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# Optimal Irrigation Schedules Based on Simulated Soil-Moisture Depletion Pattern

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**ABSTRACT:** Modelling water movement in the cropped soil can give insight into effective management practices to conserve water. Most of the existing root water uptake models solve the equation for moisture flow in the unsaturated soil coupled with a sink term representing moisture uptake by plants. Modelling requires a mathematical description of the process of the uptake of water from the soil profile by the plant root system. In the present study a numerical model, based on a mass conservative, fully implicit finite difference scheme has been formulated, wherein Richards equation coupled with a non-linear root water uptake term has been subject to appropriate boundary conditions. The non linear system of equations is linearized using Picard's iterations and resulting system of equations are solved using Thomas algorithm. The model yields spatial distribution of pressure head and moisture content at successive advancing times in the soil. From the model computed moisture contents, the moisture depletion values at different zones of crop root at different times are computed by numerical integration. The simulated moisture depletions have been compared with field observed data. The field experiments on 'Maize' under controlled conditions have been carried out at Roorkee. The soil moisture variation in the crop root zone has been continuously observed. Comparison of simulated and field observed values indicates high values of statistical parameters, which validates the reliability of the simulation formulated. Hypothetical irrigation schedules based on the simulated root zone soil moisture depletion have been developed to illustrate the applicability of the numerical model.

**Keywords:** Irrigation, Root-uptake, Crop Water Requirement, Evapotranspiration.

## INTRODUCTION

Many irrigation systems throughout the world are operated by distributing water sequentially amongst a group of users, based upon the crop water requirements indicated by the users. Irrigation scheduling requires four essential components: an estimation of water extracted from the available root zone supply, a projected rate of depletion of the remaining soil water, an accurate measure of the water supplied by precipitation, an accurate estimate of the amount of water applied through irrigation. Bishop and Long (1983) developed an irrigation schedule taking various factors into consideration to improve the efficiency of schedules. Khepar *et al.* (2000) in their study of the Kotkapura distributary of the Sirhind System in India modified the duration of schedules to take care of the seepage losses in the field channels. In both these cited studies, water is essentially the primary factor which was aimed to be conserved. For effective scheduling, opportunity should be provided to the user to specify how much water is required, duration, flow rate, or the time at which the water is required (start time of irrigation). Wang *et al.* (1995) used integer programming to develop a schedule for a canal whereby the duration of flow at an outlet could be specified by a user.

Anwar and Clarke (2001) expanded the work by Wang *et al.* (1995) and incorporated both duration and start time into the model. Santhi and Pundarikanthan (2000) have criticized the work by Wang *et al.* (1995) as hypothetical. The simulation based scheduling problem has found applications in a vast variety of fields. The paper shows how the demand based irrigation scheduling be modeled using the computational techniques. An analogy can be drawn between a water extraction process from soil by the plant roots and a sequential irrigation provision based on moisture

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requirement of the root zone soil. The duration of water required by plants is determined based on seepage and evapotranspiration losses. Alternatively, duration may be specified independently by users based on their assessment of crop requirements, weather, etc. Through a scheduling model as proposed in this work, significant irrigation water saving can be promoted.

A true scientific irrigation scheduling program also provides forecasts of future irrigation dates and amounts so that other farm operations can be planned around irrigation events. When carefully used, irrigation scheduling saves water, energy, labour, and fertilizer, and in many cases improves crop yields and crop quality. To adequately answer when and how much water to apply, root zone moisture depletion based irrigation scheduling using simulation modeling is an approach that takes into account the plant, soil, and the irrigation system. Moisture uptake based irrigation scheduling requires characteristic soil data, daily ET data and root length data along with moisture depletion data from the root zone.

In this paper, simulation has been used to develop operational irrigation schedules for "Maize" based on numerically modeled root zone moisture depletion. A fully implicit mass conservative finite difference based numerical model for predicting moisture depletion from root zone has been formulated and a non-linear root water uptake term has been used to represent the moisture extraction. The paper shows how the accurate prediction of moisture depletion leads to precise irrigation schedule, thus saving significant irrigation water which can be utilized for irrigating additional agricultural commands in case of water scarcity.

## MATERIALS AND METHODS

### Governing Equation

The mixed form of Richards equation governing water flow in the unsaturated zone, considering root water uptake can be written as,

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial Z}\left[K(\psi)\left(\frac{\partial\psi}{\partial Z}+1\right)\right] - S(Z,t) \qquad \ldots (1)$$

where $\theta$ is the volumetric moisture content of soil, $\psi$ is the pressure head, $t$ is the time, $z$ is the vertical coordinate taken positive upwards, $K$ is hydraulic conductivity, and $S(z, t)$ is the water uptake by roots expressed as volume of water per unit volume of soil per unit time.

Richards equation is highly non linear due to pressure head dependencies in the soil moisture capacity and hydraulic conductivity terms. In order to solve Richards equation, it is required to specify constitutive relations between the dependent variable (moisture content in this case) and the non linear terms (soil matric potential and hydraulic conductivity). Van Genuchten (1980) closed form equation for the soil water retention curve and unsaturated hydraulic conductivity function is used to describe the soil hydraulic properties.

### Constitutive Relationships

Van Genuchten Relationship (1980) has been used to determine the soil hydraulic characteristics in present study,

$$\Theta = \left[\frac{1}{1+\left\|\alpha\psi\right\|^{n}}\right]^{m} \quad \text{For } \psi \le 0 \qquad \ldots (2)$$

$$=1 \quad \text{for } \psi > 0 \qquad \ldots (3)$$

Where, $\psi$ is the soil matric potential, $\alpha$ and $n$ are unsaturated soil parameters with $m = 1-(1/n)$ and $\Theta$ is the effective saturation defined as,

$$\Theta = \frac{\theta-\theta_r}{\theta_S-\theta_r} \qquad \ldots (4)$$

Based on Mualem's (1976) model the relation between moisture content and hydraulic conductivity is given by (Van Genuchten, 1980),

$$K = K_{sat}\,\Theta^{1/2}[1-(1-\Theta^{1/m}{}_{v})^{m}{}_{v}]^{2} \qquad \ldots (5)$$

Where $\theta$ is the actual volumetric moisture content (cm$^3$ cm$^{-3}$), the subscripts $s$ and $r$ represents saturation and residual values of the same and $K_{sat}$ is saturated hydraulic conductivity of soil.

### Root Water Uptake

The sink term in Richards equation is represented by a non-linear root water uptake term (Ojha and Rai (1996). According to Ojha and Rai (1996), for potential transpiration conditions, the potential rate of soil water extraction $S_{max}$ is given by the relation,

$$S_{max} = \alpha\left[1-\left(\frac{z}{z_{rj}}\right)\right]^{\beta} \qquad 0 \le z \le z_{rj} \qquad \ldots (6)$$

Where $\alpha$, $\beta$ = model parameters; $z$ = depth below soil surface; and $z_{rj}$ = root depth on the $j^{th}$ day. For $z = z_{rj}$, $S_{max}$ is zero as per (6) and at $z$ = 0, $S_{max}$ attains a maximum value. Thus Eqn. (6) satisfies the desired extraction conditions, that extraction is maximum at the top and zero at the bottom of the root. Also $S_{max}$ has to satisfy the following equation,

$$\int_0^{z_{rj}} S_{max} dz = T_j \qquad \dots (7)$$

where $T_j$, is the potential rate of transpiration on the $j^{th}$ day.

Substituting for $S_{max}$ from (6) into (7) yields the following equation for $T_j$,

$$T_j = \frac{\alpha z_{rj}}{(\beta+1)} \qquad \dots (8)$$

From which α is obtained as,

$$\alpha = \frac{T_j(\beta+1)}{z_{rj}} \qquad \dots (9)$$

Using (9) in (6), $S_{max}$ is obtained as,

$$S_{max} = \left[\frac{T_j}{z_{rj}}(\beta+1)\left(1-\frac{z}{z_{rj}}\right)^{\beta}\right]; 0 \le z \le z_{rj} \qquad \dots (10)$$

It is to be noted that for $\beta = 0$, (10) reduces to a constant rate extraction model of Feddes *et al.* (1978) with $S_{max} = T_j/z_{rj}$ while for $\beta = 1$, (10) reduces to linear extraction model of Prasad (1988) with $S_{max} = 2T_j/z_{rj} - 2T_j\,(z/z_{rj}^2)$.

## Initial and Boundary Conditions

Initially the soil is assumed to be at a uniform pressure throughout its length, i.e.

$$\psi = \psi_0(Z,0) \;\; 0 \le Z \le L, t = 0 \qquad \dots (11)$$

The upper boundary condition is a prescribed flux boundary condition accounting for the evapo-transpiration taking place from the top soil and a Drichilet boundary condition, when the soil is irrigated. Thus,

$$\psi = \psi_i \qquad Z = L, \text{ during irrigation} \qquad \dots (12)$$

$$-K(\psi)\left(\frac{\partial\psi}{\partial z}+1\right) = ET \;\; Z = L, \qquad \dots (13)$$

in absence of irrigation

Where $\psi_i$ is the depth of water during irrigation and $ET$ is the potential evapotranspiration from the top soil. At lower boundary gravity drainage type condition has been assumed, where a unit hydraulic gradient is considered,

$$-K(\psi)\left(\frac{\partial\psi}{\partial z}+1\right) = K(\psi) \text{ for } t \ge 0, Z = 0 \qquad \dots (14)$$

## Numerical Model

A numerical model has been developed to solve equation (1) along with the sink term being considered, subjected to initial and boundary conditions (11) to (14), and employing the constitutive relationships (Eqns. (2) to (5)). The numerical model is based on a mass conservative, fully implicit finite difference scheme proposed by Celia *et al.* (1990). The non linear system of equations is linearized using Picard's methods (Paniconi *et al.*, 1991) and resulting system of equations are solved using Thomas algorithm. The model yields spatial distribution of pressure head and moisture content at successive advancing times in the soil. From the model computed moisture contents, the irrigation intervals are predicted based on the moisture depletion occurring from field capacity to AMC.

## Field Crop Experiments

Field crop experiments have been conducted at the field experimental station of Civil Engineering Department, Indian Institute of Technology, Roorkee, India, from April to September, 2006. The average annual rainfall at Roorkee is 1032 mm, of which about 75% is usually received between July and September. The required meteorological data for the computation of corresponding crop evapotranspiration using crop coefficient approach is obtained from the Department of Hydrology, Indian Institute of Technology Roorkee. For measuring the soil moisture profile throughout the crop season soil moisture measurement sensors have been embedded at 0.15, 0.30, 0.45, 0.60, 0.75, 0.90, 1.05 and 1.20 m, however at the ground surface the moisture content is measured using TDR soil moisture meter.

### *Crop Details*

Maize (Variety K-99 HYBRID) was sown in open fields. Crop period of Maize lasted from May 20th to September 1st, 2006 (105 days). Different plant parameters such as Leaf Area Index (LAI) and root depth were carefully observed. The entire crop growth period for the crops is divided into four stages; I–Initial, II–Crop Development, III–Mid Season and IV–Late Season. Growth stages have been considered on the basis of study by Doorenbos and Pruit (1977). Initial stage corresponds to the germination and early growth when the soil surface is not or is hardly covered by the crop (ground cover < 10%). Crop development stage starts from the end of initial stage to attainment of effective full ground cover (ground cover: 70–80%). Mid season commences from the attainment of effective full ground cover to time of start of maturing as indicated by discoloring of leaves or leaves falling off and late season stage begins from end of mid-season until full maturity or harvest. Duration of stage I, II, III and IV accordingly has been found to be 17, 30, 34 and 24 days respectively. Irrigations have been provided on 24th, 33rd and 42nd day of the crop period.

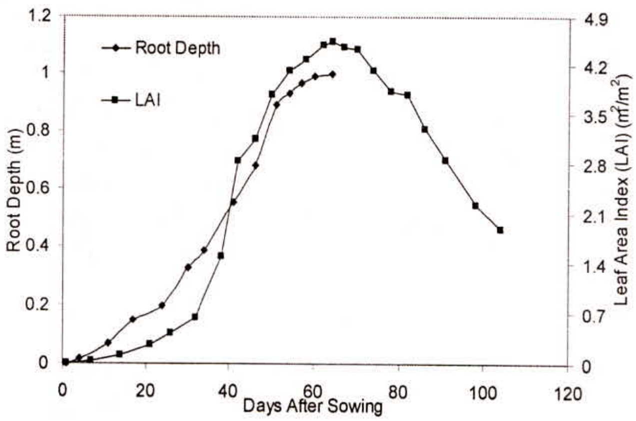


**Fig. 1:** Field Observed Plant Parameters for the Maize

Leaf Area Index (LAI) required for the partitioning of the crop evapotranspiration into plant transpiration and soil evaporation, was measured by direct method suggested by Jesus *et al.* (2001). Root depth has been measured by trench profile method described by Wolfgang (1979). Figure 1 shows the variation of root depth and LAI with crop growth period for maize.

### *Soil Parameters*

Representative soil samples were obtained from the 0–0.3 m, 0.3–0.6 m, 0.6–0.8 m, 0.8–1.0 m and 1.0–1.2 m depths, in the experimental site for testing the soil properties. USDA soil textural class for the experimental field soil is sandy loam. The bulk density, particle density and porosity for the field soil are 1.62 g/cm$^3$, 2.61 g/cm$^3$ and 0.38 respectively.

In-situ determination of soil moisture characteristic curve (SMC) has been performed, which involves simultaneous measurement of soil matric potential ($\psi$) and moisture content ($\theta$) at 0.3, 0.6, 0.9 and 1.2 m depths below the ground level. Van Genuchten Relationship (1980) has been used to determine the soil hydraulic characteristics.

The saturated moisture content $\theta_s$ in Eqn. (4) is assumed to be equal to the measured soil porosity (0.38 cm$^3$ cm$^{-3}$). A standard residual moisture content value equal to 0.065 cm$^3$ cm$^{-3}$ (Carsel and Parrish, 1988) for sandy loam soil (soil type for experimental plot) has been considered. A non linear optimization algorithm E04FDF (N.A.G., 1990) has been used to estimate the Van Genuchten parameters $\alpha$ and n, which are 6.2 m$^{-1}$ and 1.68 respectively. The value of average field saturated hydraulic conductivity ($K_{sat}$) determined at different depths using Guelph type Permeameter is 3.9 cm/hour. Experimentally obtained value of field capacity ($\theta_{fc}$ = 0.208) and SMC deduced value of wilting point ($\theta_{pwp}$ = 0.068) has been used in the present study. The available moisture which is the difference of $\theta_{fc}$ and $\theta_{pwp}$ is 0.14. The irrigation has been provided at 50% depletion of the available moisture in the effective root zone.

### Computation of Crop Evapotranspiration ($ET_c$)

Crop evapotranspiration has been determined as the product of daily crop coefficient and reference evapotranspiration. During the study period $ET_0$ (mm/day), has been computed by Penman Monteith method. The Penman-Monteith equation for the $ET_0$ is given as (Allen *et al.*, 1998),

$$ET_0 = \frac{0.408\Delta(R_n - G) + \gamma \frac{900}{T+273} u_2 (e_s - e_a)}{\Delta + \gamma(1 + 0.34u_2)} \quad \dots (15)$$

Where, $R_n$ = net radiation at the crop surface [$MJ$ m$^{-2}$ day$^{-1}$], $G$ = soil heat flux density [$MJ$ m$^{-2}$ day$^{-1}$], $T$ = mean daily air temperature at 2 m height [°C], $u_2$ = wind speed at 2 m height [m s$^{-1}$], $e_s$ = saturation vapour pressure [kPa], $e_a$ = actual vapour pressure [kPa], ($e_s - e_a$) = saturation vapour pressure deficit [kPa], $\Delta$ = slope vapour pressure curve [kPa °C$^{-1}$], $\gamma$ = psychrometric constant [kPa °C$^{-1}$].

Different parameters involved have been computed using the mathematical formulations provided by Allen *et al.* (1998). Figure 2 shows the daily $ET_0$ (mm/day) computed using Penman-Monteith method for the study period. Comprehensive list of stage-wise crop coefficients is available in literature (Allen *et al.*, 1998). The crop coefficients for initial, development, mid-season and end-season stages are denoted as $K_{c\ ini}$, $K_{c\ dev}$, $K_{c\ mid}$ and $K_{c\ end}$ respectively. FAO proposed $K_{c\ ini}$, $K_{c\ mid}$ and $K_{c\ end}$ values are 0.3, 1.2 and 0.6 for Maize. These values have been modified for the local climatic, crop and soil characteristics according to the procedure outlined in FAO guidelines.

The modified values of $K_{c\ ini}$, $K_{c\ mid}$ and $K_{c\ end}$ are 0.33, 1.126 and 0.55 respectively. Following Allen *et al.* (1998), the crop coefficient for an $i^{th}$ day in a particular stage is computed as,

$$K_{ci} = K_{c,prev} + \left[ \frac{i - \sum(L_{prev})}{L_{stage}} \right] \left( K_{c,next} - K_{c,prev} \right) \quad \dots (16)$$

where, $i$ is the day number within the growing season, $K_{c\,i}$ crop coefficient on day $i$, $L_{stage}$ is length of the stage under consideration [days], and $L_{prev}$ is the sum of the lengths of all previous stages [days]. Using equation (16) daily crop coefficients for Maize are determined.

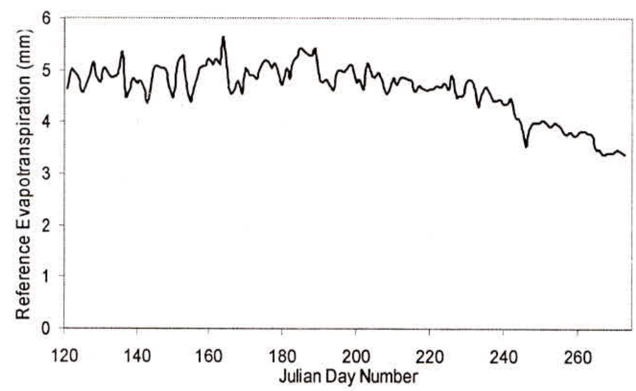


**Fig. 2:** Daily Reference Evapotranspiration during Study Period

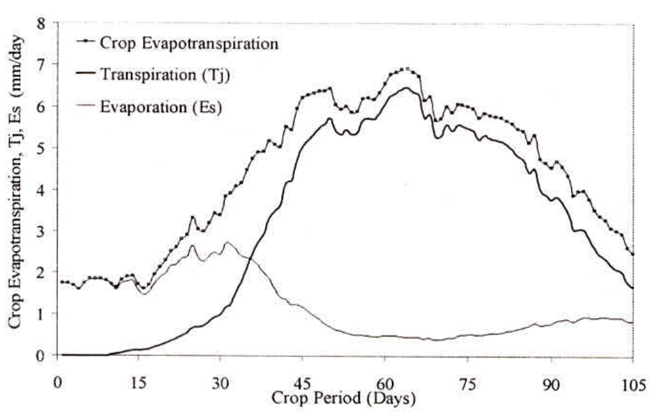


**Fig. 3:** Daily Crop Evapotranspiration, Evaporation and Transpiration for Maize

Daily crop evapotranspiration is determined as the product of daily $K_c$ value and reference evapotranspiration. Further, the daily crop evapotranspiration is partitioned into plant transpiration and soil evaporation using Eqn. (17), proposed by Belmans *et al.* (1983), where soil evaporation ($E_s$) is calculated as a fraction of the $ET_c$ using the LAI of the soil surface.

$$E_s = f \times \text{EXP}(-c * \text{LAI})\, ET_c \quad \dots (17)$$

where, $f$ and $c$ are regression coefficients, with $f = 1.0$, and $c = 0.6$. This relation gives an acceptable estimation of soil evaporation (Belmans *et al.*, 1983). Plant transpiration is part of the $ET_c$, and it can be calculated after $E_s$ is determined. Since $ET_c = E_s + T_p$, plant transpiration ($T_p$) is $T_p = ET_c - E_s$.

The plant transpiration is used as the sink term in the Richards equation and the soil evaporation is used as the boundary condition at the ground surface. Figure 3 shows the variation of crop evapotranspiration and its components, evaporation and transpiration for Maize throughout the crop period. The average daily crop evapotranspiration of Maize varied from a range of 1.4 to 3.4 mm day$^{-1}$ in the early growing period to 7.2 mm day$^{-1}$ at peak that occurred 9 Weeks After Sowing (WAS) at the silking stage of maize, when Leaf Area Index (LAI) was 4.54. Average daily $ET_c$ declined sharply to 2.57 mm day$^{-1}$ during late season stage of crop.

## RESULTS AND DISCUSSION

The obtained soil moisture characteristics, crop evapotranspiration and root depth variation over the crop period applied to the numerical model formulated by coupling Richards equation with O-R model to simulate plant moisture uptake. Initially the optimal value of the non-linearity parameter β of O-R model is determined using observed and simulated soil moisture depletion pattern. The optimal value of β for Maize has been found to be 1.5. Observed and simulated soil moisture profiles in the vadoze zone on discrete days and soil moisture status during the crop period of Maize has been compared and found to show good agreement. This indicates that numerical model involving O–R model coupled with soil moisture flow equation, when applied to precisely determined soil parameters, crop data and crop evapotranspiration accurately simulates the soil moisture dynamics in the crop root zone. This provides the exact soil moisture availability for the plant moisture uptake in the crop root zone. Generally the irrigation is practiced when the average moisture content with in the root zone depth attains certain value between the field capacity and permanent wilting point (Prasad, 1988). This value of moisture content is called the allowable depletion level.

For different depletion levels required scheduling of irrigation is carried out. For optimal scheduling, adequate scheduling criterion is an important parameter in determining the frequency of irrigation events. The two parameters which contribute to assigning an adequate scheduling criterion are; allowable moisture depletion level and root depth considered for accounting the average soil moisture level. The hypothetical condition

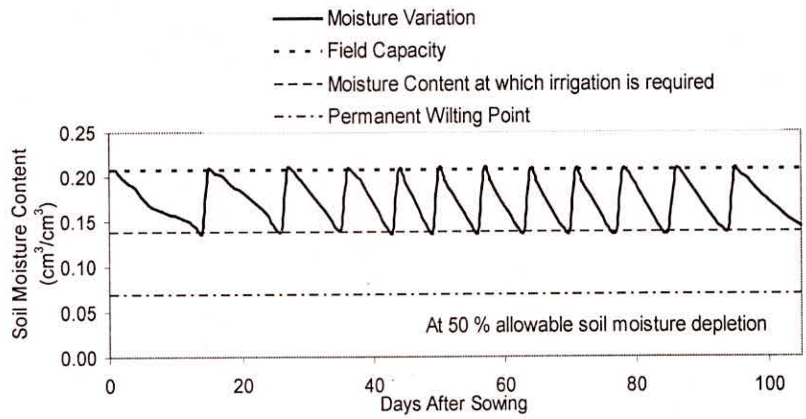


*Fig. 4*

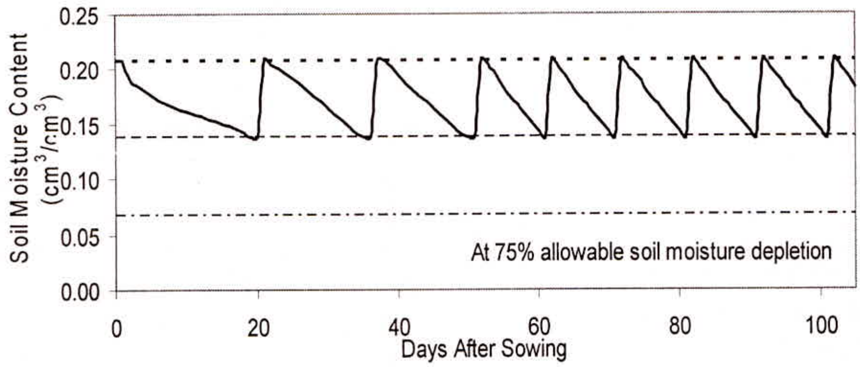


*Fig. 5*

**Figs. 4 & 5:** Irrigation Schedule for Maize at Different Allowable Moisture Depletion Levels

of no-rainfall is considered during the crop period of Maize. Though, allowable moisture depletion level is dependent on the type of crop and the moisture retention capacity of the soil, 50% and 75% moisture depletion levels are considered in the present study. The effective root depth considered for accounting the average soil moisture status is 0.3 m. The optimal irrigation schedule at 50 and 75% allowable moisture depletion level are given in Figures 4 and 5.

## SUMMARY AND CONCLUSIONS

A numerical model has been formulated to predict the soil moisture content profiles under transient field conditions. A non-linear root water uptake model has been used as sink term to represent plant moisture uptake. Numerical model takes into account a variable transpiration rate and non-uniform initial soil moisture content. Rainfall, irrigation and evaporation are treated as sources of non-uniform potential surface flux. Plant control on water uptake when soil moisture is a limiting factor is not considered. The input parameters have been precisely determined using the field crop experiments. The model formulated satisfactorily simulates the field conditions.

Application of the numerical model to field conditions and comparison of the results with field measured data showed very agreement. Precisely determined crop evapotranspiration is the dominant factor in predicting soil moisture dynamics. The practical significance of the study lies in the design of optimal irrigation schedules for simulated field conditions using the numerical model coupled with adequate scheduling criterion. Accurately predicted soil moisture profiles result in generating optimal frequency of the irrigation and hence, results in irrigation water saving.

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