URBAN STORM WATER RUNOFF AND FLOOD ROUTING

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INTRODUCTION

As man continues to build on the flood plain, an increased understanding of the nature of floods remains an ever present challenge. For Hydraulic engineers, an assessment of the characteristics of flood waves is the logical starting point at the planning and design stages of flood control works. This lecture presents an overview of flood propagation phenomena in open channels. Some fundamental concepts on the nature of flood waves are introduced. This leads naturally into flood routing and a classification of flood routing methods. Further the lecture introduces general aspects of hydrologic flood routing methods. The conventional Muskingum method which is most commonly used in practice is discussed in detail.

The objective of the lecture is to introduce the participants with the urban storm runoff and various aspects involved in the application of Muskingum method, one of the most widely used method of flood routing. The latest developments and experiences of various researchers are also discussed. This will be useful in indigenous use of the method and help in providing better understanding of limitations of the method.

1.0 COMPLEXITY AND INTERACTIONS IN URBAN SURFACE DRAINAGE

It is now generally acknowledged that urban flooding and pollution are frequently the result of multiple urban land use sources associated with a combination of overland flow, sewer surcharging and receiving watercourse overloading. Figure 1 provides detail on the sources, pathways, receptors and return flows for the urban drainage system and emphasises the complexity of interactions between the various above and below ground sources. The intraurban response to flooding operates through various process mechanisms and acts on differing spatial scales, combining above and below ground systems, storage facilities and flow routes. Four individual but interlinked drainage systems can be identified from Figure 1:

- a foul (combined) sewerage system with combined sewer overflows (CSOs) discharging to the receiving water,
- a separate surface water sewer system with surface water outfalls (SWOs) discharging to the receiving water,
- the receiving water (normally "heavily modified"),
- and exceedance surface flows during extreme wet weather conditions.

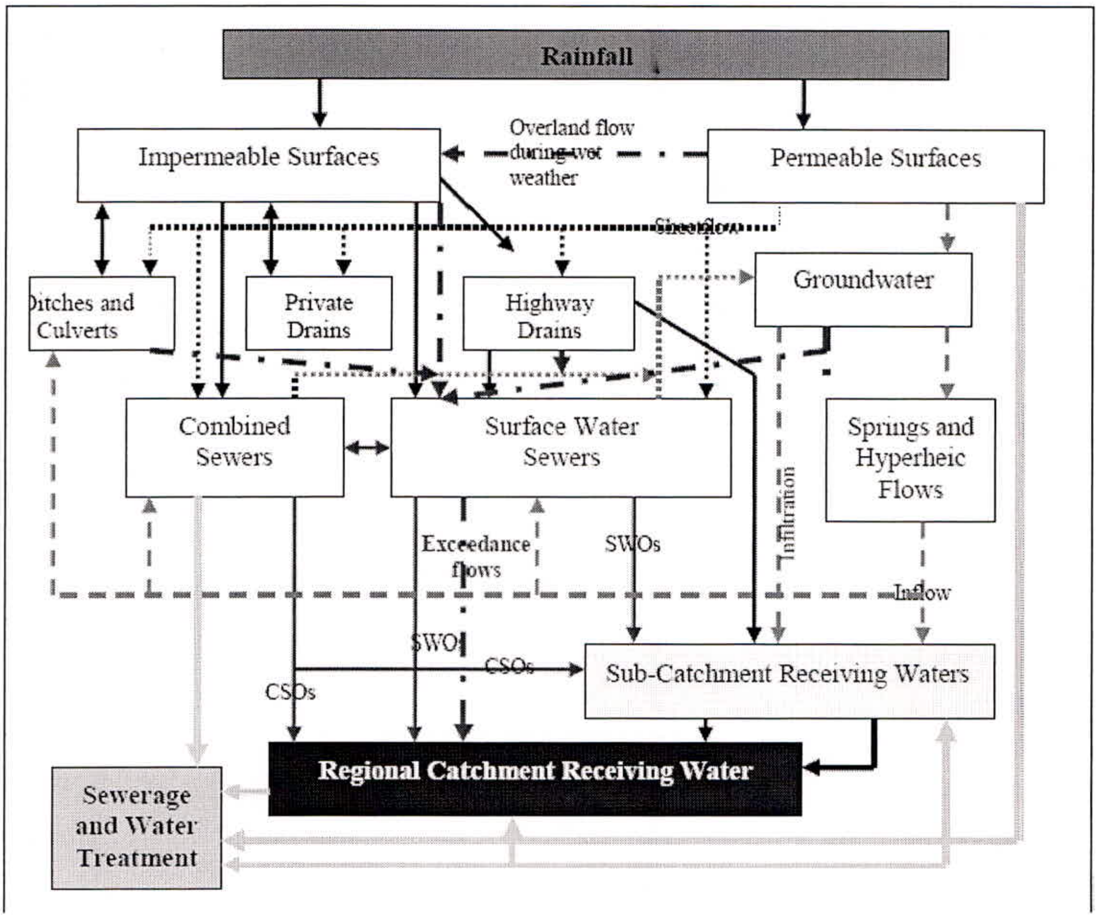

Figure 1. Interactions Between Urban Drainage Sources, Pathways and Receptors.

Interconnections, including system cross- (or mis-) connections, infiltration and inflow pathways as well as system abstractions further complicate the process interactions. A theoretical distinction can be made between pluvial flooding caused by rainfall-runoff over impermeable surfaces and exceedance flooding caused by a combination of surcharging, overland flow and sheetflow from sewers, impermeable and saturated permeable surfaces, as well as over-top flooding from ditches and culverts during extreme wet weather conditions.

These distinct source contributions are identified in Figure 1 although it is practically impossible to separately identify them under actual field conditions. During extremely intense rain storms, pluvial flooding may occur on the urban surface even when the sewer network is only subject to free surface (non-surcharging) conditions i.e. under non-exceedance conditions as the roadside gullies cannot pass the surface runoff fast enough into the belowground sewer system. However, the interactive nature of urban drainage systems demands a fully integrated, GIS-based modelling approach to simulate a replication of the real flooding and pollution situation during extreme events (>1:30 RI). Potential responses to the flood and associated pollution driver mechanisms must take into consideration this complexity of sources and scales of operation. Control and management approaches should therefore consider the level of the individual building (and curtilage), through the plot, site and subcatchment levels with initial data input to the modelling process giving indications of flood mechanisms and interactions, the areal distribution and frequency of flooding as well as damage consequences. A crucial data component is that relating to road gully and manhole location, spacing and surcharging contributions to surface overland flow during storm events. Figure 2 illustrates the complex hydraulic interactions between major (overland) and minor (sewer) drainage systems that can occur under exceedance flow conditions during a storm event and which can lead to "coincident" flooding. The varying storm design standards shown for the differing parts of the sewer system further illustrate how the hydraulic capacity of the minor system is readily overcome during extreme events with roadside gully chambers, normally designed to a 1:1 - 1:2 RI capacity, being rapidly drowned out and contributing to exceedance flows in the highway cross-section.

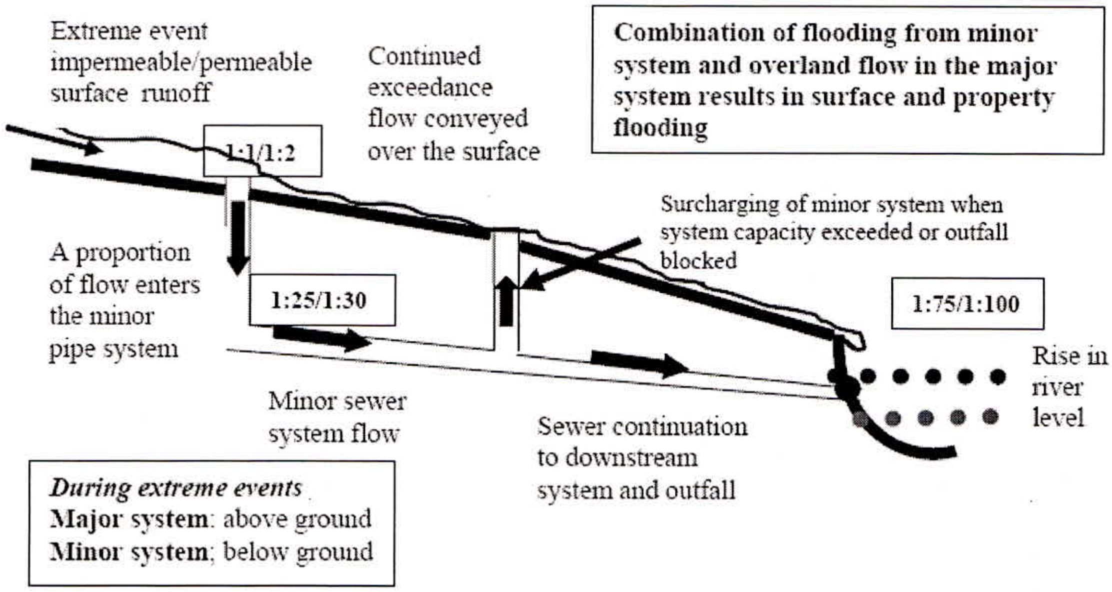

Figure 2. Urban Drainage System Extreme Event Interactions
(Based on Digman *et al.*, 2006)

The key control interaction for management action is the ability to discharge excess flows generated within the development site to appropriate temporary storage and/or infiltration facilities in order to reduce surface flooding and to improve the final discharge quality. In addition, above-ground flood routes and temporary storage for extreme event exceedance flows need to be delineated rather than seeking to expand or enlarge traditional below-ground conveyance systems given the prohibitive costs of system rehabilitation and enlargement.

2. NATURE OF FLOOD WAVE

It is perhaps an oddity that the nature of the flood waves is most readily grasped by looking at the system rather than at the flood wave itself. The system could be either a channel, a reservoir (or lake), or a channel-reservoir combination. This distinction is of fundamental importance, as will be shown here. The channel case is usually associated with the existence of a finite (nonzero) water surface slope. The reservoir case is normally taken to imply a zero water surface slope.

Flood waves traveling downstream in a channel or reservoir, in general, are subject to attenuation. The rate of travel (flood wave velocity) and the rate of attenuation depend on the system in which the flood waves are moving.

2.1 Flood waves in Stream Channels

The attenuation rate of waves is a function of the magnitude of the various forces involved in the motion. Kinematic waves do not attenuate, diffusive waves attenuate at a small to moderate rate, and dynamic waves are subject to very strong attenuation. Strictly speaking, St. Venant equations are valid only for flood waves which do not attenuate, i.e. kinematic waves. However, it can also be used as an approximation for flood waves subject to moderate attenuation, i.e. diffusion waves.

Flood waves in Lakes and Reservoirs

Flood waves in lakes and reservoirs travel at an infinite velocity i.e., there is an Instantaneous response (outflow hydrograph) to the excitation (inflow hydrograph). However, the system exerts a diffusive effect on the flood wave with the result that the peak of the outflow hydrograph is attenuated and delayed. A significant characteristics of flood routing through reservoirs is that when the inflow and outflow coincide, the outflow is a maximum (Fig. 3).

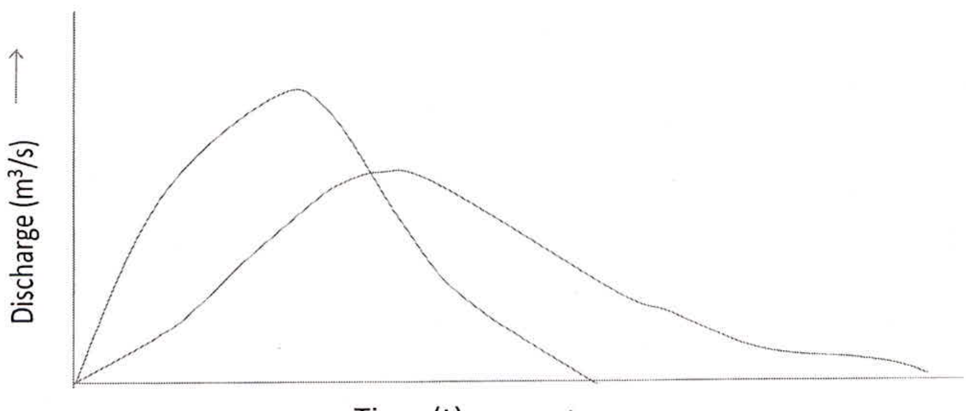

Figure 3: Linear Reservoir Routing

3.0 FLOOD ROUTING

Although flood waves appear to have well-defined properties, in practice it is often necessary to carry-out elaborate calculations in order to determine these properties. The reason for this is the variability of the natural environment, manifested in the need to handle large amount of data. Flood routing is defined as the process of tracking by calculating the movement of a flood wave. Chow (1964) defined 'Flood Routing' as the procedure whereby the time and magnitude of flood wave at a point on a stream is determined from the known or assumed data at one or more points upstream. The calculations can proceed along one of the following two lines, either by considering only temporal variations (lumped case), or by considering both temporal and spatial variations (distributed case). The lumped case is the classical reservoir routing situation, formulated in terms of one first order ordinary differential equation (the starage equation). The distributed case corresponds to stream channel routing, formulated in terms of two first order partial differential equations. The latter are commonly referred to as the equations of gradually varied unsteady open channel flow, or also, as the St. Venant equations.

4.0 CLASSIFICATION OF FLOOD ROUTING METHODS

A knowledge of the nature of flood waves provides a good basis on which to develop a classification of flood routing methods. The most general classification is that of (1) reservoir (or lake) routing and (2) stream channel routing. The essential difference between these two is that in the reservoir routing the water surface slope is zero; while in stream channel routing it is a non-zero value. Several other criteria could be used to classify flood routing methods. Among these, the following are readily identified: (1) the equations used to formulate the problem; (2) the overall approach to data collection and (3) the approach to obtain a solution.

4.1 Classification Based on Equations Used

According to the equations used to formulate the problem, flood routing methods can be classified as (1) Mass-balance methods and (2) Mass-and-momentum-balance-methods. The mass-balance methods use the ordinary differential equation of storage plus an auxiliary storage-outflow relationship. The mass and momentum-balance methods use the Saint Venant equations or appropriate simplifications. The use of the complete St. Venant equations leads to the dynamic wave, while the simplified forms lead to kinematic and diffusive waves.

4.2 Classification Based on Approach to Data Collection

According to the approach to data collection, flood routing methods can be classified as: (1) hydrologic, in which the parameter estimation is based on hydrologic observations for individual reaches; and (2) hydraulic, in which the parameter estimation is based on actual measurements of channel characteristics at individual cross-sections.

4.3 Classification Based on the Solution Technique

According to the approach based on the solution technique, flood routing methods can be classified as: (1) analytical and (2) numerical. The analytical methods are based on the solution of differential equations specified on a continuous domain of space and time. The numerical methods are based on the algebraic equations on a discrete domain. Analytical solutions use the tools of classical mathematics such as linear analysis and Laplace transforms, while numerical solutions use characteristics or finite difference methods.

Table-1 provides a summary of the classification of flood routing methods presented herein.

TABLE-1 CLASSIFICATION OF FLOOD ROUTING METHODS

Based on equations used	Mass-balance: Storage equation and an auxiliary storage-outflow relationship Mass-and-momentum-balance: Saint Venant equations (dynamic wave) or appropriate simplifications (kinematic and diffusive waves)
Based on Approach to Data Collection	Hydrologic Routing: observations for channel reaches. Hydraulic Routing: Measurements of channel characteristics at individual X-sections.
Based on Solution Technique	Analytical Routing: differential equations; continuous domain. Numerical routing: Algebraic equations: discrete domain.

5.0 HYDROLOGIC FLOOD ROUTING METHOD

The hydrologic routing method of routing can be broadly classified as (1) storage Routing method; and (2) complete linearized method and its simplifications. As complete linearized models are not very much used in practice only storage routing methods have been dealt with. The storage routing method may deal with linear, quasi-linear and non-linear flood routing problems. In linear routing the parameters of the model are kept constant throughout the routing operation. Examples are the conventional Muskingum flood routing method, Lag and route method etc. In quasi-linear routing some or all the parameters of the model change from one time step to another.

5.1 Storage Routing Mode

All the storage routing models are based on the continuity equation in the lumped form which can be written for a channel reach as:

$$\frac{dS}{dt} = I(t) - Q(t) \tag{1}$$

where, I(t) and Q(t) are inflow and outflow respectively, and S(t); the storage in the reach under study at time 't'. Since there are two unknowns viz. Q(t) and S(t) and only one equation, the solution for Q(t) can not be obtained. In order to eliminate one of the unknowns, expression for storage S(t) in terms of I(t) and Q(t) or Q(t) is used. The storage equation may be linear or non-linear in form. The following are the commonly used forms of storage equations in flood routing.

$$S(t) = K\,Q(t) \tag{2}$$

$$S(t) = K\,Q(t+\tau) \tag{3}$$

$$S(t) = K\,[\,XI(t) + (1-X)\,Q] \tag{4}$$

$$S(t) = a_o\,Q(t) + a_1 dQ(t)/dt + a_2\,d\,Q(t)/dt \tag{5}$$

$$S(t) = a_o\,Q(t) + a_1 dQ(t)/dt + b_1\,I(t) \tag{6}$$

$$S(t) = a_o\,Q(t) + a_1 dQ(t)/dt + b_1 I(t) + b_2 dI\,(t)/dt \tag{7}$$

$$S(t) = K\,(Q(t))^m \tag{8}$$

For the sake of brevity the time functions attached with the notations for inflow, outflow and storages would be dropped here afterwards. Equation (2) represents the storage of a single linear reservoir (SLR) model proposed by Zoch (1934). Using a series of n-SLRs Nash (1957) conceptualized the catchment behaviour for a unit impulse input and derived the Instantaneous Hydrograph (IUH) for the catchment. Dooge (1973) pointed out the same can also be used for modeling the flood in a river reach. Equation (3) forms the basis of the Lag and Route model proposed by Meyer (1941). It relates the outflow of time ($t+\tau$) to the storage at time ‘t’. The term represents the response delay time or the time taken for the leading edge of the flood wave to reach the outflow section. Equation (4) forms the basis of the classical Muskingum flood routing method proposed by McCarthy (1938). Equation (5) to (8) were studied by Kulandaiswamy et.al. (1957) as particular cases of general storage routing model applied to route floods in channels and river reaches. Equation (8) represents the non-linear relationship between storage and discharge and it has been employed by Rockwood (1958), and Mein et.al. (1974) for channel routing.

6.0 CONVENTIONAL MUSKINGUM METHOD

The water surface in a channel reach is not only not parallel to the channel bottom but also varies with time as given in Figure 4. Considering a channel reach having a flood flow, the total volume in storage can be considered as prism storage and wedge storage

Prism Storage

It is the volume that would exist if uniform flow occurred at the downstream depth, i.e. the volume formed by an imaginary plane parallel to the channel bottom drawn at the outflow section water surface.

Wedge Storage

It is the wedge-like volume formed between the actual water surface profile and the top surface of the prism storage.

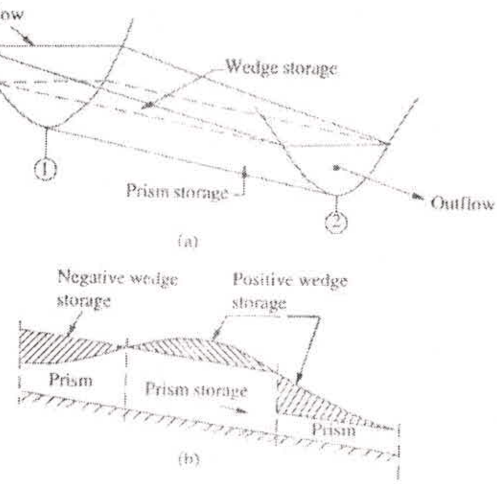

Figure 4: Storage in a channel reach

At a fixed depth at a downstream section of a river reach the prism storage is constant while the wedge storage changes from a positive value at an advancing flood to a negative value during a receding flood. The prism storage S_p is similar to a reservoir and can be expressed as a function of the outflow discharge, $S_p = f(Q)$. The wedge storage can be accounted for by expressing it as $S_w = f(I)$. The total storage in the channel reach can then be expressed as

$$S = K\left[x\,I^m + (1-x)Q^m\right] \tag{9}$$

where K and x are coefficients and m = a constant exponent. It has been found that the value of m varies from 0.6 for rectangular channels to a value of about 1.0 for natural channels.

Using m = 1.0 Eq. (9) reduces to a linear relationship for S in terms of I and Q as

$$S = K[xI + (1-x)Q] \tag{10}$$

and this relationship is known as the *Muskingum equation*. In this the parameter x is known as *weighting factor* and takes a value between 0 and 0.5. It accounts for the storage portion of the routing. When x = 0, the storage is a function of discharge only and the Eq. (10) reduces to

$$S = KQ \tag{11}$$

Such storage is known as *linear storage* or *linear reservoir*. When $x = 0.5$ both the inflow and outflow are equally important in determining the storage.

The coefficient K is known as *storage-time constant* and has the dimensions of time. It is a function of the flow and channel characteristics. It is approximately equal to the time of travel of a flood wave through the channel reach.

Estimation of K and x

Figure 2 shows a typical inflow and outflow hydrograph through a channel reach. Note that the outflow peak does not occur at the point of intersection of the inflow and outflow hydrographs. Using the continuity equation for time element Δt,

$$(I_1 + I_2)\frac{\Delta t}{2} - (Q_1 + Q_2)\frac{\Delta t}{2} = \Delta S \tag{12}$$

the increment in storage at any time t and can be calculated. Summation of the various incremental storage values enables to find the channel storage S v_s time relationship (Fig.5).

If an inflow and outflow hydrograph set is available for a given reach, values of S at various time intervals can be determined by the above equation. By choosing a trial value of x, values of S at any time t are plotted against the corresponding $[xI + (1-x)Q]$values. If the value of x is chosen correctly, a straight-line relationship as given by Eq. (4) will result. However, if an incorrect value of x is used, the plotted points will trace a looping curve. By trail and error, a value of x is so chosen that the data describe a straight line (Fig. 6). The inverse slope of this straight line will give the value of K. Normally, for natural channels, the value of x lies between 0 and 0.3. For a given reach, the values of x and K are assumed to be constant. A calibration can be performed with several flood events to arrive at constant values of x and K for a particular reach.

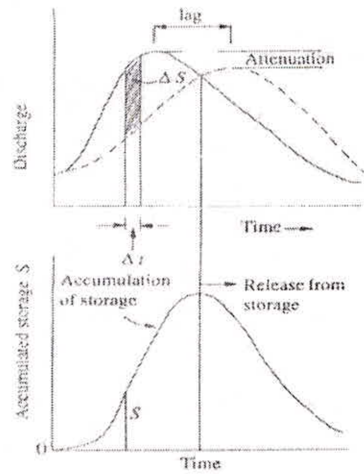

Figure 5 Hydrographs and storage in channel routing

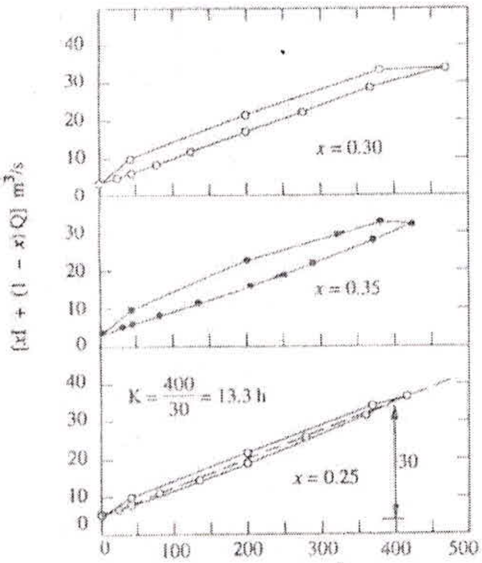

Figure 6 Determination of *K* and *x* for a channel reach

Example 1.

The following inflow and outflow hydrographs were observed in a river reach. Estimate the values of *K* and *x* applicable to this reach for use in the Muskingum equation.

Time (h)	0	6	12	18	24	30	36	42	48	54	60	66
Inflow (m^3/s)	5	20	50	50	32	22	15	10	7	5	5	5
Outflow (m^3/s)	5	6	12	29	38	35	29	23	17	13	9	7

Solution: Using a time increment Δt = 6h, the calculations are performed in a tabular manner as in Table 1. The incremental storage ΔS and S are calculated in columns 6 and 7 respectively. It is advantageous to use the units $[(m^3/s).h]$ for storage terms.

As a first trial x = 0.35 is selected and the value of $[xI + (1-x)Q]$ evaluated (column 8) and plotted against S in Fig. 3. Since a looped curve is obtained, further trials are performed with x = 0.30 and 0.25. It is seen from Fig. 3 that for x = 0.25 the plot describe a straight line and x = 0.25 is taken as the appropriate value for the reach. From Fig. 3, K = 13.3 h

***Table 1 Determination of* K *and* x**

Δt = 6h Storage in (m^3/s).h

Time (h)	I (m^3/s)	Q (m^3/s)	(I-Q)	Average (I-Q)	ΔS= Col.5 x Δt (m^3/s.h)	$S=\Sigma\Delta S$ (m^3/s.h)	$[xI + (1-x)Q]$ (m^3/s)		
							x= 0.35	x= 0.30	x= 0.25
1	2	3	4	5	6	7	8	9	10
0	5	5	0			0	5.0	5.0	5.0
				7.0	42				
6	20	6	14			42	10.9	10.2	9.5
				26.0	156				
12	50	12	38			198	25.3	23.4	21.5
				29.5	177				
18	50	29	21			375	36.4	35.3	34.3
				7.5	45				
24	32	38	-6			420	35.9	36.2	36.5
				-9.5	-57				
30	22	35	-13			363	30.5	31.1	31.8
				-13.5	-81				
36	15	29	-14			282	24.1	24.8	25.5
				-13.5	-81				
42	10	23	-13			201	18.5	19.1	19.8
				-11.5	-69				
48	7	17	-10			132	13.5	14	14.5
				-9.0	-54				
54	5	13	-8			78	10.2	10.6	11.0
				-6.0	-36				
60	5	9	-4			42	7.6	7.8	8.0
				-3.0	-18				
66	5	7	-2			24	6.3	6.4	6.5

Methodology

The Muskingum method of flood routing was introduced by McCarthy and others (U.S. Army Corps of Engineers, 1960) in connection with the flood control studies of the Muskingum River basin in Ohio, U.S.A. Since its development, this method has been widely used in river engineering practice.

For a given channel reach by selecting a routing interval Δt and using the Muskingum equation, the change in storage is

$$S_2 - S_1 = K[x(I_2 - I_1) + (1-x)(Q_2 - Q_1)] \tag{13}$$

Where, suffixes 1 and 2 refer to the conditions before and after the time interval Δt. The continuity equation for the reach is

$$S_2 - S_1 = \left(\frac{I_1 + I_2}{2}\right)\Delta t - \left(\frac{Q_1 + Q_2}{2}\right)\Delta t \tag{14}$$

From Eqs. (9) and (10), Q_2 is evaluated as

$$Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1 \tag{15}$$

where
$$C_0 = \frac{-Kx + 0.5\Delta t}{K - Kx + 0.5\Delta t} \tag{16a}$$

$$C_1 = \frac{Kx + 0.5\Delta t}{K - Kx + 0.5\Delta t} \tag{16b}$$

$$C_2 = \frac{K - Kx - 0.5\Delta t}{K - Kx + 0.5\Delta t} \tag{16c}$$

Note that $C_0 + C_1 + C_2 = 1.0$ Eq. (15) can be written in a general form for the n th time step as

$$Q_n = C_0 I_n + C_1 I_{n-1} + C_2 Q_{n-1} \tag{17}$$

Eq. (16) is known as *Muskingum Routing Equation* and provides a simple linear equation for channel routing. It has been found that for best results the routing interval Δt should be so chosen that $K > \Delta t > 2Kx$. If $\Delta t < 2Kx$, the coefficient C_0 will be negative. Generally, negative values of coefficients are avoided by choosing appropriate values of Δt.

To use the Muskingum equation to route a given inflow hydrograph through a reach, the values of K and x for the reach and the value of the outflow, Q_1, from the reach at the start are needed. The procedure is as follows:

a) Knowing K and x, select an appropriate value of Δt.

b) Calculate C_0, C_1 and C_2.

c) Starting from the initial conditions I_1, Q_1 and I_2 at the end of the first time step Δt calculate Q_2 by Eq. (15).

d) The outflow calculated in step (c) becomes the known initial outflow for the next time step. Repeat the calculations for the entire inflow hydrograph.

Example 2 illustrates the computation procedure.

Example 2: Route the following hydrograph through a river reach for which K=12.0h and x=0.20. At the start of the inflow flood, the outflow discharge is 10 m^3/s.

Time (h)	0	6	12	18	24	30	36	42	48	54
Inflow (m^3/s)	10	20	50	60	55	45	35	27	20	15

Solution: Since K = 12 h and $2Kx = 2 \times 12 \times 0.2 = 4.8$ h, Δt should be such that 12 h > Δt > 4.8 h. In the present case Δt = 6 h is selected to suit the given inflow hydrograph ordinate interval.

Using Eqs. (16a, 16b, 16c) the coefficients C_0, C_1 and C_2 are calculated as

$$C_0 = \frac{-12 \times 0.20 + 0.5 \times 6}{12 - 12 \times 0.2 + 0.5 \times 6} = \frac{0.6}{12.6} = 0.048$$

$$C_1 = \frac{12 \times 0.2 + 0.5 \times 6}{12.6} = 0.429$$

$$C_2 = \frac{12 - 12 \times 0.2 - 0.5 \times 6}{12.6} = 0.523$$

For the first time interval, 0 to 6 *h,*

I_1=10.0	C_1I_1=4.29
I_2=20.0	C_0I_2=0.96
Q_1=10.0	C_2Q_1=5.23

From Eq. 16 $Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1$ = 10.48 m^3/s

For the next time step, 6 to 12 h, Q_1 = 10.48 m^3/s. The procedure is repeated for the entire duration of the inflow hydrograph. The computations are done in a tabular form as shown in Table 2. By plotting the inflow and outflow hydrographs the attenuation and peak lag are found to be 10 m^3/s and 12 h respectively.

Table 2 Muskingum method of routing

Δt= 6 *h*

Time (h)	I (m^3/s)	$0.048I_2$	$0.429I_1$	$0.523Q_1$	Q (m^3/s)
1	2	3	4	5	6
0	10				10.00
		0.96	4.29	5.23	
6	20				10.48
		2.40	8.58	5.48	
12	50				16.46
		2.88	21.45	8.61	
18	60				32.94
		2.64	25.74	17.23	
24	55				45.61
		2.16	23.60	23.85	
30	45				49.61
		1.68	19.30	25.95	
36	35				46.93
		1.30	15.02	24.55	
42	27				40.87
		0.96	11.58	21.38	
48	20				33.92
		0.72	8.58	17.74	
54	15				27.04

Advantages and disadvantages of hydrologic routing method

(a) Advantages

(1) Hydrologic methods may provide answers in much less time than the solution procedures based on the complete equations.

(2) The channel geometry need not be described in detail.

(3) Computation cost is less and programming for solution is simpler.

(4) Hydrologic flood routing models probably can be more easily integrated with rainfall-runoff models.

(5) The use of the results from mathematical modeling often does not require the accuracy provided by the complete model.

(b) Disadvantages

(i) Hydrologic methods do not have the accuracy of a solution procedure based on the complete equations. Probably a hydrologic routing model may give sufficiently accurate results for a particular application but there is often considerable doubt as to how accurate the results are for any application.

(ii) Considerable amount of past data, especially of inflow and outflow is required for reliable estimation of the parameters involved in the model.

(iii) Backwater effect cannot be accounted for by hydrologic methods as they are single characteristic passing only the upstream disturbance to downstream.

(iv) The impact of lateral inflow on the parameters of the hydrologic models can not be taken into account due to longer length of reach usually considered.

(v) Solutions of simplified equations may lack desired generality.

Sometimes it happen that routing by Muskingum method may lead to negative values of the outflow generally occurring in the end ordinates of the outflow hydrograph. The negative values of the flow are physically impossible but mathematically correct. These negative values are taken as zero.

6.0 MUSKINGUM-CUNGE METHOD

To overcome the shortcomings of Muskingum method a flood routing method was developed by considering channel characteristics by Cunge (1969). He discretized the kinematic wave equation on the *x-t* plane in such a way that parallels the Muskingum method and came out with a physically based alternative to the Muskingum method. The alternative method is popularly known as Muskingum-Cunge method.

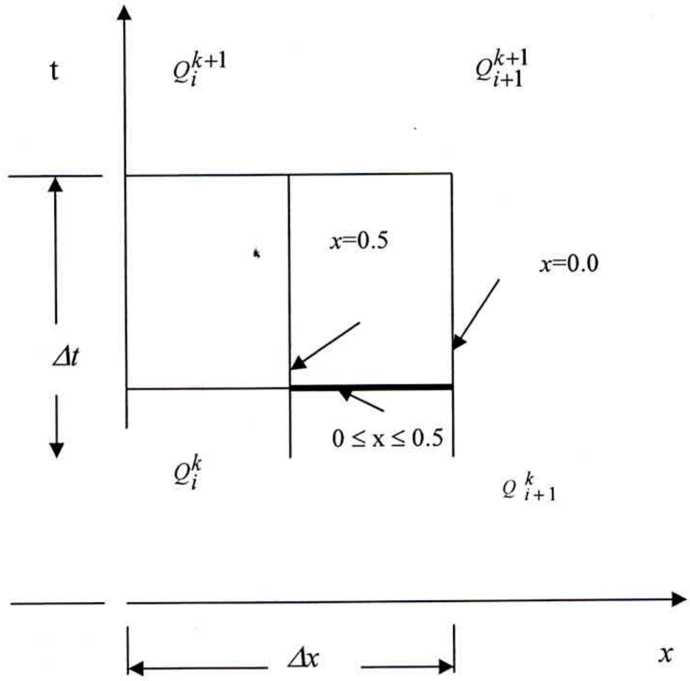

Fig. 5 Space-time discretization of Kinematic wave equation

The kinematic wave equation is given as:

$$\frac{\partial Q}{\partial t} + c\frac{\partial Q}{\partial x} = 0.0 \qquad (18)$$

in which $c = \beta V$ is the kinematic wave celerity.

Eq. (18) was discretized by Cunge (1969) on the *x-t* plane shown in Fig.4 wherein the spatial derivative was centered and the temporal derivative was off centered by means of a weighting factor *X*. The resulting equation is given as:

$$\frac{X(Q_i^{k+1}-Q_i^k)+(1-X)(Q_{i+1}^{k+1}-Q_{i+1}^k)}{\Delta t}+c\frac{(Q_{i+1}^k-Q_i^k)+(Q_{i+1}^{k+1}-Q_i^{k+1})}{2\Delta x}=0 \quad (19)$$

Solving Eq. (19) for the unknown discharge leads to the following equation:

$$Q_{i+1}^{k+1}=C_0Q_i^{k+1}+C_1Q_i^k+C_2Q_{i+1}^k \quad (20)$$

The routing coefficients are:

$$C_0=\frac{c(\Delta t/\Delta x)-2X}{2(1-X)+c(\Delta t/\Delta x)} \quad (21a)$$

$$C_1=\frac{c(\Delta t/\Delta x)+2X}{2(1-X)+c(\Delta t/\Delta x)} \quad (21b)$$

$$C_2=\frac{2(1-x)+c(\Delta t/\Delta x)}{2(1-x)+c(\Delta t/\Delta x)} \quad (21c)$$

By defining

$$K=\frac{\Delta x}{c} \quad (22)$$

it is observed that the Eqs. 11a to 11c and 15a to 15c are the same. *K* is the flood wave travel time i.e. the time taken for a given discharge to travel the reach length Δx with the kinematic celerity *c*. In a linear mode, *c* is constant and equal to a reference value, and in non-linear mode, it varies with discharge

The weighting factor *X* is derived by matching physical ($\nu_h=\frac{q_o}{2S_o}$) and numerical ($\nu_n=c\Delta x\left(\frac{1}{2}-X\right)$) diffusion as follows:

$$X=\frac{1}{2}(1-\frac{q_0}{S_0c\Delta x}) \quad (23)$$

where Δx = reach length; q_0 = discharge per unit width; *c* = kinematic wave celerity; S_0 = bottom slope.

The Courant number, C is defined as the ratio of wave celerity (c) to grid celerity $\Delta x/\Delta t$ i.e.

$$C = c\frac{\Delta t}{\Delta x} \tag{24}$$

The Cell Reynolds number is defined as the ratio of hydraulic diffusivity ($V_h = \frac{q_o}{2S_o}$) to grid diffusivity ($V_g = \frac{c\Delta x}{2}$). This leads to

$$D = \frac{q_0}{S_0 c\Delta x} \tag{25}$$

in which, D = Cell Reynolds number. Therefore, from Eqs. (17) and (19)

$$X = \frac{1}{2}(1 - D) \tag{26}$$

Eq.(25) and (26) imply that for very small values of Δx, D may be greater than 1, leading to negative values of X. In fact, for the characteristic reach length

$$\Delta x_c = \frac{q_0}{S_0 c} \tag{27}$$

the cell Reynolds number is D=1, and $X = 0$. Therefore, in the Muskingum-Cunge method, reach length shorter than the characteristic reach length result in negative values of X. This should be contrasted with classical Muskingum method, in which X is restricted in the range 0.0 – 0.5. In the classical Muskingum, X is interpreted as a weighting factor. As shown by Eq.(25), and (26) non negative values of X are associated with long reaches (Δx more than characteristic length Δx_c given by Eq.(27), typical of the manual computation used in the development and early application of the Muskingum method.

In the Muskingum-Cunge method, however, X is interpreted in a moment matching sense or diffusion-matching factor. Therefore, negative values of X are entirely possible. This feature allows the use of shorter reaches than would otherwise be possible if X were restricted to non-negative values.

The substitution of Eq.(23) and (25) into Eq.(21a) to (21c) leads to routing co-efficients expressed in terms of Courant and Cell Reynolds numbers:

$$C_0 = \frac{-1+C+D}{1+C+D} \tag{28a}$$

$$C_1 = \frac{1+C-D}{1+C+D} \tag{28b}$$

$$C_2 = \frac{1-C+D}{1+C+D} \tag{28c}$$

Thus C and D are the two routing parameters required to be estimated for Muskingum-Cunge method.

Estimation of Routing Parameters

(a) Estimation of parameter *C* (Courant number)

The parameter C can be estimated using Eq.(24). It requires an estimate for wave celerity (c) in addition to grid size (Δx, Δt). The wave celerity can be calculated with either

$$c = \beta V \tag{29}$$

or

$$c = \frac{1}{T}\frac{dQ}{dy} \tag{30}$$

where β is an exponent in the discharge area rating equation given as

$$Q = \alpha (A)^{\beta} \tag{31}$$

The calculation of β is a function of frictional type and cross sectional shape.

Theoretically Eq.(30) and (31) are the same. For practical applications, if a stage-discharge rating and cross sectional geometry are available (i.e. stage – discharge – top width tables), Eq.(31) is preferred over Eq. (30) because it accounts directly for cross sectional shape. In the absence of a stage-discharge rating and cross-sectional data, Eq.(30) can be used to estimate flood wave celerity. The velocity V in Eq. (30) can be taken as the velocity at reference flow. The choice of reference flow has bearing on the calculated results although the overall effect is likely to be small. The peak flow value has the advantage that it can be readily ascertained, although a better approximation may be obtained by using an average value.

(b) Estimation of parameter *D* (Cell Reynolds number)

Cell Reynolds numbers (D) can be calculated using the reach length (Δx), reference discharge per unit width (q_0), kinematic wave celerity (c), and bottom slope (S_0) in Eq. (25).

Resolution Requirements

When using the Muskingum-Cunge method sufficiently small values of Δx and Δt should be taken in order to approximate closely the actual shape of the hydrograph. For smoothly rising hydrographs, a minimum value of $t_p/\Delta t=5$ is recommended. This requirement usually results in the hydrograph time base being resolved into at least 15 to 25 discrete points, considered adequate for Muskingum routing.

Unlike temporal resolution, there is no definite criterion for spatial resolution. A criterion borne out by experience is based on the fact that courant and cell Reynolds numbers are inversely related to reach length Δx. Therefore, to keep Δx sufficiently small, Courant and cell Reynolds numbers should be kept sufficiently large. Thus leads to the practical criterion:

$$C + D \geq 1 \qquad (32)$$

Which can be written as $-1 + C + D \geq 0$.

This confirms the necessity of avoiding negative values of C_0 in Muskingum-Cunge routing (Eq.20). Experience has shown that negative values of either C_1 or C_2 do not adversely affect the methods over all accuracy.

Notwithstanding Eq.(32), the Muskingum Cunge method works best when the numerical dispersion is minimized, that is, when C is kept close to 1.Values of C sufficiently different from one are likely to cause the notorious dips, or negative outflows, in portions of the calculated hydrograph. This computational anomaly is attributed to excessive numerical dispersion and should be avoided.

Methodology

The steps involved in flood routing through a channel reach using the Muskingum-Cunge Method are given as follows:

(i) Estimate the parameter C (Courant number) using the following equation:

$$C = c\frac{\Delta t}{\Delta x}$$

The wave celerity c is computed using the procedure described in section 4.1. The temporal and spatial resolutions (Δt and Δx) should be such that the routing co-efficient C_0 should not be negative as well as the value of Courant number (C) should be close to one in order to minimize the numerical dispersion.

(ii) Estimate the parameter D (Cell Reynolds number) using the following equation

$$D = \frac{q_0}{S_0 c\Delta x}$$

The wave celerity (c) and reference discharge, $q_0 = \frac{Q_0}{T}$ per unit width are used together with channel slope S_0 and reach length Δx in the above equation to provide the parameter D (Cell Reynolds number). Ensure whether $-1 + C + D \geq 0$, which is the practical criterion to avoid the negative values of C_0 in Muskingum-Cunge routing.

(iii) Estimate the routing co-efficients C_0, C_1 and C_2 using the following equation:

$$C_0 = \frac{-1+C+D}{1+C+D}$$

$$C_1 = \frac{1+C-D}{1+C+D}$$

$$C_2 = \frac{1-C+D}{1+C+D}$$

(iv) Route the inflow hydrograph (Q) using the following equation in order to have the outflow hydrograph (Q_{i+1}):

$$Q_{i+1}^{k+1} = C_c Q_i^{k+1} + C_1 Q_i^k + C_2 Q_{i+1}^k$$

(v) If the channel is divided into sub-reaches, the steps (i) to (iv) should be repeated for all the sub-reaches considering the outflow from the first sub-reach as inflow to second sub-reach and so on.

Example 3 : Use the Muskingum-Cunge method to route a flood wave with the following flood and channel characteristics :

Peak flow Q_p = 1000 m^3/s

Base flow Q_b = 0 m^3/s

Channel bottom slope S_0 = 0.000868

Flow area at peak discharge A_p = 400 m^2

Top with at peak discharge T_p=100 m

Rating exponent β=1.6

Reach length Δx = 14.4 Km

Time interval Δt = 1 hr.

Time (h)	0	1	2	3	4	5	6	7	8	9	10
Flow (m^3/s)	0	200	400	600	800	1000	800	600	400	200	0

Solution:

(i) Compute the wave celerity (c)

$$c = \beta V$$

here, β = 1.6 and the mean velocity (based on the peak discharge is $V = Q_p/A_p$= 1000/400 = 2.5 m/s

$$c = 1.6 \times 2.5 = 4 \text{ m/s}$$

(ii) Compute the Courant number (C)

$$C = c\frac{\Delta t}{\Delta x} = \frac{4 \times 1 \times 3600 \times 10^{-3}}{14.4} = \frac{14.4}{14.4} = 1.0$$

(iii) Compute the Cell Reynold number (D)

$$D = \frac{q_0}{S_0 c \Delta x}$$

here q_0 = the flow per unit width (based on the peak discharge)

$$= \frac{Q_p}{T_p} = \frac{1000}{100} = 10 m^2/s$$

$$D = \frac{10}{0.000868 \times 4 \times 14.4 \times 10^3} = \frac{10}{0.868 \times 57.6} = 0.2$$

(iv) Compute the routing co-efficients

$$C_0 = \frac{-1 + C + D}{1 + C + D} = \frac{-1 + 1 + 0.2}{1 + 1 + 0.2} = 0.091$$

$$C_1 = \frac{1 + C - D}{1 + C + D} = \frac{1 + 1 - 0.2}{1 + 1 + 0.2} = 0.818$$

$$C_2 = \frac{1 - C + D}{1 + C + D} = \frac{1 - 1 + 0.2}{1 + 1 + 0.2} = 0.091$$

(iv) Compute the outflow hydrograph using the following routing equation :

$$Q_{i+1}^{k+1} = C_0 Q_i^{k+1} + C_1 Q_i^k + C_2 Q_{i+1}^k = 0.091 Q_i^{k+1} + 0.818 Q_i^k + 0.091 Q_{i+1}^k$$

The routing calculations are shown in Table 3

Table3 : Channel Routing by Muskingum Method

Time	Inflow (Q_i^{k+1})	Partial Flows $C_0 Q_i^{k+1}$	$C_1 Q_i^k$	$C_2 Q_{i+1}^k$	Outflow (Q_{i+1}^{k+1})
(hr)	(m^3/s)	(m^3/s)	(m^3/s)	(m^3/s)	(m^3/s)
(1)	(2)	(3)	(4)	(5)	(6)=(3)+(4)+(5)
0	0	-	-	-	0
1	200	18.2	0	0	18.2
2	400	36.4	163.6	1.66	201.66
3	600	54.6	327.2	18.35	400.15
4	800	72.8	490.8	36.41	600.01
5	1000	91.0	654.4	54.60	800.00
6	800	72.8	818.0	72.80	963.60
7	600	54.6	654.4	87.69	796.69
8	400	36.4	490.8	72.50	599.70
9	200	18.2	327.2	54.57	399.97
10	0	0	163.6	36.40	200.00
11	0	0	0	18.20	18.20
12	0	0	0	1.66	1.66
13	0	0	0	0.16	0.16

Advantages and limitations of Muskingum-Cunge method

(i) The Muskingum-Cunge method is a physically based alternative to the Muskingum method. Unlike Muskingum method where the parameters are calibrated using stream flow data, in the Muskingum Cunge Method the parameters are calculated based on flow and channel characteristics. This makes possible channel routing without the need for time consuming and cumbersome parameter calibration. More importantly, it makes possible extensive channel routing in ungauged streams with a reasonable expectation of accuracy.

(ii) With the Muskingum method, the Muskingum-Cunge method is limited to diffusion waves. Furthermore, Muskingum-Cunge method is based on a single valued rating and does not take into account strong flow non uniformity or unsteady flows exhibiting substantial loops in discharge-stage rating (i.e. dynamic wave). Thus the Muskingum-Cunge method is suited for channel routing in natural streams without significant backwater effects and for unsteady flows that classify that the diffusion wave criterion.

(iii) An important difference between the Muskingum and Muskingum-Cunge methods is that the former is based on the storage concept and, therefore, the parameters *K* and *X* are reach averages. The later method, however, is kinematic in nature, with the parameters *C* and *D* being based on values evaluated at channel cross sections rather than being reach average. Therefore, for the Muskingum-Cunge method to improve on the Muskingum method, it is necessary that the routing parameters evaluated at channel cross sections be representative of the channel reach under consideration.

(iv) Historically, the Muskingum Method has been calibrated using stream flow data. On the contrary, the Muskingum-Cunge method relies on physical characteristics such as rating curves, cross sectional data and channel slope. The different data requirements reflect the different theoretical bases of the method i.e. storage concept in the Muskingum method, and kinematic wave theory in the Muskingum-Cunge method.

7.0 REMARKS

The hydrologic approach to flood routing is based on the consideration that in a large number of practical cases, the inertia terms of the equations of motion play an exceedingly small role. Such methods have advantages from the practical consideration. Miller and Cunge (1975) have listed the advantages and disadvantages in detail. Some of these are listed below:

Advantages

(1) Hydrologic methods may provide answers in much less time than the solution procedures based on the complete equations.

(2) The channel geometry need not be described in detail.

(3) Computation cost is less, programming for solution is simpler.

(4) Hydrologic flood routing models probably can be more easily integrated with rainfall-runoff models.

(5) The use of the results from mathematical modeling often do not require the accuracy provided by the complete model.

Disadvantages

(1) Hydrologic methods do not have the accuracy of a solution procedure based on the complete equations. Probably a hydrologic routing model may give sufficiently accurate results for a particular application but there is often considerable doubt as to how accurate the results are for any application.

(2) Considerable amount of past data, especially of inflow and outflow is required for reliable estimation of the parameters involved in the model.

(3) Backwater effect can not be accounted for by hydrologic methods as they are single characteristic passing only the upstream disturbance to downstream.

(4) The impact of lateral inflow on the parameters of the hydrologic models can not be taken into account due to longer length of reach usually considered.

(5) Solutions of simplified equations may lack desired generality.

Sometimes it happen that routing by Muskingum method may lead to negative values of the outflow generally occurring in the end ordinates of the outflow hydrograph. The negative values of the flow are physically impossible but mathematically correct. These negative values are taken as zero.

Although current interest in the field of flood routing seems to be in the methods utilizing numerical solutions of the complete equations of continuity and momentum, hydrologic methods are still useful and may be preferable in some circumstances. The limitations of each technique must be thoroughly understood so that an intelligent choice of method, to be used, may be made.

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