

## Water Resources Planning Using Multiobjective Differential Evolution

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**ABSTRACT:** Multiobjective formulations are realistic models for many complex engineering optimization problems. In this paper, the applicability of Multiobjective Differential Evolution (MODE) in irrigation planning is demonstrated for a case study of Mahi Bajaj Sagar Project, India. Three objectives, namely, net benefits, agricultural production and labour employment are analyzed in the multiobjective environment. Multiobjective model is subjected to constraints such as continuity equation, land area limitations, crop diversion requirements, minimum and maximum area of crops, live storage and canal capacity restrictions. Seventy five percent dependable inflow level is considered for the planning problem. Four variations (strategies) of Differential Evolution, namely, DE/rand/1/bin, DE/rand/2/bin, DE/best/1/bin and DE/best/2/bin are assessed in multiobjective environment. The parameters that control the functioning of MODE are population size, crossover and mutation probabilities along with number of generations. Various combinations of above parameters are also tested to assess the robustness of the methodology. It is observed that DE/rand/1/bin and DE/rand/2/bin generate similar trend of results with a reasonably good pareto-optimal front as compared to that of DE/best/1/bin and DE/best/2/bin. The results obtained prove the efficiency of the proposed MODE. Effect of parameters is significant, indicating the necessity of careful selection of parameters and strategies.

### INTRODUCTION

Most real world problems involve the simultaneous optimization of two or more (often conflicting) objectives. The solution of such problems (called "multi-objective") is different from that of a single objective optimization problem. The main difference is that multi-objective optimization problems normally have not one but a set of solutions which are all equally good. Non-traditional optimization methods are gaining importance due to their advantage of handling non-differentiable, nonlinear, multimodal functions having complex search space with many local optimal solutions in a systematic and effective way (Ranjithan, 2005). On the other hand, demand for water is increasing day by day due to more requirements from irrigation, municipal and industrial sectors. This necessitates effective methodology for increasing productivity of water for irrigation. Efficient irrigation strategies can be formulated in this regard in multi-objective environment with the help of mathematical models and utilization of effective methodologies.

However, difficulty of using non-traditional methods as envisaged by researchers is number of input parameters and determination of their precise values, thus making the solution process rather complex (Vasan, 2005). This complexity along with fuzziness/ uncertainty in available resources necessitated to find robust solution that can be used by irrigation managers with ease while planning the cropping pattern and reservoir scheduling for the command area. In the present study, a non-traditional optimization method, namely, Differential Evolution is applied in multiobjective environment to a case study of Mahi Bajaj Sagar Project, India. The study is divided into description of Multiobjective Differential Evolution, literature review, case study and mathematical model for irrigation planning, results and discussion followed by conclusions.

### MULTIOBJECTIVE DIFFERENTIAL EVOLUTION (MODE)

Differential Evolution (DE) algorithm is a recent evolutionary algorithm designed to optimize problems

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over continuous domains. The advantages of DE are its simple structure, ease of use, speed and robustness. The method is a population based search technique which utilizes NP as population of D dimensional parameter vectors for each generation. The innovative idea behind DE is a new scheme for generating trial parameter vectors. DE generates new parameter vectors by adding the weighted difference vector between two population members to a third member. If the resulting vector yields a fitter/better objective function value(s) than a predetermined population member, the newly generated vector replaces the vector with which it was compared. In addition, the best parameter vector is evaluated for every generation in order to keep track of the progress that is made during the minimization process. Extracting distance and direction information from the population to generate random deviations result in an adaptive scheme with excellent convergence properties.

Price and Storn (1997) initially gave the working principle of DE/rand/1/bin (explained as above) and

later extended to nine more strategies. In the present study, 4 DE strategies i.e., DE/rand/1/bin, DE/rand/2/bin, DE/best/1/bin and DE/best/2/bin (equivalent to DE/x/y/z in notation) are applied in multiobjective environment. Here DE/x/y/z indicates DE for Differential Evolution,  $x$  represents a string denoting the vector to be perturbed,  $y$  is the number of difference vectors considered for perturbation of  $x$  and  $z$  stands for the type of crossover being used.

In the present study, DE is extended in multi-objective environment (MODE) and uses non-dominated sorting for sorting the members in different fronts, penalty function approach, crowding distance operator for finding out the members having maximum diversity. More information on multiobjective methodologies is available in Deb (2001) and Deb *et al.* (2002). A flowchart indicating the working of MODE algorithm is shown in Figure 1. Important parameters that control evolution under DE are, the population size, crossover and mutation probabilities along with number of generations.

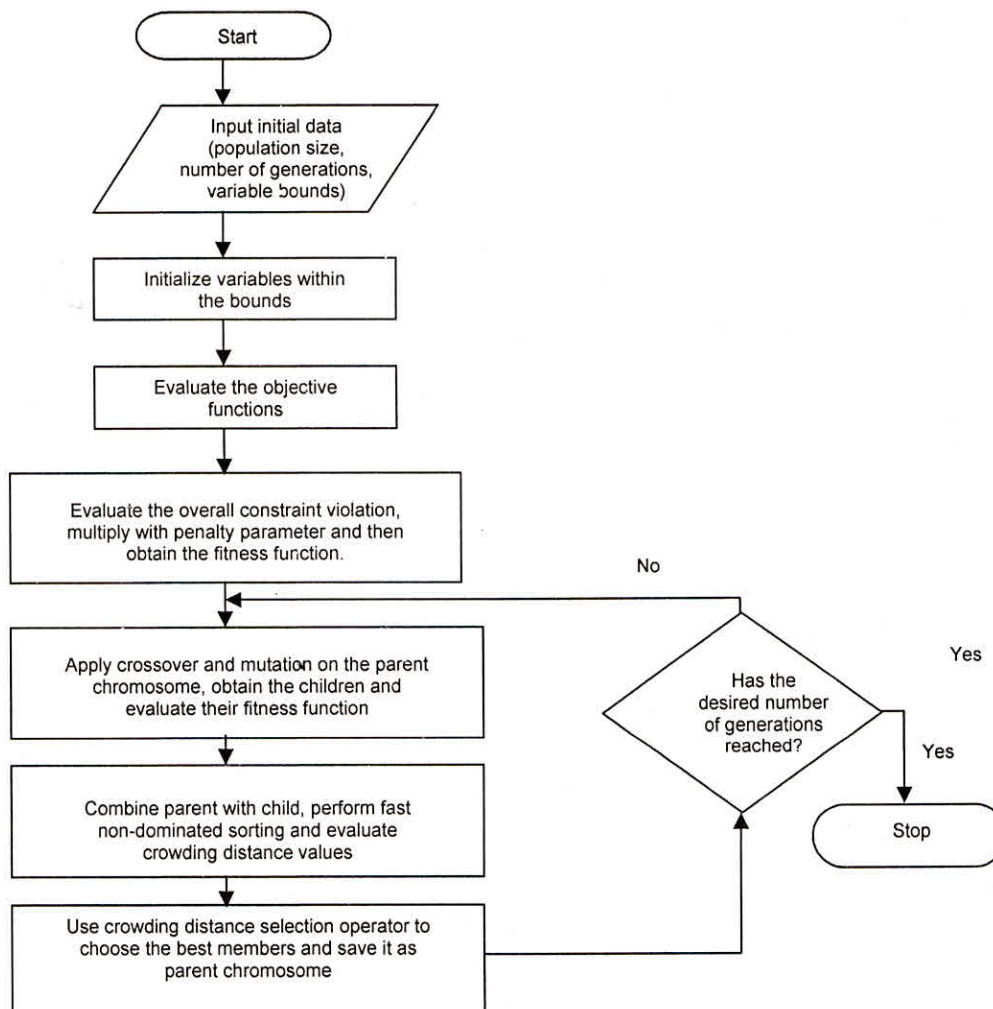


Fig. 1: Flow chart of MODE algorithm

## LITERATURE REVIEW

Numerous literature is available on Differential Evolution. However, a brief and relevant literature is presented in this paper. Lampinen and Storn (2004) explained Differential Evolution with three numerical examples. They performed sensitivity analysis on various parameters such as population size, crossover constant and weighting factor. It is concluded that the method is easy to implement and use, effective, efficient and robust. Ranjithan (2005) stressed on the role of evolutionary computation in environmental, water resources systems analysis and discussed the various methods in brief, namely, Simulated Annealing, Tabu Search, Genetic Algorithm, Evolutionary Strategies, Particle Swarm technique and Ant Colony Optimization. He concluded that new areas that shape the direction of a beneficial integration of evolutionary computation into environmental and water resources systems are essential (Vasan, 2005).

Janga Reddy and Nagesh Kumar (2007) presented Multiobjective Differential Evolution (MODE) with an application to a case study of Hirakund Reservoir, Orissa. The developed algorithm is first tested on a few bench-mark test problems and validated with standard performance measures by comparing them with Non dominated Sorting Genetic Algorithm-II (NSGA-II). They used five different DE strategies. Two objectives, maximize hydro power production and minimize the annual sum of squared deficits of irrigation release from demands were considered. The initial population for MODE was set to 200 with crossover of 0.3 and a mutation constant of 0.5. For NSGA-II, the initial population was set to 200, crossover of 0.9 and mutation probability of  $1/n$ , where  $n$  is the number of variables ( $n = 108$ ). SBX and real parameter mutation were set at 10 and 20 respectively. The best compromise solution is also found. It is concluded that MODE outperformed NSGA-II for the given planning problem. It is inferred from above literature review that very little work has been reported in the field of irrigation planning in multiobjective framework involving three objectives.

## CASE STUDY AND MATHEMATICAL MODELING FOR IRRIGATION PLANNING

Mahi Bajaj Sagar Project (MBSP) is located in Banswara district in southern part of Rajasthan. The project is situated near a village Borkhera about 16 km northeast of Banswara town. The project includes a dam with gross and live storage capacity of the reservoir 2180.39 Mm<sup>3</sup>, 1829.27 mm<sup>3</sup>, (here mm<sup>3</sup> = 10<sup>6</sup> m<sup>3</sup>),

Kagdi pick up weir, three canals, namely, Left Main Canal (LMC), Right Main Canal (RMC), Bhungra Canal (BC) and two hydroelectric power houses (*PH 1* located near Banswara with installed capacity of two 25 MW each, and *PH 2* near Lilvani village with installed capacity of two 45 MW each). Culturable Command Area (CCA) and cropping intensity of the project is 80000 hectare (ha) and 89% respectively (MBSP Report on Status June 2002 at a Glance, 2002). Mathematical model is formulated in three objective environment, namely, (1) net benefits (2) agricultural production and (3) labour employment (Raju, 1995). These are explained briefly below:

### Annual Net Benefits

The Annual Net Benefits (BEN) from planning region under different crops after meeting the costs of seeds, fertilizer, labour, surface water, ground water and plant protection are to be maximized and can be described as,

$$\text{Max } BEN = \sum_{i=1}^{36} B_i \cdot A_i - P_{GW} \sum_{t=1}^{12} GW_t \quad \dots (1)$$

where  $i$  is crop index (1 to 12 represents Maize, Paddy, Cotton, Pulses, Sugarcane, Zaid Crop (Kharif/Summer season), Wheat, Barley, Gram, Barseen, Mustard, Fruits & Vegetables (Rabi/Winter season) respectively for LMC; 13 to 24 for RMC (same crops as above); 25 to 36 for BC (same crops as above);  $t$  is time index (1 for January and 12 for December).  $A_i$  is area of crop  $i$  grown in the command area (ha);  $B_i$  is net return from  $i^{\text{th}}$  crop including ground water cost (Rs/ha);  $GW_t$  is monthly ground water withdrawal (10<sup>6</sup> m<sup>3</sup>);  $P_{GW}$  is ground water cost (Rs./10<sup>6</sup> m<sup>3</sup>).

### Agricultural Production

Agricultural production of all the crops for the whole planning region is to be maximized.

### Labour Employment

Labour employment for each crop  $i$  annually for the whole planning region is to be maximized.

The mathematical model is subjected to constraints such as (Vasan, 2005):

- Monthly continuity equation for the reservoir storage
- Crop area restrictions in Kharif and Rabi seasons
- Water releases from the reservoir and ground water that satisfy the irrigation demands of the command area
- Reservoir storage
- Canal capacity restrictions

- Monthly releases for hydropower production in power house 1 and 2 that must be greater than or equal to minimum specified value
- Crop diversification
- Total monthly ground water withdrawal in a year should be less than or equal to the estimated annual ground water potential of the aquifer.

**RESULTS AND DISCUSSION**

MATLAB code for NSGA-II, developed by Seshadri (2008), has been suitably modified for the working of MODE. Penalty function approach is employed in the present study to handle the constraints. The applicability of MODE is demonstrated with a test example.

**Test Example**

A test problem named “CONSTR” which is reported as difficult to be solved by general multiobjective solvers is considered to evaluate the efficiency of MODE (Deb *et al.*, 2002). CONSTR is a two variable problem that involves two objective functions and two constraints which is presented below,

$$\text{Min } f_1(x) = x_1; \text{ Min } f_2(x) = (1 + x_2) / x_1$$

$$\text{Subject to: } g_1(x) = x_2 + 9x_1 \geq 6; \quad g_2(x) = -x_2 + 9x_1 \geq 1$$

$$x_1 \in [0.1, 1.0]; \quad x_2 \in [0, 5]$$

The problem is analyzed with four strategies DE/rand/1/bin, DE/rand/2/bin, DE/best/1/bin and DE/best/2/bin. The complexity involved in the CONSTR example is the non-smooth Pareto-optimal curve. Pareto curve obtained by Deb *et al.* (2002) and Janga Reddy and Nagesh Kumar (2007) is almost similar as compared to the pareto curve obtained by two strategies DE/rand/1/bin, DE/rand/2/bin with 100 generations and a penalty of 2 and 20 for the first objective function and second objective function respectively. Pareto curve is not same for other two strategies indicating that selection of suitable DE strategy and penalty values play a major role. Figures 2(a) to 2(d) presents pareto-optimal curves for four strategies, namely, DE/rand/1/bin, DE/best/1/bin, DE/best/2/bin, and DE/rand/2/bin respectively.

**CASE STUDY**

Inflow levels with seventy five percent dependability are considered for the planning problem. Dependable inflow values into the reservoir for the month of June, July, August, September and October are considered.

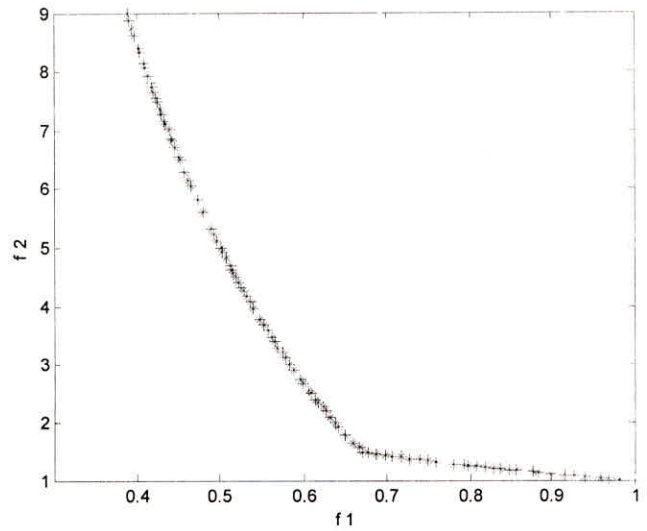


Fig. 2(a): Pareto front for strategy DE/rand/1/bin

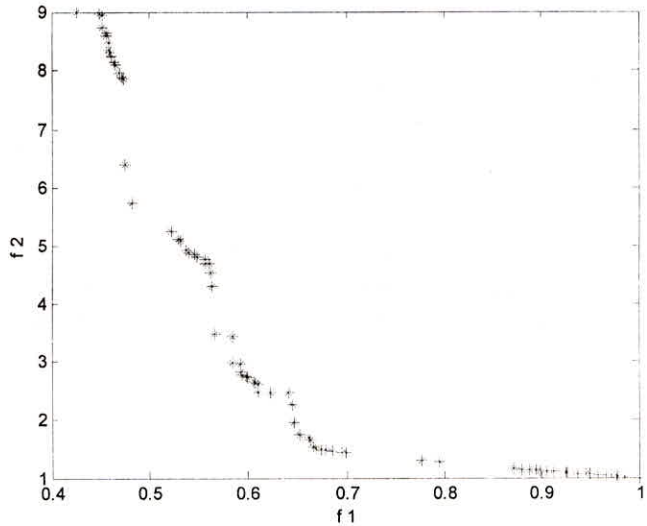


Fig. 2(b): Pareto front for strategy DE/best/1/bin

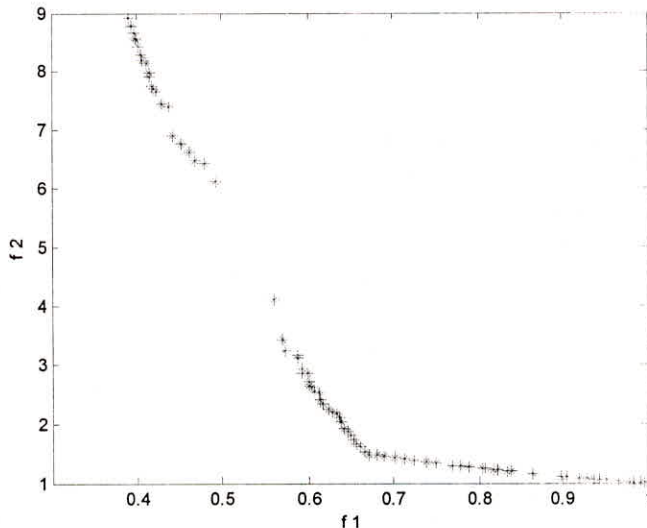


Fig. 2(c): Pareto front for strategy DE/best/2/bin

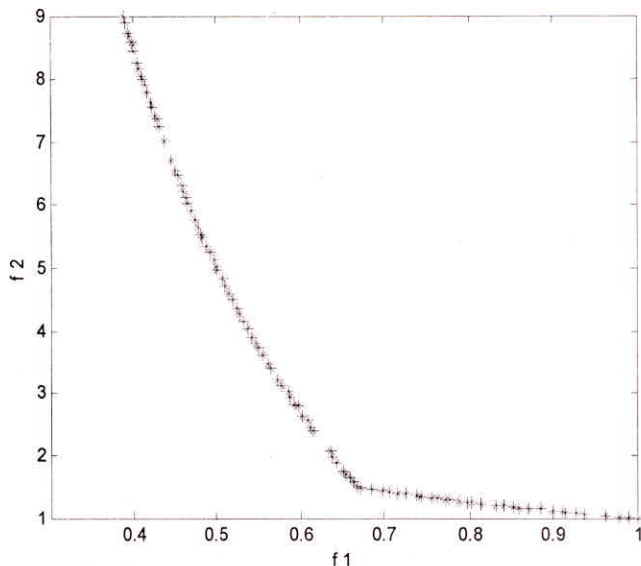


Fig. 2(d): Pareto front for strategy DE/rand/2/bin

Inflows into the reservoir for other months are not significant. The originally proposed CCA of MBSP is 80,000 ha (Phase 1) and the revised (proposed) CCA in Phase 2 is 1,23,500 ha. Out of these, 57,531 ha have been opened for irrigation. The model is validated for CCA that is opened for irrigation and proposed as per phase 1. Accordingly the lower and upper limits are fixed based on cropping intensity of 89% i.e., 89% of 57,531 ha and 89% of 80,000 ha with existing live storage of reservoir 1829.27 mm<sup>3</sup>. This enables to determine the lower and upper bounds for all the decision variables. The model includes a set of 67 constraints and 121 variables.

In the present study, a dynamic penalty method is used where the penalty parameter is changed with the generation counter. MODE is run for 500 generations with population size of 100, crossover and mutation probability to 0.9 and 0.1. Figure 3 presents results for strategy DE/rand/1/bin. It can be observed from Figure 3 that the Pareto-optimal curve is reasonably continuous. The annual net benefits are varying from Rs.  $1.134 \times 10^9$  to Rs.  $1.1355 \times 10^9$ ; agricultural production from  $2.62 \times 10^6$  to  $2.645 \times 10^6$  tonnes and labour employment from  $4.96 \times 10^6$  to  $4.985 \times 10^6$  Man-days. It is inferred that there is reasonably good variation. Figure 4 presents pareto-optimal front for DE/rand/2/bin with variation of annual net benefits (Rs.  $1.14 \times 10^9$  to Rs.  $1.16 \times 10^9$ ), agricultural production ( $2.764 \times 10^6$  to  $2.772 \times 10^6$  tonnes) and labour employment ( $5.159 \times 10^6$  to  $5.165 \times 10^6$  Man-days). It is observed that pareto-optimal front converges to a single point, when MODE is run with strategy DE/best/1/bin, with values of net benefits

(Rs.), agricultural production (tonnes) and labour employment (Man-days) as  $9.891 \times 10^8$ ,  $1.698 \times 10^6$  and  $3.9324 \times 10^6$  respectively. Similar trend is observed for DE/best/2/bin.

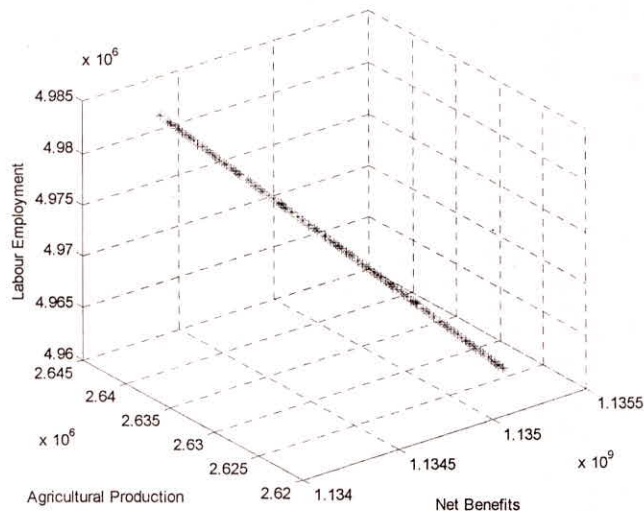


Fig. 3: Pareto-optimal front after 500 generations (DE/rand/1/bin)

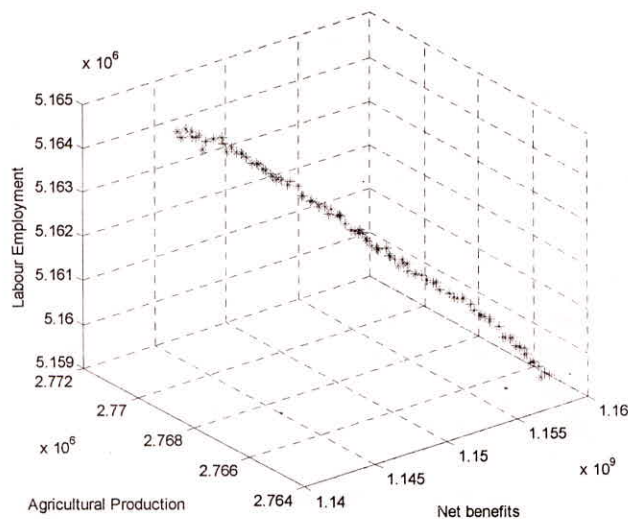


Fig. 4: Pareto-optimal front after 500 generations (DE/rand/2/bin)

It is observed that strategies DE/rand/1/bin and DE/rand/2/bin are showing comparatively similar trend of results with a reasonably good pareto-optimal front as compared to that of DE/best/1/bin and DE/best/2/bin. The behaviour of MODE for strategies DE/best/1/bin and DE/best/2/bin have made the authors to investigate the theory of multiobjective optimization in depth. The analysis of the study is still in progress and interested readers may contact authors for further information. Similar inferences are drawn with test example CONSTR where DE/rand/1/bin and DE/rand/2/bin

are showing similar result pattern. Among DE/rand/1/bin and DE/rand/2/bin, DE/rand/1/bin can be considered for further analysis, due to significant variations in three objectives as compared to DE/rand/2/bin.

An effort is also made to study the effect of crossover and mutation probability values for various population sizes and generations. It is observed that significant effect of number of generations is observed for given population size, crossover and mutation probability values on three objectives for the chosen strategy. The present study can be extended to other similar situations with suitable modifications.

## CONCLUSIONS

Multiobjective Differential Evolution is applied for a case study of Mahi Bajaj Sagar Project (MBSP) and following conclusions are drawn based on chosen set of parameters and case study:

1. DE/rand/1/bin and DE/rand/2/bin generated a better pareto-optimal curve as compared to that of DE/best/1/bin and DE/best/2/bin.
2. It is observed that significant effect of number of generations is observed for given population size, crossover and mutation probabilities on three objectives for the chosen strategy.

It is suggested that MODE with DE/rand/1/bin can be used as an efficient multiobjective optimization solver. Further study is in progress, to investigate the reason for the convergence of pareto-optimal front to a single point using DE/best/1bin and DE/best/2/bin.

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