RECHARGE FROM LARGE WATER BODIES

BY
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1.0 INTRODUCTION

1.1 General

Lakes are a highly visible features on earth surface in many parts of the world, because they are pleasing aesthetically and important economically for water supply etc., and are foci for urban development. The other benefits that might occur from a lakes/reservoirs are flood cushion, pisiculture, etc. Hence owing to its importance in various hydrologic and economic fields, it is necessary to maintain their quality and existance in its pure form. Understanding of the water balance is imperative the lake management. Hydrologic studies on lakes include lake manipulation, general hydrologic description, lake lavel fluctuation analysis and effect of floods and evaporation on lakes and the interaction with the aquifer system underneath i-

Inspite of early recognition of lake management studies, very few attempts have made to the understanding of the interaction of lake and ground-water reservoir. Even with the well established science of limnology, groundwater system around la have not received the attention it deserves. Since the ground water is often a very significant component of water budget of lake, the quantification of groundwater flow system around a lake is generally necessary for the accurate preparation of lakes hydrologic, chemical or energy budget. In most of the studies of lake water balance, groundwater is often calculated as residual of the water balance equation, which lead to inaccurate water balance due to the reasons that overland run-off and non channelised surface flow in the lake, is often lumped into the residual with groundwater and that the errors in the measurements of different components are not considered. The process of estimating lake seepage rate, based on the observed flow system, makes use of the simple application of Darcy's law in one or two dimensions, thus, over simplyfying most real lakes system. Spacial variability of groundwater flow is a function of nearly boundary, variable lake depth thickness of lake sediment, hydraulic conductivity, aquifer homogenity and isotropy and hence, it is obvious that manual flow net calculations based on Darcy's law will be cembersom and inaccurate. Therefore, proper theoretical and field studie of lake and groundwater interaction are needed, former will define the principles underlying this interaction and later will help in varifying the theoretical models and can be used to develop practical field measurement techniques.

With the advent of very high speed computers, various numerical techniques have been evolved for solving non linear problems with complex boundary conditions. The three dimensional analysis of complex flow is now possible with the help of mathematical modelling. Thus, the mathematical modelling has been proved to be the only tool for assessing the groundwater flow pattern near the lake, as they are useful for ident fying most important parameters affecting lake-seepage rates, thereby lending direction to field studies of groundwater system around the lake.

1.2 Scope of the Present Study

The present lecture deals with the understanding of the behaviour of the groundwater system in conjunction with lake (as a recharge source) on one side and a river (as a discharging area) on the other side.

The cross sectional view of the problem of lake-aquifer-river interaction is shown in Fig.1. At one side of the lake constant head boundary (river) has been assumed and on the lateral side no flow boundary has been considered (vide fig.2). The lecture deals with the developing type curves between the non-dimensional time and non-dimensional drawdown from the lake. For the present analysis, the aquifer has been considered to be homogeneous with Kh/Kv = 500 (Kh and Kv are the hydraulic conductivity in horizontal and vertical direction respectively) and the lake was simulated to be of square cross-section in the plan having uniform depth. A three dimensional finite differenc model has been used to simulate the lake aquifer river interaction.

2.0 DESCRIPTION OF THE HYPOTHETICAL SETTING

For the analysis of the lake-aquifer-river interaction, a lake of square cross section, 300 m x 300 m has been considered. It was also assumed that the lake has constant depth of 9 m over its section. The aquifer was assumed to be homogenous with a constant head boundary (i.e. river) and a no flow boundary at either side of the lake, at a distances of 2.5 km. from the lake side. Detail description of the problem is shown in Fig.3. For the present problem Kh/Kv was assumed to be 500.

The present problem is to study the lake-aquifer-river interaction for a hypothetical setting (described earlier). It was also intended to develop a type curve between non dimensional time and non dimensional rise of water table level, using which the rate of recharge from the lake can be determined at any instant of time, provided a record of water level fluctuations in an observation well, situated anywhere within the influence area of lake, is known and vice-versa. The proper location of the observation well has also been suggested.

3.0 MODEL DESCRIPTION

3.1 Mathematical Model

The three dimensional movement of incompressible ground-water through heterogenous and anisotropic medium may be described by the partial difference equation

$$\frac{\partial}{\partial x}(K_{xx}\frac{\partial h}{\partial x})+\frac{\partial}{\partial y}(K_{yy}\frac{\partial h}{\partial y})+\frac{\partial}{\partial z}(K_{zz}\frac{\partial h}{\partial z} - W = Ss\frac{\partial h}{\partial t} \quad \ldots(1)$$

where,

x, y and z are the cartesian co-ordinates alined along the major axes of hydraulic conductivity K_{xx}, K_{yy} and K_z

h is the potentiometric head (L),

W is the volumetric flux per unit volume and represents sources and/or sinks of water (L^{-1}),

Ss is the specific storage of the porous material (L^{-1}),

t is the time (t)

The above equation when combined with specification of flow and boundary conditions of an aquifer system along with initial conditions forms a mathematical model of three dimensional groundwater flow.

Analytical solution of Eq.1 is not possible for very complex systems, hence, numerical methods are employed to obtain approximate solutions. Finite difference approach is one of such numerical methods, in which, the continuous derivatives are replaced by differences between functional values at discrete points. Thus, this process will lead to a set of simultaneous algebraic differential equations, the solution of which yield the values of head at specific points and time. These values will be an approximation to the time varying head distribution that would be given by an analytical solution of the partial differential equation of flow.

3.2 Formulation of Finite Difference Equations

The basis of the finite difference equation is the use of continuity equation of flow, which implies that the sum of all flows into and out of a cell is equal to the change in the storage within the cell, therefore, the continuity equation for a cell is given by eq.2, provided density of groundwater is contant.

$$Q_i = Ss \frac{\Delta h}{\Delta t} \Delta V \qquad \ldots(2)$$

where,

Q_i = is a flow rate into the cell (L^3T^{-1}),

Ss = is the specific storage defined as the ratio of the volume of water which can be injected per unit volume of aquifer material per unit change of head (L^{-1})

Δh = is the change of head over a time interval of length t,

ΔV is the volume of the cell (L^3)

The flow rate into a cell (i,j,k) will be the algebraic sum of flow rate from all the six adjoining cells and flow from each adjoining cell can be represented by Darcy's law. The flow from outside the aquifer may be dependent on the head in the receiving cell but independent of all other heads in the aquifer or they may be entirely independent of the head in the receiving cell, thus, the flow from outside the aquifer may be represented by the equation 3.

$$QS_{i,j,k} = \sum_{n=1}^{N} a_{i,j,k,n}$$

$$= \sum_{n=1}^{N} p_{i,j,k,n}\, h_{i,j,k} + \sum_{n=1}^{N} q_{i,j,k,n}$$

$$= p_{i,j,k}\, h_{i,j,k} + Q_{i,j,k} \qquad \ldots(3)$$

where,

(i,j,k) is array convention describing row, column and layer respectively,

$Q_{i,j,k,n}$ represents the flow from nth external source into cell (i,j,k) (L^3t^{-1}), and

$P_{i,j,k,n}$ and $Q_{i,j,k,n}$ are constant (L^2T^{-1} and L^3T^{-1} respectively)

and $P_{i,j,k} = \sum_{n=1}^{N} P_{i,j,k,n}$, and

$Q_{i,j,k} = \sum_{n=1}^{N} q_{i,k,k,n}$,

N is the total number of external sources or stresses affecting a single cell.

Substituting in eq.2, the sum of the flows from adjoining cells (obtained by the use of Darcy's law) and the flow from all external sources into the cell i,j,k one gets the finite difference equation (Eq. 4) for a particular cell after rearranging it.

$$CV_{i,j,k-1/2}\, h^m_{i,j,k-1} + CC_{i-1/2,j,k}\, h^m_{i,j,k}$$

$$+ CR_{i,j-1/2,k}\, h^m_{i,j-1/2,k} + (-CV_{i,j,k-1/2} - CC_{i-1/2,j,k}$$

$$- CR_{i,j-1/2,k} - CR_{i,j+1/2,k} - CC_{i+1/2,j,k} - CV_{i,j,k+1/2}$$

$$+ HCOF_{i,j,k})\, h^m_{i,j,k} + CR_{i,j+1/2,k}\, h^m_{i,j+1,k}$$

$$+ CC_{i+1/2,j,k}\, h^m_{i+1,j,k} + CV_{i,j,k+1/2}\, h^m_{i,j,k+1} = RHS_{i,j,k} \quad \ldots(4)$$

where,

$$HCOF_{i,j,k} = P_{i,j,k} - SCI_{i,j,k}/(t_m - t_{m-1})$$

$$RHS_{i,j,k} = -Q_{i,j,k} - SC1_{i,j,k}\, h^{m-1}_{i,j,k}/(t_m - t_{m-1}), \text{ and}$$

$$SC1_{i,j,k} = SS_{i,j,k}(\Delta r_j \Delta c_i \Delta v_k),$$

r, c and v are the spacing in row, column and vertical direction

CV, CR and CC are the conductance in vertical, row and column direction respectively between the two cells. Conductance is the product of hydraulic conductivity and cross sectional area of flow divided by the length of flow path (i.e. the distance between the nodes). Suffix j-1/2 represents the average value of the parameters between j and j-1.

Equation 4 can be written in backward difference form by specifying flow terms at time t_m, the end of the time interval, and approximating the time derivatives of head over the interval t_{m-1} to t_m, i.e.

$$CR_{i,j-1/2,k}(h^m_{i,j-1,k} - h^m_{i,j,k}) +$$

$$CR_{i,j+1/2,k}(h^m_{i,j+1,k} - h^m_{i,j,k}) +$$

$$CC_{i-1/2,j,k}(h^m_{i-1,j,k} - h^m_{i,j,k}) +$$

$$CC_{i+1/2,j,k}(h^m_{i+1,j,k} - h^m_{i,j,k}) + CV_{i,j,k-1/2}(h^m_{i,j,k-1} - h^m_{i,j,k})$$

$$CV_{i,j,k+1/2}(h^m_{i,j,k+1}-h^m_{i,j,k})+$$

$$P_{i,j,k}\, h^m_{i,j,k}+Q_{i,j,k} = SS_{i,j,k}(\Delta r_j \Delta c_i \Delta v_k)$$

$$x\ (h^m_{i,j,k}-h^{m-1}_{i,j,k})/(t_m-t_{m-1}) \qquad \ldots(5)$$

An equation of the above type may be written for each of the cells in the system. Thus, one have n equations in n unknowns for a system of n cells, and such a set of equations can be solved simultaneously to get the head value of each cell (specifically at nodes)of a system at different time. For the solutic of these equations strongly implicit procedure has been adopted. An equation of the form of eq.5 is required for each variable head cell. 'Constant head cells' are those in which head remain constant with time and, hence, do not require an equation. The equations for adjacent variable head cells, however, will contain non zero conductance term representing flow from constant head cell. 'No flow cells' are those to which there are no flow from adjacent cells and there is no equation for a no flow cell as well as for adjacent cells.

3.3 Discretization of Space

For the formulation of finite difference equations, the spacial discretization of the system, shown in Fig.3, is given in Fig.4 in which the discretization for only an quadrant bloc} is shown. The whole system is discretized into 32 rows and

32 columns and single layer, forming 32 x 32 x 1 cells/or nodes The spacing of rows and columns was kept symmetrical about the axes passing through the centre of the lake. Due to greater variation of water table near the lake, the logarithmic increa in the spacings of rows/columns away from the lake was assumed. Table 1, shows the spacing of rows/columns, away from the cent of the lake, Fig.4 shows the plan of discretized space.

Table 1

Spacing of Row/Column Away from the Lake

Spacing (m)	30	30	30	30	30	40	50	60	80	100	140	240	360	480	600	700

3.4 Discretization of Time

The total period of simulation was taken to be 364 days consisting of 52 time step of uniform span of seven days each. The volumetric budget was obtained at each time step and the distribution of head within the aquifer was obtained at every fourth time step.

4.0 ANALYSIS

Simulation of the transient seepage through lake into the aquifer and subsequent drainage into an adjoining river, for the hypothetical problem described earlier, was carried out. The head distribution in the aquifer system at discrete nodes over a period of time, also discretized into different time steps, was obtained (discretization of space and time have already been discussed in the previous chapters). The simulation was repeated for different values of head difference between the water level in the lake and river stage ($\triangle H$) i.e., 9m, 7m, 5m, and 3m. For convenience, the river stage and the water table level in the aquifer was kept at a constant level at the start of the simulation while the water level in the lake was kept varying for different simulations. The hydraulic conductivity of the aquifer was considered as 10.7 m/day, while the specific yield(s) was taken as 15%. The ratio Kh/Kv was kept equal to 500 and the bottom and sides of the lake was considered to have low permeability values due to sediment disposition and were taken as 0.038 times hydraulic conductivity of the aquifer. The difference between the water table level at the discrete points (nodes), due to seepage occuring from the lake and the river stage was computed and designated as $\triangle h$. Plots were drawn between X/L and $\triangle h/\triangle H$ for discrete time interval for first layer for each $\triangle H$. Fig.5 shows such variation for $\triangle H$=3m. Where X is the perpendicular distance from the lake to the observation well location and L is the perpendicular distance between lake and river. Values of L for the present discretization is 2500 m. Table 2 shows the difference values of X/L for present discretization.

Table 2

Values of X/L for present discretization

X	20	65	120	190	280	400	590	890	1310	1850
X/L	0.008	0.026	0.048	0.076	0.112	0.160	0.236	0.356	0.524	0.740

From the plots, it can be seen that $\Delta h/\Delta H$ increases with increase in time for a particular X/L, though the rate of increase, decreases with increase in time, suggesting that a steady state situation can be reached.

Variation of $((\Delta h)_s - (\Delta h))/ H$ with X/L and t was observed by plotting the graph between $((\Delta h)_s - (\Delta h))/ H$ vs X/L for all value of t for each simulation (i.e., ΔH = 9.0, 7.0, 5.0 and 3.0m). Figs.6 and 7 show such variation for ΔH = 5.0m and 3.0 m respectively which indicate that with increasing time, the maxima of the curve shifts in a positive X-direction and it is observed that the shifting of the maxima for all time is ranged between X/L = 0.15 to 0.25 (this range of X/L was found to be the same for different value of ΔH). Hence, it can be said that within the above range, if a observation well is located, it will observe a comparatively rapid change of head and thus it will be sensitive. Therefore, it can be concluded tha the location of the observation well within X/L of 0.15 to 0.2 will be ideal.

The further analysis of results have shown that the parameters $X^2S/T.t$ and $T.\Delta h/Q_R$ are uniquely related and the relation has found to be the same for different values ΔH, where T is the transmissivity of aquifer and t is the time fro the commencement of simulation, and Q_R is the seepage rate fro lake. Fig.8 shows the relation between the above parameters which includes the data from all simulation (i.e. ΔH=9.0m, 7.0, 5.0m, 3.0m) and the type curve has been drawn. The variation of the above parameters has been given in tabular form in the Appendix.

The type curve shown in Fig.8 can be used to estimate the recharge Q_R from the lake, knowing Δh at a certain value of X/L and t(while S and T are kept constant for the simulation).

This type curve (Fig.8) is applicable only for Kh/Kv = 500 and 1/L = 0.12, where 1 is the length of the side of the lake. In order to establish the applicability of the type curve for other value of Kh, Kv and 1/L, further study needs to be carried out.

5.0 CONCLUSIONS

Three dimensional model study of the seepage from the lake of square cross section and uniform depth in a homogeneou aquifer (Kh/Kv = 500) with constant head boundary on lateral sides and no flow boundary on the other sides of the lake, ea at a distance of 2500m from the side of the lake, has been carried out and the results have been analysed. The conclusi drawn from the study are as given below:

1. The parameter $X^2S/T.t$ is found to be uniquely relate with the parameter $T.\Delta h/Q_R$ irrespective of the value of ΔH, while Kh/Kv and 1/L are kept constant. A typ curve has been developed between the parameter $X^2S/T.$ and $T.\Delta h/Q_R$ for the specific values of Kh/Kv = 500 and 1/L = 0.12.

2. The proper location of the observation well has also been suggested, a method for estimation of recharge Q from the lake, using the water level fluctuation in t observation well, has also been presented.

REFERENCES

Anderson, M.P. and J.A. Munter (1981), 'Seasonal reversals of ground water flow around lakes and the relevance to stagnation points and lake budgets', Water Resources Research. V. 17, no.4, pp. 1139-1150.

Bues, D.J. (1983), 'The hydrologic relationship between Lake, Michigan and a shallow dolomite aquifer at Mequon, Wisconsin', Unpubl. M.S. Thesis, Univ. of Wisconsin Milwaukee, Milwaukee, Wisc., 211 pp.

Javquent,N.G. (1976), 'Groundwater and surface water relationships in the glacial province of northern wisconsin snake lake', Ground water, 14(4), 194-199.

Karnauskas, R.J., and M.P. Anderson (1978), 'Groundwater lake relationships and groundwater quality in the sand plain province of Wisconsin-Nepco lake', Groundwater. V.16, no.4, pp.273-281.

Marnauskas, R.J. and Anderson, M.P. (1978), 'Groundwater lake relationships and groundwater quality in the sand plain province of Wisconsin Nepco Lake', Groundwater, 16(4), 273-281.

Munter, J.A. (1979), 'Groundwater modelling of three lake/aquifer systems in Wisconsin', Master's Thesis, Univ. of Wisconsin Madison. 100 pp.

Munter, J.A. and Anderson, M.P. (1981), 'The use of groundwater flow models for estimating lake seepage rates', Groundwater 19(6), 608-616.

Rinaldo-Lee, M.B., M.P.Anderson, D.A.Stephenson and S.F.Huffman (1979), 'Hydrogeology and computer model of the bass lake area, St.Croix County, Wisconsin', Univ. of Wisc. Water Resources Center, Tech. Rept.WISWRC 79-01. 48 pp.

Winter, T.C. (1981), 'Uncertainties in estimating the water balance of lakes', Water Resources Bulletin, Vol. 17, No.1, 82-115.

Winter, T.C. (1980), 'Bibliography of U.S. GEOLOGICAL Survey studies of lakes and reservoirs, the first 100 years', U.S. Geological Survey Open-File Report

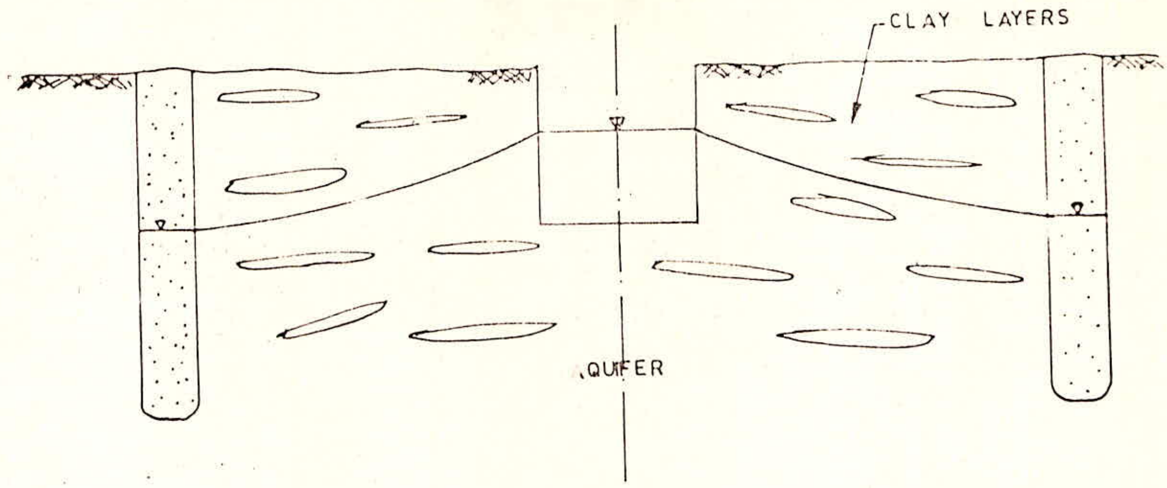


FIG. 1 – CROSS SECTIONAL VIEW OF THE PROBLEM OF LAKE RIVER – AQUIFER INTERACTION.

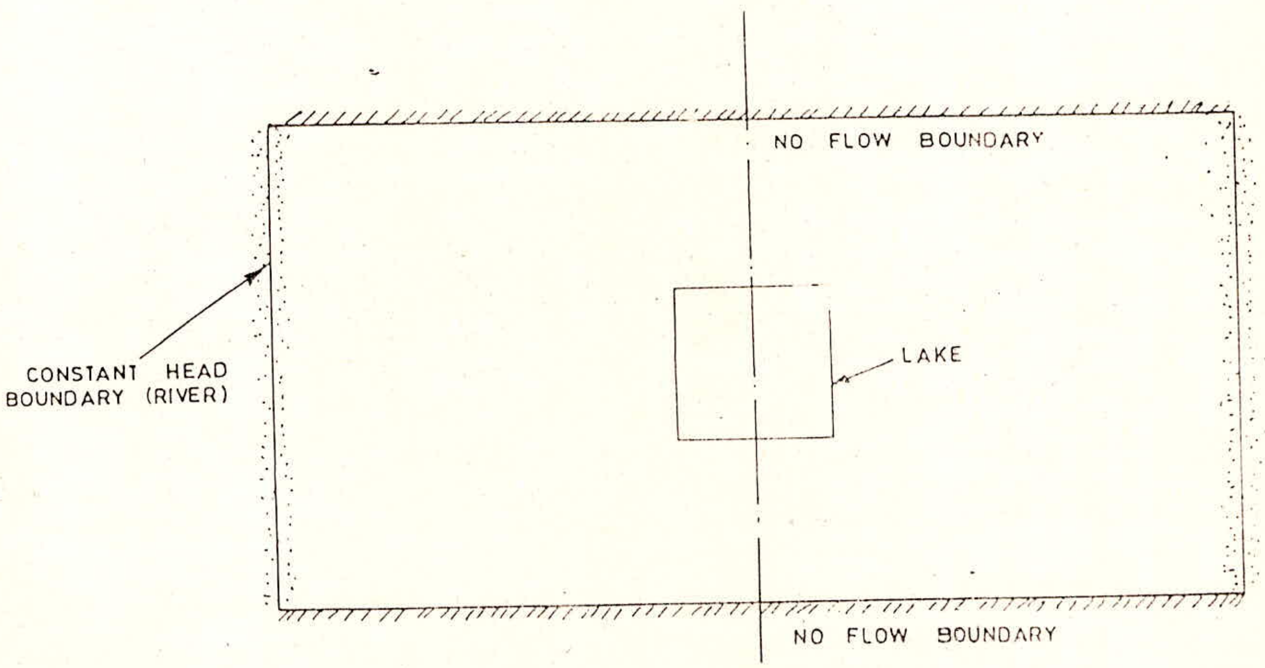


FIG. 2 - PLAN VIEW OF THE PROBLEM OF LAKE - RIVER - AQUIFER INTERACTION.

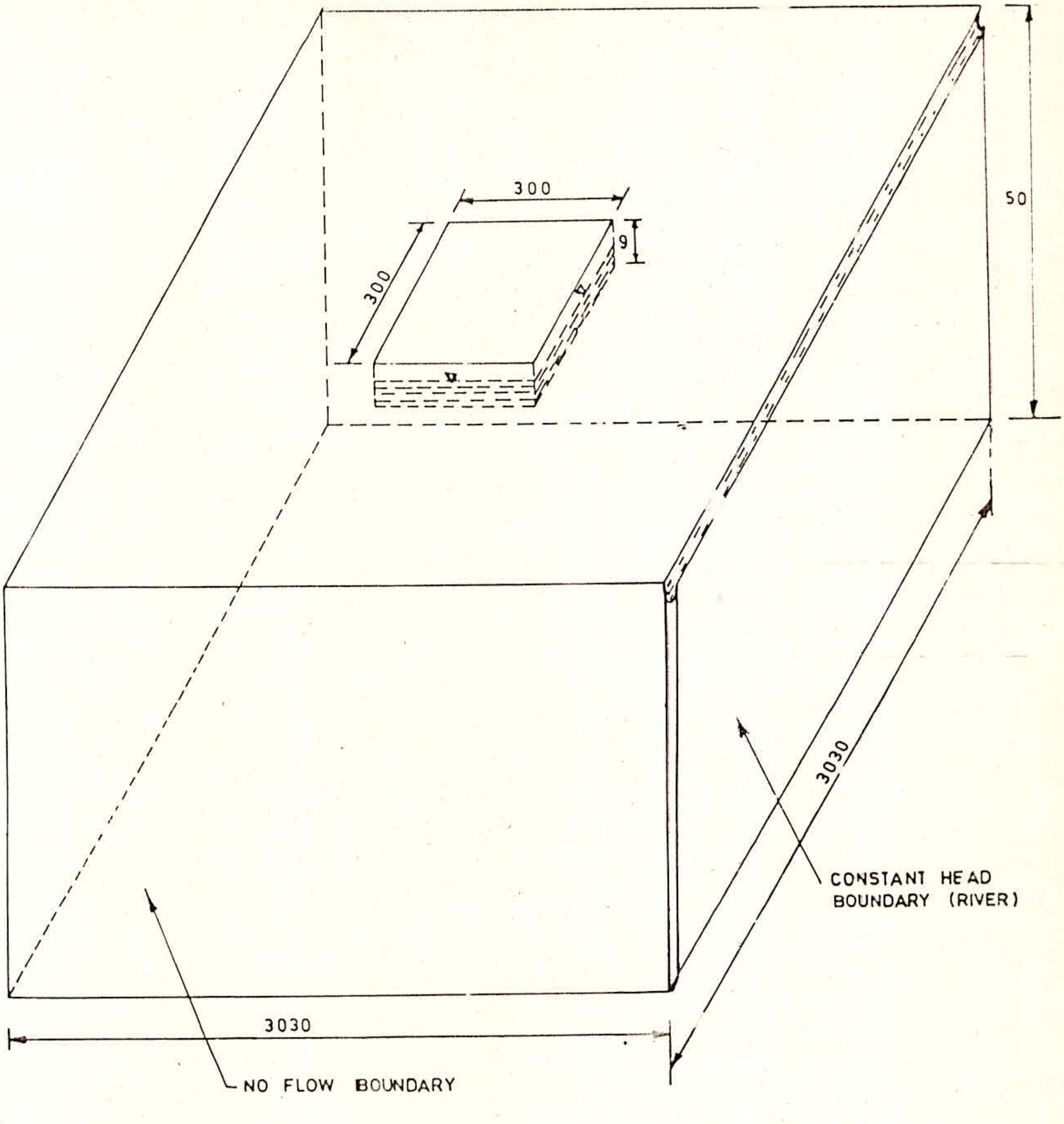
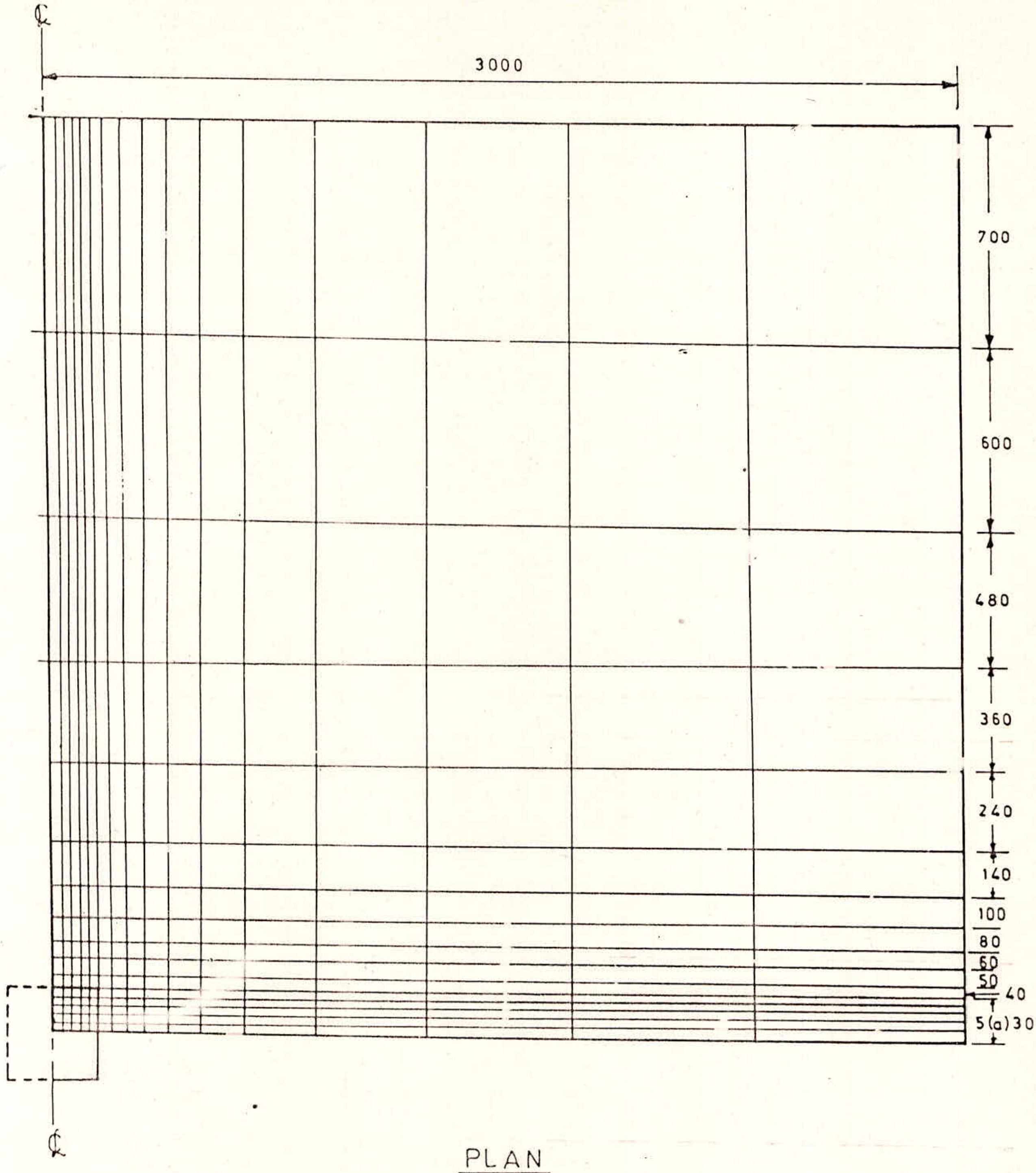


FIG. 3 - DESCRIPTION OF THE PROBLEM.



PLAN

SCALE:- 1 CM = 200 M

FIG. 4 – SPACE DISCRETIZATION PLAN

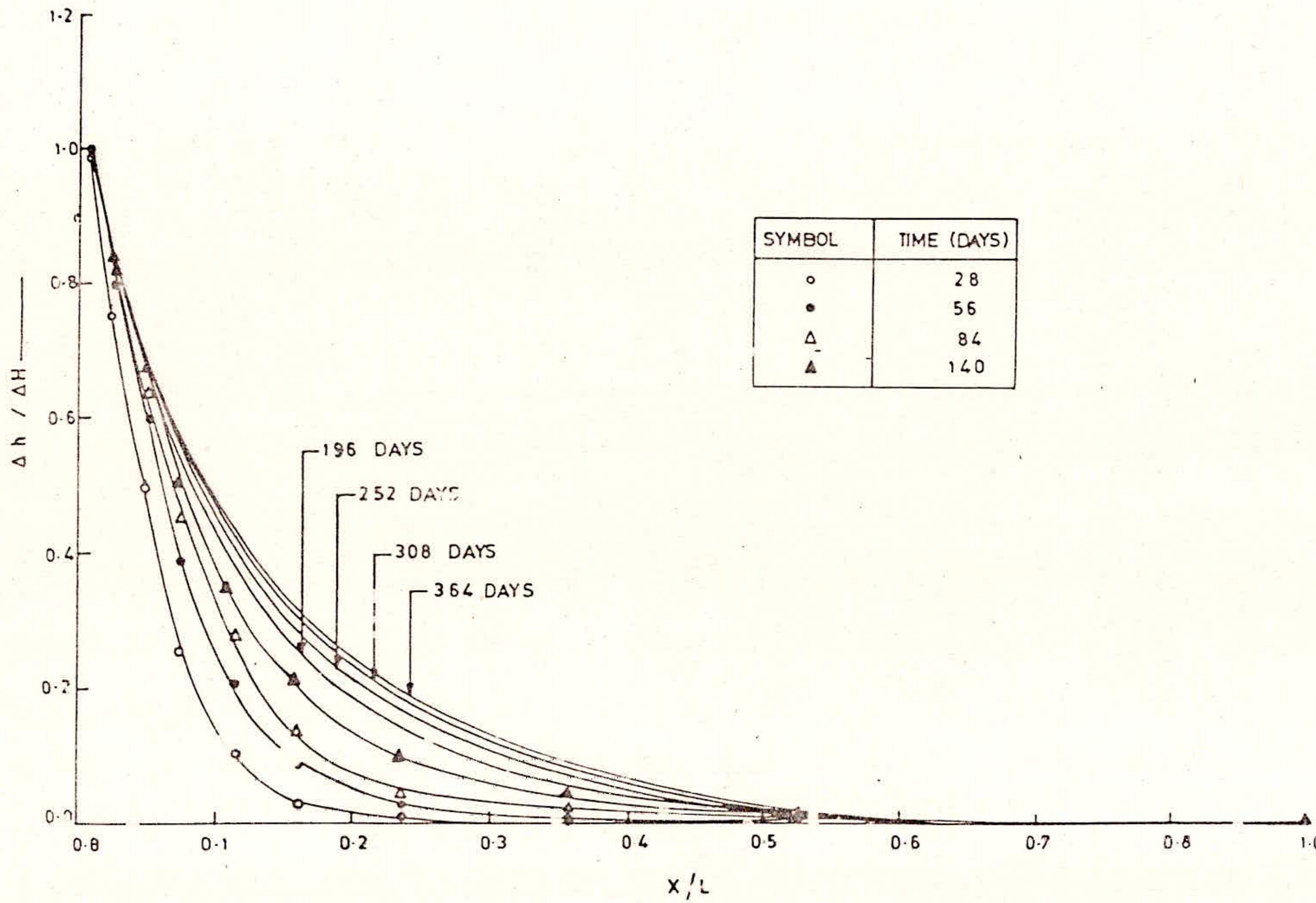


FIG. 5 - VARIATION OF $\Delta h/\Delta H$ WITH X/L AND t ($\Delta H = 3{\cdot}0$ m)

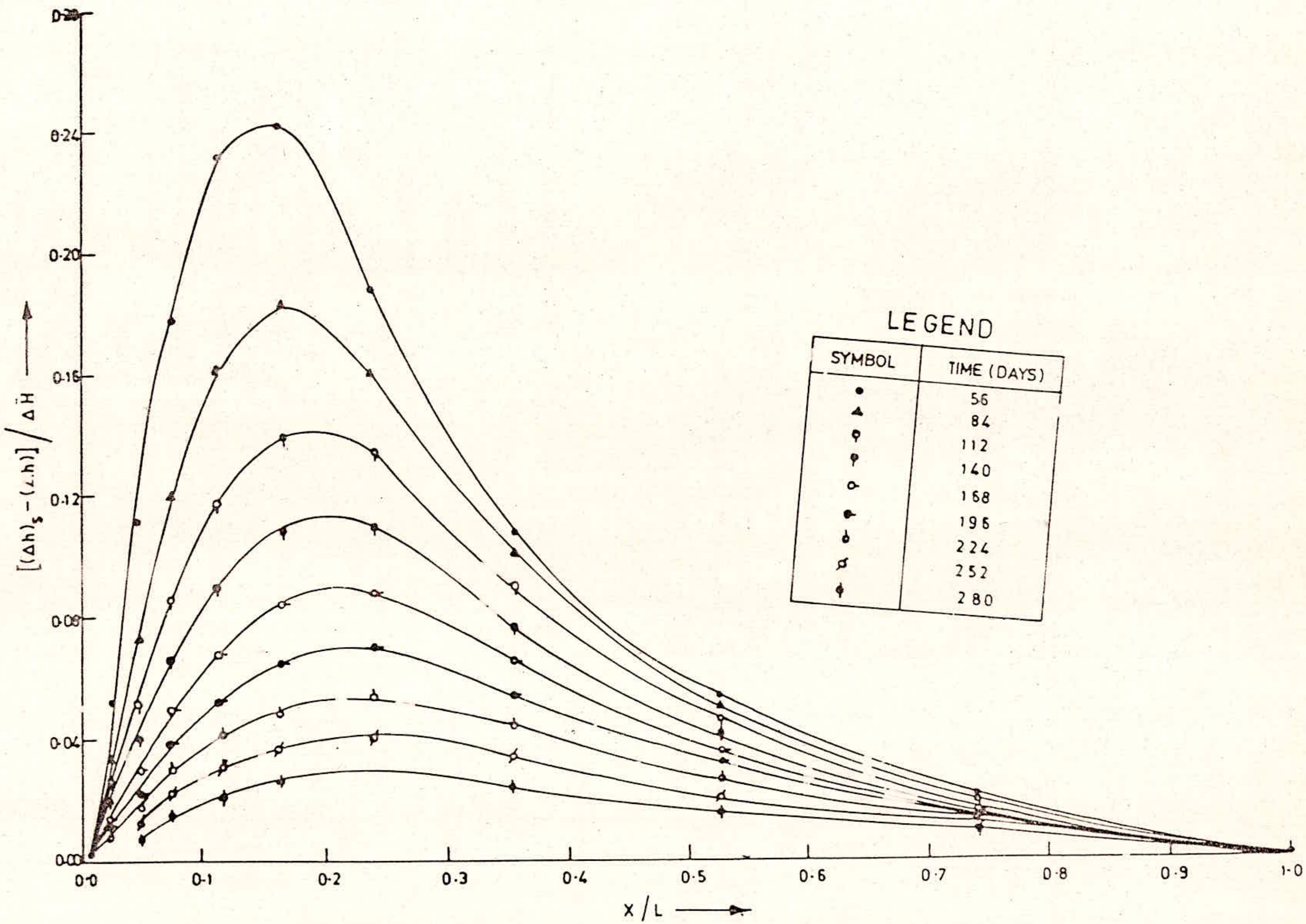


FIG. 6–VARIATION OF $[(\Delta h)_s-(\Delta h)]/\Delta H$ WITH X/L AND t (ΔH= 5·0 m)

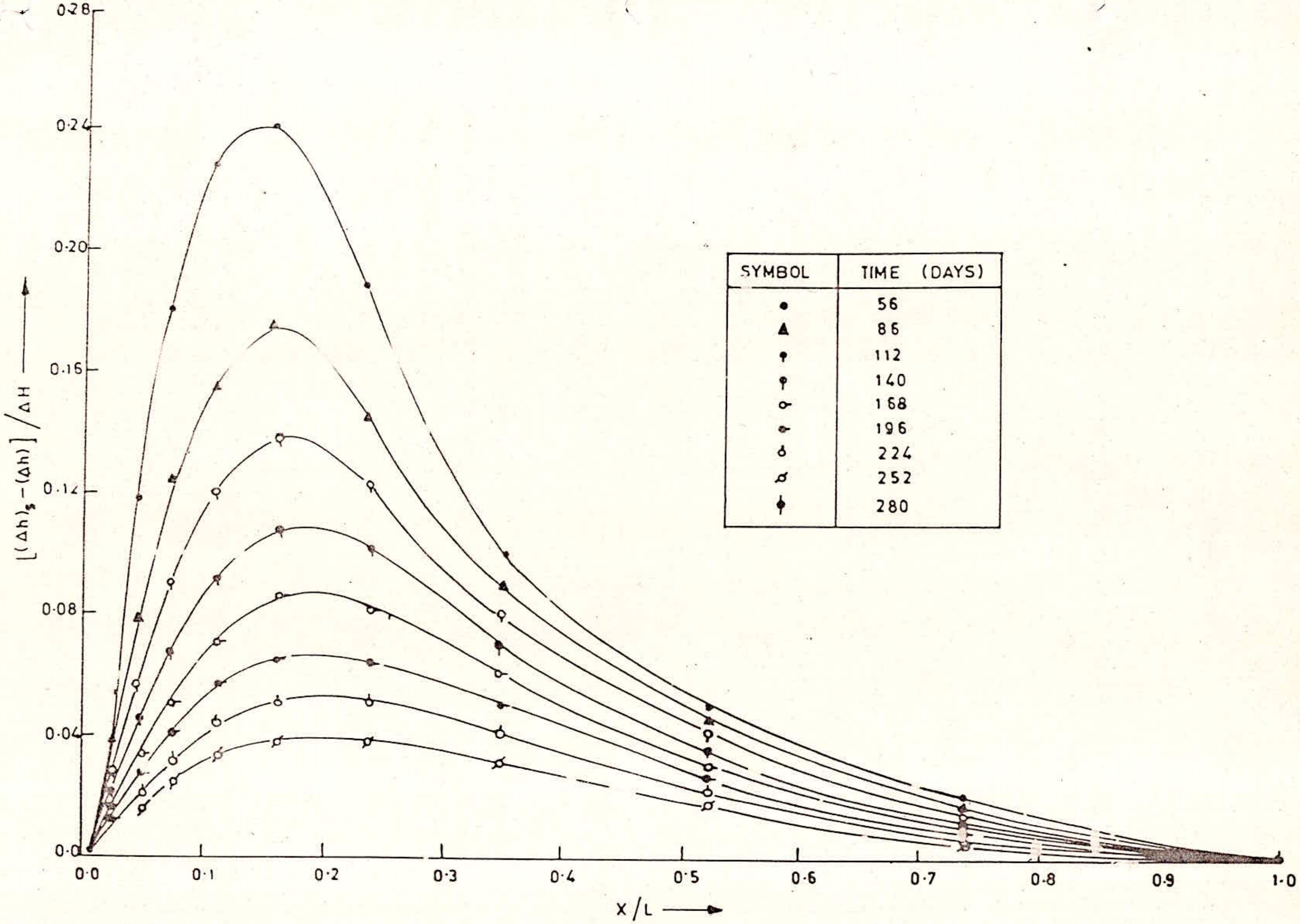


FIG. 7 - VARIATION OF $[(\Delta h)_s - (\Delta h)]/\Delta H$ WITH X/L AND t (ΔH = 3.0 m)

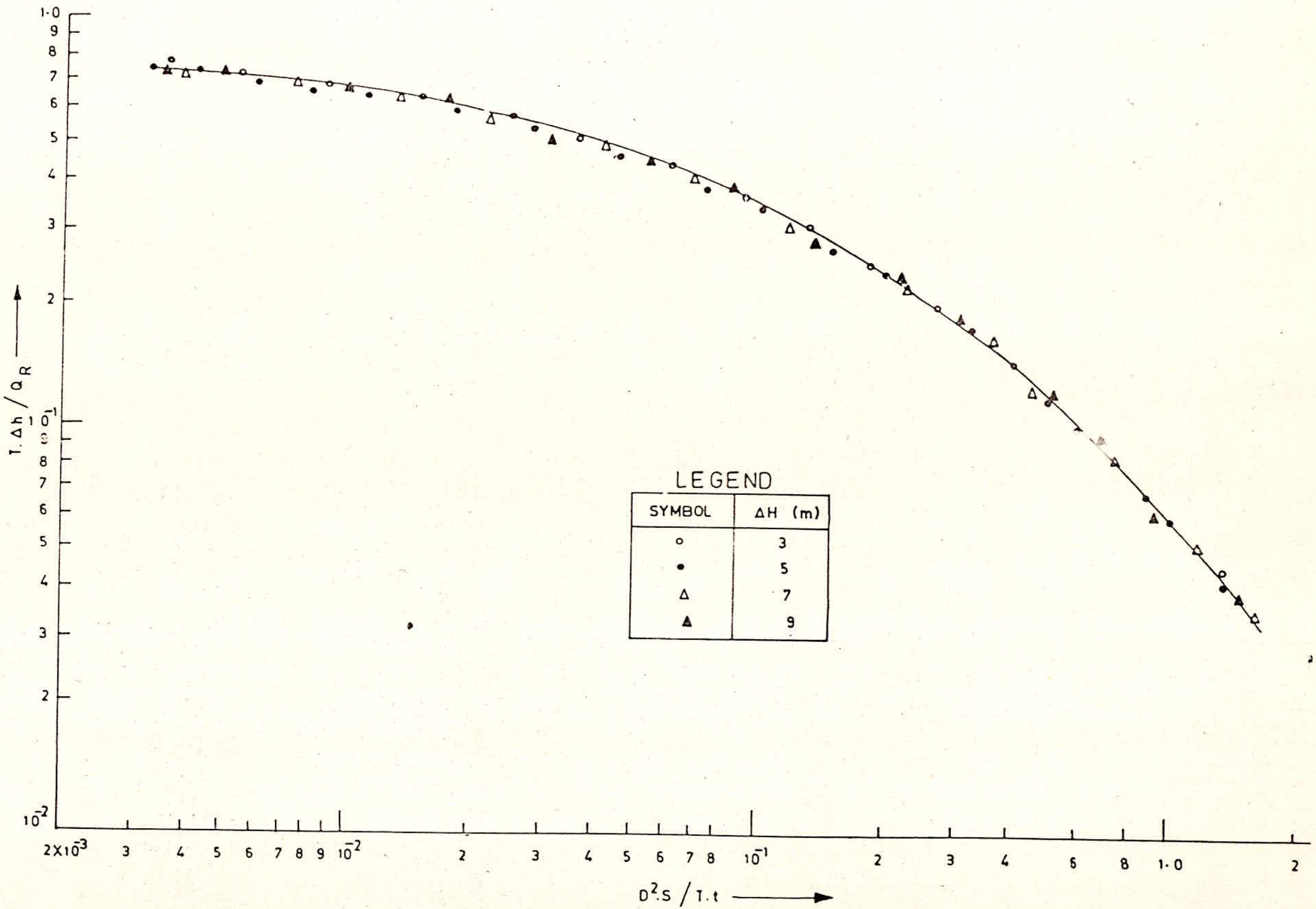


FIG. 8 - RELATION BETWEEN $X^2 S / T.t$ AND $T. \Delta h / Q_R$ (FOR $K_h / K_v = 500$ AND $l/L = 0.12$)