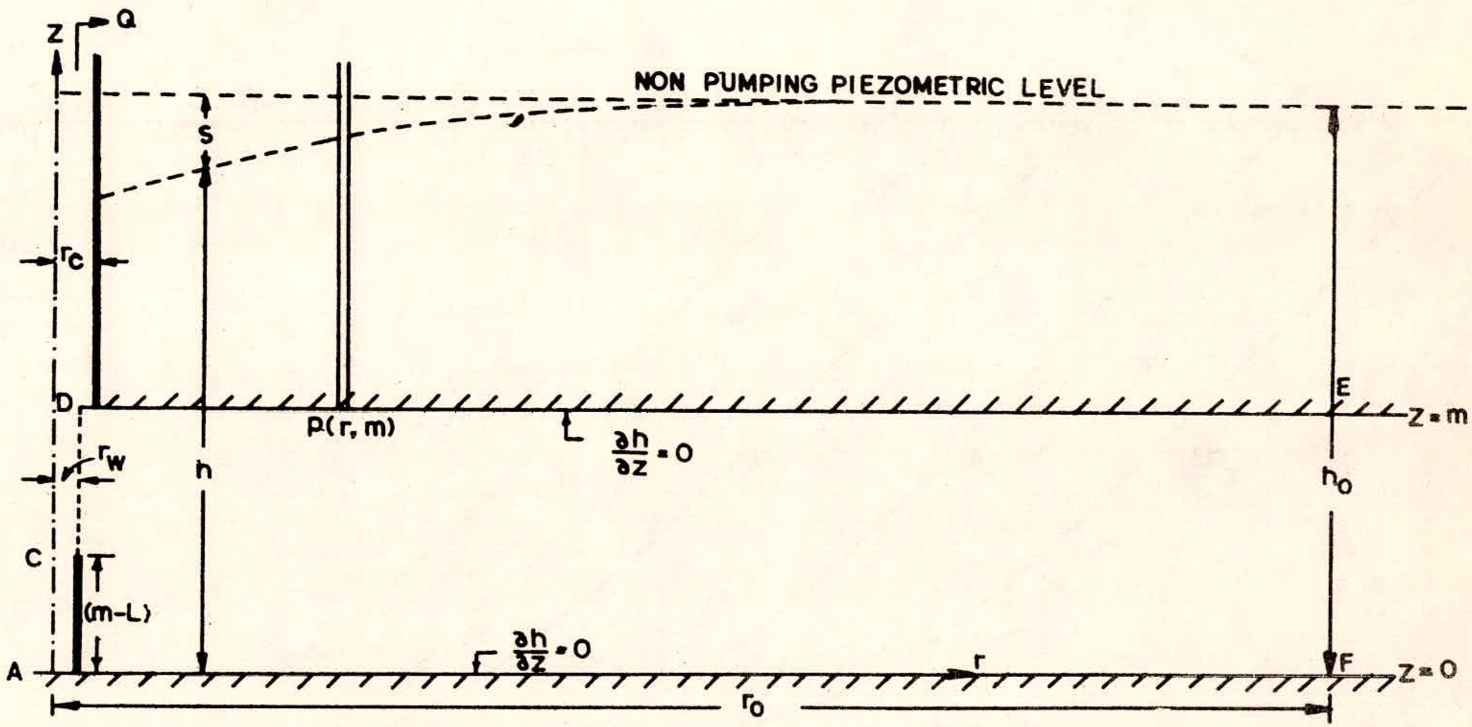

FIG. 1. PARTIALLY SCREENED WELL WITH STORAGE

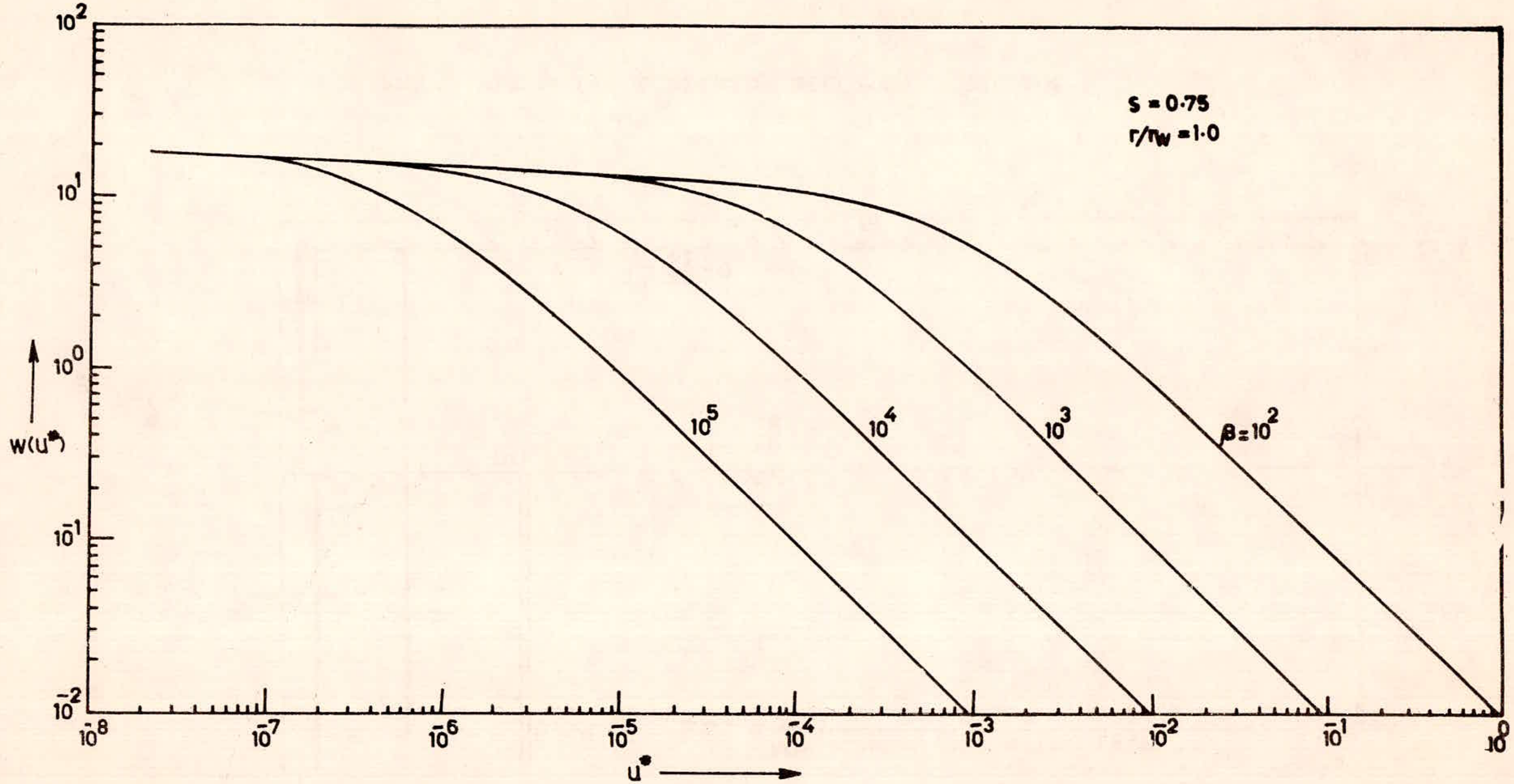

FIG. 2. TYPE CURVES FOR A PARTIALLY SCREENED WELL WITH STORAGE IN A CONFINED AQUIFER - DARCY FLOW.

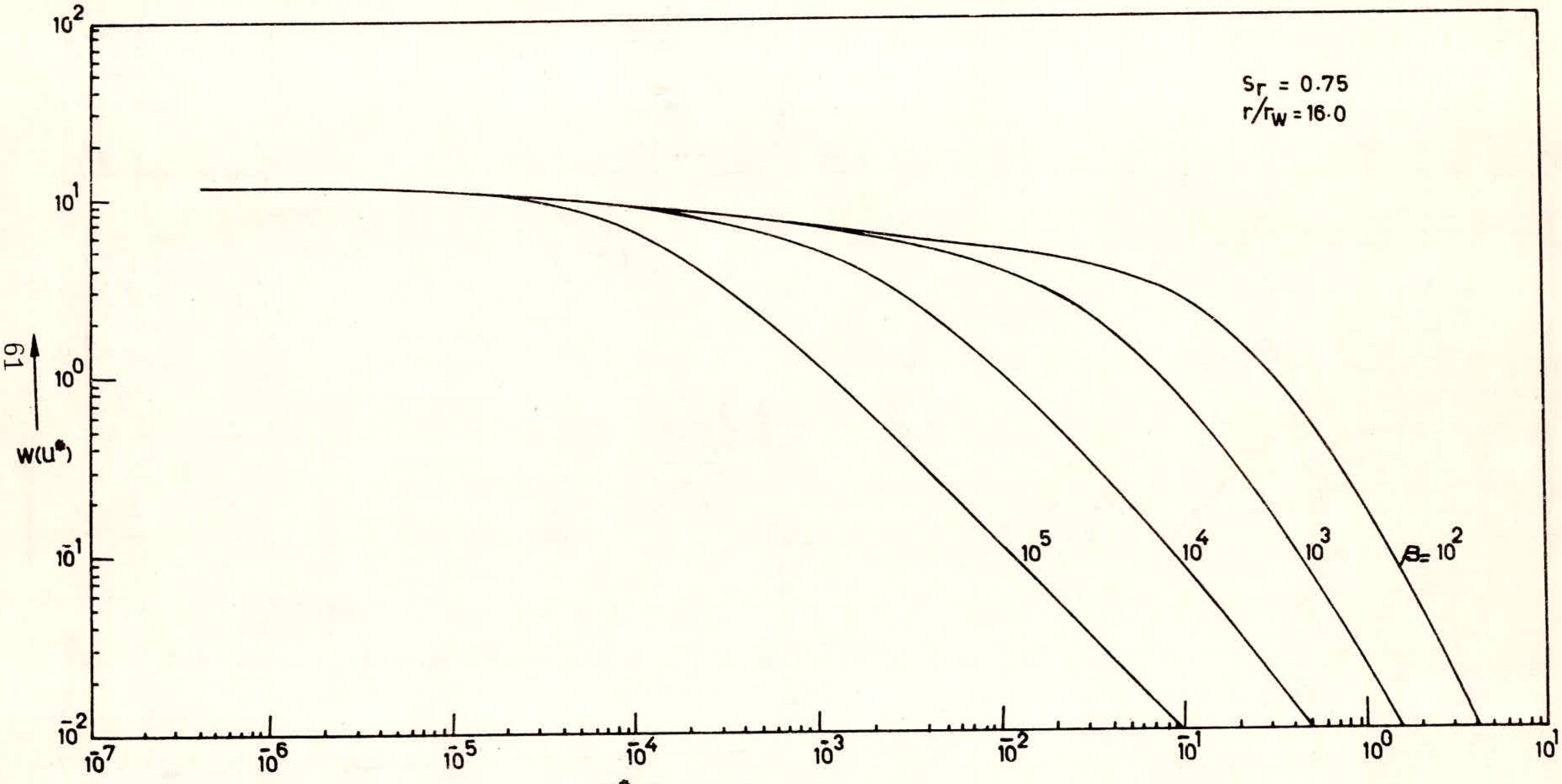

FIG 3 TYPE CURVES FOR A PARTIALLY SCREENED WELL WITH STORAGE IN A CONFINED AQUIFER - DARCY FLOW

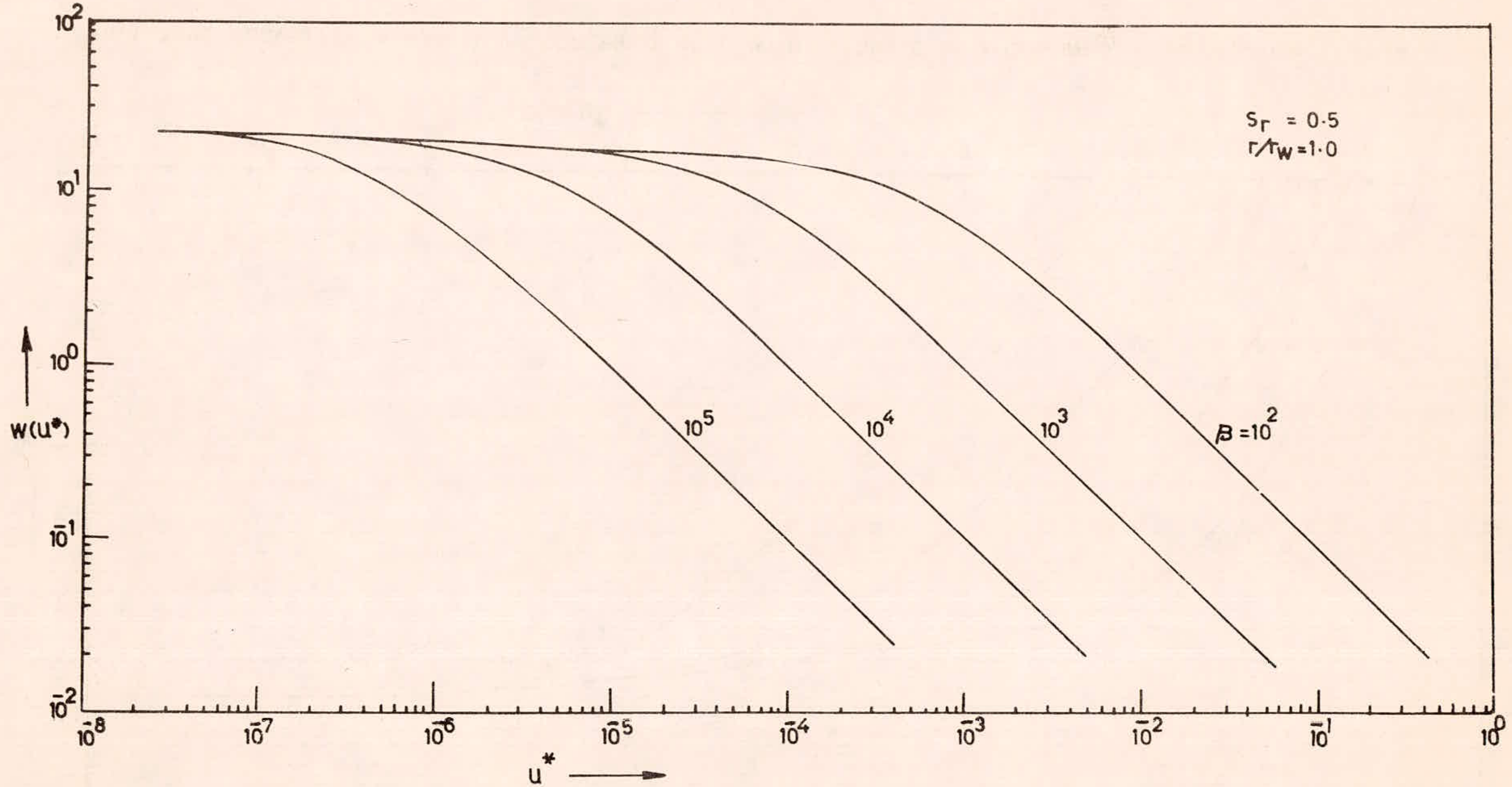

FIG. 4. TYPE CURVES FOR A PARTIALLY SCREENED WELL WITH STORAGE IN A CONFINED AQUIFER - DARCY FLOW

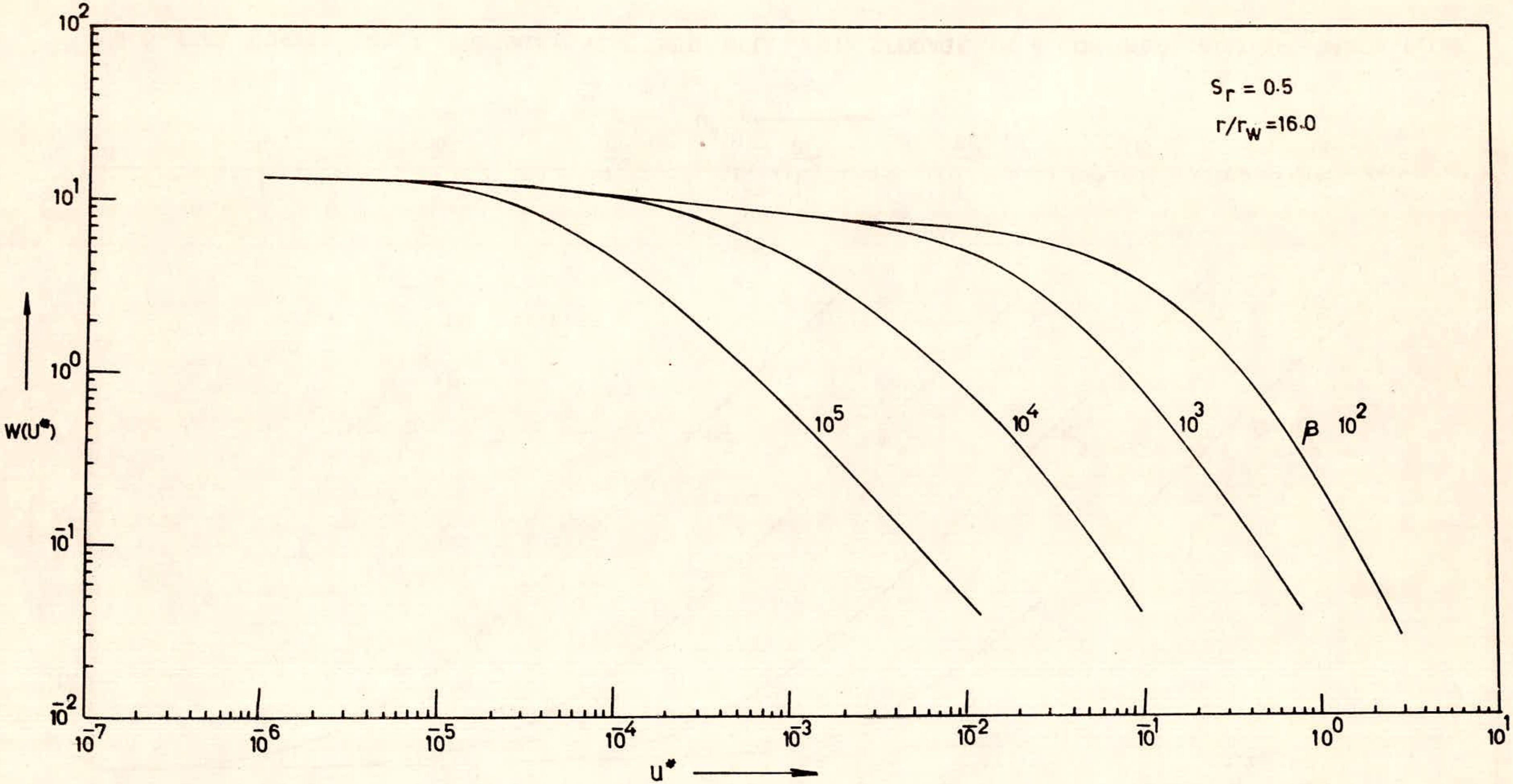

FIG. 5. TYPE CURVES FOR A PARTIALLY SCREENED WELL WITH STORAGE IN A CONFINED AQUIFER – DARCY FLOW

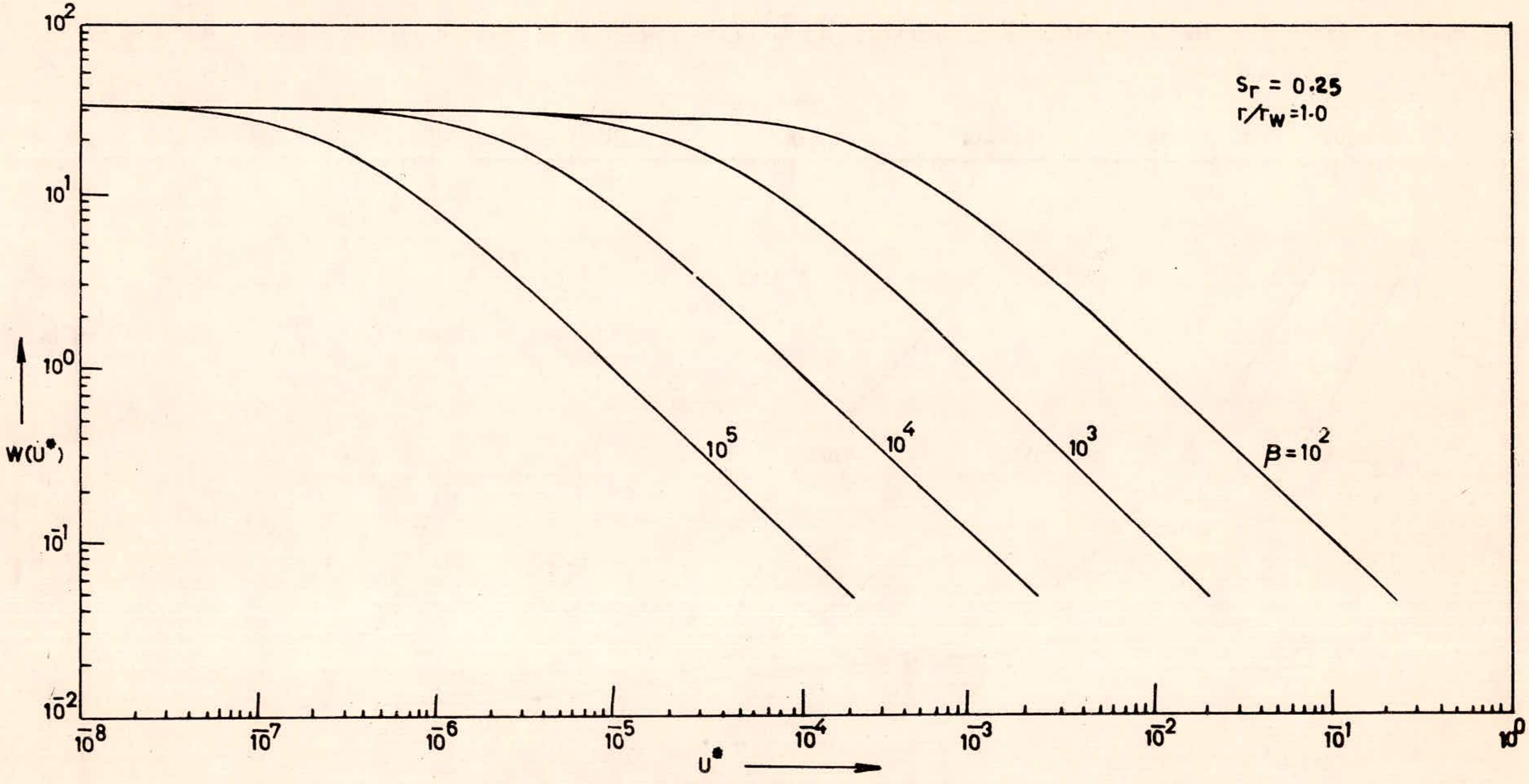

FIG. 6. TYPE CURVES FOR A PARTIALLY SCREENED WELL WITH STORAGE IN A CONFINED AQUIFER - DARCY FLOW.

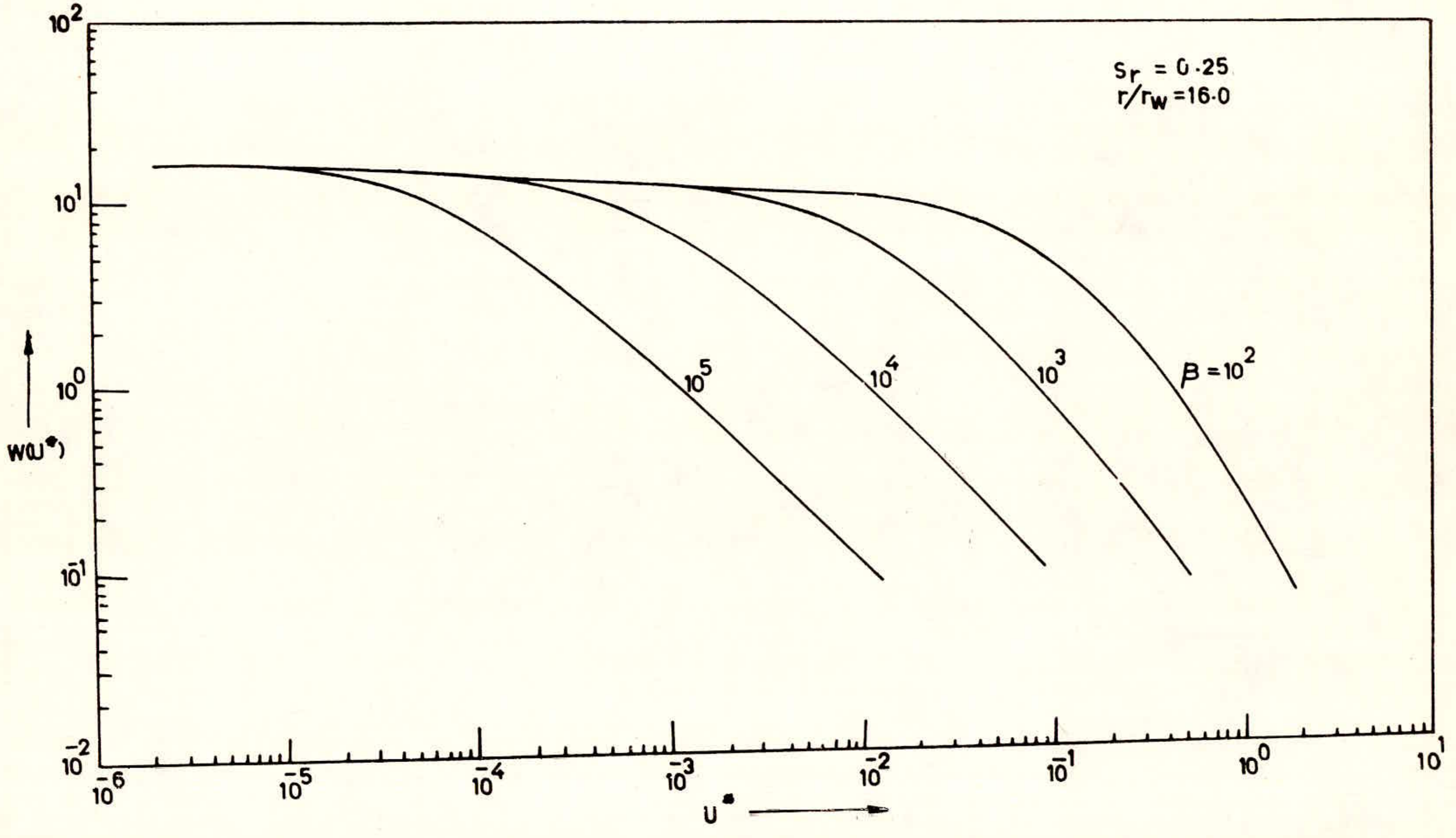

FIG. 7. TYPE CURVES FOR A PARTIALLY SCREENED WELL WITH STORAGE IN A CONFINED AQUIFER - DARCY FLOW.