Final Report

Performance Evaluation of 2D-VPMM and 2D-Explicit Schemes for Two-Dimensional Overland Flow Simulation

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PREFACE

The overland flow simulation is essential component in the distributed hydrological models (HM) widely used in various Land Surface Modelling (LSM) system to address various challenges in the water resources assessment and management including the climate change impact on the water resources. Basically, overland flow simulation is required to handle various hydrological and environmental issues such as flood estimation and inundation mapping, flood regulations, drought studies, design and management of surface drainage system, urban storm water management system, climate change impact assessment, waste water management, and transport of sediment and chemicals and so on.

These days, there are plethora of overland flow simulation models are available to simulate one-, two- or even three dimensional overland flow. However, it is essential to know in details about the hydraulic as well as simulation characteristics of each of these overland flow simulation scheme used in the HM in order to utilize these scheme effectively in the LSM system. In this context, the present study entitled as "Performance Evaluation of 2D-VPMM and 2D-Explicit Schemes for Two-Dimensional Overland Flow Simulation" which attempt to develop the computer code for simulating the two dimensional overland flow by using the two dimensional diffusion wave method based on the numerical explicit scheme and its comparison with the two dimensional Variable Parameter Muskingum McCarthy (2D-VPMM) model is very essential as well as important.

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ABSTRACT

Estimation of overland flow is essential in addressing many hydrological and environmental problems such as flood estimation, soil erosion and non-point source pollution. Modeling of overland flow can be done using the equation of continuity and motion. The literature is replete with modelling of overland flow using one-dimensional flow description. But a one-dimensional model generally may not be able to simulate runoff generation on land surfaces characterized by irregular slopes. To overcome this problem, attention is focused in the recent times on the development of two-dimensional overland flow model. Two-dimensional overland flow modelling require the use of continuity equation describing the flow variation in x and y directions and the flow depth variation in time. Generally simplified forms of the momentum equation describing momentum variation in x and y directions are employed. For all the practical purposes these equations are solved using the numerical methods. One of the solution procedures is based on the two-dimensional explicit finite difference scheme.

To address these issues, the present study attempts to develop a computer code for two dimensional diffusion wave (DW) overland flow simulations using the explicit solution scheme (2D-DW-Explicit Model) in MATLAB R2013a software. In case of V-catchment in which two overland flow plane joining the channel at middle of these two planes, the channel routing is carried out by using 1D-VPMM method. The two-dimensional Variable Parameter Muskingum McCarthy method named as 2D-VPMM method which is developed by Perumal and co-researchers (PI of this study is also main contributor in the 2D-VPMM model development) for the simulation of two-dimensional overland flow (Shakya, 2015) and computer code written in MATLAB is used to verify the results by 2D-DW-Explicit Model and to compare the predictive abilities of these two computational schemes. A tilted V-Catchment used by Di Giammarco et al. (1996) and the data collected from the rainfall - runoff study of laboratory catchment of the University of Illinois, Urbana Champaign, USA has been used to verify as well as to compare the results by these two methods. Based on the conducted study, it was found that the 2D-VPMM method is performing better than the 2D-DW-Explicit Model in terms of accuracy and execution time.

CHAPTER - 1
INTRODUCTION

1.1 General

A thin sheet of flow which occur at the upstream end of slope before concentrating in the well-defined channel at the downstream end of slope is referred as Overland flow (Chow et al., 1988). Many hydrological problems such as overland flow modelling, flood routing, soil erosion prediction, river management and civil protection work due to occurrence of meteorological event requires prediction of water levels and discharges at particular locations (Kale, 2010). To solve these hydrological problems overland flow is one of the important components. The characteristics and spatial variability of rainfall has considerable impact on hydraulic characteristics of the overland flow. The characteristics of the overland flow generation are not only influenced by rainfall characteristics but also by soil and topographic characteristics. Modeling of overland flow can be done using the governing equation of continuity and motion known as Saint-Venant equations and its various simplified variants. Mostly the literature replete with one dimensional description of the overland flow, but it has shortfall in case of the overland flow generation on the land surfaces characterized by irregular slopes (Zhang and Cundey, 1989; Liu et al. 2004).

Two-dimensional overland flow modeling is the appropriate choice to overcome this circumstance. The study of *two dimensional overland flow over a land surface refers either to flood plain inundation or overland flow resulting from intense rainfall and the mechanism governing overland and channel processes is characterized by the presence of a free surface, and the elevation of which may vary in space and time* (Di Giammarco et al., 1996). Further, due to highly non-linear nature of these governing equations, the global analytical solutions are not available except for limited simplified situations. This situation warrants the use of the appropriate numerical techniques to solve the two-dimensional dynamic wave continuity and momentum equations for overland flow modelling required for flood predictions and to decide on policies for minimization of the flood hazards. The numerical techniques used for solving the two-dimensional overland flow problem are highly complicated and always encounter the problem of instability and convergence because of nonlinear nature of the governing equations (Tayfur et al., 1993).

Therefore, over last four decades several numerical schemes are proposed to solve Saint-Venant equations and its different simplified variants viz., kinematic wave (KW)and diffusion wave (DW) models. The close review of literature suggest that the numerical techniques developed so far can broadly classified into two broad categories namely approximate and numerical methods. The KW models, storage routing schemes such as Muskingum schemes and diffusion analogy based techniques are referred to the approximate methods whereas the direct method such as finite difference, finite element, finite volume and the Method of Characteristics (MoC) falls under the numerical methods category. The direct methods are further categorized as either implicit or explicit schemes based on either the unknown values are solved simultaneously or sequentially along a time line from one predefined distance point to the next, respectively (Subramanya, 2009). The implicit scheme although unconditionally stable as compared to the explicit schemes, but their use implies solving big systems and hence the explicit scheme is widely preferred numerical scheme in case of overland flow modelling. For example, the widely used rainfall-runoff model such as CASC2D model (Downer et al., 2002; Ogden and Julien, 2002) and the Gridded Surface Subsurface Hydrologic Analysis model (GSSHA) (Downer et al. 2005) employs the explicit finite difference scheme for solving diffusion wave equations for two-dimensional overland flow simulation. Furthermore, the Rainfall-Runoff-Inundation (RRI) model developed by ICHARM-PWRI (Sayama et al., 2012) is also use explicit scheme to solve the a two-dimensional diffusion wave equation for overland flow modeling However, explicit numerical scheme possess a longer execution time, numerical stability and the mass conservation problem in the overland flow modelling.

To overcome these problems, recently, Perumal and co-researchers have developed approximate scheme to solve two-dimensional overland flow problem using the Variable Parameter Muskingum McCarthy method named as 2D-VPMM method (Shakya, 2015). However, there is no any attempt to scientifically evaluate the performance of these two schemes namely explicit as used in the CASC2D model and the approximate schemes using 2D-VPMM method for solving the two-dimensional overland flow modelling problem in the literature. Therefore, there is a need of comparing the predictive abilities of these two computational schemes using different experimental data from two-dimensional overland flow runoff cases available in the literature to emphasize the main strengths and weaknesses.

1.2 The Scope of the Study

Use of appropriate numerical model seems to be necessary to simulate the overland flow and flows in channels and rivers in order to predicting the flood prone areas necessary to prepare an efficient disaster management action plan to minimize the risk associated with potential extreme meteorological events at basin scale. By considering the very complex nature of numerical modeling of overland flow modeling due to involvement of several phenomenon, the sophisticated model based on simplifying assumption and procedure is always preferred over complex numerical models. The explicit numerical schemes are usually preferred over implicit schemes for simulating runoff in a natural catchment except when there is necessity of considering downstream boundary condition. The scope of this study is limited to development of a computer code for simulating 2D overland flow using well accepted numerical explicit scheme and testing its performance by comparing the simulation results with those obtained with GSSHA model and recently developed an accurate, efficient and numerical stable and robust method named as 2D-VPMM method which is seems to be capable of replacing the numerical methods employing smaller spatial and temporal discrtitizations and complicated boundary using observed rainfall-runoff events over small experimental catchments.

1.3 The Objectives of the Study

This study has been taken up with following objectives

a. Development of computer code for the explicit solution scheme for two dimensional overland flow simulation

b. Performance evaluation of the simulation results by 2D-VPMM and 2D-explict schemes for two-dimensional overland flow simulation using experimental plot data available in the literature.

1.4 The Limitations of the Study

As described in section 1.3, in this study an attempt has been made to develop an computer code for simulation of two dimensional overland flow based on well accepted explicit scheme and its comparison with results obtained using GSSHA model and 2D-VPMM model using observed rainfall-runoff events over small non-infiltrating experimental catchment. The main purpose is to test the performance these two numerical techniques and the development of code to apply this methodologies to larger natural catchment will be out of the scope of this study. The main limitation of this methodology is that it does not take into account the backwater effect due to exclusion of downstream boundary condition.

CHAPTER - 2
LITERATURE REVIEW

2.1 General

Surface flow routing, including the overland flow and channel routing, refers to the process of transporting precipitation-generated (due rain and other forms of precipitation) surface runoff from source area to outlet. The interest in the development of efficient hydraulic/hydrologic model Surface flow models is more than evident to accomplish many planning and management tasks such as flood-event forecasting, river swelling effect simulation, influence of urbanization and agriculture activities on water quantity, quality and availability in space and time, simulation of other phenomenon such as bank erosion, sedimentation process and ground water transport etc.

The flow that traverses across the land surfaces refereed as “overland flow runoff” plays an essential role in natural ecosystems and is the main source of transfer of living and mineral elements in the landscape. In areas modified by human activities (e.g. soil sealing), or during extreme climatic events, excess water runoff can lead to serious environmental issues such as the flooding of urban areas (Borman et al., 2006; Evarard et al., 2007), or the pollution of water bodies. In order to prevent or to mitigate such events, it is necessary to accurately predict the dynamic as well as the spatial extent of runoff production and transfer (Souch`ere et al., 2005).

Natural hillslopes on which overland flow is generated are seldom planer surfaces with homogeneous physical and hydraulic properties. Microtopography, surface roughness, and soil hydraulic properties vary over distances of centimetres to meters, and they strongly influence runoff characteristics along the hillslope, and hillslope hydrographs (Zhang and Cundy, 1989). These spatial variations have significant impacts on soil erosion and contaminant transport. To predict the hydraulic and hydrologic behavior of overland flow, soil erosion, and contaminant movement, and to examine the relationship between surface flow processes and hillslope soil and vegetation features, physically based, multidimensional models, which incorporate spatial variations in hillslope characteristics, are necessary. Such overland flow models should be capable to accurately determine the flow depths and velocities, and, hence, the capacity of the flow to entrain and transport sediment and chemicals (Moore and Foster, 1989). Considering all these essential factors influencing the flow propagation in open channel and on overland, it is important to develop simplified as well as accurate solutions for the governing equations of flood waves which plays important role in area of hydrology and hydraulics. Consequently, there have been many research studies for modeling these flows which are briefly reviewed in following sections.

2.2 Development and Limitations of Physically Based One-Dimensional Overland Flow Models

Development of rational method for estimation of peak discharge resulting from a rainfall event with uniform intensity and duration equal to or greater than the time of concentration by Mulvany in 1850 marked the development of rainfall-runoff models. The development of equations for modeling surface flow by St. Venant de (1871) now called as St. Venant equations (SVE) marked the initiation of development of physically based models. The SVE are adapted to describe channel flow as well shallow water flow such as overland flow. They are derived from the Navier-Stokes equations by averaging over depth, and assuming several hypotheses due to nonlinear terms (Gerbeau and Perthame, 2001). However, the numerical solution of SVE is very complicated and can be solved analytically or semi-analytically only under certain restrictive conditions. Further, the instabilities and convergence problems encountered during solution of SVE by numerical methods due to high nonlinear nature of the governing equations (Liggett and Woolhiser 1967) provided impetus to simplification of SVE by modifying the momentum balance equations whenever justified by the physical conditions (Vieira 1983, Singh, 1996, Singh 2017a,b,c). Most of these simplifications have involved two of the common models resulting from such simplifications are the kinematic and the diffusion wave models (Morris and Woolhiser 1980). The widespread application of diffusion and kinematic wave approximation of surface water flow routing in physics-based watershed models can be attributed to the fact that, numerical solutions of the full shallow water equations, under complex topography and transient, distributed forcing (e.g., rainfall and infiltration), are computationally intensive; furthermore, it suffers from numerical stability and convergence problems. Indeed, in rainfall-runoff/overland flow simulations, the full dynamic wave equations are rarely applied and, when applied, they are limited to small-scale geometry (experiment plots or single hillslopes) (for an example, see Chow and Ben-Zvi, 1973; Zhang and Cundy, 1989; Fielder and Ramirez, 2000). Therefore, making a careful and judicious choice regarding the numerical method for a dynamic wave model is critical.

Depending on the simplifications in the SVE, five types of flow waves are identified to simulate surface flow as: dynamic waves, steady dynamic waves, gravity waves, diffusive waves, and kinematic waves, and hence five types of models (Singh, 1996; Singh 2017a,b,c). Further, Perumal and Ranga Raju (1999) introduced new wave type known as Approximate Convection Diffusion (ACD) equations based on simplification of momentum equations in stage as well as in discharge formulation which govern the transition between the diffusion and the kinematic waves (including the latter). An attempt has been made to analyze the characteristics of these waves using various techniques by plethora of researchers (Lighthill and Whitham, 1955; Woolhiser and Liggett, 1967; and Ferrick, 1985). They found that most of natural flow cases can be sufficiently handled by using the diffusive and kinematic wave approximations. Furthermore, an analytical solution for diffusion waves in rivers is derived by Hayami (1951) using a disturbance function as the boundary condition upstream. Later, Kazezyılmaz-Alhan and Medina (2007) presented an analytical solution for diffusion waves to overland flow with variable rainfall intensity. In this analytical solution, the inverse of Laplace transform was obtained by using the Stehfest algorithm and this solution is restricted to constant hydraulic diffusivity and wave celerity. Kazezyılmaz-Alhan (2012) attempted to improved solution for diffusion waves for variable hydraulic diffusivity and wave celerity by employing the De Hoog algorithm and proposing an iterative technique.

The derivation of kinematic wave number by Woolhiser and Liggett (1967) proved to be useful in deciding on applicability limit of kinematic wave approximation and also acted as catalyst for propagation and wide acceptance of kinematic wave approximation (Singh, 1996) . An remarkable research work such revision of kinematic wave number with use of Froude number (Morris and Woolhiser, 1980), division of the Froude number versus kinematic wave number diagram into several zones by Vieira (1983), error differential equation for judging the accuracy of kinematic wave and diffusive wave approximations (Singh, 1994) and a comprehensive analysis of the accuracy of kinematic wave and diffusion wave approximations by Moramarco et al. (2008a, b) gave the real impetus to the popularity of kinematic wave and diffusion wave approximation. The studies Moramarco and Singh (2002) and Tsai (2003) are also focused on the applicability of Kinematic and Diffusion wave approximation in various conditions. revised the kinematic wave number with kinematic wave (Perumal and Sahoo, 2007, Kale, 2010) have also showed that the Variable Parameter Muskingum Discharge (VPMD) and Variable Parameter Muskingum Stage (VPMS) methods developed using ACD equations have capable to simulate the flood wave in transition between the diffusion and the kinematic waves (including the latter).

Dunne and Dietrich (1980) have shown that while one-dimensional models may successfully predict or fit the average flow depth and hydrograph on a real hillslope, they are unable to simulate the spatial variability of the flow fields. This spatial variability can be significant. For their experimental plots on Kenyan hillslopes, the coefficient of variation of the cross-slope depth ranged from 0.5-0.8. Yet they are limited by the kinematic wave assumptions. For example, the method does not allow backfacing slopes in the flow fields. Furthermore, the simulation error by the kinematic wave equation is rather significant when the kinematic number k, as defined by Woolhiser and Liggett (1967), is less than 10. Kinematic and diffusion wave equations are the simplified forms of the dynamic wave equations. Diffusion waves are obtained by neglecting the acceleration terms and kinematic waves are obtained by neglecting both the acceleration and the pressure terms in the momentum equation. The kinematic wave model represents unsteady flow through the continuity equation while it substitutes a steady uniform flow for the momentum equation (Lighthill and Whitham, 1955). A kinematic wave does not subside or disperse as it travels downstream while it changes its shape. The diffusion waves are obtained by introducing physical diffusion into the kinematic wave equation which results mathematically in a second-order term. Diffusion occurs most in natural unsteady open channel flows and in overland flow (Hayami, 1951; Lighthill and Whitham, 1955; Ponce, 1989; Perumal and Sahoo, 2007). Diffusion waves may be preferred in simulations of the flood waves in rivers and on flood plains with milder slopes that changes between 0.001 and 0.0001 (referred from Kazezyilmaz-Alhan, 2012). Solving the full Saint-Venant equations by using numerical techniques (finite difference or finite element) leads to problems of instabilities and lack of inertial terms. Diffusion wave theory applies to the milder slopes (0.00 – 0.0001), for which the kinematic wave theory is insufficient (Kazezyillaz-Alhan et al., 2005).

The solution of the fully-dynamic shallow water equations is computationally demanding, which restricts the high-resolution simulation over a large area. As a consequence, various simplifications were more commonly used in the past, including the kinematic wave, diffusive wave models and other simplified forms (Woolhiser and Liggett 1967; Di Giammarco et al. 1996; Feng and Molz 1997; Kazezyilmaz-Alhan and Medina 2007; Gottardi and Venutelli 2008; Costabile et al., 2009; Moramarco et al., 2008; Kale and Perumal, 2014). The diffusive wave model neglects the inertia of the fluid in the momentum equations. The kinematic wave model further neglects the effect of pressure gradient on the water motion. The various researchers based on verification of the accuracy and error estimation of diffusion and kinematic waves overland flow on a plane by numerical experiments concluded that diffusion wave approximation is fairly accurate for most overland flow conditions (Govindaraju et al., 1988, 1990; Singh and Aravamuthan, 1996; and Moramarco and Singh; 2002).

The plethora of research studies devoted to investigate the application of the galerkian finite element method to solve the kinematic wave problems for overland flow (Gottardi and venutelli, 1993; Motha and Wigham, 1995; Jaber, 2001). Most of these studies reported some oscillations and the necessity of using small time steps in order to get stable and accurate solutions (Jaber and Mohtar, 2003). Further, the application of Galerkian formulation of the consistent finite element method and finite difference techniques to solve steady overland flow over permeable surface using externally coupled surface (kinematic wave approximation) and a subsurface flow model to overland flow by Motha and Wigham (1995) have also reported numerical oscillations in a finite element model. Further, the study carried out by Jaber (2001) has also reported occurrence of oscillations while using conventional consistent finite element scheme for certain time step ranges. Although the lumped and upwind finite element schemes are proposed as alternatives to the consistent schemes, the upwind scheme did not show the improvement while the lumped scheme show some improvement in terms of stability and accuracy of the solution. Further, the study by Singh et al. (2002) have brought out hat routing overland flow with explicit finite difference kinematic wave model tends instability problem. Kazeyzilmaz-Alhan et al. (2005) investigated the realiability of several finite difference numerical formulations for solving one-dimensional kinematic and diffusion wave equations that describe overland flow. They compared the numerical solutions with the corresponding analytical solutions considered as the benchmark solutions. The McCormack numerical scheme has been shown to be more accurate and more efficient than the classical explicit and implicit finite difference schemes.

There are plethora of studies which deals with the simulation of overland flow in one dimension. However, an actual hillslope is not smooth as assumed in one-dimensional flow modelling for simplicity. Therefore, to model the complex flow processes over undulating topographic conditions, the two dimensional overland flow models is obvious choice over one-dimensional flow. The brief review on the development of two dimensional overland flow models is provided in the following section.

2.3 Two Dimensional Overland Flow Models

As described earlier one dimension overland flow models which assume a homogeneous plane surface and thus ignore the real, measurable spatial variation in the overland flow field due to significant errors, in the distributions of velocity, depth (Dunne and Dietrich, 1980). For their experimental plots on Kenyan hillslopes, the coefficient of variation of the cross-slope depth ranged from 0.5-0.8. In period between early 70's and 80's, various researchers such as Chow and Ben-Zvi, (1973) have applied Lax-Wendroff scheme,, a characteristic method, a finite element scheme and Lax-Wendroff scheme to model overland flow by using a two-dimensional hydrodynamic equation, respectively. All these studies had used a much simplified version of the hydrodynamic equation in which all the terms related to the convective acceleration were dropped from the hydrodynamic equation which are significant to represent spatial variations in hillslope characteristics. Zhang and Cundy (1989) were the first researchers to solve the two dimensional (2D) fully-dynamic shallow water equations with a finite difference scheme. The study by Zhang and Cundy (1989) has proved to be guiding step for the development of various two dimensional overland flow modeling schemes as it has brought out the importance of ground micro-topography. This study has shown that representation of the highly irregular microtopographic earth surface with a smooth surface does not lead to significant differences in the simulated discharge hydrographs because the continuity requirements are met in both cases. However, the grid spacing of 1 m used in their study for the simulation did not allow for detailed understanding of its effect. Practically, it is much more realistic to model the flow (flow depth and velocities) over the actual varying microtopography surface for erosion and flood inundation calculations.

Therefore, the concept of the kinematic cascade for representing real earth surface with a series of plane surfaces having different gradient has been adopted to incorporate variable slopes in an overland flow model (Borah et al., 1980). Stephenson, (1981) was probably the first researcher to present the kinematic wave model based on this concept for two-dimensional overland flow modelling. Govindaraju et al. (1992) presented a new methodology based on the simplified semi-analytical solution using the eign function expansion which is then combined with the kinematic wave approximation. They showed that the simplified semi-analytical solution procedure is advantages over the standard numerical solution of the two-dimensional overland flow equations in terms of computational effort and the amount of data required. This method predicts only an average flow depth and cannot recognizes the local variations of microtopography. Liu et al. (2004) developed a two dimensional kinematic wave model for simulating runoff generation and flow concentration on an experimental infiltrating hillaslope receiving artificial rainfall. Experimental results showed that the geometry of the topography of the slppe surface causes overland flow contrations and one-dimensional model is not able to simulate flow line concentration of the overland flow on irregular slopes. Also, a fully dynamic model is difficult to apply due to complex surface boundaries of a hillslopes, hence, a quasi-2D kinematic wave model is found adequately reflect the runoff concentration processes.

Hromadka et al. (1987) developed a diffusion hydrodynamic model, in which both the convective acceleration and local acceleration terms were dropped from the hydrodynamic equation. Further, Tayfur et al. (1993) compared numerical solutions of dynamic, diffusive and kinematic wave models for two dimensional overland flow on rough surfaces with an average steep slope of 0.086. In this study, the full St. Venant equations and the kinematic wave and diffusion wave approximations were used to route flow over experimental plots, and numerical results were compared with the observed hydrographs. The influence of the microtopography on convergence of the flow equations and deviations in local flow depths and velocities are discussed qualitatively. This study brings out the limitations of the surface flow equations when applied to irregular topography. The kinematic wave approximation is unacceptable in such cases since the characteristics move in the forward direction only. The implicit numerical procedure for the full St. Venant equations and the diffusion wave approximation also breaks down when the flow surface changes rapidly. It appears that while the numerical procedure may require a fine mesh dictated by computational accuracy, measurements need to be made at a larger scale yielding smoother surfaces to satisfy the gradually varying assumption in the flow equations. Di Giammarco et al. (1996) proposed a control volume finite element (CVFE) method to solve the mass conservation and momentum equations in simplified form (ignoring the contribution of the inertial terms) which is a locally conservative formulation of the better known finite element approach, to deal more efficiently with overland flow. From the point of view of conservation of scalar quantities such as mass and energy, the CVFE approach is an extension of the classical finite difference conservative staggered grid approach to irregular domains. The possibility of easily imposing as well as physically interpreting fluxes along lines and boundaries makes the method attractive in the overland flow generations. They have considered various hypothetical cases to conduct numerical experiments. One of the experiment was the case of the tilted V-catchment (with slopes in two directions (x and y) of overland plane and channel slope of the plane along the length of the channel. This case study were adopted in various studies conducted later on to demonstrate the capabilities of the newly developed two-dimensional overland flow models (e.g. Pandey and Huyakorn, 2004; He et al., 2008 and Lai, 2009; Sulis et al., 2010; Yu and Duan, 2014. Gottardi and venitelli (2008) presented a simplified DW and KW models for simulating overland flow over impervious surfaces, with analytical time integration of the ordinary differential equations. The one- and two-dimensional overland flow generations indicated good stability and efficiency of the numerical-analytical approach with unsteady rainfall rates and spatial variation of the surface roughness. The new solution scheme proposed by Lai (2009) based on conservative finite-volume formulation was tested with the results of analytical solution, Di Giammarco et al. (1996) and CASC2D (Sanchez, 2002) and found comparable. This study attempted to compare the stability range of explicit and implicit solver and found that the explicit solver allow more flexibility in selection of time step as compared to the implicit solver. In recent years, there has been an increasing trend of extending the application of the shallow water equations to the city and catchment scales (e.g., Unami et al. 2009; Mügler et al. 2011; Caviedes-Voullième et al., 2012; Costabile et al., 2013; Simons et al. 2014). Cea et al. (2014) have presented a validation of a 2-D overland flow models using empirical laboratory data. Instead of evaluating the performance of models to predict the observed hydrograph, they have used spatially distributed 2D water depth and velocity data to verify that how model the spatial distribution of these variables. They have considered several overland flow conditions over two impervious surfaces of the order of one square meter with different micro and macro-roughness characteristics. Based on the results of the study, Cea et al. (2014) concluded that even if the resolution of the topography data and numerical mesh are high enough to include all the small scale features of the bed surface, the roughness coefficient must account for the macro-roughness characteristics of the terrain in order to correctly reproduce the flow hydrodynamics.

Raneef (2014) has attempted to extend the Variable Parameter Muskingum Discharge (VPMD) method developed for one dimensional overland flow (Kale, 2010) to two dimensional overland flow plane sloping in both x and y directions S_{0x} and S_{0y}, respectively. They have assumed that the flow direction is controlled completely by the landform i.e the sheet of the flowing water may take the path having higher magnitude of slope, which is the resultant direction with slope of magnitude $S_0 = \sqrt{S_{0x}^2 + S_{0y}^2}$. Then the VPMD method was applied to the one-dimensional plane having a slope gradient of S_0, oriented along the direction of S_{0x} or S_{0y}, whichever is larger. The results of this approximation were also compared with the tilted V-Catchment as employed by Di Giammarco et al. (1996). Although this approximation gives the comparable results, however, the simulation results was realistic only when $S_{0x} \gg S_{0y}$. How this approximation gives erroneous resuls when $S_{0x} \approx S_{0y}$. Approximate methods such as the one based on the Variable Parameter Muskingum Discharge (VPMD) routing method is also capable of modelling the one-dimensional overland flow as it has been demonstrated by Kale (2010). But the VPMD method is not a fully mass conservative method. In order to overcome this problem, Shakya (2015) and Perumal et al. (2018) have developed a fully volume conservative two-dimensional Variable Parameter McCarthy-Muskingum (VPMM) overland flow routing method using one dimensional VPMM channel routing method (Perumal and Price, 2013). They have concluded that the method is numerically stable, substantially accurate and easy to solve at same time free from numerical convergence problem and can be applied for wide range of computational cell size without loss of accuracy.

2.4 Concluding Remarks

In last decades since 1960, the surface runoff models based on the numerical solution of the shallow water equations known as Saint-Venant equations or their approximations emerged as the effective tools for water resources planning and management. The traditional flow models have avoided using microtopography into modelling framework not only because of the complications arising in the numerical procedure, but also because of the extra effort involved in obtaining the microtopography data at the grid scale dictated by the numerical model. With advancement of remote sensing and GIS techniques obtaining spatial and temporal microptopographic data is no more a constraint while complications in the application of the numerical solution schemes particularly for two dimensional flow simulation has continued to be considerable constraint although computational facilities are dramatically improved over last four decades.

It is also fact that the model user community mostly not aware about the different hypothesis lying behind these development and simplifications and hence encounters the problem during judging its adaptability to fulfill their set objectives and test case configurations. Further, various research study emphasize the sufficiency of diffusive wave model for overland flow modeling. Therefore, there is need of a study which aims at comparing the predictive abilities of different models and evaluating potential gain by using advanced numerical scheme for modelling runoff based on diffusive wave formation in two dimensional domains to emphasize their main strengths and weaknesses.

CHAPTER - 3
METHODOLOGY

In this chapter the basic equations used for the development of the computer code for two dimensional explicit and VPMM overland flow solution schemes are presented. Firstly, the explicit scheme for solving the two dimensional diffusive wave overland model is presented which is followed by the two dimensional VPMM solution method. For the sake of brevity, these method are briefly presented however it is advised to refer the original references for more details.

3.1 Two-dimensional Explicit solution for Overland Flow Routing

The two-dimensional overland flow can be expressed by the continuity and momentum equation for each of the planar coordinate directions x and y. These equations can be written as (Zhang and Cundy 1989; Chow and Ben-Zvi 1973)

$$\frac{\partial h}{\partial t}+\frac{\partial (uh)}{\partial x}+\frac{\partial (uv)}{\partial y}=R_e(x,y,t)-i(x,y,t)\cos(\Psi)\cos(\phi) \tag{3.1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\cos(\Psi)\cos(\phi)\,g\frac{\partial h}{\partial x}= gsin(\Psi)-gS_{fx}-\frac{q_L u}{h} \tag{3.2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+\cos(\Psi)\cos(\phi)\,g\frac{\partial h}{\partial y}= gsin(\phi)-gS_{fy}-\frac{q_L v}{h} \tag{3.3}$$

where

$h(x,y,t)$ = overland flow depth;

$u(x,y,t)$ = depth − averaged flow velocity in the x − direction;

$v(x,y,t)$ = depth − averaged flow velocity in the y − direction;

$R_e(x,y,t)$ = Rainfall intensity;

$i(x,y,t)$ = infiltration rate;

$q_L(x,y,t)$ = net lateral inflow rate (Rainfall − infiltration);

Ψ = angle of the slope with respect to x − direction;

ϕ = angle of the slope with respect to y − direction;

g = acceleration due to gravity;

S_{fx} and S_{fy} = the friction slopes respectively in x and y − directions;

The friction slope in Manning's equation can be written as

$$S_{fx} = \frac{n^2 u\sqrt{u^2+v^2}}{h^{4/3}} \tag{3.4}$$

$$S_{fy} = \frac{n^2 v\sqrt{u^2+v^2}}{h^{4/3}} \tag{3.5}$$

where

$n = \text{Manning's roughness coefficient};$

In the simplified diffusive wave form the above equation (3.1) to (3.3) by ignoring the infiltration term can be expressed for each square grid cell as in a raster GIS system as (Boll, 2001)

$$\frac{\partial h}{\partial t} + \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = R_e \tag{3.6}$$

For simplification, the two acceleration terms in the energy equations (equation 3.2 and 3.3) that are typically small are neglected, then equation (3.2) and (3.3) can be reduced to

$$S_{fx} = S_{0x} - \frac{\partial h}{\partial x} \tag{3.7}$$

$$S_{fy} = S_{0y} - \frac{\partial h}{\partial y} \tag{3.8}$$

where the terms $\frac{\partial h}{\partial x}$ and $\frac{\partial h}{\partial y}$ represent the slope of water surface in the x- and y-directions, respectively.

The unit-width discharge for turbulent flow in the x- and y-directions, respectively, is described by the Manning equation

$$q_x = \frac{1}{n}\sqrt{S_{fx}}\, h^{5/3} \tag{3.9}$$

$$q_y = \frac{1}{n}\sqrt{S_{fy}}\, h^{5/3} \tag{3.10}$$

Where,

h = flow depth,

q_x and q_y = flow discharge in x and y direction,

t = time, and all other variables are previously described above.

3.1.1 Numerical Representation in Explicit Finite Difference Form

The explicit numerical scheme to solve the two-dimensional diffusive wave equation (Equation 3.6) in the finite difference form can be expressed as

$$\frac{\Delta h}{\Delta t} + \frac{\Delta q_x}{\Delta x} + \frac{\Delta q_y}{\Delta y} = R_e \tag{3.11}$$

Using above Equation (3.11), the flow at time increment t+1 can be estimated using previous known information about flow depth at time t as

$$\Delta h = \left[R_e - \frac{\Delta q_x}{\Delta x} - \frac{\Delta q_y}{\Delta y}\right] \tag{3.12}$$

The Equation (3.12) can be expressed as

$$h^{t+1} - h^t = \left[R_e - \frac{q_{xout}^t - q_{xin}^t}{\mathrm{W}} - \frac{q_{yout}^t - q_{yin}^t}{\mathrm{W}}\right]\Delta\mathrm{t} \tag{3.13}$$

where the subscripts "in" and "out" describe flow into and out of a given cell, respectively. W is the cell length in the x- or y-direction.

When the Cartesian grid cells in the watershed are defined by the i and j coordinates in x and y-direction respectively as shown in Figure 3.1, the Equation 3.12

$$h_{i,j}^{t+1} = h_{i,j}^t + R_e\Delta\mathrm{t} - \left[\frac{q_{x(i,j+1)}^t - q_{x(i,j)}^t}{\mathrm{W}} - \frac{q_{y(i,j+1)}^t - q_{y(i,j)}^t}{\mathrm{W}}\right]\Delta\mathrm{t} \tag{3.14}$$

Similarly, the equations (3.7) and (3.8) can be written as

$$S_{0x(i,j)} = \frac{E_{i,j-1} - E_{i,j}}{W} \tag{3.15}$$

$$S_{0y(i,j)} = \frac{E_{i-1,j} - E_{i,j}}{W} \tag{3.16}$$

Based on the above expansion, the energy equation of the diffusive-wave approximation can be expressed as

$$S_{fx}^t = S_{0x(i,j)} - \frac{h_{i,j}^t - h_{i,j-1}^t}{W} \tag{3.17}$$

$$S_{fy}^t = S_{0y(i,j)} - \frac{h_{i,j}^t - h_{i-1,j}^t}{W} \tag{3.18}$$

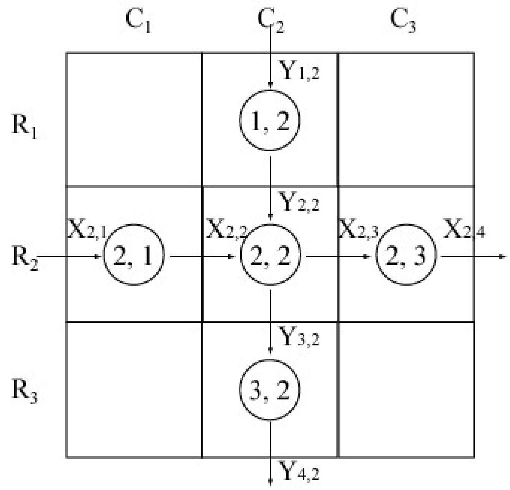

Figure 3.1. Representation of a cell grid system and notation convention for any flow or other quantity calculation that crosses cell borders (Adopted from Boll, 2001).

The computed unit discharge in x and y directions when $S^t_{fx(i,j)} \geq 0$ and $S^t_{fy(i,j)} \geq 0$ are given by the following equations

$$q_{x(i,j)} = \frac{1}{n_{i,j}}\left[h^t_{i,j}\right]^{5/3}\sqrt{S^t_{fx(i,j)}} \tag{3.19}$$

$$q_{y(i,j)} = \frac{1}{n_{i,j}}\left[h^t_{i,j}\right]^{5/3}\sqrt{S^t_{fy(i,j)}} \tag{3.20}$$

However, when $S^t_{fx(i,j)} \geq 0$ and $S^t_{fy(i,j)} \geq 0$ condition does not met then the unit discharge in x and y directions are given by the following equations

$$q_{x(i,j)} = \frac{-1}{n_{i,j}}\left[h^t_{i,j}\right]^{5/3}\sqrt{-S^t_{fx(i,j)}} \tag{3.21}$$

$$q_{y(i,j)} = \frac{-1}{n_{i,j}}\left[h^t_{i,j}\right]^{5/3}\sqrt{-S^t_{fy(i,j)}} \tag{3.22}$$

It should be noted that the stability of the explicit scheme for the finite-difference solution can be achieved by using small time steps. The selection of time interval depends upon the grid size, precipitation characteristics (rainfall intensity and duration), surface conditions (slope, roughness), and infiltration characteristics.

3.2 Two Dimensional VPMM Model for Overland Flow Modelling

The 2D VPMM model is developed using the governing equations of the two dimensional flow, the diffusive wave approximation of the Saint-Venant equations represented by continuity and momentum (energy) equations (Equations 3.6 – 3.8). The brief theoretical background is presented here in the following section.

3.2.1 Concept of Variable Parameter Muskingum Discharge (VPMD) Routing Method

The basic development of 2D-VPMM method is started based on the development of 1D-VPMD method (Kale, 2010; Kale and Perumal, 2014). The basic concept of the VPMD overland flow method is that there exists one-to-one relationship between the flow depth (or the cross sectional area) of the flow and the discharge at the same location defining the steady state flow rating curve. While in unsteady flow condition, the unique relationship exists between the above mentioned variables not at the same location but somewhere downstream from that location as shown in Figure 3.2.

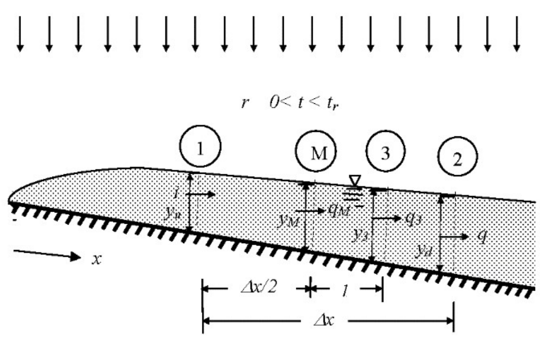

Figure 3.2. Definition sketch of the VPMD method governing the flow over an impervious surface (Kale, 2010).

3.2.2 Mathematical Development of 2D-VPMM Method

In order to reduce the mass conservation problem in the VPMD method for overland flow simulation (Kale and Perumal, 2014), the 1D-VPMM method for overland flow modelling has been developed based on the VPMM channel routing method (Perumal and Price, 2013). The extension of the 1D-VPMM method into 2D-VPMM for two dimensional overland flow modelling by Shakya (2015) and Perumal et al., (2018) is briefly presented in the following section.

The two-dimensional overland flow plane is considered to be formed by a grid network of seamlessly interconnected cells of size $\Delta x \times \Delta y$. Figure 3.3 shows the details of the considered grid network of cells.

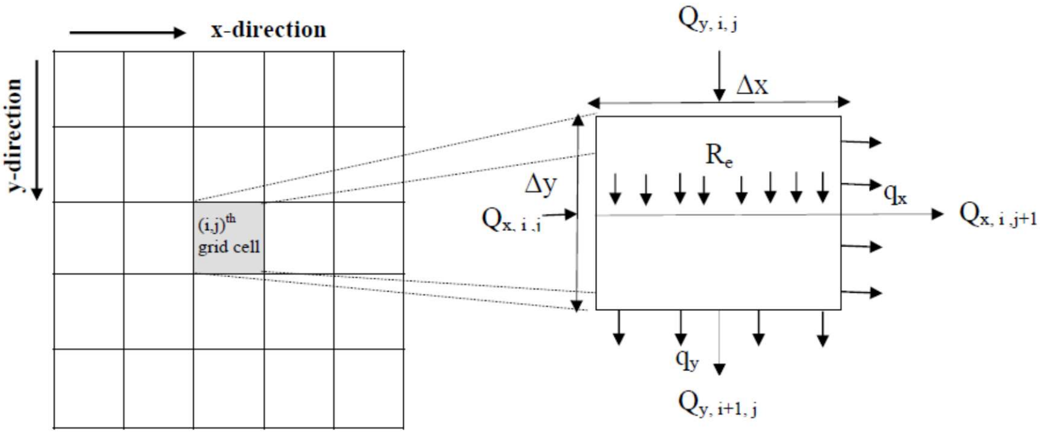

Figure 3.3. Computational grid used in the development of 2D-VPMM method for overland flow modelling (Perumal et al., 2018).

3.2.2.1 The Assumptions Employed in the 2D VPMM Method Development

The assumptions employed in the development of the 2D-VPMM model for two-dimensional overland flow propagation over a grid cell subject to rainfall excess R_e, as shown in Figure 3.3.

i. The flow plane in the respective flow direction is assumed to be a wide rectangular channel.
ii. The overland plane is assumed to be fully impervious and, therefore, the applied rainfall is considered as the rainfall excess (effective rainfall).
iii. The longitudinal gradient of water depth, $\partial h/\partial x$ and $\partial h/\partial y$ respectively, in x and y directions are not negligible relative to the bed slopes S_{0x} and S_{0y}, respectively.
iv. As the Muskingum storage equation is derived from the momentum equation of the Saint-Venant equations (*Perumal and Price*, 2013), it can be considered as the manifestation of the momentum equation governing the flow dynamics.

3.2.2.2 Brief Description of the 2D-VPMM Model

By multiplying Equation (3.6) by the cell area of grid $\Delta x \times \Delta y$, one can get

$$\Delta x\Delta y\frac{\partial h}{\partial t} = R_e\Delta x\Delta y - \Delta x\Delta y\frac{\partial q_x}{\partial x} - \Delta x\Delta y\frac{\partial q_y}{\partial y} \tag{3.23}$$

It can be inferred from equation (3.23) that both the grid discharges Q_x and Q_y are influenced by the grid storage S which is the manifestation of momentum equation governing the dynamics of flow in the direction of flow. As both Q_x and Q_y are influenced by the same grid storage in conjunction with the grid cell rainfall during the routing time interval Δt, the two-dimensional flow continuity equation expressed by equation (3.23) can be considered as the equation governing the proposed two-dimensional overland flow model. If steady flow prevails over the two-dimensional overland flow plane, then the storage S over the grid cell can be expressed as

$$S_x = \Delta x A_{Mx} \; S_y = \Delta x A_{My} \tag{3.24 a, b}$$

where the notations S_x and S_y individually denote the same grid storage S of the grid cell and A_{Mx} and A_{My} denote the cross-sectional flow area at the mid-section of the grid cell along x and y directions, respectively. The subscripts x and y attached with the storage notation S, is simply meant for qualifying the storage controlling the flow in x and y directions which in turn represent the momentum equations governing the flow dynamics in x and y directions.

The grid storage S given by equation (3.24) may be expressed in terms of normal discharge and normal velocity along x and y directions as

$$S_x = \frac{\Delta x Q_{0x}}{v_{0x}} \; S_y = \frac{\Delta y Q_{0y}}{v_{0y}} \tag{3.25 a, b}$$

Again, the normal discharges Q_{0x} and Q_{0y} may be re-expressed in terms of unsteady discharges and its variations in the respective flow directions as

$$S_x = \frac{\Delta x}{v_{0x}}\left(Q_x + \frac{a_{0x}}{c_{0x}}\frac{\partial Q_x}{\partial x}\right) \; S_y = \frac{\Delta y}{v_{0y}}\left(Q_y + \frac{a_{0y}}{c_{0y}}\frac{\partial Q_y}{\partial y}\right) \tag{3.26 a, b}$$

where

$$a_{0x} = \frac{Q_{0x}}{2S_{0x}\Delta y} \; a_{0y} = \frac{Q_{0y}}{2S_{0y}\Delta x} \tag{3.27 a, b}$$

The notations a_{0x} and c_{0x}, and a_{0y} and c_{0y}, respectively, denote the diffusion coefficient and normal wave celerity along x and y flow directions. They are estimated at the midpoint of the grid cell corresponding to the flow depth h_M.

Accordingly, the equations governing the dynamics of flow in x and y directions expressed by equation (3.23) are modified using equation (3.26) as

$$\frac{\partial}{\partial t}\left[\frac{\Delta x}{v_{0x}}\left(Q_x + \frac{a_{0x}}{c_{0x}}\frac{\partial Q_x}{\partial x}\right)\right] = R_e \Delta x \Delta y - \Delta x \frac{\partial Q_x}{\partial x} - \Delta y \frac{\partial Q_y}{\partial y} \tag{3.28 a}$$

$$\frac{\partial}{\partial t}\left[\frac{\Delta y}{v_{0y}}\left(Q_y + \frac{a_{0y}}{c_{0y}}\frac{\partial Q_y}{\partial y}\right)\right] = R_e \Delta x \Delta y - \Delta y \frac{\partial Q_y}{\partial y} - \Delta x \frac{\partial Q_x}{\partial x} \tag{3.28 b}$$

In order to arrive at the overland flow routing equation in the classical Muskingum method formulation, Equation (3.28) is applied at the center point M of the finite difference cube as shown in Figure 3.4, formed by encompassing the grid cells at the time levels of t and t+Δt (denoted, henceforth, as t+1), where Δt is the routing time interval. The (i, j)th grid cell corresponding to a given time level with inflow and outflow discharges, respectively, denoted as $Q_{x,i,j}$ and $Q_{x,i,j+1}$, and $Q_{y,i,j}$ and $Q_{y,i+1,j}$, for flow along x and y directions , respectively, is also shown in Figure 3.3. The subscripts (*i, j*) denote the spatial index variation along y and x directions, respectively. First, the routing equation governing the estimation of Q_x at the outlet of any grid cell is developed in detail using the VPMM routing method (Perumal and Price, 2013).

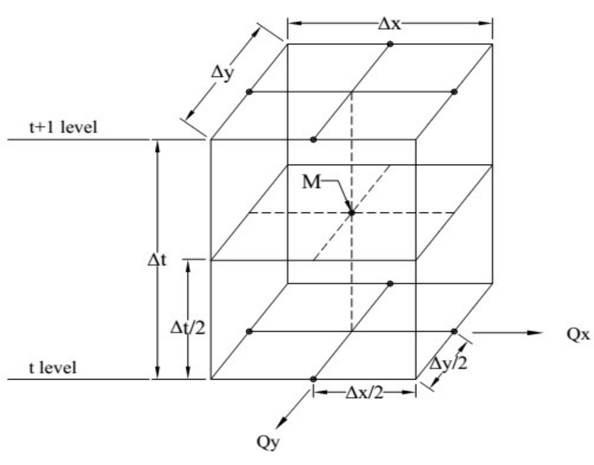

Figure 3.4. Finite difference scheme employed for discretization of the governing equation of 2D-VPMM method (Shakya, 2015).

Accordingly Equation (3.28 a) is expressed in finite difference form to arrive at the equation governing overland flow propagation along x as

$$\frac{\Delta x}{v_{0x,M}^{t+1}}\left[\frac{Q_{x,i,j+1}^{t+1}+Q_{x,i,j}^{t+1}}{2}+\left.\frac{a_{0x}}{c_{0x}}\right|_M^{t+1}\left(\frac{Q_{x,i,j+1}^{t+1}-Q_{x,i,j}^{t+1}}{\Delta x}\right)\right]$$

$$-\frac{\Delta x}{v_{0x,M}^{t}}\left[\frac{Q_{x,i,j+1}^{t}+Q_{x,i,j}^{t}}{2}+\left.\frac{a_{0x}}{c_{0x}}\right|_M^{t}\left(\frac{Q_{x,i,j+1}^{t}-Q_{x,i,j}^{t}}{\Delta x}\right)\right]$$

$$=\Delta t R_e \Delta x \Delta y-\Delta t\left(\frac{Q_{x,i,j+1}^{t+1}+Q_{x,i,j+1}^{t}}{2}-\frac{Q_{x,i,j}^{t+1}+Q_{x,i,j}^{t}}{2}\right) \tag{3.29}$$

$$-\Delta t\left(\frac{Q_{y,i+1,j}^{t+1}+Q_{y,i+1,j}^{t}}{2}-\frac{Q_{y,i,j}^{t+1}+Q_{y,i,j}^{t}}{2}\right)$$

The variables with superscripts t and t+1 represent that variable at t and t+1 time level. In a similar manner, the equation governing flow along y direction may be expressed. Equation (3.29) may be reformulated to express it in the form of governing equation of the Muskingum method, respectively, as

$$\frac{\Delta x}{v_{0x,M}^{t+1}}\left[\left(\frac{1}{2}-\left.\frac{a_{0x}}{c_{0x}}\right|_M^{t+1}\frac{1}{\Delta x}\right)Q_{x,i,j}^{t+1}+\left(\frac{1}{2}+\left.\frac{a_{0x}}{c_{0x}}\right|_M^{t+1}\frac{1}{\Delta x}\right)Q_{x,i,j+1}^{t+1}\right]$$

$$-\frac{\Delta x}{v_{0x,M}^{t}}\left[\left(\frac{1}{2}-\left.\frac{a_{0x}}{c_{0x}}\right|_M^{t}\frac{1}{\Delta x}\right)Q_{x,i,j}^{t}+\left(\frac{1}{2}+\left.\frac{a_{0x}}{c_{0x}}\right|_M^{t}\frac{1}{\Delta x}\right)Q_{x,i,j+1}^{t}\right]$$

$$=\Delta t R_e \Delta x \Delta y-\Delta t\left(\frac{Q_{x,i,j+1}^{t+1}+Q_{x,i,j+1}^{t}}{2}-\frac{Q_{x,i,j}^{t+1}+Q_{x,i,j}^{t}}{2}\right) \tag{3.30}$$

$$-\Delta t\left(\frac{Q_{y,i+1,j}^{t+1}+Q_{y,i+1,j}^{t}}{2}-\frac{Q_{y,i,j}^{t+1}+Q_{y,i,j}^{t}}{2}\right)$$

Using the following notations,

$$K_x^{t+1}=\frac{\Delta x}{v_{0x,M}^{t+1}}\;K_x^{t}=\frac{\Delta x}{v_{0x,M}^{t}} \tag{3.31 a,b}$$

$$\theta_x^{t+1} = \left(\frac{1}{2} - \frac{Q_{0x,M}^{t+1}}{2S_{0x}c_{0x,M}^{t+1}} \frac{1}{\Delta x \Delta y} \right) \quad \theta_x^{t} = \left(\frac{1}{2} - \frac{Q_{0x,M}^{t}}{2S_{0x}c_{0x,M}^{t}} \frac{1}{\Delta x \Delta y} \right) \tag{3.32 a,b}$$

The flow propagation equation (3.30) can be reformulated in the form of the governing equation of the Muskingum method as

$$\begin{aligned} &K_x^{t+1}\left[\theta_x^{t+1}Q_{x,i,j}^{t+1} + \left(1-\theta_x^{t+1}\right)Q_{x,i,j+1}^{t+1}\right] - K_x^{t}\left[\theta_x^{t}Q_{x,i,j}^{t} + \left(1-\theta_x^{t}\right)Q_{x,i,j+1}^{t}\right] \\ &= \Delta t R_e \Delta x \Delta y - \Delta t\left(\frac{Q_{x,i,j+1}^{t+1} + Q_{x,i,j+1}^{t}}{2} - \frac{Q_{x,i,j}^{t+1} + Q_{x,i,j}^{t}}{2}\right) \\ &\quad -\Delta t\left(\frac{Q_{y,i+1,j}^{t+1} + Q_{y,i+1,j}^{t}}{2} - \frac{Q_{y,i,j}^{t+1} + Q_{y,i,j}^{t}}{2}\right) \end{aligned} \tag{3.33}$$

Since two independent momentum equations are employed to represent the flow dynamics along x and y directions of the two-dimensional overland flow process, it may be considered that Equation (3.33) is the equation governing flow along x direction duly accounting for its dynamics. The parameters K_x and θ_x denote the travel time and the weighting factor of the Muskingum storage equation governing flow dynamics in the x direction. It may be noted that there are two unknowns $Q_{x,i,j+1}^{t+1}$ and $Q_{y,i+1,j}^{t+1}$ in the equation (16) and, therefore, to arrive at the explicit solution of $Q_{x,i,j+1}^{t+1}$ using the Muskingum routing equation, the inflow and outflow of Q_y corresponding to time $(t+1)$ are assumed to be the same as the known estimates of Q_y corresponding to time t. Based on this consideration, the Muskingum routing equation for estimation of flow at the outlet of the grid cell along x direction is arrived at from equation (3.33) as

$$Q_{x,i,j+1}^{t+1} = C_{1x}Q_{x,i,j}^{t+1} + C_{2x}Q_{x,i,j}^{t} + C_{3x}Q_{x,i,j+1}^{t} + C_{4x}\left(2R_e\Delta x\Delta y\right) - C_{4x}\left[2\left(Q_{y,i+1,j}^{t} - Q_{y,i,j}^{t}\right)\right] \tag{3.34}$$

where,

$$\begin{aligned} C_{1x} &= \frac{\Delta t - 2K_x^{t+1}\theta_x^{t+1}}{\Delta t + 2K_x^{t+1}\left(1-\theta_x^{t+1}\right)} \quad C_{2x} = \frac{\Delta t + 2K_x^{t}\theta_x^{t}}{\Delta t + 2K_x^{t+1}\left(1-\theta_x^{t+1}\right)} \\ C_{3x} &= \frac{-\Delta t + 2K_x^{t}\left(1-\theta_x^{t}\right)}{\Delta t + 2K_x^{t+1}\left(1-\theta_x^{t+1}\right)} \quad C_{4x} = \frac{\Delta t}{\Delta t + 2K_x^{t+1}\left(1-\theta_x^{t+1}\right)} \end{aligned} \tag{3.35 a,b,c,d}$$

Similarly the routing equation for estimation of flow at the outlet of grid cell along y direction is obtained as

$$Q_{y,i+1,j}^{t+1} = C_{1y} Q_{y,i,j}^{t+1} + C_{2y} Q_{y,i,j}^{t} + C_{3y} Q_{y,i+1,j}^{t} + C_{4y} \left(2 R_e \Delta x \Delta y\right) - C_{4y} \left[2\left(Q_{x,i,j+1}^{t} - Q_{x,i,j}^{t}\right)\right] \quad (3.36)$$

where,

$$C_{1y} = \frac{\Delta t - 2K_y^{t+1}\theta_y^{t+1}}{\Delta t + 2K_y^{t+1}\left(1-\theta_y^{t+1}\right)} \quad C_{2y} = \frac{\Delta t + 2K_y^{t}\theta_y^{t}}{\Delta t + 2K_y^{t+1}\left(1-\theta_y^{t+1}\right)}$$

$$C_{3y} = \frac{-\Delta t + 2K_y^{t}\left(1-\theta_y^{t}\right)}{\Delta t + 2K_y^{t+1}\left(1-\theta_y^{t+1}\right)} \quad C_{4y} = \frac{\Delta t}{\Delta t + 2K_y^{t+1}\left(1-\theta_y^{t+1}\right)} \quad (3.37\ a,b,c,d)$$

where,

$$K_y^{t+1} = \frac{\Delta y}{v_{0y,M}^{t+1}} \quad K_y^{t} = \frac{\Delta y}{v_{0y,M}^{t}} \quad (3.38\ a,b,c,d)$$

$$\theta_y^{t+1} = \left(\frac{1}{2} - \frac{Q_{0y,M}^{t+1}}{2 S_{0y} c_{0y,M}^{t+1}} \cdot \frac{1}{\Delta x \Delta y}\right) \quad \theta_y^{t} = \left(\frac{1}{2} - \frac{Q_{0y,M}^{t}}{2 S_{0y} c_{0y,M}^{t}} \cdot \frac{1}{\Delta x \Delta y}\right) \quad (3.39\ \ a\ b)$$

3.2.3 Runoff Routing Procedure

The step-by-step 2D-VPMM model overland flow routing procedure of overland flow propagation in x and y directions of each cell is presented in the flow chart shown in Figure 3.5. The runoff due to effective rainfall over the cell is considered as the lateral flow in the overland flow routing process along x and y directions. But the runoff entering into the cell from the adjacent cell or cells and leaving from the cell to the adjacent cell or cells are treated as inflow and outflow, respectively, for the routing equations employed in the model. The steps 1 to 14 as shown in Figure 3.5 are repeated for the next grid cell and subsequently for all the grid cells at the current time level. After estimating outflow discharges g for all the grid cells of the overland flow plane grid, the computation advances to the next time level. Following the same computational steps until the total simulation time.

3.3 Performance Evaluation Measures

The following performance criteria has been used to evaluate the performance of the 2D-VPMM method and 2D- DW explicit method.

Step 1: Use constant rate value of R_e as a input over the routing time interval Δt

Step 2: Assuming initially dry condition of overland plane: $K_x^t = K_y^t = 0$ *and* $\theta_x^t = \theta_y^t = 0$ *at* $t = 0$

Step 3: The routing computations are carried out grid cell wise, i.e., for each individual grid cell, first in the x-direction and then subsequently in the y-direction or vice versa

Step 4: Compute unrefined estimate of travel time K and weighting factor θ at current time level (t +1) by assuming $K_x^{t+1} = K_x^t$; $K_y^{t+1} = K_y^t$ and $\theta_x^{t+1} = \theta_x^t$; $\theta_y^{t+1} = \theta_y^t$

Step 5 Compute the coefficient of routing equation (17), i.e. C_{1x}, C_{2x}, C_{3x} and C_{4x} using equation (18) and that of equation (19), i.e. C_{1y}, C_{2y}, C_{3y} and C_{4y} using equation (20)

Step 6: Compute outflow discharge $Q_{x,i,j+1}^{t+1}$ at $(t+1)$ by using equation (17) and subsequently outflow discharge $Q_{y,i+1,j}^{t+1}$ using equation (19)

Step 7: Compute the normal discharges $Q_{0,M}$ along x and y directions corresponding to unsteady flow depth h_M using $Q_{0M,x}^{t+1} = \theta_x^{t+1} Q_{x,i,j}^{t+1} + \left(1-\theta_x^{t+1}\right) Q_{x,i,j+1}^{t+1}$ and $Q_{0M,y}^{t+1} = \theta_y^{t+1} Q_{y,i,j}^{t+1} + \left(1-\theta_y^{t+1}\right) Q_{y,i+1,j}^{t+1}$, respectively.

Step 8: Compute the normal flow depths along x and y directions corresponding to the computed $Q_{0M,x}^{t+1}$ and $Q_{0M,y}^{t+1}$ using $h_{M,x,i,j}^{t+1} = \left[\left(Q_{0,M,x}^{t+1} \times n\right) / \left(\Delta y \times \sqrt{S_{0x}}\right)\right]^{\frac{3}{5}}$ and $h_{M,y,i,j}^{t+1} = \left[\left(Q_{0,M,y}^{t+1} \times n\right) / \left(\Delta x \times \sqrt{S_{0y}}\right)\right]^{\frac{3}{5}}$, respectively.

Step 9: Computed velocities $v_{0M,x}$ and $v_{0M,y}$ corresponding to these normal depths using $v_{0M,x} = \frac{1}{n}\left(h_{M,x,i,j}\right)^{\frac{2}{3}} S_{0x}^{\frac{1}{2}}$ and $v_{0M,y} = \frac{1}{n}\left(h_{M,y,i,j}\right)^{\frac{2}{3}} S_{0y}^{\frac{1}{2}}$, respectively.

Step 10: Computed the celerities $c_{0M,x}$ and $c_{0M,y}$ using $c_{0M,x} = 1.67 v_{0M,x}$ and $c_{0M,y} = 1.67 v_{0M,y}$, respectively.

Step 11: Using equations at step 9, compute the refined values K_x^{t+1} and K_y^{t+1} using equations (14a) and (21a), respectively.

Step 12: Using equations at steps 7 and 10, compute the refined values θ_x^{t+1} and θ_y^{t+1} using equations (15a) and (22a), respectively.

Step 13: Using refined values of K_x^{t+1} and K_y^{t+1}, *and* θ_x^{t+1} *and* θ_y^{t+1} at steps 11 and 12 and those corresponding values at the previous time step (t), the refined routing coefficients for x-direction outflow and those for y-direction outflow are computed using equations (18a-d) and (20a-d), respectively.

Step 14: Compute refined outflow discharges using equation (17) and (19) for x and y-directions, respectively. These outflow discharges now become the inflow discharges to the adjoining grid cells of the considered grid.

Figure 3.5. The flow chart of the step-by-step 2D-VPMM overland flow routing procedure of overland flow propagation in x and y directions of each cell.

Nash-Sutcliffe Efficiency (NSE):

$$NSE = \frac{\sum_{i=1}^{N}\left(Q_{oi} - Q_{Avg}\right)^2 - \sum_{i=1}^{N}\left(Q_{oi} - Q_{ci}\right)^2}{\sum_{i=1}^{N}\left(Q_{oi} - Q_{Avg}\right)^2} \tag{3.40}$$

where, Q_{oi} = ith ordinate of the observed discharge hydrograph; Q_{Avg} = Average of the observed discharge hydrogrh; N = number of the ordinates of the hydrograph; Q_{ci} = ith ordinate of the computed discharge hydrograph.

The percentage error in peak discharge, Q_{per} (in %):

$$Q_{per} = \left(\frac{Q_{pc}}{Q_{po}} - 1\right) \times 100 \tag{3.41}$$

where, Q_{pc} is the peak discharge computed by the proposed 2D-VPMM model, Q_{po} is the peak discharge of the benchmark solution or the observed hydrograph.

The normalized error in time to peak discharge, t_{qper}:

$$t_{qper} = \left(\frac{t_{qpc}}{t_{qpo}} - 1\right) \tag{3.42}$$

where, t_{qpc} is the time to peak discharge of the computed hydrograph by the proposed 2D-VPMM model, t_{qpo} is the time to peak discharge of the hydrograph of benchmark or the observed hydrograph.

The mass conservation error, *EVOL* (in %):

$$EVOL(in\%) = \left[\left\{\frac{\sum_{i=1}^{NQ} Q_{ci}}{\sum_{j=1}^{NR} R_{ej} A}\right\} - 1\right] \times 100 \tag{3.43}$$

where, R_{ej} = jth ordinate of the effective rainfall intensity, A = Area of the overland flow plane, NQ = Number of simulated hydrograph ordinates, NR = Number of effective rainfall ordinates generating the hydrograph. A negative value of EVOL indicates a loss of mass and a positive value of EVOL indicates a gain of mass. A value of EVOL = 0 implies that the method is fully volume conservative.

CHAPTER - 4

ANALYSIS AND RESULTS

In this chapter, the simulation results obtained using the two dimensional diffusive wave explicit model (2D-DW-explicit model) which employ the explicit finite difference scheme (similar to the explicit numerical scheme used in the CASC2D model developed by the Colorado State University, USA) and the 2D-VPMM model which employ the storage-based Muskingum-McCarthy schemes are compared. This study uses the observed rainfall-runoff events datasets which are widely considered in the literature to test the source code written for the 2D overland flow modelling methods. The first case considered in this study uses a hypothetical case of a tilted V-Catchment used by Di Giammarco et al. (1996), in which two planes each of size 1000m x 800m are joined with a rectangular channel in between. The second case considers the experimental rainfall-runoff data collected from the study of laboratory catchment of the University of Illinois, Urbana Champaign (Maksimović and Radojković, 1986). The obtained results by using the 2D-DW-explicit model and the 2D-VPMM model and their comparison to evaluate performance of these individual method is described in following section.

4.1 Performance Evaluation Using Hypothetical V-Catchment Data

The tilted V-catchment rainfall-runoff simulation event is studied by Di Giammarco et al. (1996) to verify the performance of their proposed the control volume finite element method (CVFEM) based on the simplified form of the mass conservation and momentum equation by ignoring the contribution of inertial term. In the study of Di Giammarco et al. (1996) the experiments was carried out on the various slope cases. One of the case was of the tilted V-Catchment (with slopes in two directions i.e x and y –direction of the overland flow plane and channel slope equal to the slope of the plane along the length of the channel) as shown in Figure 4.1. We have attempted to use this 2D tilted V-catchment case of overland flow generation by a rainfall event to examine the suitability of the 2D-DW-explicit and 2D-VPMM models in reproducing the rainfall-runoff phenomena as well as their comparative performance. In this tilted V-catchment experiment, the simulation was conducted using a rainfall excess input with an intensity of 10.8mm/hr for the duration of 90 minutes. The runoff generated from the precipitation over the channel area is not considered. The Manning's n for the planes is taken as 0.015, whereas for the channel it is taken as 0.15. Di

Giammarco et al. (1996) have also attempted to compare the outflow hydrograph at the channel outlet obtained using the CVFEM with the various methods like the Integrated Finite Difference (IFD) method and the results of the SHE model (Abbott et al., 1986). The tilted V-catchment numerical experiment conducted by Di Giammarco et al. (1996) is further used by numerous researchers such as Panday and Huyakorn (2004), He et al. (2008), Lai (2009), CASC2D (digitized from Lai, 2009), 2D fully dynamic wave equation (Costabile et al., 2013) and MODHMS (digitized from Yu and Duan, 2014). The simulated discharge hydrograph at the outlet of channel of tilted V-catchment by using 2D-VPMM, 2D-DW-Explicit and Gridded Surface Subsurface Hydrologic Analysis model (GSSHA) (Downer et al. 2005), all other method are presented in Figure 4.2 and their performance criteria is presented in Table 4.1. Note that the 1D-VPMM channel routing method (Perumal and Price, 2013) applicable for routing flow in rectangular channel is applied to routing channel flow in both of these 2D-VPMM, 2D-DW-Explicit models.

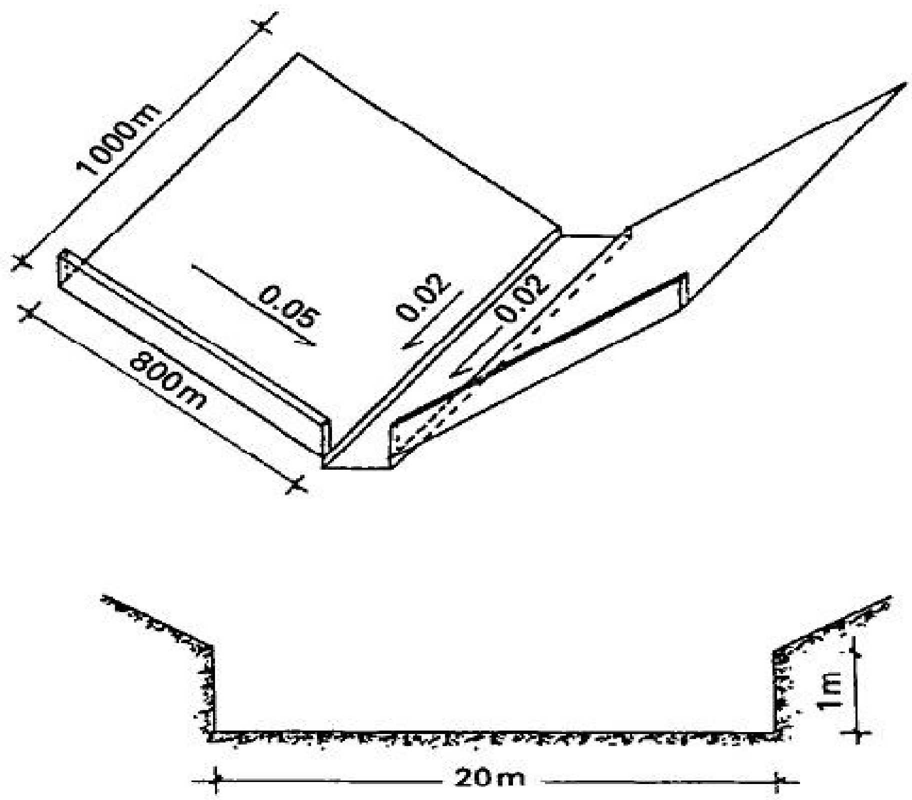

Figure 4.1. Geometry of the Tilted V-Catchment (Di Giammarco et al., 1996).

Table 4.1 The 2D-VPMM model simulation results comparison with 2D-DW-Explicit model and other model results (for tilted-v-cathment).

Model	**NSE (%)**	**Q_{per} (%)**
2D-VPMM	99.36	-0.19
2D-DW-Explicit	98.83	-0.19
CVFEM (*Di Giammarco et al.*, 1996)	98.04	1.98
Panday and Huyakorn (2004)	95.56	0.4
He et al., (2008)	98.12	0.09
Lai, (2009)	93.86	-0.12
MODHMS (digitized from *Yu and Duan,* 2014)	95.90	1.23
CASC2D (digitized from *Lai,* 2009)	90.49	-0.22

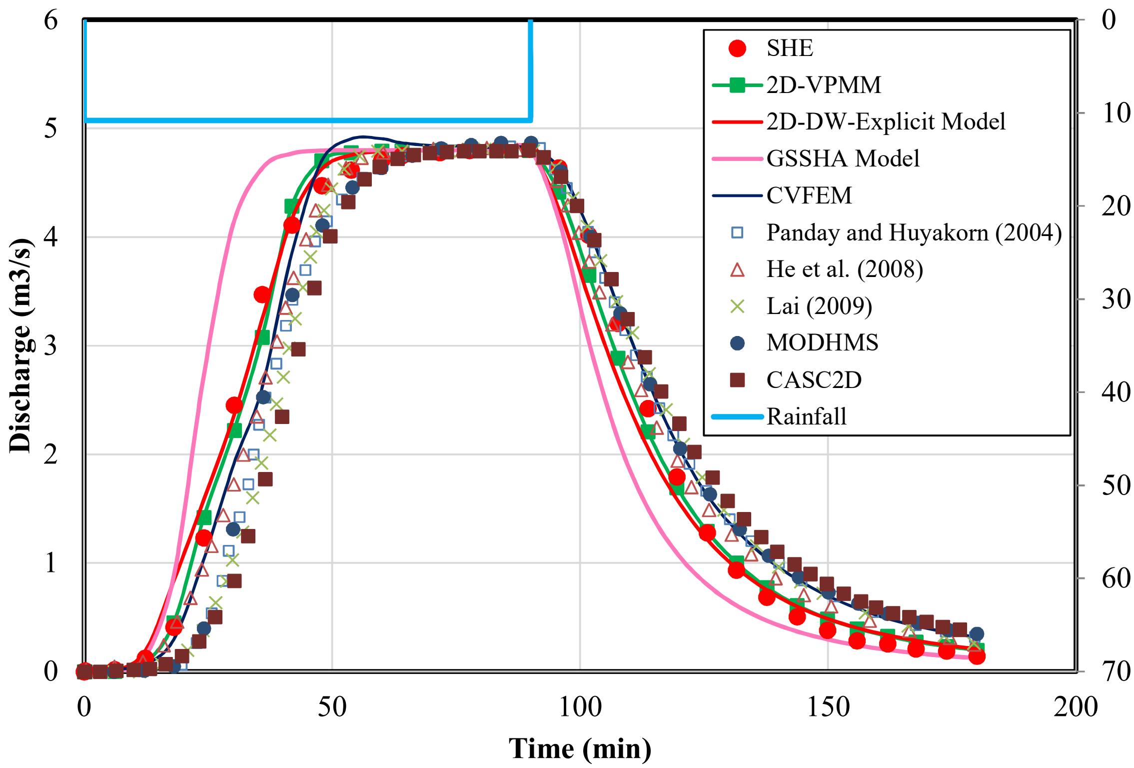

Figure 4.2. Comparison of discharge hydrograph simulated by the 2D-VPMM and 2D-DW-Explicit models at the channel outlet of the tilted V-catchment along with the simulations of various methods as well as benchmark solution (SHE model).

The discharge hydrograph simulation results presented in the Figure 4.2 are obtained by using the grid size i.e. $\Delta x = \Delta y = 50$ m and $\Delta t = 6$ sec for both the 2D-VPMM, 2D-DW-Explicit and GSSHA models. The Di Giammarco et al. (1996) used grid size, $\Delta x = \Delta y = 50$ m and $\Delta t = 5\ to\ 100$ sec (adaptive time step) in their CVFEM model whereas He et al. (2008) model used grid size i.e. $\Delta x = \Delta y = 50$ m and $\Delta t = 20$ sec. Lai, (2009) analyzed the stability range of implicit and explicit schemes for this particular problem and found that the explicit solver allow use of time step Δt up to 30 seconds. However, we have used digitized results of all these above methods. Note that in this particular case, the model results of the SHE model are considered as the benchmark solution to compare the performance of various models as presented in Table 4.1. From the results shown in Table 4.1 and Figure 4.2, it can be revealed that the results of the 2D-VPMM method are slightly better as compared to the all other considered rainfall-runoff simulation models. Particularly, the 2D-VPMM model perform slightly better than the 2D-DW-Explicit model. Further, it could be observed that the execution time required by the explicit model is slightly higher as compared to the 2D-VPMM model by keeping all the input variables same. In case of GSSHA model, we have created DEM for this V-catchment using Water Modelling Software (WMS, https://www.aquaveo.com/software/wms-watershed-modeling-system-introduction). However, this software is designed to create input data sets of natural watershed for the GSSHA model and hence it is expected that while creating the input data set for this small catchment, there may some error and hence, the simulation results by GSSHA model are not really matching well with the SHE model results and the other models used in this particular case.

The sensitivity analysis of the 2D-VPMM and 2D-DW-Explicit models results for the conservation of the mass is carried out when various spatial and temporal time steps are used for simulation by keeping the same input conditions of the SHE model simulation and is presented in Figure 4.3. From Figure 4.3, it can be inferred that the 2D-VPMM is slightly higher volume conservative than the 2D-DW-Explicit method. In case of 2D-VPMM model though the mass conservation error increases with increase with increase grid size, it can be considered that it is negligible for all practical cases. However, the 2D-DW-Explicit method exhibit different mass conservation characteristics, though it shows marginally higher error in mass conservation as compared to the 2D-VPMM model, but this model is least affected by the use different spatial and temporal time steps. The performance evaluation of the 2D-VPMM and 2D-DW-Explicit models simulations is studied using the performance evaluation criteria explained in Section 3.3, by comparing with the simulation results of the SHE model. The sensitivity analysis carried out for the 2D-VPMM and 2D-DW-Explicit models is shown in Table 4.2. From the results presented in Table 4.2, it can be inferred that the 2D-VPMM model is performing slightly better as compared to the 2D-DW-Explicit model based on used performance criteria.

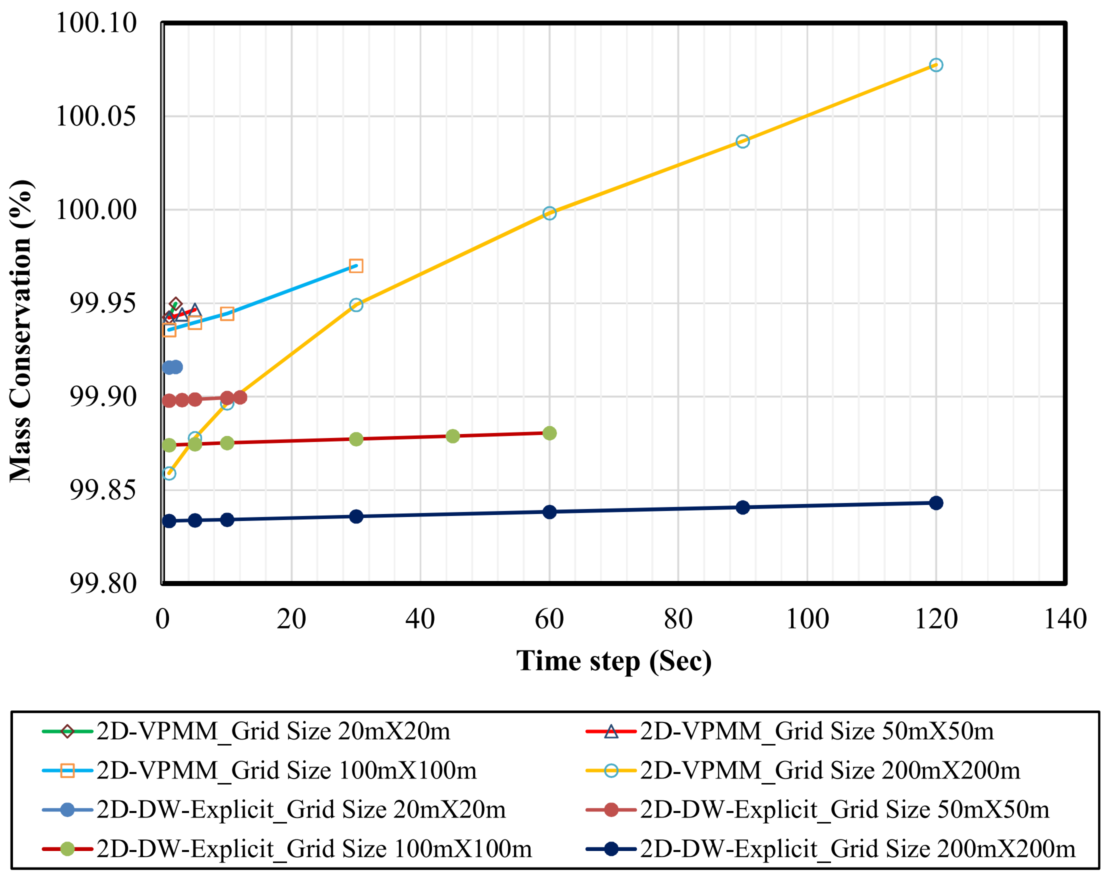

Figure 4.3. Sensitivity analysis of the 2D-VPMM and the 2D-DW-Explicit models for the tilted V-catchment.

Table 4.2 Performance evaluation of 2D-VPMM and 2D-DW-Explicit models for tilted V-catchment.

	2D-VPMM			2D-DW-Explicit Model		
Test Case	**NSE (%)**	Q_{per} **(%)**	**EVOL (%)**	**NSE (%)**	Q_{per} **(%)**	**EVOL (%)**
$\Delta x = \Delta y = 25$ m $\Delta t = 3$ sec	99.25	-0.19	-0.22	98.65	-0.19	-0.25
$\Delta x = \Delta y = 50$ m $\Delta t = 6$ sec	99.36	-0.19	-0.24	98.83	-0.19	-0.28
$\Delta x = \Delta y = 100$ m $\Delta t = 10$ sec	99.54	-0.2	-0.27	99.06	-0.19	-0.33
$\Delta x = \Delta y = 200$ m $\Delta t = 30$ sec	99.59	-0.22	-0.31	99.00	-0.22	-0.41

4.2 Performance Evaluation Using University of Illinois Experimental Laboratory catchment Data

In order to verify the performance of the 2D-VPMM and 2D-DW-Explicit models, these models are applied to simulate the observed hydrographs of the two-dimensional overland flow experiments conducted by *Shen et al.* (1974) on the impervious laboratory catchment of the Watershed Experimentation System at the University of Illinois, Urbana-Champaign. The observed hydrographs of eight different events of these laboratory experiments subjected to different rainfall inputs for different laboratory catchment configurations are reported by *Maksimović and Radojković* (1986). The geometry of this laboratory catchment is shown in Figure 4.4.

The experimental catchment consists of two symmetric planes each of length L and width W/2 characterized with two slopes Sox and Soy. The geometric characteristics and rainfall information for different events are shown in Table 4.3. The hydrographs obtained for this case by the 2D-VPMM and 2D-DW-Explicit models are compared with the experimental results shown in Figure 4.5 to Figure 4.12. The numerical grid size of 3.0355 m (x) × 3.0480 m (y) and simulation time step dt = 6 sec is used in this case for overland flow simulation. The performance evaluation criteria obtained while simulating the outflow discharge hydrograph for each considered events for both of these 2D-VPMM and 2D-DW-Explicit models are presented in Table 4.4.

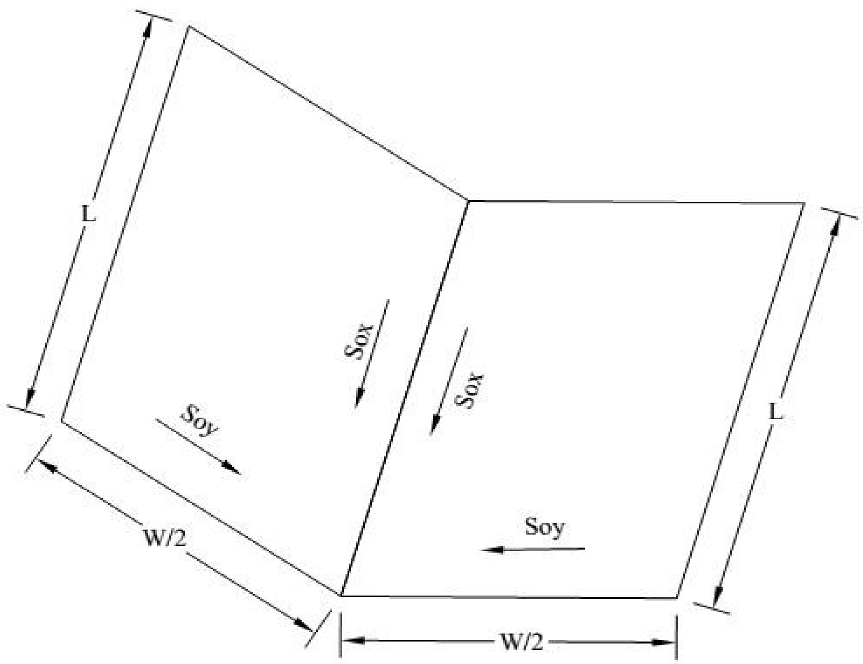

Figure 4.4. Geometry of the University of Illinois laboratory catchment.

Table 4.3. Geometrical characteristics and the rainfall input for different events considered in the University of Illinois laboratory catchment.

Event Number	L (m)	W (m)	S_{0x} (%)	S_{0y} (%)	Rainfall Intensity (mm /min)	Rainfall Duration (sec)
5	12.192	12.142	0.5	0.5	2. 97	240
6	12.192	12.142	0.5	0.5	2. 97	120
7	12.192	12.142	0.5	0.5	2. 97	60
8	12.192	12.142	0.5	0.5	2. 97	30
9	12.192	12.142	1.0	0.5	1. 90	240
10	12.192	12.142	1.0	0.5	1. 90	120
11	12.192	12.142	1.0	0.5	1. 90	60
12	12.192	12.142	1.0	0.5	1. 90	30

Note that the to simulate the outflow hydrographs of all these events using the 2D-VPMM and 2D-DW-Explicit models, the 1D-VPMM method (Perumal and Price, 2013) is incorporating for routing flow in the triangular channel while accounting for the contribution of lateral flow. From Table 4.3, it can be seen that the overland planes of events 5 to 8 of these V-catchment system are characterized by equal slopes in both x and y directions (S_{0x} = S_{0y} = 0.005) subjected to an uniform rainfall intensity of 2.967mm/min for a duration of 240, 120, 60 and 30 secs, respectively, corresponding to these events; whereas, the overland planes of this system corresponding to the events 9 to 12 are characterized by different slopes in x and y directions (S_{0x} = 0.01; S_{0y} = 0.005), subjected to an uniform rainfall intensity of 1.90mm/min for a duration of 240, 120, 60 and 30 secs, respectively, corresponding to these events. For the events 5 to 8, the calibrated roughness is 0.016, whereas for the events 9 to 12, the calibrated roughness is considered as 0.015. This is because these latter events are subjected to reduced uniform rainfall intensity in comparison with the events 5 to 8 causing the overall roughness to be lesser than that of the former events. For those events (events 7, 8, 11 and 12), where the rainfall duration is smaller than the time of concentration, the roughness decreased as a result of cessation of the rainfall and for such events the decreased roughness of the plane is considered as 0.011 after the cessation of rainfall.

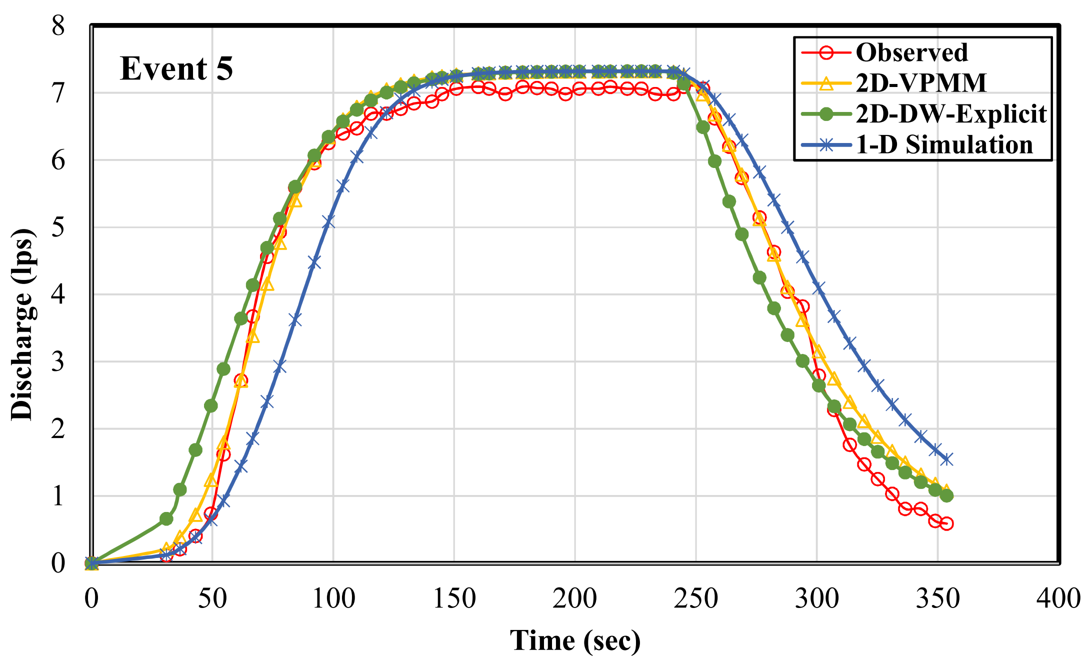

Figure 4.5. Comparison of the simulated hydrograph by the 2D-VPMM and 2D-DW-Explicit models with the observed hydrograph (University of Illinois, Event 5).

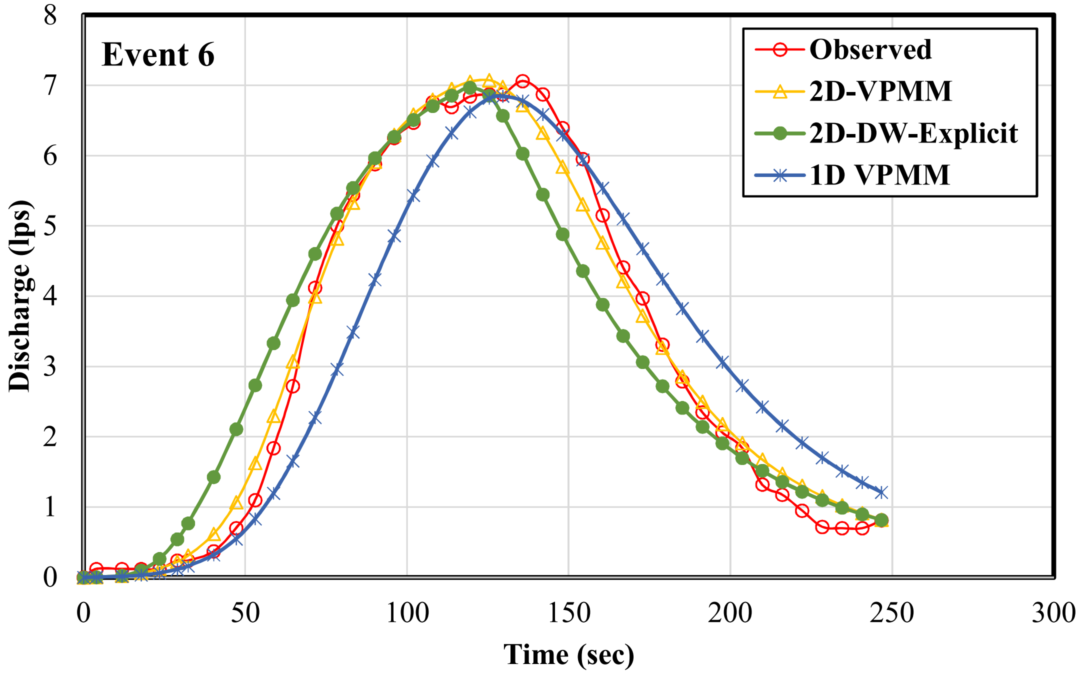

Figure 4.6. Comparison of the simulated hydrograph by the 2D-VPMM and 2D-DW-Explicit models with the observed hydrograph (University of Illinois, Event 6).

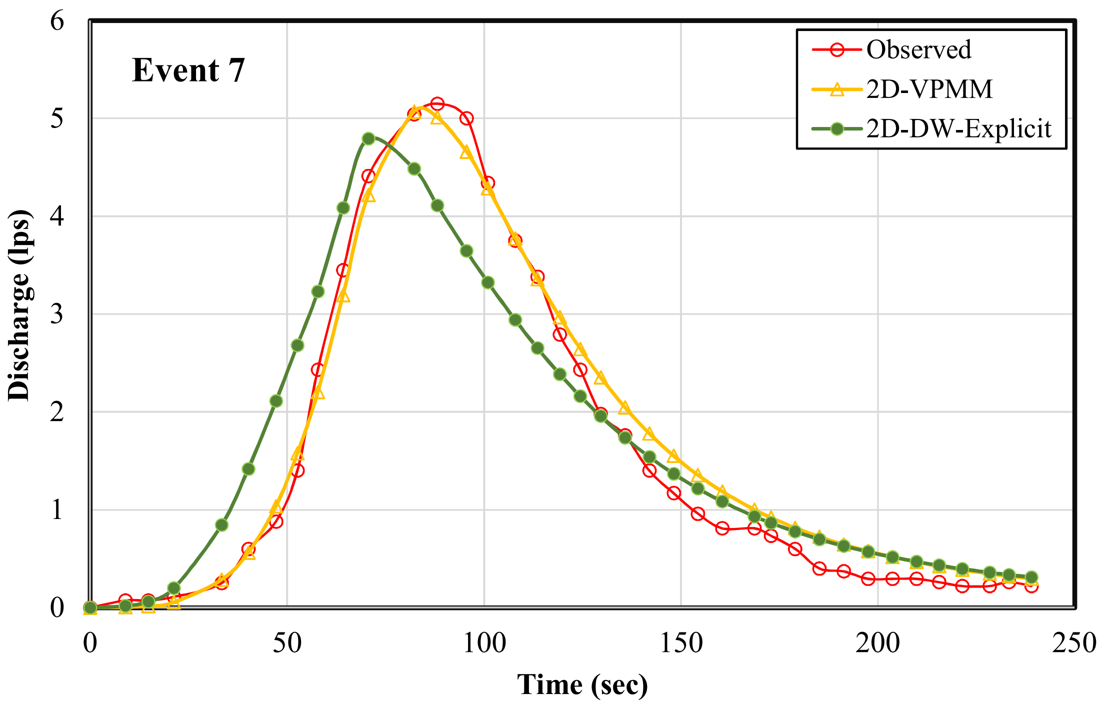

Figure 4.7. Comparison of the simulated hydrograph by the 2D-VPMM and 2D-DW-Explicit models with the observed hydrograph (University of Illinois, Event 7).

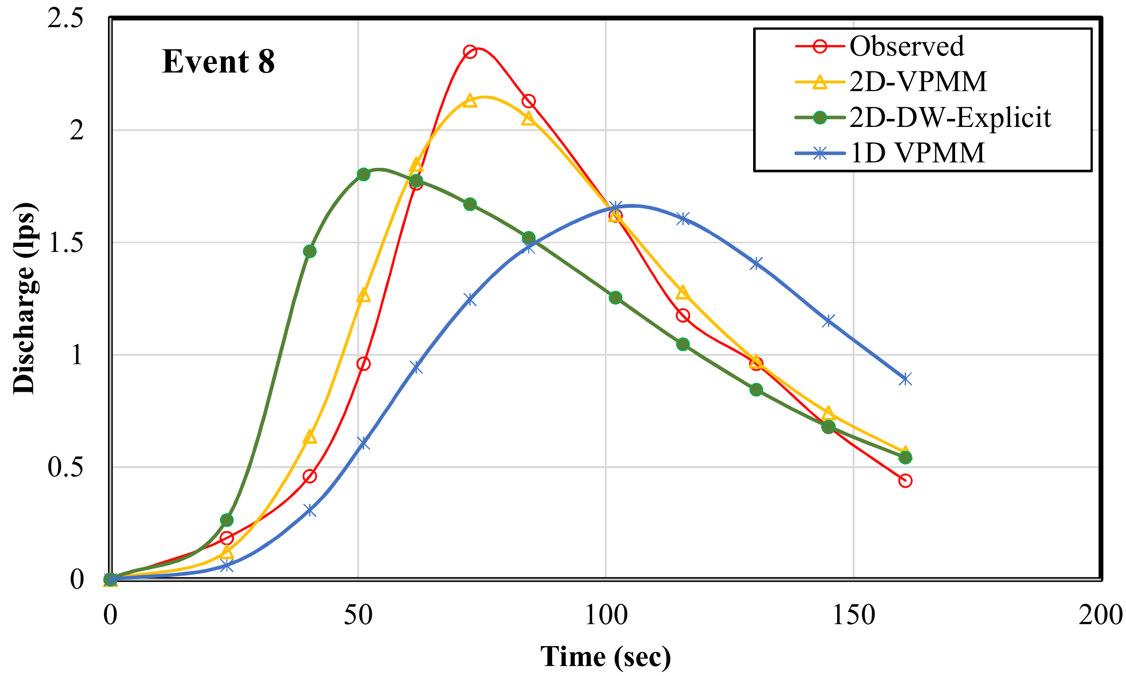

Figure 4.8. Comparison of the simulated hydrograph by the 2D-VPMM and 2D-DW-Explicit models with the observed hydrograph (University of Illinois, Event 8).

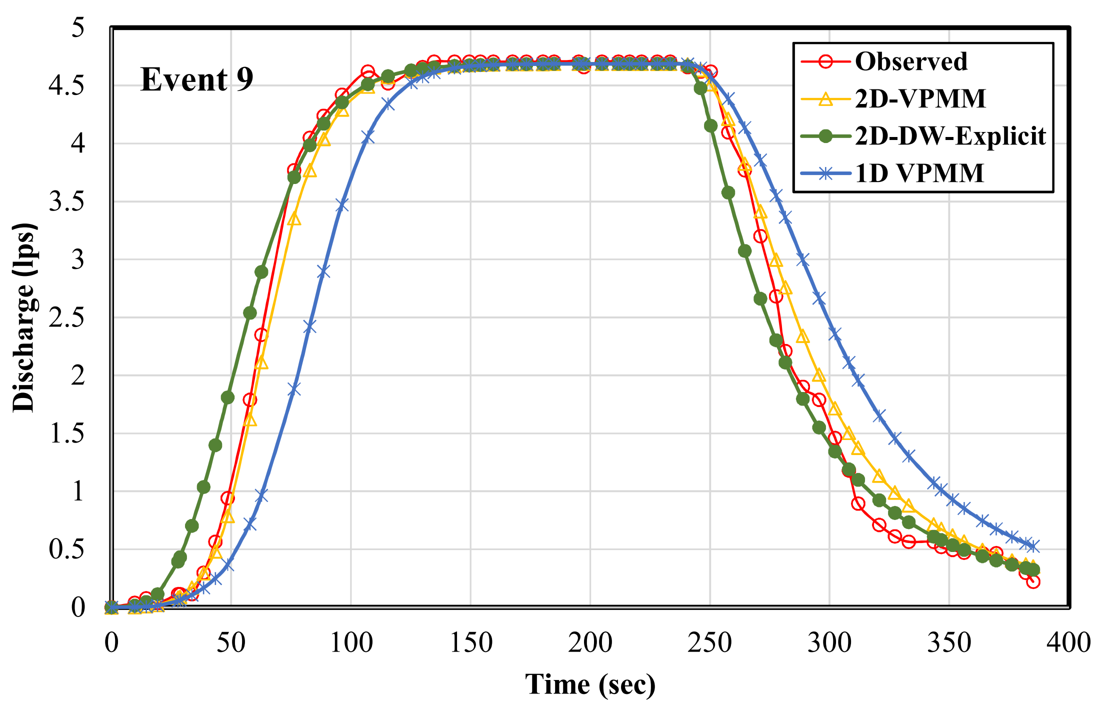

Figure 4.9. Comparison of the simulated hydrograph by the 2D-VPMM and 2D-DW-Explicit models with the observed hydrograph (University of Illinois, Event 9).

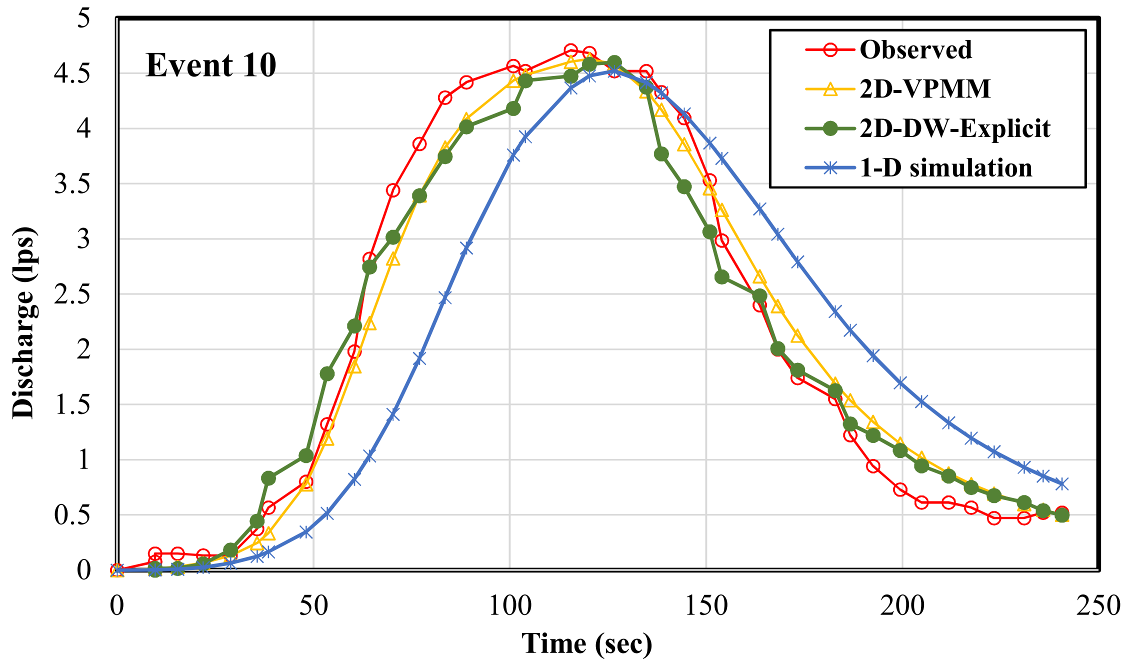

Figure 4.10. Comparison of the simulated hydrograph by the 2D-VPMM and 2D-DW-Explicit models with the observed hydrograph (University of Illinois, Event 10).

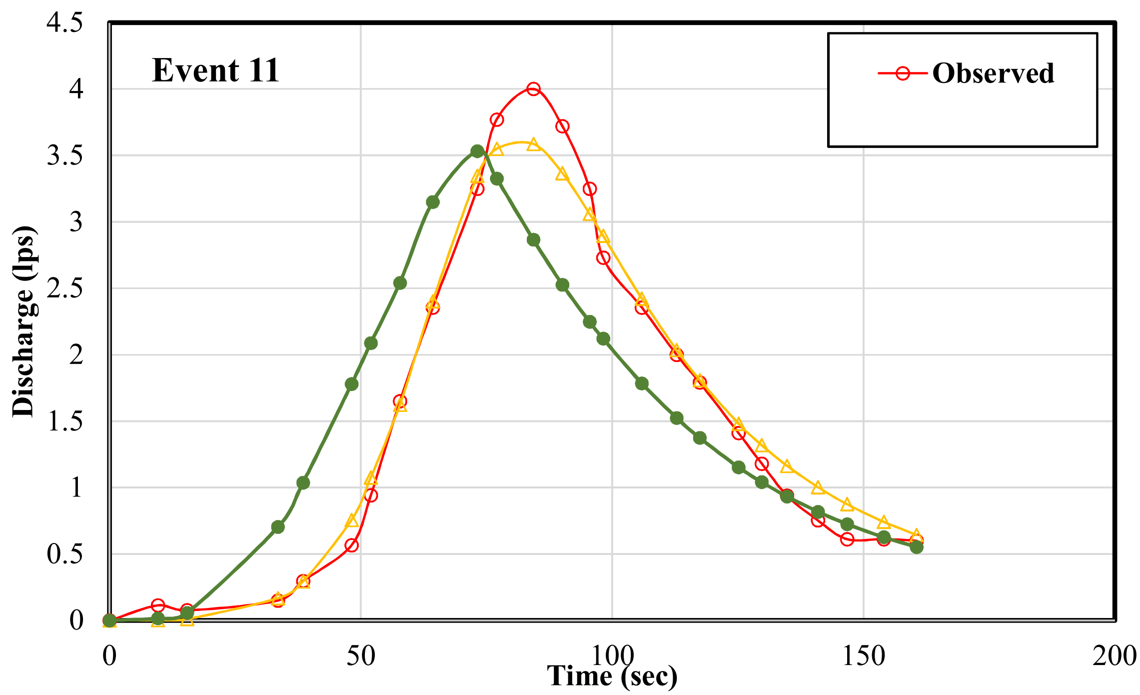

Figure 4.11. Comparison of the simulated hydrograph by the 2D-VPMM and 2D-DW-Explicit models with the observed hydrograph (University of Illinois, Event 11).

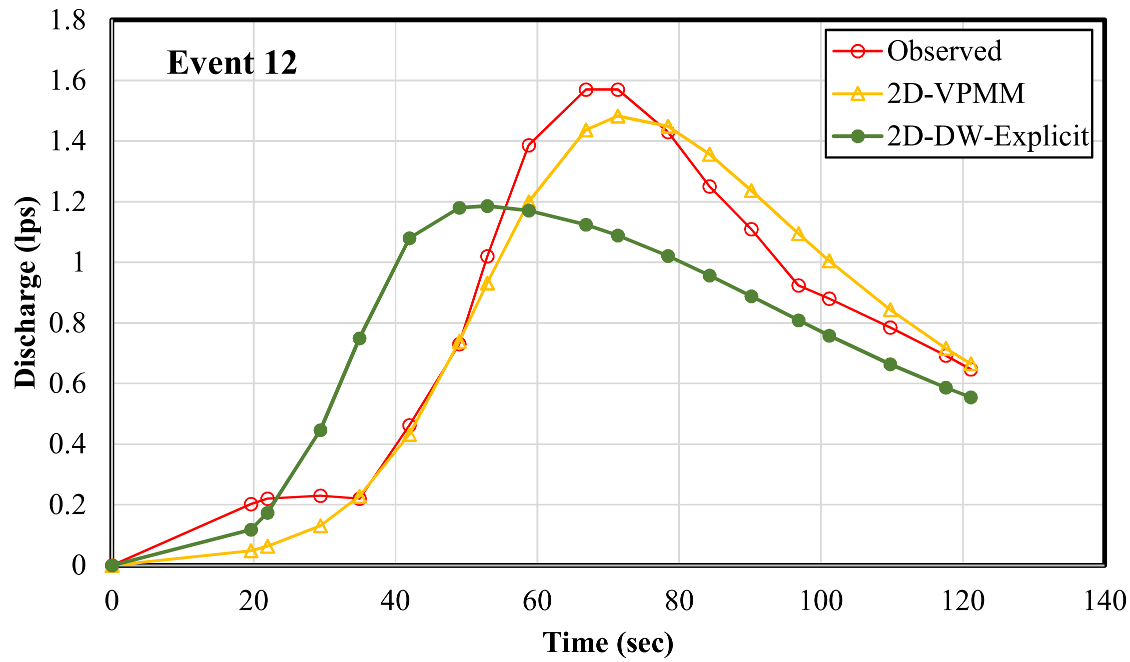

Figure 4.12. Comparison of the simulated hydrograph by the 2D-VPMM and 2D-DW-Explicit models with the observed hydrograph (University of Illinois, Event 12).

Table 4.4. Performance evaluation of 2D-VPMM and 2D-DW-Explicit models for University of Illinois laboratory catchment rainfall-runoff events.

Event Number	2D-VPMM dt=6sec			2D DW Explicit dt=6sec		
	η (%)	Q_{per} (%)	tq_{per} (sec)	η (%)	Q_{per} (%)	tq_{per} (sec)
5	98.36	3.24	73.1	95.71	3.24	73.10
6	98.87	0.16	-10.37	91.95	-1.35	-16.29
7	98.11	-1.48	0	88.18	-6.98	-17.53
8	96.57	-9.18	0	57.60	-23.22	-21.48
9	99.09	-0.41	98.56	97.83	-0.41	98.56
10	97.61	-1.66	4.81	94.08	-2.30	4.81
11	98.12	-9.61	0	74.15	-11.67	-11.16
12	95.41	-5.55	4.5	59.73	-24.45	-13.92

Referring to Figures 4.5 to Figure 4.12 and Table 4.4, it can be seen that the 2D-VPMM model is able to produce the outflow discharge hydrographs with quite high accuracy almost above 97% except for events 8 and 12. For the events 8 and 12, the efficiency of reproduction for the 2D-VPMM model given by η is around 96.57 % and 95.41 %, respectively, somewhat lower than that of other events. However, the efficiency of reproduction for the 2D-DW-Explicit model given by η were reasonably very good in case of events 5, 9 and 10 (η > 94%), acceptable in case of events 6 and 7 (η = 91.95 % and 88.18 %, respectively). However, in case of events 8, 11and 12, the 2D-DW-Explicit model has performed very worst as the efficiency of reproduction given by η were 57.60 %, 74.15 % and 59.73 %, respectively. From Figures 4.5 to 4.12, it can be concluded by visualization of 1D-VPMM and 2D-VPMM model simulation results that there is necessity to use the two dimensional model as it is presenting the actual geometry and flow process. It can be revealed from all these simulation results for events 5 to 12 (also see Figures 4.5 to 4.12), the 2D-VPMM model is performed far better than the 2D-DW-Explict model. These simulations demonstrate that the 2D-VPMM model is highly efficient for all the cases of simulation, except for the events 8 and 12. However, the performance of the 2D-DW-Explicit model is very worst particularly for events 8, 11 and 12. The plausible reason one can attribute for such behavior of these two events could be due to shorter duration of rainfall input applied for these events. It may also be noticed that there are discharge measurement errors in all these experimental events as the values of discharges at a given time are not the same for different events during the same period of rainfall application with the same intensity of rainfall. Furthermore, it was revealed that while the use of a larger time interval for the explicit scheme computation leads to unstable solutions, stable solutions are obtained using a relatively small time intervals. However, among the stable solutions obtained using a relatively larger time intervals for numerical solutions result in early arrival of the runoff hydrographs in comparison with the benchmark hydrographs. Even with the use of smaller time interval, close reproduction of the benchmark hydrographs could not be achieved. It is inferred from the study that one has to apply caution while accepting the stable solutions of the two-dimensional overland flow model obtained using the explicit finite difference scheme.

CHAPTER - 5
CONCLUSIONS AND RECOMMENDATIONS

This study was conducted with aim to achieve mainly two objectives: (1) development of computer code for the two dimensional overland flow simulation by using two dimensional diffusive wave explicit model (2D-DW-explicit model) which employing the explicit finite difference scheme (similar to the explicit numerical scheme used in the CASC2D model developed by the Colorado State University, USA) and (2) to carry out the performance evaluation by comparing the simulation results of the 2D-DW-explicit model with those obtained by the 2D-VPMM model (which employ the storage-based Muskingum-McCarthy schemes) using experimental plot data available in the literature.

The performance of the 2D-VPMM and 2D-DW-Explicit model for two dimensional overland flow simulation is carried out using hypothetical tilted V-catchment rainfall-runoff simulation event studied by Di Giammarco et al. (1996) and University of Illinois experimental laboratory catchment data comprised of observed hydrographs of the two-dimensional overland flow experiments from eight events conducted by *Shen et al.* (1974) on the impervious laboratory catchment of the Watershed Experimentation System at the University of Illinois, Urbana-Champaign. *Maksimović and Radojković* (1986) have presented rainfall-runoff data of these eight different events subjected to different rainfall inputs for different laboratory catchment configurations in the tabular form. Out of these two data sets, rainfall-runoff simulation event studied by Di Giammarco et al. (1996) is widely used in the literature to demonstrate the suitability as well as accuracy of the newly developed 1D and 2D numerical models while the University of Illinois experimental laboratory catchment data is first time introduced by the PI and Co-PI's to verify the 2D-VPMM model and in the present study. Based on these conducted study following conclusions are drawn:

1. The computer code developed for two dimensional overland flow simulation by using 2D-DW-explicit model is able to reproduce the benchmark solution as well as other available 2D overland flow models simulation results including the 2D-VPMM overland flow simulation model results with higher Nash-Sutcliffe efficiency (η) for the tilted V-catchment. Note that in order to simulate the flow in rectangular channel, 1D-VPMM channel routing method applicable for flow routing in the rectangular channel is used for simulation of outflow discharge hydrograph at the end of channel by the 2D-DW-Explicit and 2D-VPMM models.

2. Further, the performance evaluation of these two overland flow simulation models based on the use of titled V-Catchment data suggests that the 2D-VPMM method perform better than the 2D-DW-Explicit model and all other considered 2D overland flow simulation models. Further, it was observed that the execution time required by the 2D-DW-Explicit model is slightly higher as compared to the 2D-VPMM model by keeping all the input variables same.
3. Furthermore, the sensitivity analysis conducted based on the use of tilted V-catchment data suggests that the 2D-VPMM model is able to preserve slightly higher volume than the 2D-DW-Explicit model. In case of the 2D-VPMM model though the mass conservation error increases with increase with increase grid size, it can be considered that it is negligible for all practical cases. However, the 2D-DW-Explicit model exhibit different mass conservation characteristics, as it is least affected by the use different spatial and temporal time steps.
4. The performance evaluation of these two models based on the University of Illinois experimental laboratory catchment data use suggests that the 2D-VPMM model is able to produce the outflow discharge hydrographs with quite high accuracy (with $\eta > 97\%$) except for events 8 and 12. For the events 8 and 12, the 2D-VPMM model able to simulate hydrographs with η values 96.57 % and 95.41 %, respectively. The 2D-DW-Explicit model is able to simulate the outflow discharge hydrographs reasonably well in case of events 5, 9 and 10 ($\eta > 94\%$), acceptable in case of events 6 and 7 (η = 91.95 % and 88.18 %, respectively) and performed very worst in case of events 8, 11 and 12 (η = 57.60 %, 74.15 % and 59.73 %, respectively). These results clearly brought out the accuracy, robustness, stability, flexibility and practicability of the 2D-VPMM overland flow model.
5. Further, it can be revealed that the 2D-VPMM model can be easily applied to one-dimensional overland flow simulation also. The comparison of 1D-VPMM and 2D-VPMM model brought out the necessity to use the two dimensional overland flow model for simulating the rainfall-runoff cases on spatially varied topographic land surfaces.

Based on this study, it is recommended that the 2D-VPMM overland flow simulation model is an viable option that can be applied for its use in large scale hydrological models coupled with atmospheric models to study the water resources problems and climate change impact assessment due to flexibility offered by this model in use of very large spatial and temporal computational grid size.

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