EXCHANGE OF FLOW BETWEEN RIVER AND AQUIFER SYSTEM

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CONTENTS

LIST OF SYMBOLS

H_o	Water level rise above the initial stream stage
m	Time step
$Q_r(n)$	Recharge rate from the r^{th} reach during n^{th} time step
$q(n)$	Recharge rate per unit length per unit time taking place during n^{th} unit time step
S	Water table rise in the aquifer
t	Time
T	Transmissivity of an aquifer
W	Width of the river
$W(\gamma)$	Width of the river during time step γ
x	Co-ordinate
ϕ	Storage coefficient of an aquifer
α	T/ϕ
$\partial(x,W,n)$	Rise in piezometric surface in the aquifer at distance x at the end of n^{th} unit time step due to unit recharge per unit length taken place during the first unit time period when the river width is W
$\Gamma_r(n)$	Reach Transmissivity during n^{th} unit time step
τ	Time, duration of flood
$\sigma(n)$	River stage during time step n

LIST OF FIGURES

ABSTRACT

A mathematical ground water flow model has been developed to predict the exchange of flow between a partially penetrating river and a homogeneous infinite aquifer. The model considers the changes in river stage and corresponding changes in river width. Given the values of aquifer parameters, the transmissivity and the storage coefficient, the saturated thickness below the river bed, saturated thickness far away from the river, the initial width of river at the water surface and depth of water in the river, the model can predict the exchange flow rate between the aquifer and the river consequent to passage of a single or several successive floods. From the study it is found that in case of a partially penetrating river the exchange flow rates are reduced significantly in comparison to those of a fully penetrating river due to river resistance. In case of a partially penetrating river the peak inflow occurs simultaneously with the occurrence of peak stage. It is found that about 25% of the aquifer recharge comes back to river after the recession of a typical flood. A five times increase in river width during the passage of a flood may cause the maximum inflow rate from river to increase by two times in comparison to the maximum inflow rate from a river whose width does not change abruptly.

1.0 INTRODUCTION

The interaction between a river and an aquifer has been examined in some detail in recent years. There are two main aspects of this process: 1) the flow from the aquifer to support river flow and the flow from river to the aquifer. Recharge may occur whenever the stage in a river is above that of the adjacent ground water table, provided that the bed comprises permeable or semi-permeable material. This type of ground water recharge may be temporary, seasonal or continuous. Also it may be a natural phenomenon or induced by man. Man can induce ground water recharge from rivers by lowering the water table adjacent to rivers through ground water abstraction.

A river in general penetrates fully or partially the upper aquifer. When the river stage rises during the passage of a flood, the upper aquifer is recharged through the bed and banks of the river. A single aquifer river interaction problem has been studied analytically by several investigators (Morel-Seytoux and Daly,1977,Todd,1955,Cooper and Rorabough, 1963) for finite and infinite aquifer. The expressions for aquifer recharge in the time of varying river stages have been derived by these investigators. The analysis made by Cooper and Rorabough, is for a fully penetrating river. Therefore the influence of the river width on river aquifer interaction cannot be ascertained from their analysis. Morel-Seytoux and Daly (1977) have analysed the river aquifer interaction problem for varying river stage in a partially penetrating river. In the analysis presented by Morel-Seytoux and Daly the river width has been assumed to remain in variant during the variation of stage. During passage of a flood, the stage as well as the river width change. It is therefore pertinent to analyse the river aquifer interaction during passage of a flood incorporating both the changes in river stage and width.

2.0 REVIEW

A review of aquifer recharge studies due to varying river stage has been presented here.

Cooper and Rorabaugh (1963) have studied flow into and out of the aquifer of finite length, L, in response to changes in river stage of a fully penetrating river. They solved one dimensional Boussinesq's equation under the conditions

$$H(x,0) = 0, \quad 0 \leq x \leq 1$$

$$\frac{\partial H(1,t)}{\partial x} = 0, \quad t \geq 0$$

and

$$H(0,t) = \begin{cases} NH_o \, (1-\cos\omega t)e^{-\partial t} & \text{for } 0 \leq t \leq \tau \\ 0 & \text{for } t > \tau \end{cases}$$

where τ is the duration of the flood wave ($\omega = 2\pi/\tau$)

$$\partial = \omega \cot(0.5\,\omega t_c)$$

determines the asymmetry of the flood wave, t_c is the time of the flood crest, and

$$N = \frac{e^{\partial t_c}}{1-\cos \omega t_c}$$

adjusts all curves of a given ∂ to peak at the same H_o. The solution has been carried out in two steps, one for $t \leq \tau$ and another for $t \geq \tau$. For a semi-infinte aquifer, ($1=\infty$), excited by a symmetrical flood wave ($\partial = 0$), Cooper and Rorabough have also solved the Boussinesq's equation satisfying the following initial and boundary conditions:

$$H(x,0) = 0 \qquad x \geq 0$$

$$\lim_{x \to \infty} H(x,t) = 0 \qquad t \geq 0$$

$$H(0,t) = \begin{cases} 0.5H_o(1-\cos\omega t) & t \leq \tau \\ 0 & t > \tau \end{cases}$$

The following expression for exchange of flow between a partially penetrating river and an aquifer has been derived by Morel-Seytoux and Daly. In the derivation the width of a river reach does not change with change in river stage. Starting from the relation $Q_r(n) = \Gamma_r[\sigma_r(n)\ S_r(n)]$, in which Γ_r is the reach transmissivity, $\sigma_r(n)$ is the river stage measured from a high datum during time n, $S_r(n)$ is the depth to piezometric surface below the river at time n measured from the same high datum, the following integral equation has been obtained by them:

$$Q_r(t) + \Gamma_r \int_0^t Q_r(\tau) k_{rr}(t-\tau)\, d\tau = \Gamma_r\, \sigma_r(t) \tag{2.1}$$

where $k_{rr}(.)$ is the reach kernel (Morel-Seytoux 1975). The above expression is valid for the case in which the interaction is taking place through a single reach. In case of several pervious reaches the generalized equation has been given as

$$Q_r(t) + \Gamma_r \sum_{\rho=1}^{R} \int_0^t Q_\rho(\tau) k_{r\rho}(t-\tau)\, d\tau = \Gamma_r\, \sigma_r(t) \tag{2.2}$$

where R is the number of reaches. Equation(2.2) is a system of R integral equation to be solved simultaneously. Discretising the time parameter and assuming the river flow to be uniform within a time step the following solution to the integral equation(2.2) has been given by Morel-Seytoux and Daly:

$$Q_r(n) + \Gamma_r \sum_{\rho=1}^{R} \sum_{\gamma=1}^{n} \partial_{r\rho}(n-\gamma+1) Q_\rho(\gamma) = \Gamma_r\, \sigma_r(n) \tag{2.3}$$

in which

$$\underset{r\neq\rho}{\partial_{r\rho}(n)} = \frac{1}{4\pi T}\left[E\left\{\frac{\phi d^2_{r\rho}}{4Tn}\right\} - E\left\{\frac{\phi d^2_{r\rho}}{4T(n-1)}\right\}\right] \tag{2.4}$$

$$\partial_{rr}(n) = \frac{1}{\phi ab}\int_0^1 \operatorname{erf}\left\{\frac{a}{2}\left[\frac{\phi}{4T(n-\tau)}\right]^{\frac{1}{2}}\right\} . \operatorname{erf}\left\{\frac{b}{2}\left[\frac{\phi}{4T(n-\tau)}\right]^{\frac{1}{2}}\right\} d\tau \tag{2.5}$$

ϕ = storage coefficient, T = transmissivity, a = length of the river reach, b = width of the river reach, $d_{r\rho}$ = distance from centre of the r^{th} reach to ρ^{th} reach, Γ_r = reach transmissivity of the r^{th} reach, and $\sigma_r(n)$ = river stage of the rth reach during time period n measured from a high datum.

3.0 STATEMENT OF THE PROBLEM

A schematic section of a partially penetrating river in a homogeneous and isotropic aquifer of infinite areal extent is shown in Fig.1. The river and the aquifer are initially at rest condition. Due to passage of a flood, the river stages changes with time. The changes are identical over a long reach of the river. The width of the river changes with change in river stage. The change may be gradual or abrupt. It is required to find the recharge from the river to the aquifer and the flow from the aquifer to the river after the recession of the flood.

4.0 METHODOLOGY

The following assumptions are made for the analysis:

i) The flow in the aquifer is in horizontal direction and one dimensional Boussinesq's equation governs the flow in the aquifer.

ii) The time parameter is discretised. Within each time step, the river stage, width and the exchange flow rate between the river and the aquifer are seperate constant but they vary from step to step.

iii) The exchange of flow between the river and the aquifer is linearly proportional to the difference in the potentials at the river boundary and in the aquifer below the river bed.

5.0 ANALYSIS

The differential equation which governs the flow in the aquifer is

$$T \frac{\partial^2 S}{\partial x^2} = \phi \frac{\partial S}{\partial t} \qquad (1)$$

in which

S = the water table rise in the aquifer, T = transmissivity of the aquifer, ϕ = storage coefficient of the aquifer.

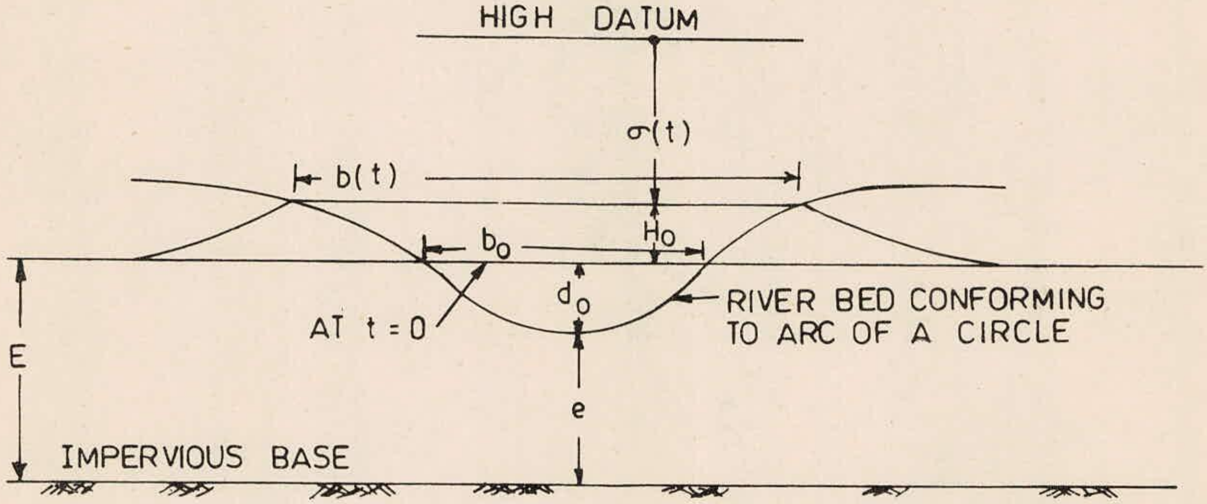

Fig.1(a) Schematic section of a river whose width changes gradually during the passage of a flood

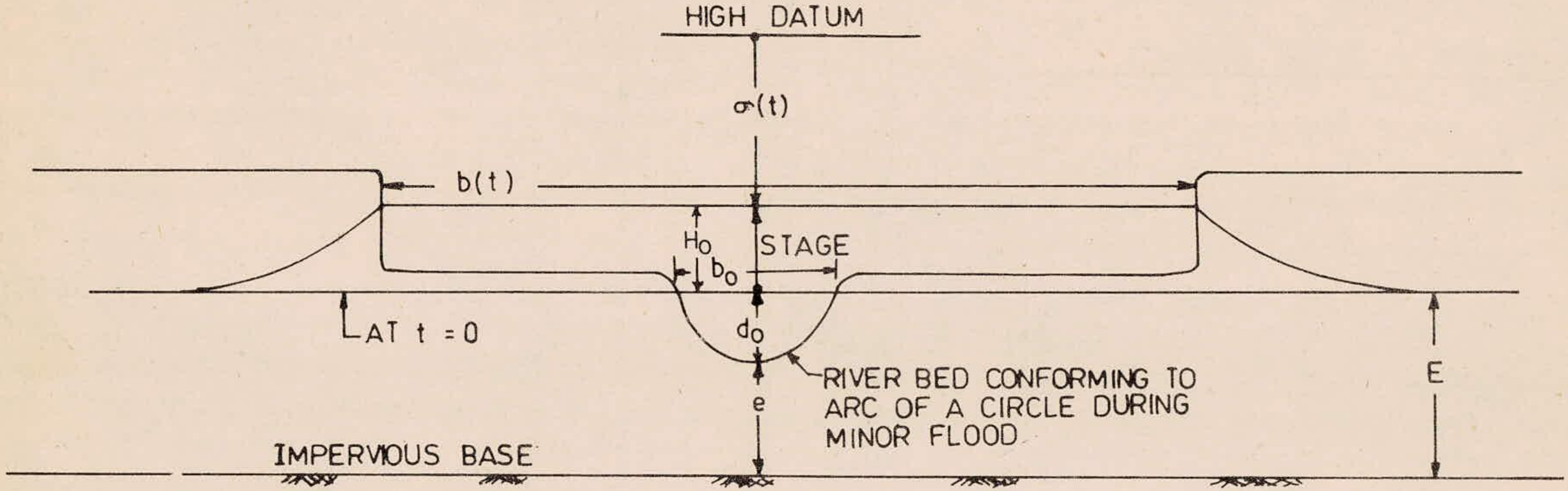

Fig.1(b) Schematic section of a river whose width changes abruptly during the passage of a high flood

If the aquifer and the river were initially at rest, the initial condition to be satisfied is:

$S(x,0) = 0$

The boundary conditions to be satisfied are:

$S(\pm \infty,t) = 0$

At the river and the aquifer interface recharge from the river to the aquifer takes place in a manner similar to that from an overlying bed source to an underlying aquifer through an intervening aquitard. The river resistance and the aquitard resistance are analogous. If the river fully penetrates the upper aquifer, then $S(0,n)=\sigma(n)$ i.e. the rise in water table height at the river is equal to rise in river stage, $\sigma(n)$. For a partially penetrating river $S(0,n)\neq\sigma(n)$ and $S(0,n)$ is to be determined as a part of the solution. The recharge which can be assumed to be linearly proportional to the potential difference, $[\sigma(n)-S(r,n)]$, is to be incorporated at the river boundary

The solution to the problem has been obtained following the principle of superposition.

If the recharge takes place at unit rate per unit length of the river and if the width of the river is 'W', the rise in piezometric surface at distance x from the centre of the river would be as given below:

$$S(x,t)=F(x,T,\phi,W,t)-\frac{[x^2+0.25W^2]}{2TW} \quad \text{for } |x|<\frac{W}{2}$$

$$=F(x,T,\phi.W,t)-\frac{\sqrt{x}^2}{2T} \quad \text{for } |x|>\frac{W}{2} \qquad (2)$$

in which

$$F(x,T,\phi,W,t)= \frac{t}{2\phi W}[\mathrm{erf}\{\frac{x+0.5W}{\sqrt{4\alpha t}}\}-\mathrm{erf}\{\frac{x-0.5W}{\sqrt{4\alpha t}}\}]$$

$$+ \frac{1}{4TW}[\{x+0.5W\}^2\mathrm{erf}\{\frac{x+0.5W}{\sqrt{4\alpha t}}\}- \{x-0.5W\}^2\mathrm{erf}\{\frac{x-0.5W}{\sqrt{4\alpha t}}\}]$$

$$+ \frac{\sqrt{\alpha t}}{2TW\sqrt{\pi}}[\{x+0.5W\}\exp\{-\frac{(x+0.5W)^2}{4\alpha t}\}-\{x-0.5W\}\exp\{-\frac{(x-0.5W)^2}{4\alpha t}\}], \qquad (3)$$

$\alpha = T/\phi$,

W = width of the river, and

x = distance measured from the centre of the river to the point of observation.

Let the rise in piezometric surface at a distance x from the centre of of the river at the end of n^{th} unit time step if unit recharge takes place from unit length of the river during the first unit time period, in which the river width is $W(1)$, be designated as $\partial[x,W(1),n]$. Hence

$$\partial[x,W(1),n]=F[x,T,\phi,W(1),n] - F[x,T,\phi,W(1),n-1] \qquad (4)$$

$$\text{for } n > 2$$

$$\partial[x,W(1),1]=F[x,T,\phi,W(1),1] - \frac{\sqrt{(x)^2}}{2T}$$

$$\text{for } |x| > W(1)/2$$

$$\partial[x,W(1),1]=F[x,T,\phi,W(1),1]- \frac{1}{2TW(1)}[0.25W^2(1)+x^2]$$

$$\text{for } |x| < W(1)/2$$

Dividing the time span into discrete time steps, and assuming that, the recharge per unit length is constant within each time step but varies from step to step, the rise in piezometric surface below the centre of the river due to time variant recharge taking place through varying width can be written as

$$S(0,n)= \sum_{\gamma=1}^{n} q(\gamma)\partial[0,W(\gamma),n-\gamma+1] \qquad (5)$$

in which $q(\gamma)$ is the recharge rate per unit length per unit time which is taking place through a width of $W(\gamma)$ during time step γ.

Substituting for $S(0,n)$ in the expression

$$q(n) = \Gamma_r(n)[\sigma(n)-S(0,n)] \quad (6)$$

and simplifying

$$\frac{q(n)}{\Gamma_r(n)} = \sigma(n) - \sum_{\gamma=1}^{n} q(\gamma)\partial[0,W(\gamma),n-\gamma+1] \quad (7)$$

in which $\Gamma_r(n)$ is the reach transmissivity.

Splitting the temporal summation into two parts and rearranging

$$q(n)\{\frac{1}{\Gamma_r(n)} + \partial[0,W(\gamma),1]\} = \sigma(n) - \sum_{\gamma=1}^{n-1} q(\gamma)\partial[0,W(\gamma),n-\gamma+1] \quad (8)$$

Hence,

$$q(n) = \frac{\sigma(n) - \sum_{\gamma=1}^{n-1}[q(\gamma)\partial[0,W(\gamma),n-\gamma+1]}{\frac{1}{\Gamma_r(n)} + \partial[0,W(n),1]} \quad (9)$$

$q(n)$ can be solved in succession starting from time step 1.

If the river is divided into a number of reaches to account for variation in stage and width from reach to reach the following equation similar to equation (2.3) could be derived:

$$Q_r(n)+\Gamma_r(n) \sum_{\rho=1}^{R} \sum_{\gamma=1}^{n} \partial_{r\rho} [b_r(\gamma),n-\gamma+1]Q_\rho(\gamma) = \Gamma_r(n)\sigma_r(n) \quad (10)$$

in which,

$Q_r(n)$=exchange of flow per unit time between the r^{th} reach and the river,

$$\underset{r\neq\rho}{\partial_{r\rho}}\{b_r(\gamma),m\} = \frac{1}{4\pi T}[E\{\frac{\phi d_{r\rho}^2}{4Tm}\} - E\{\frac{\phi d_{r\rho}^2}{4T(m-1)}\}], \quad (11)$$

$$\partial_{rr}\{b_r(\gamma),m\} = \frac{1}{\phi ab_r(\gamma)} \int_0^1 \text{erf}\{\frac{a}{2}[\frac{\phi}{4T(m-\tau)}]^{\frac{1}{2}}\}\text{erf}\{\frac{b_r(\gamma)}{2}[\frac{\phi}{4T(m-\tau)}]^{\frac{1}{2}}\}d\tau \quad (12)$$

$d_{r\rho}$ =distance of the centre of r^{th} reach from the centre of the ρ^{th} reach and $b_r(\gamma)$=width of the r^{th} reach during time step γ.

6.0 RESULTS AND DISCUSSIONS

Exchange of flow between the river and the aquifer consequent to passage of a flood has been presented for the following cases:

Case 1: Change in river width is gradual with change in river stage.

Case 2: Change in river width is gradual during low flood and the width abruptly attains a high value during high flood.

The width of the river for Case 1 corresponding to any stage has been determined assuming that the cross section of the river conforms to part of a circle.

For numerical computation the following data are required:

i) Initial saturated thickness at large distance from the river

ii) Initial width of the river at the water surface and the initial depth of water in the river

iii) Thickness of aquifer below the river bed

iv) Storage coefficient, and transmissivity of the aquifer

v) Time to flood peak

vi) Highest river stage during passage of the flood

vii) Duration of flood

viii) Width of the river in case the width changes abruptly.

In case the river cross section is irregular, the cross section for each stage has to be assigned for numerical computation.

The reach transmissivity constant which changes with change in river width has been evaluated using the following relation given by Bouwer (1969)

$$\Gamma_r(n) = \frac{K\pi}{\log_e[(e+d)/w.p.]+0.5\pi L/(e+d)} \qquad (13)$$

The distance L specifies the zone of influence on each side of the river and it has been assumed to be b(n)/2 +200m. The wetted perimeter, w.p., and the characteristic length, L, change with change in river stage.

The variations of $Q(t)/[0.5H_o\sqrt{2\pi T\phi/\tau}]$ with $Kt/(2\phi E)$ are presented in Fig.2(a) for three different durations of flood. The dimensionless exchange flow rate term, $Q(t)/[0.5H_o\sqrt{2\pi T\phi/\tau}]$, has been formed following Cooper and Rorabaugh. The time to peak $Kt_c/(2\phi E)$, for each flood has been assumed to be 0.015. It is seen that with increase in the duration of flood, the peak exchange flow rate increases though the time to peak and maximum depth of water for all the three floods are same. This is because, the river stages at any time except at peak is higher for a flood of longer duration. The time to peak discharge matches with the time to peak stage. The variation for $\tau/t_c=2$ corresponds to a symmetrical flood wave. Variations of $Q(t)/[0.5H_o\sqrt{2\pi T\phi/\tau}]$ with t/τ during passage of a symmetrical flood wave have been presented by Cooper and Rorabaugh (1963) for a fully penetrating river. It is found that there is distinct difference between the variations in exchange flow rates for a partially penetrating and a fully penetrating river. In case of a partially penetrating river the occurrence of peak in-flow rate coincides with time of peak stage. In case of a fully penetrating river the time of peak flow rate from the river to the aquifer preceeds the occurrence of maximum river stage. In case of a partially penetrating river, the magnitude of maximum inflow rate to the aquifer is greatly reduced. For example in case of a fully penetrating river,the dimensionless peak flow rate from one side of the river to the aquifer is about 1.35, whereas in case of a partially penetrating river the peak flow rate from both sides of the river to the aquifer

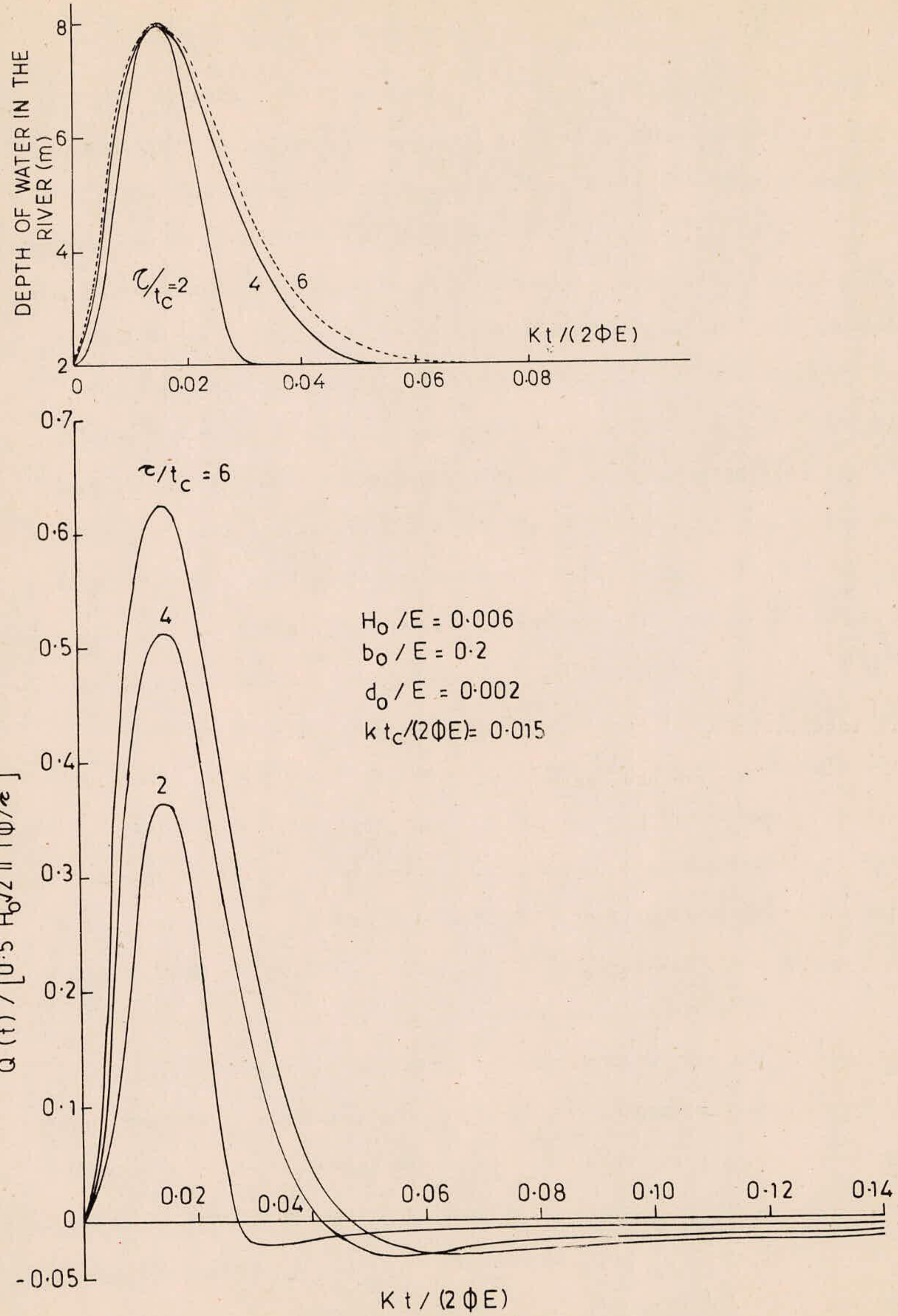

Fig.2(a) Exchange of flow between a river and an aquifer during the passage of floods of different durations; the flood peak occurs at $Kt/(2\Phi E)=0.015$; initial width of the river $b_o/E=0.2$

is 0.365. The river resistance reduces the flow rate significantly. It is further seen that, the peak flow rate from the aquifer to the river during the recession of the flood is comparable to the maximum inflow rate for fully penetrating river. For a fully penetrating river the maximum out flow rate, $Q(t)_{max}/[0.5H_o\sqrt{2\pi T\phi/\tau}]$ is about 1.15. The maximum outflow rate for a partially penetrating river obtained from the present analysis is 0.02. Thus the inflow and outflow are reduced significantly due to river resistance in case of partially penetrating river.

The exchange of flow between the river and the aquifer for a river having a larger cross section with $b_o/E=0.5$ and $d_o/E=0.002$ has been presented in Fig.2(b). It could be seen that between two rivers, the one with larger width at the same depth of water, though contributes more towards the aquifer recharge, the increase in recharge is not proportionate to the increase in the width. For the river with $b_o/E=0.2$ and $d_o/E=0.002$ the maximum recharge rate, $Q(t)_{max}/[0.5H_o\sqrt{2\pi T\phi/\tau}]$ is 0.365. For the river with $b_o/E=0.5$, the maximum recharge rate is 0.525.

The variation of exchange flow rate with time has been presented in Fig.2(c) for a flood wave whose peak occurs at $Kt_c/(2\phi E)=0.025$. Comparing the results presented in Figs.2(a) and 2(c), it is found that for symmetrical flood waves with same peak stage, if the time of occurrence of the peak stage increases, the maximum inflow rate decreases marginally. For example for $Kt_c/(2\phi E)=0.015$, $H_o/E=0.006$, $b_o/E=0.2$, $d_o/E=0.002$, $\tau/t_c=2$, the peak inflow rate is $11.24m^3/day$ corresponding to $E=1000m$, $T=1000m^2/day$, $\phi=0.1$, and $t_c=3$ days. For $Kt_c/(2\phi E)=0.025$, the corresponding peak inflow rate is $10.92m^3/day$.

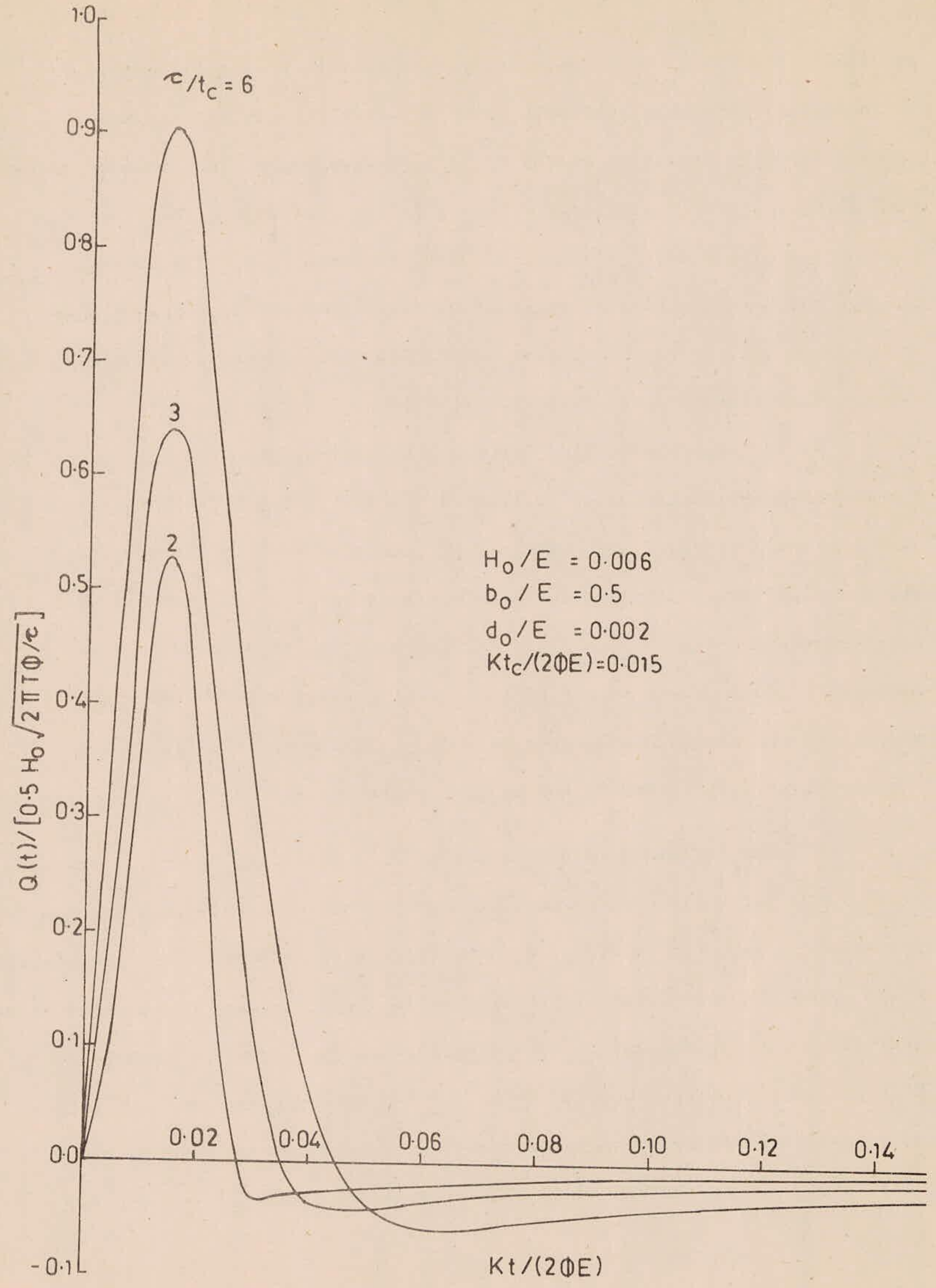

Fig.2(b) Exchange of flow between a river and an aquifer during passage of a flood for different durations, the flood peak occurs at $Kt/(2\phi E)=0.015$ initial width of the river, $b_o/E=0.5$

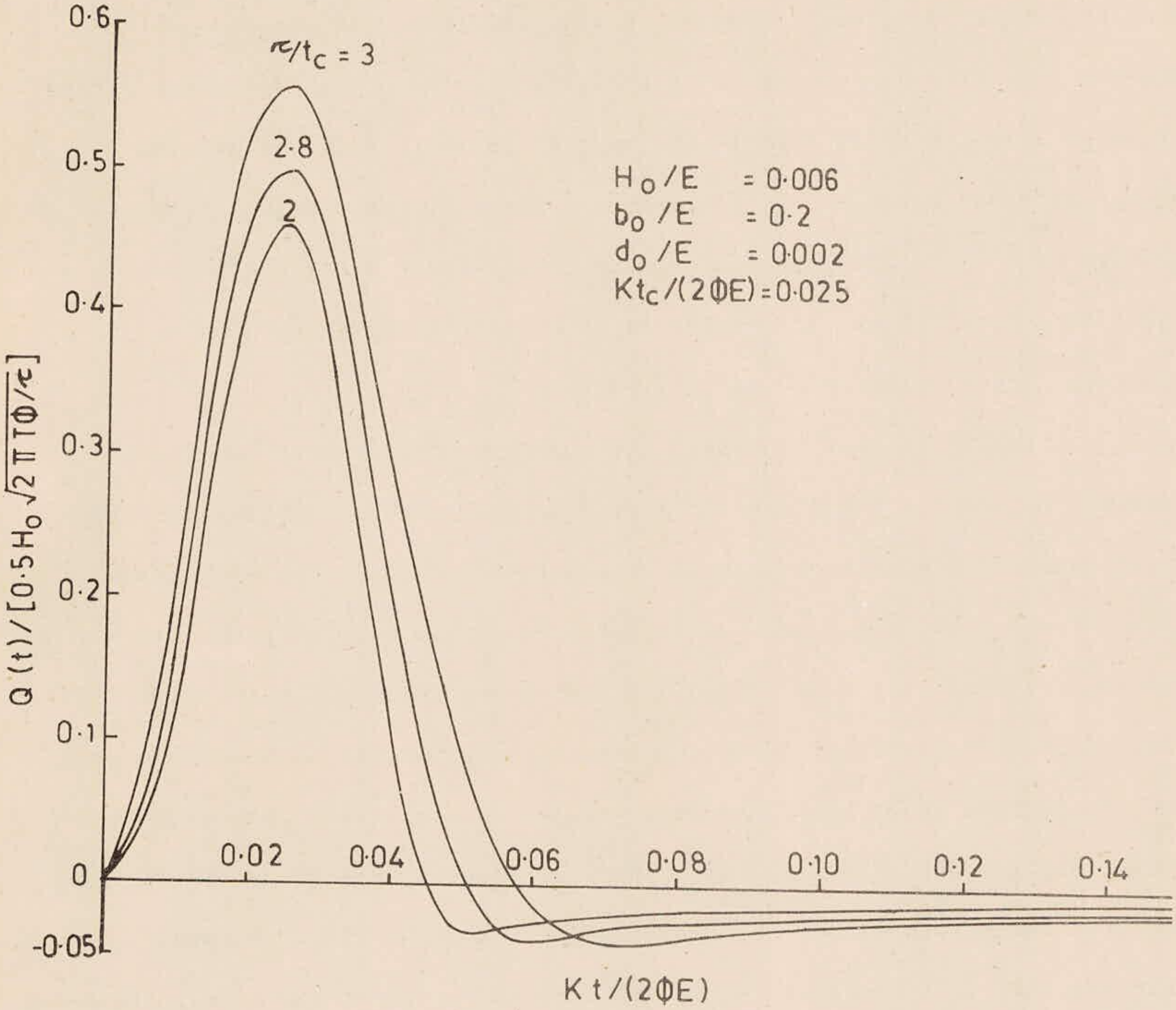

Fig.2(c) Exchange of flow between a river and an aquifer during the passage of a flood for different durations, the flood peak occurs at $Kt/(2\phi E)=0.025$, and initial width of the river, $b_o/E=0.2$

The variation of cumulative flow from the river to the aquifer with time has been shown in Fig.3 for different values of τ/t_c. The cumulative flow reaches a maximum value rapidly and then decreases sluggishly. The commencement of decline in cumulative flow indicates the commencement of reverse flow from the aquifer to the river. The monotonic decreasing trend at large time indicates that if the aquifer is of infinite length, the volume of water which would flow from the river to the aquifer during the passage of a flood will not return back to the river after the recession of the flood. For $\tau/t_c=2$, 26% of the total aquifer recharge returns to the river by dimensionless time $Kt/(2\phi E)=0.2$.

Using the present model, the exchange of flow that takes place between the aquifer and the river during several successive floods could be ascertained from continuous record of the river stages. The exchange of flow between a river and an aquifer has been determined for three successive floods occurring over a time span of 72 days and the results are shown in Fig.4. The river stages and the corresponding river widths are also shown in the figure. The river stage has been assumed to attain the same maximum height during each of the floods. There is marginal difference in the peak recharge rates from flood to flood. During the first flood the peak recharge rate is 6.3 m^3/day. During the second and the third flood, the peak recharge rates are 6 m^3/day and 5.8 m^3/day respectively. The recharge that would take place for the same river stages indicated in Fig.4, if the river width changes abruptly to attain a width of 1000m, when its stage cross a height of 5m, has been shown in Fig.5. Though the width changes by 5 times, the changes in peak discharges are about two times. The recharge rates being governed primarily by the difference in potentials between the river the aquifer do not change in proportion to the change in river width. The computer programme developed for calculating the river aquifer interaction is given in Appendix I.

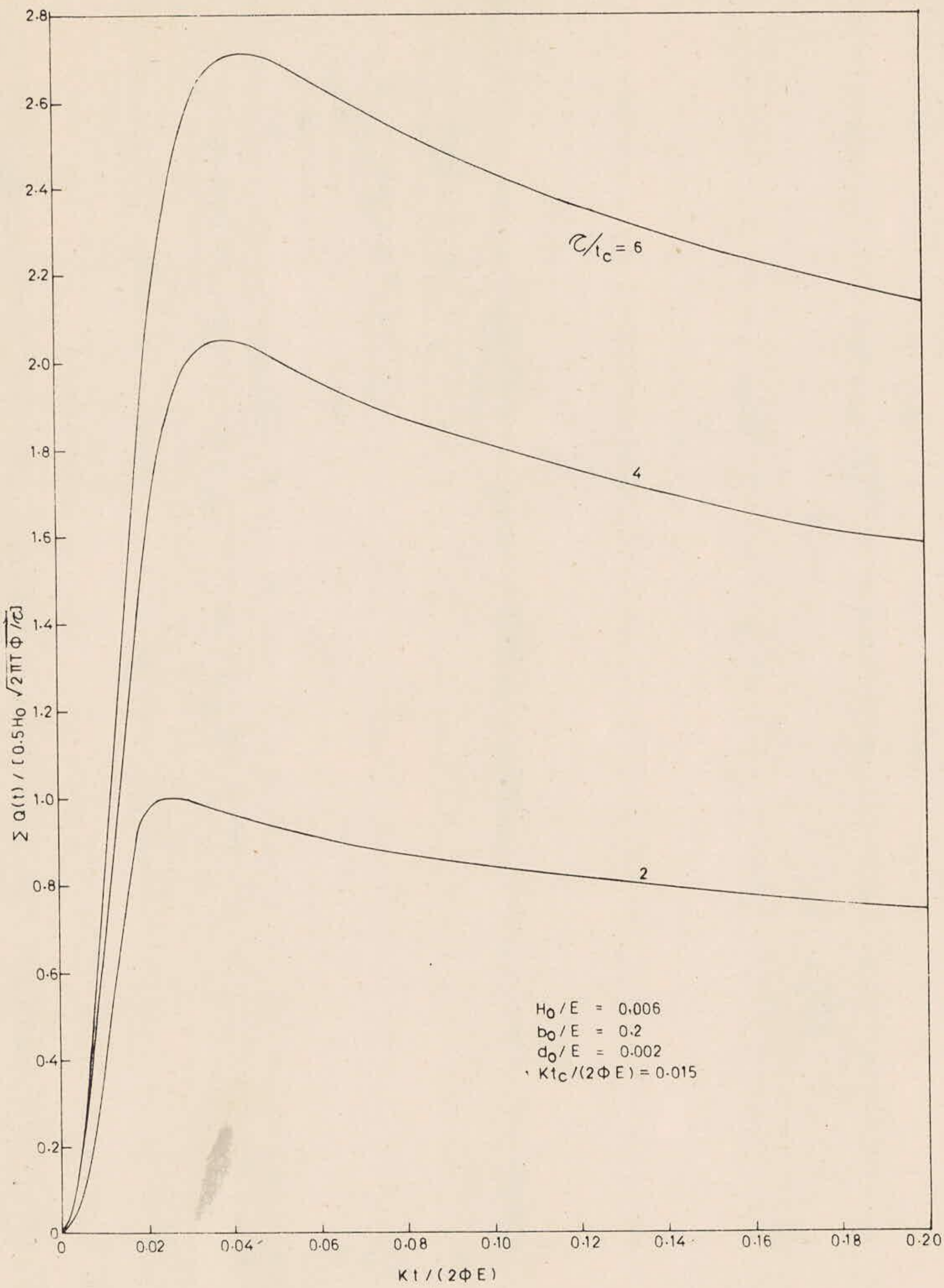

Fig.3 Variation of cumulative flow with time consequent to passage of floods of different duration

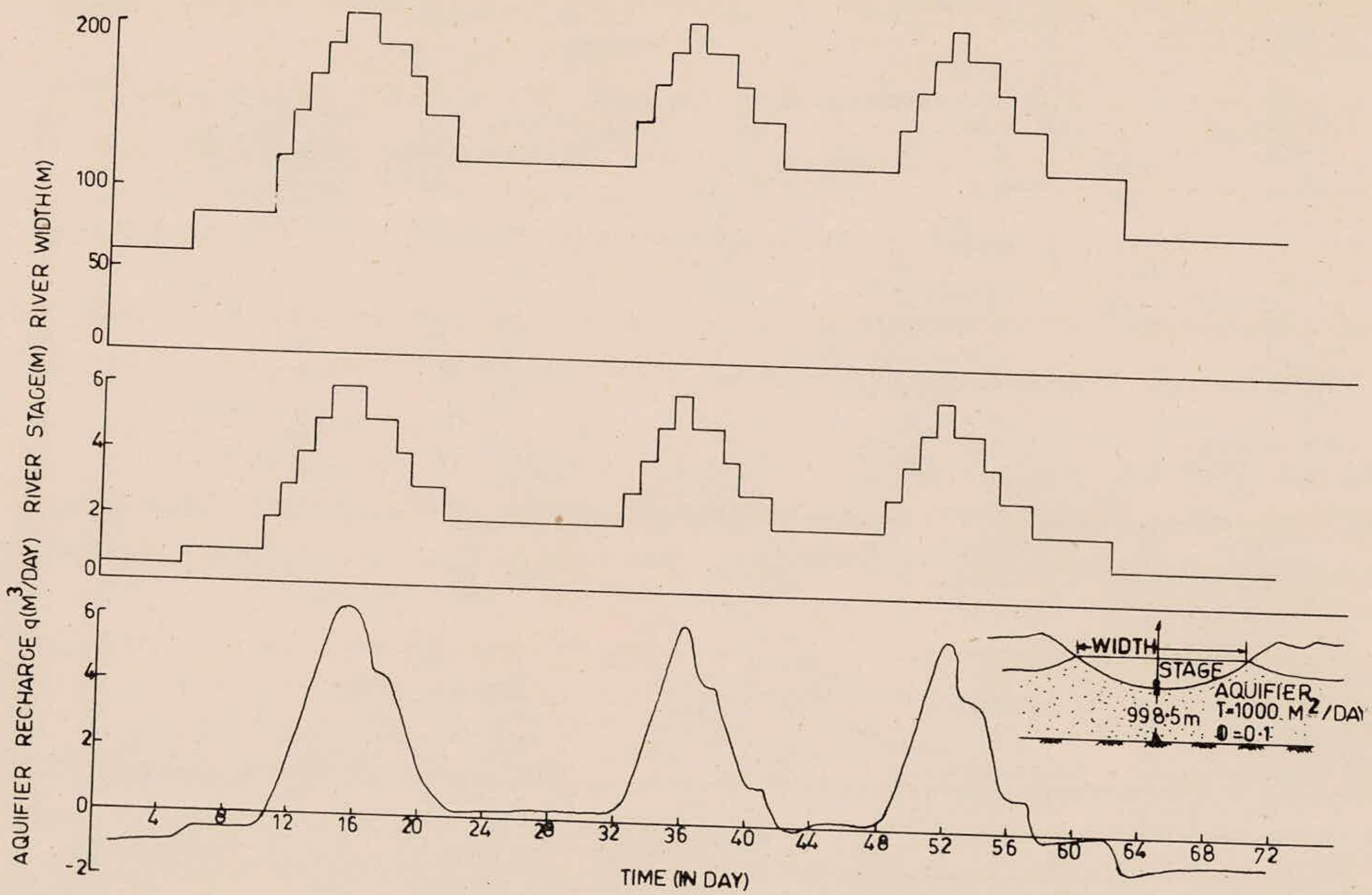

Fig.4 Variation of flow from a river whose stage and width vary with time

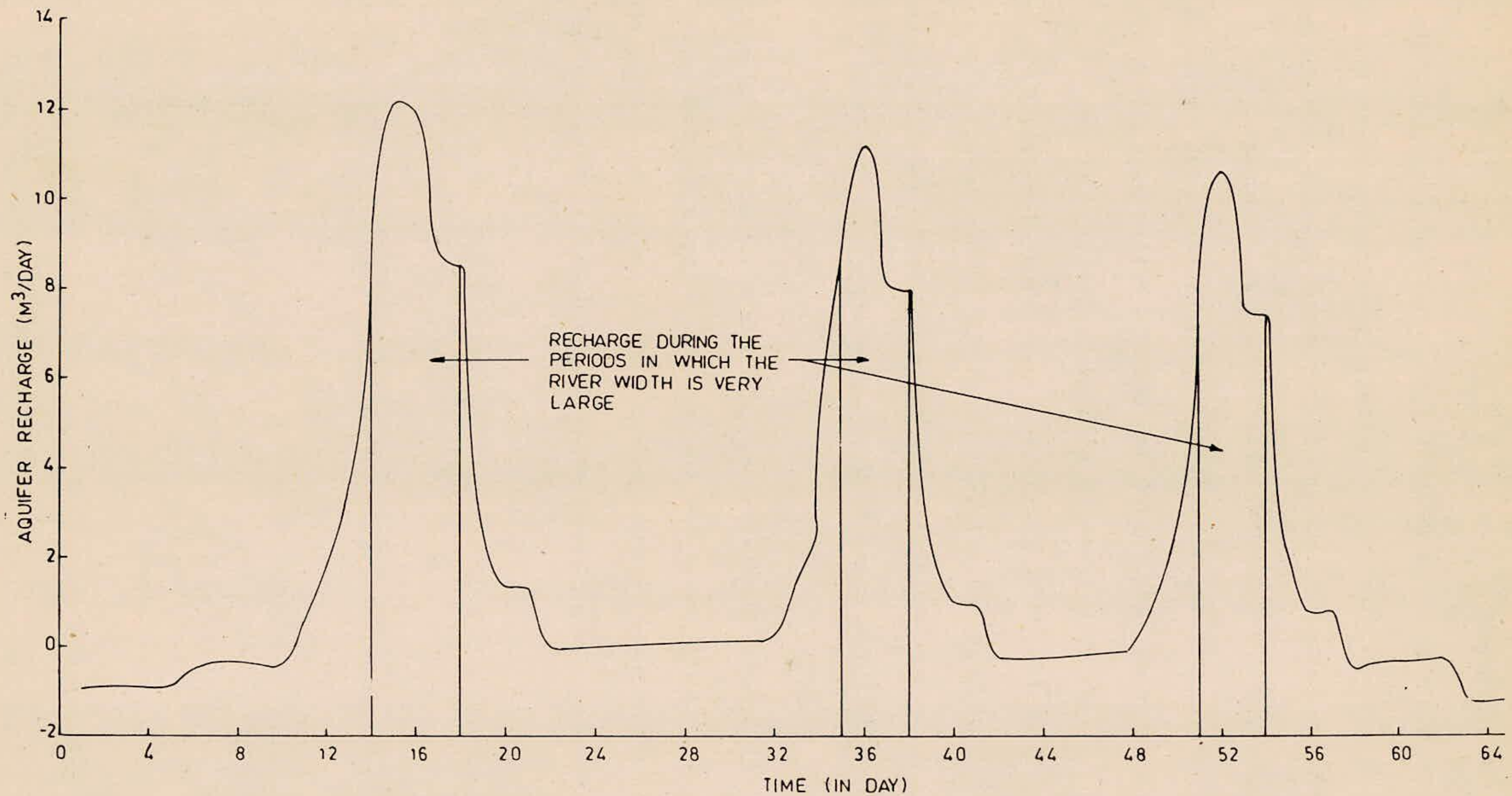

Fig.5 Variation of flow from a river whose width vary abruptly with stage

7.0 CONCLUSIONS

A mathematical model has been developed to predict the exchange of flow between an aquifer and a partially penetrating river whose width changes with the change in its stage during the passage of a flood. Based on the study the following conclusions have been derived:

1) For partially penetrating river, the river resistance reduces the maximum inflow and outflow rates which are very much less than those of a fully penetrating river.
2) In case of a partially penetrating river the peak inflow occurs simultaneously with the occurrence of peak stage.
3) In case of aquifer of infinite length about 25% of the aquifer recharge comes back to river after the recession of a typical flood.
4) A five times increase in river width during the passage of a flood may cause the maximum inflow rate from river to increase by two times in comparison to the maximum inflow rate from a river whose width does not change abruptly.

REFERENCES

Bouwer , H. 1969. Theory of seepage from open channels. Advances in Hydroscience, edited by V.T. Chow. Academic Press, V. 5, pp.121-172.

Cooper, H.H., Jr., and M.I. Rorabough, 1963. Ground water Movements and Bank Storage Due to Flood Stages in Surface Streams, Geol. Surv. Water Supply Paper 1536-J.

Morel-Seytoux, H.J. 1975. Optimal Legal Conjunctive Operation of Surface and Ground Waters. Proceedings Second World Congress, International Water Resources Association, New Delhi.

Morel-Seytoux, H.J. and C.J. Daly, 1977. A Discrete Kernel Generator for Stream Aquifer Studies. Water Resources Research Journal.

Todd, D.K., 1955. Ground Water flow in relation to a flooding stream, Proc. Amer. Soc. Civil Engineers, Vol.81, Sep. 628, pp.20.

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      DIMENSION DEL(5,365,365),CORD(5),Q(365),RISE(5,365),UE(365)
     1,SIGMA(365),B(1),RDEPTH(365),RWIDTH(365),THETA(365)
     2,WP(365),RICHT1(365),RICHT2(365),RICHT3(365),RICHT4(365)
     3,SUMQ(365),QQ(365),SSUMQ(365),UUE(365)
      OPEN(UNIT=1,FILE='VRSTAGE1.DAT',STATUS='OLD')
      OPEN(UNIT=2,FILE='VRSTAGE1.OUT',STATUS='NEW')
      READ(1,1)AK,E,PHI,DI,TC,TAI,HO,DEPTHI
1     FORMAT(8F10.3)
      READ(1,4),WIDTHI
4     FORMAT(F10.5)
      READ (1,2)NTIME,NTAI
2     FORMAT(2I4)
      READ(1,5)(CORD(IOB),IOB=2,5)
5     FORMAT(4F10.5)
C     IOB IS NUMBER OF OBSERVATION POINTS AT WHICH WATER TABLE RISE
C     ARE TO BE CALCULATED
C     AK=CO EFFICIENT OF PERMEABILITY IN M/DAY
C     E= SATURATED THICKNESS AT A LARGE DISTANCE FROM THE RIVER
C     PHI=STORAGE COEFFICIENT
C     DI=DEPTH TO IMPERVIOUS STRATUM FROM RIVER BED
C     TC= TIME TO FLOOD PEAK
C     TAI=DURATION OF FLOOD
C     NTAI=DURATION OF FLOOD
C     NTIME= TIME DURATION OVER WHICH EXCHANGE IS CALCULATED
C     HO=MAXIMUM HEIGHT OF FLOOD  ABOVE THE  INITIAL WATER LEVEL
C     DEPTHI=DEPTH OF WATER IN THE RIVER BEFORE ONSET OF FLOOD
C     WIDTHI=WIDTH OF RIVER AT THE WATER SURFACE BEFORE ONSET OF FLOOD
      T=AK*E
      WRITE(2,10)
10    FORMAT(6X,'AK',8X,'E',9X,'T',9X,'PHI',5X,'DI',9X,'TC',9X,'TAI',7X,
     1'HO',7X,'DEPTHI')
      WRITE(2,11),AK,E,T,PHI,DI,TC,TAI,HO,DEPTHI
11    FORMAT(9F10.3)
C
C
C     DEPTH OF WATER IN THE RIVER IS BEING CALCULATED FOR VARIOUS TIME
C     DURING PASSAGE OF THE FLOOD
C
C
      PAI=3.14159265
      OMEGA=2.*PAI/TAI
      DELTA=OMEGA*COS(.5*OMEGA*TC)/SIN(.5*OMEGA*TC)
      ANN=EXP(DELTA*TC)/(1.-COS(OMEGA*TC))
      CONST=.5*HO*SQRT(OMEGA*PHI*T)
      DO 31 I=1,NTAI
      AI=I
      RDEPTH(I)=ANN*HO*(1.-COS(OMEGA*AI))*EXP(-DELTA*AI)+DEPTHI
31    CONTINUE
      INT=NTAI+1
```

```
      DO 32 I=INT,NTIME
32    RDEPTH(I)=DEPTHI
      WRITE(2,12)
12    FORMAT(10X,'RIVER DEPTH')
      WRITE(2,3)(RDEPTH(I),I=1,NTIME)
3     FORMAT(8F10.3)
C
C
C     INTIAL WIDTH OF RIVER AT THE WATER SURFACE IS  READ AND
C     WIDTHS WETTED PERIMETER & SIGMA AT DIFFERENT TIME ARE CALCULATED
C     THE RIVER CROSS SECTION CONFORMS TO PART OF ACIRCLE
C
C
      RADIUS=(WIDTHI**2*.25+DEPTHI**2)/(2.*DEPTHI)
      DO 20 I=1,NTIME
      SIGMA(I)=RADIUS-RDEPTH(I)
      RWIDTH(I)=SQRT(8.*RDEPTH(I)*RADIUS-4.*RDEPTH(I)**2)
      THETA(I)=ATAN(.5*RWIDTH(I)/SIGMA(I))
      WP(I)=2.*RADIUS*THETA(I)
20    CONTINUE
      WRITE(2,13)
13    FORMAT(10X,'RIVER WIDTH')
      WRITE(2,3),(RWIDTH(I),I=1,NTIME)
      UE(1)=AK/(2.*PHI*E)
C
C     UE(N) IS DIMENSIONLESS TIME FACTOR
C
      UUE(1)=1./TAI
C
C
C     UUE(N) IS ANOTHER FORM OF DIMENSIONLESS TIME FACTOR
C
C
C
C     REACH  TRANSMISSIVITY ARE BEING DETERMINED
C     THERE ARE DIFFERENT FORMULIE FOR REACH TRANSMISSIVITY COMPUTION
C     FORMULA 1 HAS BEEN USED IN THE PRESENT ANALYSIS
C
C
C
C
      WRITE(2,15)
15    FORMAT(4X,'RICHT1',4X,'RICHT2',3X,'RICHT3',5X,'RICHT4')
      DO 21 I=1,NTIME
      RICHT1(I)=AK*PAI/(ALOG((DI+RDEPTH(I))/WP(I))+PAI*.5*(RWIDTH(I)*.5+200.)
     1/(DI+RDEPTH(I)))
      RICHT4(I)=(PAI*AK)/(ALOG((DI+RDEPTH(I))/WP(I)))
      RR=WP(I)/PAI
      RICHT3(I)=(PAI*AK)/ALOG(.5*(DI+RDEPTH(I))/RR)
      RICHT2(I)=AK*(2.*DI+WP(I))/(DI+10.*WP(I))
```

```
      WRITE(2,14)RICHT1(I),RICHT2(I),RICHT3(I),RICHT4(I)
14    FORMAT(4F10.5)
21    CONTINUE
C
C
C     DISCRETE KERNELS ARE GENERATED
C
C
C
      NTIMEF=NTIME
      DO 25 NW=1,NTIME
      DO 24 N=1,NTIMEF
      AN=N
      R=0.
      B1=RWIDTH(NW)
      CALL DPQ(B1,AN,T,PHI,R,DN)
      DEL(1,NW,N)=DN
24    CONTINUE
      NTIMEF=NTIMEF-1
25    CONTINUE
      H=SIGMA(1)+RDEPTH(1)+DI-E

C
C
C
      CORD(1)=0.
C
C
C
C
C
      B(1)=SIGMA(1)-H
      Q(1)=B(1)/(-1./RICHT1(1)-DEL(1,1,1))
      QQ(1)=Q(1)/CONST
C     QQ(N) DIMENSIONLESS FLOW RATE=Q(N)/(.5*HO*SQRT(OMEGA*PHI*T))
C
C
C
C     CALCULATION OF EXCHANGE OF FLOW  FROM 2ND TIME STEP ON WARD
C
C
      DO 300 N=2,NTIME
      AN=N
      UE(N)=UE(1)*AN
      UUE(N)=UUE(1)*AN
      SND11=0.
      JJ=N-1
      DO 301 NGAMA=1,JJ
      SND11=SND11+Q(NGAMA)*DEL(1,NGAMA,N-NGAMA+1)
301   CONTINUE
```

```
      B(1)=(RDEPTH(N)+DI-E-SND11)*(-1.)
      Q(N)=B(1)/(-1./RICHT1(N)-DEL(1,N,1))
      QQ(N)=Q(N)/CONST
C
C
C
C
C
C     CUMULATIVE FLOWS ARE BEING CALCULATED
C     SUMQ IS CUMULATIVE FLOWS
C     SSUMQ IS CUMULATIVE DIMENSIONLESS FLOW
C
C
      DO 40 I=1,NTIME
      SUMQ(I)=0.
      SSUMQ(I)=0.
      DO 41 L=1,I
      SSUMQ(I)=SSUMQ(I)+QQ(L)
41    SUMQ(I)=SUMQ(I)+Q(L)
40    CONTINUE
300   CONTINUE
      WRITE(2,290)
290   FORMAT(8X,'Q(N)',10X,'UE(N)',10X,'SUMQ(N)'10X,'SIGMA(N)',5X,'N',7X,
      1'QQ(N)',7X,'SSUMQ(N)',7X,'UUE(N)')
      WRITE(2,33),((Q(J),UE(J),SUMQ(J),SIGMA(J),J,QQ(J),SSUMQ(J),UUE(J)),
      1J=1,NTIME)
33    FORMAT(4E16.7,I4,3E13.4)
C
C
C     CALCULATION OF EVOLUTION OF WATER TABLE
C
C
C
      DO 240 IOB=2,5
      R=CORD(IOB)
      NTIMEF=NTIME
      DO 245 NW=1,NTIME
      DO 242 N=1,NTIMEF
      AN=N
      B1=RWIDTH(NW)
      CALL DPQ(B1,AN,T,PHI,R,DN)
242   DEL(IOB,NW,N)=DN
      NTIMEF=NTIMEF-1
245   CONTINUE
240   CONTINUE
      DO 250 IOB=1,5
      DO 263 N=1,NTIME
      RISE(IOB,N)=0.
263   CONTINUE
250   CONTINUE
```

```
      DO 265 IOB=1,5
      DO 260 N=1,NTIME
      DO 261 NGAMA=1,N
      RISE(IOB,N)=RISE(IOB,N)+DEL(IOB,NGAMA,N-NGAMA+1)*Q(NGAMA)
261   CONTINUE
260   CONTINUE
265   CONTINUE
      WRITE(2,271),CORD(1),CORD(2),CORD(3),CORD(4),CORD(5)
271   FORMAT(2X,5E17.2)
      DO 280 N=10,NTIME,10
      WRITE(2,270),(RISE(1,N),RISE(2,N),RISE(3,N),RISE(4,N),RISE(5,N),N)
270   FORMAT(3X,5E16.7,I5)
280   CONTINUE
      STOP
      END
C
C
C
C
      SUBROUTINE DPQ(B1,AN,T,PHI,R,DN)
      IF(AN.LT.1.1) GO TO 1
      CALL DPQ1(B1,AN,T,PHI,R,RES)
      RESN=RES
      AM=AN-1
      CALL DPQ1(B1,AM,T,PHI,R,RES)
      RESNM1=RES
      DN=RESN-RESNM1
      GO TO 2
1     CALL DPQ1(B1,AN,T,PHI,R,RES)
      DN=RES
2     CONTINUE
      RETURN
      END
      SUBROUTINE DPQ1(B1,AN,T,PHI,R,RES)
      DIMENSION Y(2), YY(2),YYY(2)
      IF(R.GE.(B1*.5)) GO TO 600
      C10=.5/T*(R**2+(.5*B1)**2)
      GO TO 703
600   C10=.5/T*(B1)*R
703   CONTINUE
      ALPHA=T/PHI
      PAI=3.14159265
      C1=ALPHA/(2.*T)
      C2=1./(4.*T)
      C3=SQRT(ALPHA/PAI)/(2.*T)
      C4=1./(2.*T)
      X1=R
      C5=X1+.5*B1
      C6=X1-.5*B1
      ST=AN
```

```
      C9=SQRT(4.*ALPHA*ST)
      Y(1)=C5/C9
      Y(2)=C6/C9
      DO 20 INDEX=1,2
      X=Y(INDEX)
      CALL ERF(X,ERFX)
      YY(INDEX)=ERFX
      XX=-Y(INDEX)**2
20    YYY(INDEX)=EXP(XX)
      RES=C1*ST*YY(1)-C1*ST*YY(2)-C10+C2*C5**2*YY(1)-C2*C6**2*YY(2)
     1+C3*SQRT(ST)*C5*YYY(1)-C3*SQRT(ST)*C6*YYY(2)
      RES=RES/B1
      RETURN
      END
      SUBROUTINE ERF(X,ERFX)
      XINDEX=X
      IF(X)4,5,5
4     X=-X
5     CONTINUE
      IF(X-9.)1,2,2
1     CONTINUE
      T=1.0/(1.0+0.3275911*X)
      ERFX=1.0-(0.25482959*T-0.28449673*T**2+1.4214137*T**3-1.453152*T
     1**4+1.061405*T**5)*EXP(-X**2)
      GO TO 3
2     ERFX=1.
3     CONTINUE
      IF(XINDEX)6,7,7
6     ERFX=-ERFX
7     CONTINUE
      RETURN
      END
```

VRSTAGE1.DAT

```
1.0    1000.   .10    998.0   3.    21.    6.   2.
500.
00720021
10.    20.    100.    200.
```

AK	E	T	PHI	DI	TC	TAI	HO	DEPTHI
1.000	1000.000	1000.000	0.100	998.000	3.000	21.000	6.000	2.000

RIVER DEPTH

4.453	7.154	8.000	7.434	6.256	5.021	3.991	3.236
2.728	2.409	2.220	2.113	2.055	2.026	2.011	2.005
2.002	2.001	2.000	2.000	2.000	2.000	2.000	2.000
2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000
2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000
2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000
2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000
2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000
2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000

RIVER WIDTH

746.032	945.576	999.904	963.869	884.242	792.193	706.328	636.003
583.994	548.796	526.778	513.925	506.868	503.205	501.406	500.572
500.211	500.067	500.017	500.002	500.000	500.000	500.000	500.000
500.000	500.000	500.000	500.000	500.000	500.000	500.000	500.000
500.000	500.000	500.000	500.000	500.000	500.000	500.000	500.000
500.000	500.000	500.000	500.000	500.000	500.000	500.000	500.000
500.000	500.000	500.000	500.000	500.000	500.000	500.000	500.000
500.000	500.000	500.000	500.000	500.000	500.000	500.000	500.000
500.000	500.000	500.000	500.000	500.000	500.000	500.000	500.000

RICHT1	RICHT2	RICHT3	RICHT4
2.63285	0.32416	4.20604	10.63714
2.82430	0.28136	6.12955	51.54420
2.85903	0.27240	6.86702	531.81189
2.83678	0.27824	6.36409	74.69209
2.77647	0.29267	5.42851	24.71016
2.68653	0.31256	4.56998	13.31978
2.58193	0.33521	3.92129	8.98675
2.48075	0.35769	3.47022	6.92411
2.39670	0.37729	3.17295	5.83360
2.33530	0.39234	2.98639	5.23262
2.29501	0.40262	2.87499	4.89995
2.27082	0.40895	2.81170	4.71891
2.25733	0.41255	2.77747	4.62328
2.25027	0.41445	2.75985	4.57465
2.24678	0.41539	2.75122	4.55101
2.24517	0.41583	2.74723	4.54010
2.24446	0.41602	2.74551	4.53539
2.24418	0.41609	2.74482	4.53352
2.24409	0.41612	2.74458	4.53286
2.24406	0.41613	2.74451	4.53267
2.24405	0.41613	2.74450	4.53264
2.24405	0.41613	2.74450	4.53264
2.24405	0.41613	2.74450	4.53264
2.24405	0.41613	2.74450	4.53264
2.24405	0.41613	2.74450	4.53264
2.24405	0.41613	2.74450	4.53264
2.24405	0.41613	2.74450	4.53264
2.24405	0.41613	2.74450	4.53264

Q(N)	UE(N)	SUMQ(N)	SIGMA(N)	N	QQ(N)	SSUMQ(N)	UUE(N)
0.6238853E+01	0.5000000E-02	0.6238853E+01	0.1562155E+05	1	0.3802E+00	0.3802E+00	0.4762E-01
0.1391277E+02	0.1000000E-01	0.2015162E+02	0.1561885E+05	2	0.8478E+00	0.1228E+01	0.9524E-01
0.1606108E+02	0.1500000E-01	0.3621270E+02	0.1561800E+05	3	0.9788E+00	0.2207E+01	0.1429E+00
0.1394760E+02	0.2000000E-01	0.5016030E+02	0.1561857E+05	4	0.8500E+00	0.3057E+01	0.1905E+00
0.1010467E+02	0.2500000E-01	0.6026497E+02	0.1561974E+05	5	0.6158E+00	0.3673E+01	0.2381E+00
0.6300771E+01	0.3000000E-01	0.6656573E+02	0.1562098E+05	6	0.3840E+00	0.4056E+01	0.2857E+00
0.3343083E+01	0.3500000E-01	0.6990882E+02	0.1562201E+05	7	0.2037E+00	0.4260E+01	0.3333E+00
0.1362551E+01	0.4000000E-01	0.7127137E+02	0.1562276E+05	8	0.8303E-01	0.4343E+01	0.3810E+00
0.1710783E+00	0.4500000E-01	0.7144245E+02	0.1562327E+05	9	0.1043E-01	0.4354E+01	0.4286E+00
-0.4857986E+00	0.5000000E-01	0.7095665E+02	0.1562359E+05	10	-0.2960E-01	0.4324E+01	0.4762E+00
-0.8158836E+00	0.5500000E-01	0.7014077E+02	0.1562378E+05	11	-0.4972E-01	0.4274E+01	0.5238E+00
-0.9591038E+00	0.6000000E-01	0.6918166E+02	0.1562389E+05	12	-0.5845E-01	0.4216E+01	0.5714E+00
-0.1001147E+01	0.6500000E-01	0.6818052E+02	0.1562394E+05	13	-0.6101E-01	0.4155E+01	0.6190E+00
-0.9913153E+00	0.7000000E-01	0.6718920E+02	0.1562397E+05	14	-0.6041E-01	0.4094E+01	0.6667E+00
-0.9577065E+00	0.7500000E-01	0.6623150E+02	0.1562399E+05	15	-0.5836E-01	0.4036E+01	0.7143E+00
-0.9150068E+00	0.8000000E-01	0.6531649E+02	0.1562400E+05	16	-0.5576E-01	0.3980E+01	0.7619E+00
-0.8704433E+00	0.8500000E-01	0.6444604E+02	0.1562400E+05	17	-0.5304E-01	0.3927E+01	0.8095E+00
-0.8275971E+00	0.9000000E-01	0.6361845E+02	0.1562400E+05	18	-0.5043E-01	0.3877E+01	0.8571E+00
-0.7877144E+00	0.9500000E-01	0.6283073E+02	0.1562400E+05	19	-0.4800E-01	0.3829E+01	0.9048E+00
-0.7510262E+00	0.1000000E+00	0.6207970E+02	0.1562400E+05	20	-0.4577E-01	0.3783E+01	0.9524E+00
-0.7173787E+00	0.1050000E+00	0.6136232E+02	0.1562400E+05	21	-0.4372E-01	0.3739E+01	0.1000E+01
-0.6865893E+00	0.1100000E+00	0.6067574E+02	0.1562400E+05	22	-0.4184E-01	0.3698E+01	0.1048E+01
-0.6582807E+00	0.1150000E+00	0.6001746E+02	0.1562400E+05	23	-0.4012E-01	0.3657E+01	0.1095E+01
-0.6321616E+00	0.1200000E+00	0.5938530E+02	0.1562400E+05	24	-0.3852E-01	0.3619E+01	0.1143E+01
-0.6079804E+00	0.1250000E+00	0.5877732E+02	0.1562400E+05	25	-0.3705E-01	0.3582E+01	0.1190E+01
-0.5855085E+00	0.1300000E+00	0.5819181E+02	0.1562400E+05	26	-0.3568E-01	0.3546E+01	0.1238E+01
-0.5645854E+00	0.1350000E+00	0.5762722E+02	0.1562400E+05	27	-0.3441E-01	0.3512E+01	0.1286E+01
-0.5450312E+00	0.1400000E+00	0.5708219E+02	0.1562400E+05	28	-0.3321E-01	0.3479E+01	0.1333E+01
-0.5267226E+00	0.1450000E+00	0.5655547E+02	0.1562400E+05	29	-0.3210E-01	0.3446E+01	0.1381E+01
-0.5095569E+00	0.1500000E+00	0.5604591E+02	0.1562400E+05	30	-0.3105E-01	0.3415E+01	0.1429E+01
-0.4934020E+00	0.1550000E+00	0.5555251E+02	0.1562400E+05	31	-0.3007E-01	0.3385E+01	0.1476E+01
-0.4781907E+00	0.1600000E+00	0.5507431E+02	0.1562400E+05	32	-0.2914E-01	0.3356E+01	0.1524E+01
-0.4638312E+00	0.1650000E+00	0.5461048E+02	0.1562400E+05	33	-0.2827E-01	0.3328E+01	0.1571E+01
-0.4502615E+00	0.1700000E+00	0.5416022E+02	0.1562400E+05	34	-0.2744E-01	0.3300E+01	0.1619E+01
-0.4373936E+00	0.1750000E+00	0.5372283E+02	0.1562400E+05	35	-0.2665E-01	0.3274E+01	0.1667E+01
-0.4252189E+00	0.1800000E+00	0.5329761E+02	0.1562400E+05	36	-0.2591E-01	0.3248E+01	0.1714E+01
-0.4136403E+00	0.1850000E+00	0.5288397E+02	0.1562400E+05	37	-0.2521E-01	0.3223E+01	0.1762E+01
-0.4026392E+00	0.1900000E+00	0.5248133E+02	0.1562400E+05	38	-0.2454E-01	0.3198E+01	0.1810E+01
-0.3921576E+00	0.1950000E+00	0.5208917E+02	0.1562400E+05	39	-0.2390E-01	0.3174E+01	0.1857E+01
-0.3821807E+00	0.2000000E+00	0.5170699E+02	0.1562400E+05	40	-0.2329E-01	0.3151E+01	0.1905E+01
-0.3726360E+00	0.2050000E+00	0.5133436E+02	0.1562400E+05	41	-0.2271E-01	0.3128E+01	0.1952E+01
-0.3635481E+00	0.2100000E+00	0.5097081E+02	0.1562400E+05	42	-0.2215E-01	0.3106E+01	0.2000E+01
-0.3548432E+00	0.2150000E+00	0.5061597E+02	0.1562400E+05	43	-0.2162E-01	0.3085E+01	0.2048E+01
-0.3465130E+00	0.2200000E+00	0.5026945E+02	0.1562400E+05	44	-0.2112E-01	0.3063E+01	0.2095E+01
-0.3385296E+00	0.2250000E+00	0.4993092E+02	0.1562400E+05	45	-0.2063E-01	0.3043E+01	0.2143E+01
-0.3308933E+00	0.2300000E+00	0.4960003E+02	0.1562400E+05	46	-0.2016E-01	0.3023E+01	0.2190E+01
-0.3235312E+00	0.2350000E+00	0.4927650E+02	0.1562400E+05	47	-0.1972E-01	0.3003E+01	0.2238E+01
-0.3164819E+00	0.2400000E+00	0.4896001E+02	0.1562400E+05	48	-0.1929E-01	0.2984E+01	0.2286E+01
-0.3097067E+00	0.2450000E+00	0.4865031E+02	0.1562400E+05	49	-0.1887E-01	0.2965E+01	0.2333E+01
-0.3031848E+00	0.2500000E+00	0.4834712E+02	0.1562400E+05	50	-0.1848E-01	0.2946E+01	0.2381E+01
-0.2969107E+00	0.2550000E+00	0.4805021E+02	0.1562400E+05	51	-0.1809E-01	0.2928E+01	0.2429E+01
-0.2908701E+00	0.2600000E+00	0.4775934E+02	0.1562400E+05	52	-0.1773E-01	0.2910E+01	0.2476E+01
-0.2850346E+00	0.2650000E+00	0.4747431E+02	0.1562400E+05	53	-0.1737E-01	0.2893E+01	0.2524E+01

-0.2794225E+00	0.2700000E+00	0.4719489E+02	0.1562400E+05	54	-0.1703E-01	0.2876E+01	0.2571E+01
-0.2739909E+00	0.2750000E+00	0.4692089E+02	0.1562400E+05	55	-0.1670E-01	0.2859E+01	0.2619E+01
-0.2687450E+00	0.2800000E+00	0.4665215E+02	0.1562400E+05	56	-0.1638E-01	0.2843E+01	0.2667E+01
-0.2636937E+00	0.2850000E+00	0.4638845E+02	0.1562400E+05	57	-0.1607E-01	0.2827E+01	0.2714E+01
-0.2588060E+00	0.2900000E+00	0.4612965E+02	0.1562400E+05	58	-0.1577E-01	0.2811E+01	0.2762E+01
-0.2540560E+00	0.2950000E+00	0.4587560E+02	0.1562400E+05	59	-0.1548E-01	0.2796E+01	0.2810E+01
-0.2494947E+00	0.3000000E+00	0.4562610E+02	0.1562400E+05	60	-0.1520E-01	0.2780E+01	0.2857E+01
-0.2450565E+00	0.3050000E+00	0.4538104E+02	0.1562400E+05	61	-0.1493E-01	0.2765E+01	0.2905E+01
-0.2407589E+00	0.3100000E+00	0.4514028E+02	0.1562400E+05	62	-0.1467E-01	0.2751E+01	0.2952E+01
-0.2365905E+00	0.3150000E+00	0.4490369E+02	0.1562400E+05	63	-0.1442E-01	0.2736E+01	0.3000E+01
-0.2325606E+00	0.3200000E+00	0.4467113E+02	0.1562400E+05	64	-0.1417E-01	0.2722E+01	0.3048E+01
-0.2286342E+00	0.3250000E+00	0.4444250E+02	0.1562400E+05	65	-0.1393E-01	0.2708E+01	0.3095E+01
-0.2248345E+00	0.3300000E+00	0.4421766E+02	0.1562400E+05	66	-0.1370E-01	0.2695E+01	0.3143E+01
-0.2211509E+00	0.3350000E+00	0.4399651E+02	0.1562400E+05	67	-0.1348E-01	0.2681E+01	0.3190E+01
-0.2175604E+00	0.3400000E+00	0.4377895E+02	0.1562400E+05	68	-0.1326E-01	0.2668E+01	0.3238E+01
-0.2140858E+00	0.3450000E+00	0.4356487E+02	0.1562400E+05	69	-0.1305E-01	0.2655E+01	0.3286E+01
-0.2107112E+00	0.3500000E+00	0.4335416E+02	0.1562400E+05	70	-0.1284E-01	0.2642E+01	0.3333E+01
-0.2073985E+00	0.3550000E+00	0.4314676E+02	0.1562400E+05	71	-0.1264E-01	0.2629E+01	0.3381E+01
-0.2042128E+00	0.3600000E+00	0.4294255E+02	0.1562400E+05	72	-0.1244E-01	0.2617E+01	0.3429E+01

0.00E+00	0.10E+02	0.20E+02	0.10E+03	0.20E+03	
0.6174480E+00	0.6173045E+00	0.6168721E+00	0.6032090E+00	0.562740[illegible]E+00	10
0.3346731E+00	0.3346681E+00	0.3346525E+00	0.3341958E+00	0.3331566E+00	20
0.2270706E+00	0.2270817E+00	0.2271205E+00	0.2283782E+00	0.2323747E+00	30
0.1703081E+00	0.1703195E+00	0.1703685E+00	0.1719084E+00	0.1767192E+00	40
0.1351058E+00	0.1351195E+00	0.1351689E+00	0.1366766E+00	0.1413978E+00	50
0.1111804E+00	0.1111912E+00	0.1112375E+00	0.1126471E+00	0.1170401E+00	60
0.9389752E-01	0.9391019E-01	0.9395075E-01	0.9523747E-01	0.9926510E-01	70