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Scale-Invariant Analysis of Hydrological Processes: A Case Study from Portugal

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ABSTRACT: Scale-invariant theories offer frameworks that can assist bringing together the study of different hydrological processes, and tools that can be used to better understand their complex non-linear structure. Scaling theories offer an alternative to ('traditional') approaches that study one scale independent of the other. This work discusses examples of application of spectral analyses to studying the internal temporal structure of precipitation and river-discharge using daily data from Portugal. Precipitation was measured with non recording gauges, over a period of 54 years, at 61 locations in Mainland Portugal. River discharge data were obtained also for different locations, but record lengths are different. The discharge rates were obtained using rating curves. The data have different origins: e.g., climate, size of the drainage basins, topography. The analyses showed that scaling is present in the data, over a considerable range of scales. Empirical exponents describing the scaling of the energy spectra of the precipitation intensity and river discharge rates were determined. Results showed that spectral analysis is able to quantify differences in these processes variability, taking into account the behaviour observed across scales.

INTRODUCTION

Quite often, combined work on different components of the earth system requires bridging across scales, in space and/or time. Specifically, hydrological processes are often observed and modelled at different scales. This involves some sort of extrapolation or transfer of information across scales which is not always fully supported by our understanding of these processes.

During the last decades, hydrological processes have started being studied with scale-invariant. This framework is believed to offer the possibility to bring together the study of different hydrological processes, and tools that can be used to better understand their complex non-linear structure. Scaling theories offer an alternative to ('traditional') approaches that study one scale independent of the other. Scaling (or scale invariance) is a well known concept in physics. It is based on the invariance of properties across scales. The characterization of scale invariant behaviour in hydrological processes can assist in the choice of models to be used at different scales and contribute to improving the selection of the resolution for data collection, and the type of measuring device. Empirical scale truncations are made often (i.e. one scale is studied independently of the others) and many models are often misused because their restricted applicability to different scales is not taken into consideration. It is pertinent, for example, to know whether there are intrinsically different phenomena, as one move from one scale to the next; and whether results obtained on one scale can be extrapolated to another.

Many physical processes are involved in the formation of precipitation and surface runoff. Atmospheric processes that produce precipitation operate over a variety of time and space scales, and interact, for example, with surface topography, soil moisture and vegetation. Hence, precipitation exhibits wide variability over a broad range of scales: in time, over intervals of minutes to years; and in space, from less than one to several thousand square kilometres. At the ground surface, this variability is a consequence, for example, of the different precipitation-generating mechanisms (e.g. related to cloud formation and to the different cooling mechanisms) and the general patterns of atmospheric circulation. Local factors are also important: latitude; altitude; distance from moisture sources; prevailing wind direction (towards or away from the source of moisture) and wind intensity; relative temperatures of land and bordering oceans (see e.g. Eagleson, 1970). Apart from variations in precipitation quantities, their patterns of occurrence also differ, depending on the climate regime. In general, the greater the annual precipitation is the lower variation from year to year. Seasonal variation in

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precipitation is pronounced where the annual oscillation in the atmospheric circulation changes the amount of moisture inflow over those regions (e.g. Chow *et al.*, 1988).

The process of river discharge is known to be extremely non-linear and variable in time and space, since it depends very much on climatic regimes (particularly rainfall) and complex rainfall-runoff processes occurring over a variety of time scales and across drainage basins. The variability in these hydrological processes is an important issue in many studies and areas of research e.g. hydrology, hydraulics, water resources, land planning, and urban development. It involves a large dynamic range, which in certain cases leads to catastrophic events related, for example, to both flood and drought situations.

This paper aims at discussing the use of simple scaling tools, such as spectral methods, to perform exploratory analysis of hydrological series and to contribute to understanding its dynamics across scales. It investigates the scale-invariant temporal structure of precipitation and river discharge. Both processes are highly non-linear hydrological processes that exhibit wide variability over a broad range of time and space scales. This study uses daily data from Portugal. Precipitation was measured with non recording gauges, over a period of 54 years, at 61 locations in Mainland Portugal. River discharge data were obtained also for different locations, but record lengths are different. The discharge rates were obtained using rating curves. The data have different origins: e.g. climate, physiographic characteristics of drainage basins and river networks.

The analyses showed that scaling is present in the data, over a considerable range of scales. Empirical exponents describing the scaling of the energy spectra of the precipitation intensity and river discharge rates were determined. Results showed that spectral analysis is able to quantify variability differences in these processes, taking into account the behaviour observed across scales.

SPECTRAL AND SCALE-INVARIANT METHODS

The term scaling (or scale-invariance) is used to indicate that certain features of a dynamic system are independent of scale. Scaling is expected to hold from some large (outer or upper) scale down to a small (inner or lower) scale. This behaviour leads to a class of scaling rules (power laws) characterized by scaling exponents. Statistical properties of scale-invariant systems at different scales (i.e., on large and small scales) are related by a scale-changing operation that involves only scale ratios.

One can use standard spectral methods and analysis to test for scale-invariance. Spectral methods are also known as Fourier transform methods (see e.g. Wu, 1973; Box and Jenkins, 1976; Press *et al.,* 1989; Hastings and Sugihara, 1993; Rajagopalan and Lall, 1998). The idea behind these methods is that a physical process can be described either in the time domain (by the values of some quantity as a function of time) or in the frequency domain (where the process is specified by giving its amplitude as a function of frequency). The two representations are linked by means of the Fourier transform equations. The Fourier transform can be an efficient computational tool for accomplishing certain manipulations of the data. The related power spectrum, which can be defined as the distribution of variance or power across wave-length or frequency, can itself be of intrinsic interest (see e.g. Press *et al.,* 1989).

Spectral analysis is one approach to the study of the statistical properties of time series. It provides a useful exploratory analysis tool for examining time-series data. In general, spectral analysis can be used to investigate structured temporal variations embedded in hydro-climatic data. Spectral analysis can provide an intuitive frequency-based description of the time series and indicate interesting features such as long memory, presence of high frequency variation and cyclical behaviour (e.g. Wu, 1973; McLeod and Hipel, 1995). This type of analysis is useful for detecting the periodicity of short-interval precipitation sequences at a specific point. If a process contains periodic terms, the frequencies of these terms exhibit a number of high, sharp peaks in the spectrum. This indicates that a significant amount of variance is contained in these frequencies.

The most familiar consequence of scaling is the power-law behaviour that is expected in the energy (power) spectra of scaling processes (e.g. Mandelbrot, 1982; Schertzer and Lovejoy, 1985, 1987),

$$E(\omega) \approx \omega^{-\beta} \qquad \ldots (1)$$

where ω is the wave-number, $E(\omega)$ is the energy, and β is the spectral exponent. For temporal processes, the wave-number ω can be approximated by $\omega \sim 1/\tau$, τ being the magnitude of any time interval. For spectra in rain see e.g. Ladoy *et al.* (1991), Fraedrich and Larnder (1993), Tessier *et al.* (1993), Lovejoy and Schertzer (1995), Olsson (1995), Tessier *et al.* (1996), Burlando and Rosso (1996), de Lima (1998); for spectra in streamflow discharge see e.g. Tessier *et al.* (1996), Pandey *et al.* (1998), Dahlstedt and Jensen (2005).

The type of behaviour expressed in Eqn. (1) is expected to occur over a range of wave-numbers and might not be observed for small samples. The energy spectrum is only second-order statistics (i.e. the spectrum is related to the Fourier transform of the autocorrelation function), and is thus not particularly robust. When applied to highly intermittent data, large samples may be needed to obtain good estimates of the overall average spectra (see e.g. Lovejoy and Schertzer, 1991).

Spectral and scale invariance analyses can be used to characterize the internal structure of precipitation and river discharge: the spectral analysis locates the dominant wave-numbers (i.e. frequencies), while the scale invariance analysis determines the upper and lower scales that bound the scale-invariant regimes in these processes.

PRECIPITATION AND RIVER DISCHARGE DATA

The precipitation and river discharge data used in this work are from Portugal. The precipitation data were collected by the Portuguese Meteorological Institute and the river discharge data were collected by the Portuguese Institute for Water. Both are the national agencies holding responsibility for this type of data collection.

The Precipitation Data

The precipitation data analysed in this study were recorded at 10 locations in Mainland Portugal (see Table 1). The precipitation measuring devices have horizontal openings of 200 cm^2 at 1.5 m above the ground surface. The resolution of the measurements is 0.1 mm of precipitation. Trace precipitation of less than 0.1 mm is disregarded and such days are considered dry (zero precipitation days).

Mainland Portugal is located in the transitional region between the sub-tropical anticyclone and the sub-polar depression zones. The most significant factors conditioning climate in Mainland Portugal are its latitude, orography (Figure 1, left, shows an altimetry map of Mainland Portugal) and the influence of the Atlantic Ocean. Although the variation in climate factors is quite small, it is still sufficient to justify significant variations in precipitation (see Figure 1, right). There are marked differences between the northern and southern regions, as well as between the littoral belt and the more inland regions. While the northwest region of Portugal is one of the wettest spots in Europe, with mean annual precipitation in excess of 2000 mm, average rainfall in the interior of the Alentejo (in the southern part of Portugal) is of the order of 500 mm and shows large interannual variability. In common with other southern European regions, Portugal has a mild Mediterranean climate, with a warm and dry summer period, more pronounced in the southern regions, but with well-known vulnerability to climate variability, especially to droughts and desertification in the southern sector. All the data sets analysed exhibit a marked seasonal distribution of precipitation during the year. The sites were selected to get five pairs of data sets, covering five latitudes, from north to south; each pair includes a western and an eastern site.

Table 1: Basic Information on the 10 Selected Precipitation Measuring Sites in Mainland Portugal, Including Descriptive Statistics of Precipitation Data for the Period 1941–1994

Measuring Stations	*Code*	*Lat. (N)*	*Long. (W)*	*Altitude (m)*	*Mean Annual Prec. (mm)*	*Wettest day (mm)*	*Coef. Var. (daily prec.)*
Ponte de Lima	1554	41°46'	8°36'	15	1643.6	137.2	2.287
Outeiro do Gerês	1470	41°47'	7°58'	800	2233.5	242	2.587
Coimbra/Geofísico	549	40°12'	8°25'	141	976.9	79.3	2.454
Penhas Douradas	568	40°25'	7°33'	1380	1692.2	159.8	2.525
Lisbon/Geofísico	535	38°43'	9°09'	77	727.6	95.6	2.983
Évora	557	38°34'	7°54'	309	620.7	104	3.131
Cercal do Alentejo	6142	37°48'	8°41'	176	778.7	118.5	2.98
Algodor	6172	37°45'	7°49'	163	506.2	111.2	3.525
Bravura/Barragem	6634	37°12'	8°42'	75	692.8	97.7	3.167
Tavira	282	37°07'	7°39'	25	563.1	186	3.946

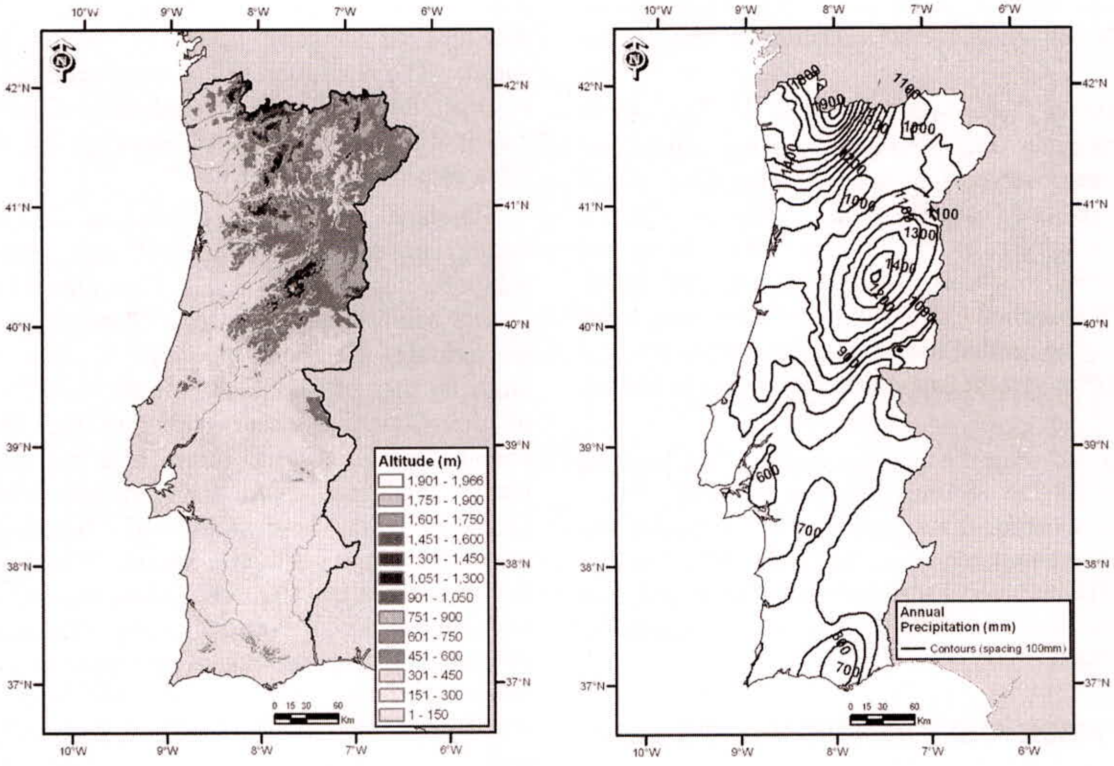

Fig. 1: Left: an altimetry map of Mainland Portugal. Right: annual precipitation distribution over the territory, for the period 1941–1994, obtained with the data from the 61 measuring stations indicated in Figure 1

Convective and frontal storms dominate the occurrence of precipitation in Mainland Portugal. Convective storms are frequent during the summer and in early and mid autumn, and are more frequent in the southern regions; frontal storms occur principally in the winter, and tend to affect the northern regions more.

Figure 2 shows the daily precipitation at 10 locations in the years 1941 to 1994. The corresponding station information (name, reference number, coordinates and altitude above mean sea level) is given in Table 1, which also contains some descriptive statistics relative to the precipitation at those locations. The sites were selected to get five pairs of data sets, covering five latitudes, from north to south; each pair includes a western and an eastern site.

The River Discharge Data

The river discharge data analysed in this study were also recorded at various locations in Mainland Portugal, in the Douro river drainage basin, in the north of the country. Douro is an international river, shared between Portugal and Spain in the Iberian Peninsula (Figure 3). Basic information on the four daily time series selected is given in Table 2. Their location is given approximately in Figure 3; the stations are in 4 tributaries of the river Douro. The discharge rates were obtained from limnimetric scale data and using rating curves. The observation period is not the same for all the series.

Table 2: Basic Information on the Selected Hydrometric Stations in the River Douro Drainage Basin. Altitude Refers to the Zero of the Limnimetric Scale

Code	*Station*	*Lat. (N)*	*Long. (W)*	*Altitude (m)*	*River*	*Drainage Area (km^2)*	*Series Length (years)*	*Period*	*Aver. Yearly Discharge (dam^3)*
06K/01	Ermida Corgo	41°14′	7°45′	120.00	Corgo	294.23	46	1956–2002	264485
04J/04	Cunhas	41°32′	7°51′	198.87	Beça	337	48	1949–1997	289427
03P/01	Vinhais	41°49′	7°00′	416.16	Tuela	478	41	1956–1997	357540
08O/02	Cidadelhe	40°55′	7°06′	253.84	Côa	1743	41	1956–1997	489494

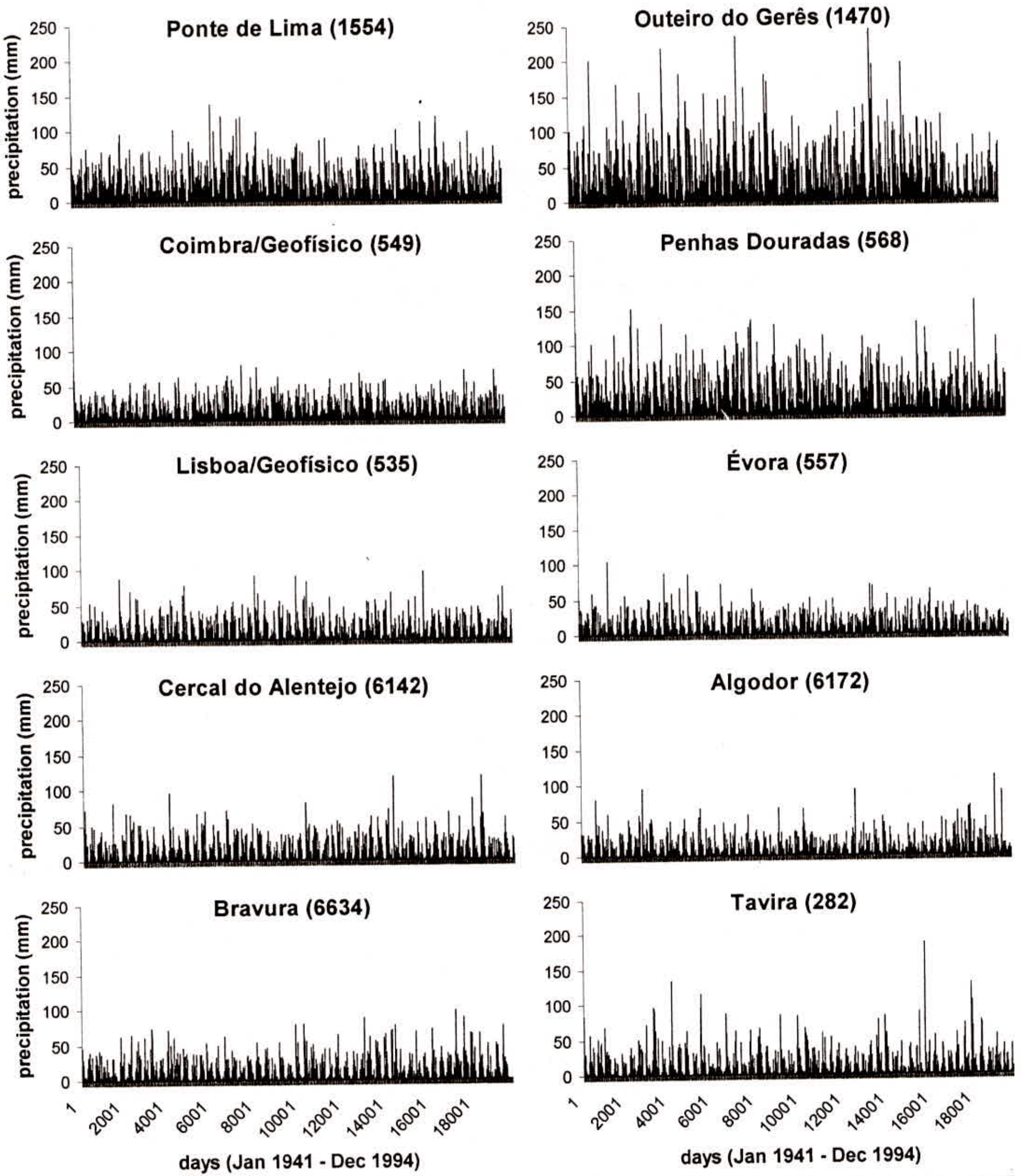

Fig. 2: Daily precipitation in 10 selected locations in Mainland Portugal, for the years 1941 to 1994. The plots were arranged according to the coordinates of the measuring sites: from north to south (top to bottom) and from west to east (left to right)

The drainage basins have different characteristics (e.g. geometric, geological, and land use). Their area ranges from about 294 to 1743 km^2. The rainfall regime in the territory and its strong seasonality (see above) make the surface flow regimes observed in the drainage network highly irregular (Figure 4).

RESULTS FROM SPECTRAL ANALYSES

This section presents results of the investigation of scaling in the temporal structure of precipitation and river discharge from Mainland Portugal (see previous section) using spectral analysis. Prior to the calculation of the spectra the data were normalized by dividing the daily rates by the mean daily rate at each location. The spectra have been smoothed for high frequencies.

Precipitation Analysis

The energy spectra obtained for the daily precipitation from Ponte de Lima, Outeiro do Gerês, Coimbra, Penhas Douradas, Lisboa, Évora, Cercal do Alentejo, Algodor, Bravura and Tavira are plotted on log-log axes in Figure 5. The plots were also arranged here according to the coordinates of the measuring sites: from north to south (top to bottom) and from west to east (left to right).

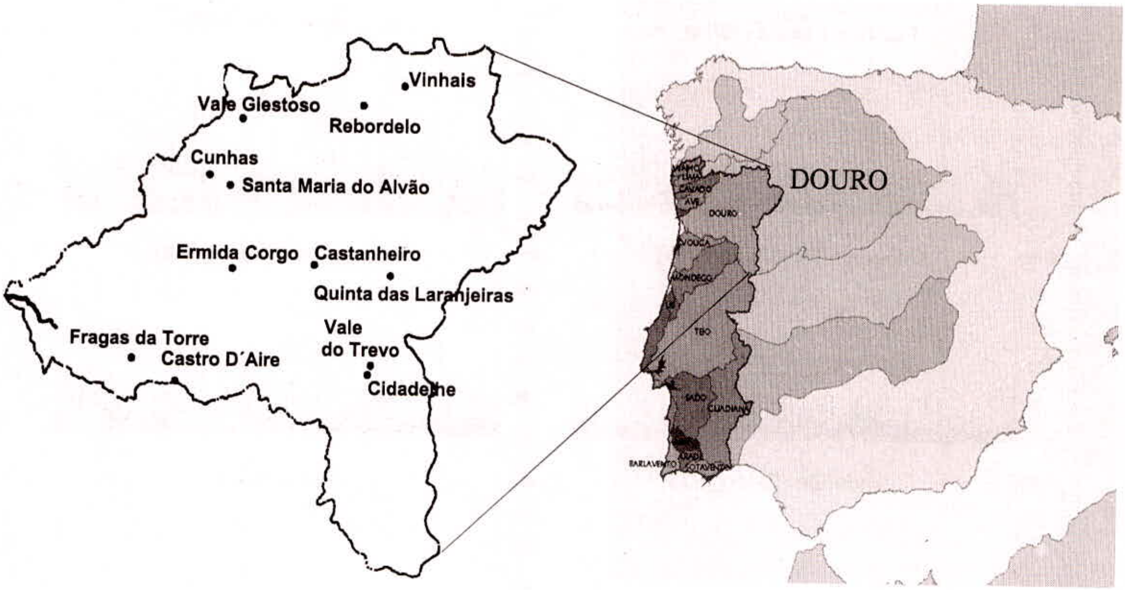

Fig. 3: Main drainage basins in Mainland Portugal, and schematic location of conventional hydrometric stations in the Douro River drainage basin

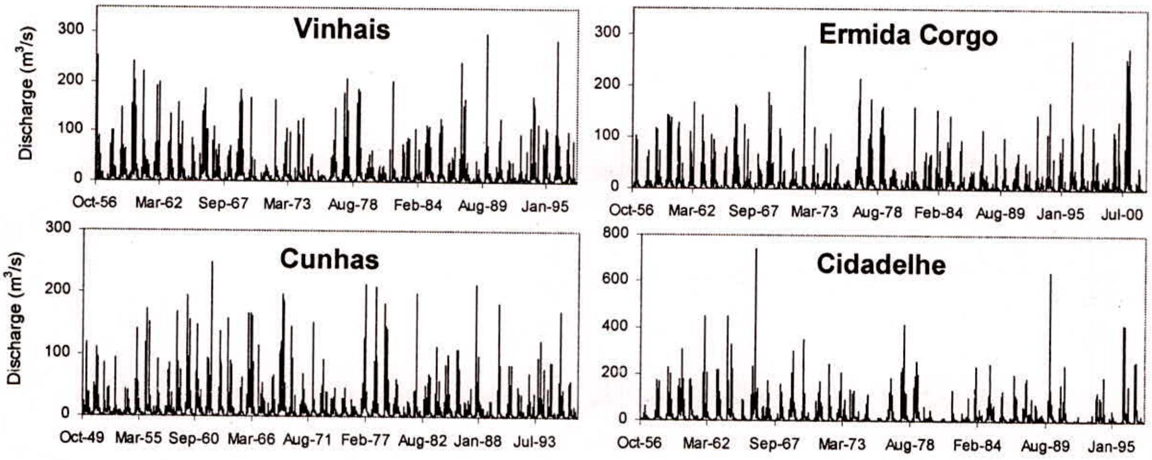

Fig. 4: River discharge daily time series from the Douro river drainage basin (see Table 2). Note that vertical axes scales are different

The spectra exhibit power-law behaviour, confirming the presence of scale invariance in the temporal structure of precipitation over a wide range of scales. This behaviour extends from 1 day up to about one month. In certain cases the behaviour observed suggests that scaling extends to even larger scales (see, e.g. the spectrum obtained for the data from Lisboa).

To obtain estimates for the spectral exponents β, in Eqn. (1), power laws were fitted to the energy spectra with linear regressions of $\log(E(\omega))$ versus $\log(\omega)$, over the range of scales exhibiting scale-invariant (power-law) behaviour. The spectral exponents β that correspond to the right-hand side scaling regions of the spectra in Figure 5 were estimated as indicated in Table 3. As expected, the variability present in the data varies strongly with location. The values of β are in the range 0.431, for the data from Outeiro do Gerês, to 0.205, for the data from Algodor. For other studies addressing the climatic effects on precipitation variability essentially from a scaling point of view see e.g. Svensson *et al.* (1996), Harris *et al.* (1996), Sivakumar (2000).

The spectral peaks at $\omega \approx 0.0027\ \text{day}^{-1}$, clearly observed in the spectra (see Figure 5), correspond to the annual frequency cycle associated with the precipitation process. This oscillation seems to 'emerge' on a scaling background, since the spectral slope remains unchanged on both sides of the annual peak. The

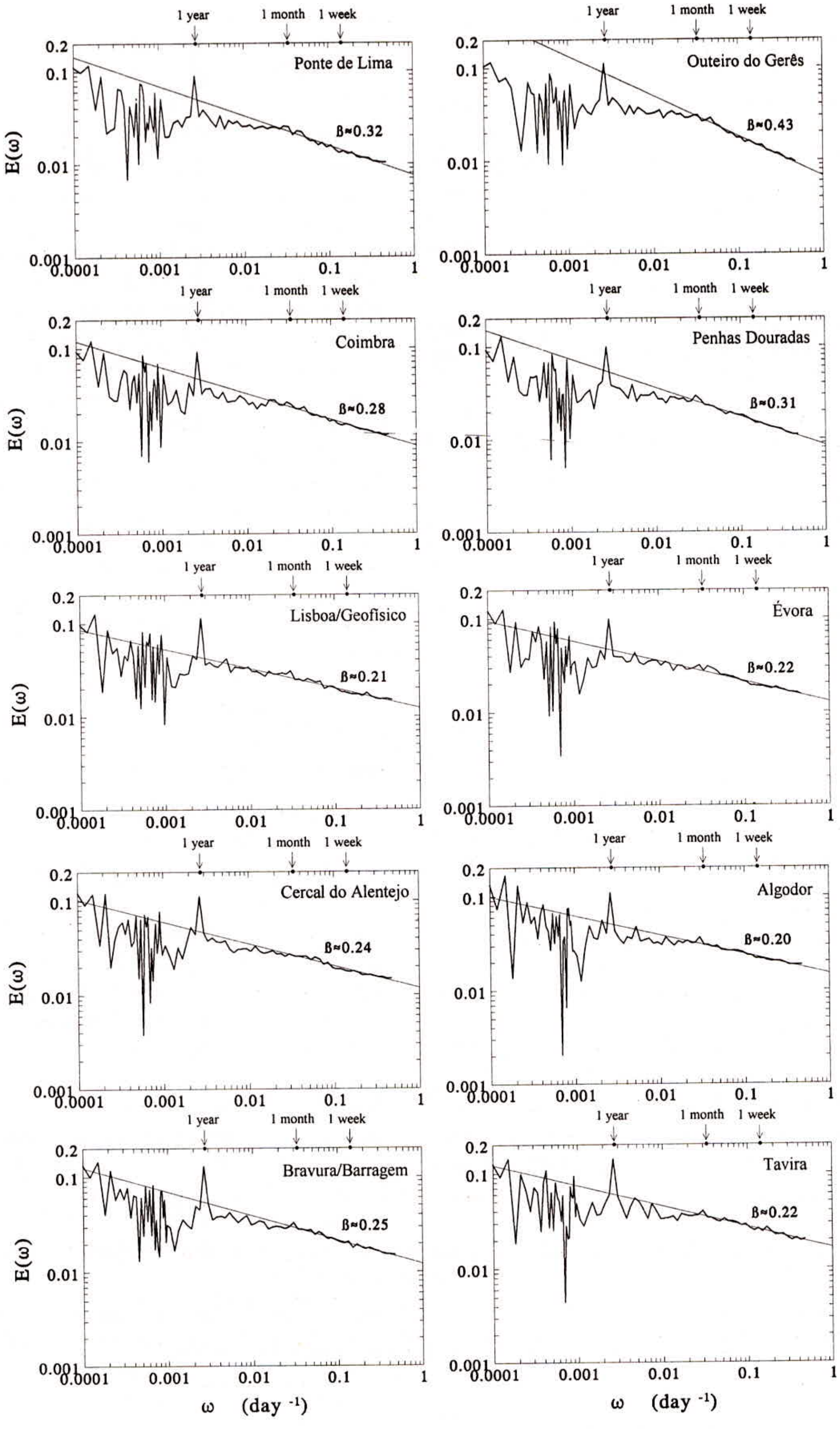

Fig. 5: Energy spectra obtained for daily precipitation recorded at 10 locations in Mainland Portugal, from 1941 to 1994

adjacent region of some spectra exhibits a nearly flat-power behaviour (i.e. $E(\omega) \approx \omega^0$); this behaviour extends up to roughly one decade, in some cases. The scaling regime associated with the range of scales characterized by a flat spectral plateau is expected to govern inter- and intra-seasonal variability, as discussed by, among others, Fraedrich and Larnder (1993). For rainfall in Europe, these authors reported a flat spectral plateau for time scales from about 3 years down to 3 days.

Table 3: Spectral Exponents Obtained for the Daily Precipitation Data Recorded at 10 Locations in Mainland Portugal, from 1941 to 1994

Measuring Stations	*Spectral Exponent β*
Ponte de Lima	0.32
Outeiro do Gerês	0.43
Coimbra/Geofísico	0.28
Penhas Douradas	0.31
Lisboa/Geofísico	0.21
Évora	0.22
Cercal do Alentejo	0.24
Algodor	0.20
Bravura/Barragem	0.25
Tavira	0.22

The presence of a break in the scaling at about 3 days have been also reported by e.g. Ladoy *et al.* (1991), Olsson (1995), Svensson *et al.* (1996), Tessier *et al.* (1996). Other studies reported other critical scale: at around 5 days (see e.g. de Lima, 1999) and 2 weeks (see e.g. Ladoy *et al.,* 1993), for example. Several authors associate this high frequency critical scale observed in the rainfall spectra with the duration of synoptic events (see e.g. Ladoy *et al.,* 1991; Fraedrich and Larnder, 1993; Tessier *et al.,* 1993).

In this study, in general, scaling is observed to hold from 1 day up to approximately 3 weeks, sometimes up to roughly one-and-a-half months. Other studies have shown that the scaling behaviour of precipitation extends down to scales of the order of minutes (e.g. Olsson, 1995; de Lima, 1998; de Lima and Grasman, 1999; Kiely and Ivanova, 1999; Peters and Christensen, 2002). This is of particular interest for extrapolating statistics.

The flat plateau discussed above is followed by another section (i.e. for even larger time scales), indicating large-scale climate variability. In the lowest frequency range (i.e. for the largest time scales), some of the spectra start rising with decreasing frequency (e.g. spectra obtained for the data from Coimbra, Penhas Douradas, Algodor, Bravura). The behaviour observed for the lowest frequencies is expected to be related to climatic fluctuations (see e.g. Fraedrich and Larnder, 1993; Rajagopalan and Lall, 1998). It describes long-term variability. Several studies report that global and regional climate variability is well organized on inter-annual and inter-decadal time scales (see e.g. Butler *et al.,* 1998; Rajagopalan and Lall, 1998). This has important implications for the interpretation and utility of hydro-climatic records. For the data studied in this work the analysis of the low frequency range should be approached with care because these estimates are based on a small number of long-period cycles.

River Discharge Analysis

The energy spectra obtained for the daily river discharge from the Douro basin, in the northern region of Portugal (Figure 3), are plotted on log-log axes in Figure 6.

Several general features are usually observed in river flow and they can be easily seen in the data plotted in Figure 4: (1) as a result of the periodicity in precipitation, river flow has also strong seasonal character (see also the spectral peaks at $\omega \approx 0.0027$ day^{-1} in Figure 6); (2) the seasonal cycle of river flow is asymmetric, i.e. river flow increases rapidly and decreases gradually; (3) the fluctuations in river flow are large for large river flow and small for small river flow (see examples in Table 4).

Table 4: Some Descriptive Statistics on the River Discharge Data and Spectral Exponent β (see Table 2 and Figure 6)

Measuring Stations	*Mean Daily Discharge (m^3/s)*	*Coef. Variation (daily discharge)*	*Maximum Daily Discharge (m^3/s)*	*Max. Daily Specific Discharge (mm)*	*Spectral Exponent β*
Ermida Corgo	8.4	1.94	289	84.9	0.51
Cunhas	9.2	1.77	249	63.8	0.61
Vinhais	11.3	1.75	295	53.3	0.60
Cidadelhe	15.5	2.11	740	36.7	0.70

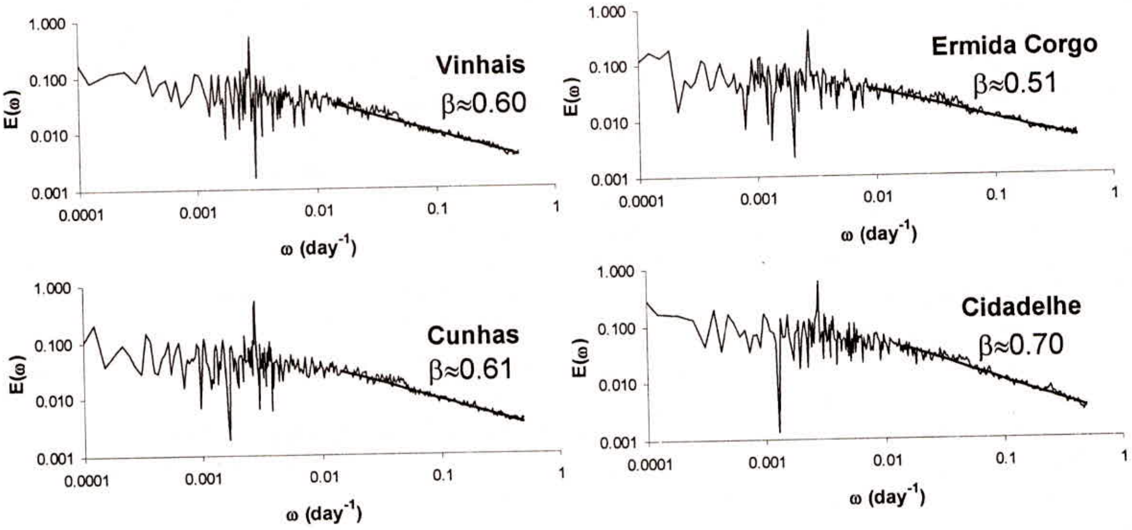

Fig. 6: Energy spectra obtained for daily river discharge recorded at 4 locations in the Douro basin, in the north of Mainland Portugal (see Table 3)

Similarly to precipitation (see discussion above), the spectra of river discharge also reveal scale-invariant behaviour. The spectral exponents in Figure 6 and Table 4 were determined for the range of scales up to 20 days ($\omega \approx 0.05$ day^{-1}). However, the scaling region seems to extend up to larger scales in some plots (Figure 6). The behaviour at lower frequencies also suggests that river discharge is well organized at inter-annual and inter-decadal time scales. However, once more, the behaviour observed could be biased by the small number of long-period cycles that is investigated here.

River flow reflects the interactions between the precipitation input and the local drainage basin characteristics that determine the hydrologic response of the basin. For the data analysed, the spectra of river flow exhibit a spectral slope β larger than the corresponding value found for precipitation in the region. This indicates more regularity in the river flow data: the precipitation signal is thus modified and smoothed by the complexity and variety of processes involved in the precipitation-runoff processes.

Several studies of both precipitation and river discharge indicate that the scaling behaviour of these natural processes depend on physical and geographical factors. In certain cases these are dominant local factors contributing to the dynamics of these processes. For precipitation, the synoptic maximum has been held responsible, by various authors, for the observation of a break in the scaling behaviour at scales that range from a few days to 2 weeks, depending on location. For the local conditions in Mainland Portugal, a break in the scale is usually observed at somewhat larger scales: 3 weeks, 1 month, or even larger scales. For the river discharge data, further analyses have to be carried out to explain the behaviour observed for the Portuguese data discussed in this study. There is still the need for a more complete understanding and characterization of the processes involved in the rainfall-runoff processes at different basin scales, under different climatic conditions and physiographic characteristics of the drainage basin (see e.g. Morin *et al.*, 2001; Labat *et al.*, 2002; Gupta, 2004; Dahlstedt and Jensen, 2005). Please note that the results of the analyses conducted in this study should be interpreted as the product of natural sample-to-sample variability. Rather long time series (of about 54 years for the precipitation data, and on average 44 years for the river flow data) were used; analyses of smaller samples can yield different results as they can therefore capture more localized characteristics of the processes involved, namely precipitation.

CONCLUDING REMARKS

Spectral methods can be used as exploratory and additional tools for investigating variability and scale-invariant properties in the temporal structure of hydrological processes such as precipitation and river discharge. Spectral analysis, although based only on second order statistics, gives important information about the structure and dynamics of these highly intermittent hydrological processes. It is able to quantify differences in variability, taking into account the behaviour observed across scales.

In this study, spectral analyses of daily precipitation and river discharge data from Mainland Portugal show that the temporal structure of these processes exhibits scaling behaviour across a wide range of scales. Scaling frameworks are useful tools for understanding the variability in hydrological processes over wide ranges in scale, which is a basis for extrapolating information across scales. However, various factors can affect the size of the scaling range and the characteristics of the scaling regime.

For precipitation, local climate and synoptic conditions introduce upper breaks in the scaling; the data analysed in this study are from both semiarid and humid regions and this leads to differences in the data statistics. For river discharge the size of the drainage area and other physiographic factors may lead to the identification of different scaling regimes. However, the limited diversity of river flow data explored here does not allow us to take conclusions on these issues.

Moreover, certain features of the recording devices and data processing methods can introduce a bias in the data which may affect the characterization of the processes, including the low-limit of the scaling range. This was not discussed in this work but it has already been reported by different studies (e.g. Harris *et al.*, 1997, 2001; de Lima and Grasman, 1999).

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