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Bivariate Flood Frequency Analysis for the Hydrological Safety of Dams

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ABSTRACT: A large number of dams, built in Italy in the last century, have been dimensioned using hydrological data sample limited in length of time series. Therefore, the design flood was affected from the scarcity of data and a poor knowledge of the flood formation processes. Nowadays, besides the data availability, sophisticated statistical methods addressed for extreme events have been developed and can be fundamental to review the hydrological safety of old dams.

Based on the previous contents, in this paper a bivariate statistical approach, describing the dependence structure between flood peak and flood volume, is proposed. The analysis is carried out through the application of a one-parameter Archimedean copula, particularly useful in the practical hydrology. Since the Archimedean copula represents a family of functions, a procedure based on the goodness-of-fit statistics for selecting the best one has been carried out. The chosen copula function provided an estimate of variables flood peak and volume for a given joint return period. The bivariate probability distribution has been expressed in terms of marginal probability distributions of the two considered variables, that can have different form. Pairs of flood peak and volume with a fixed joint return period have been selected and a hypothetical shape of the flood hydrograph has been surmised. The proposed method has been applied to the Calcione dam, a small reservoir located in Central Italy, in order to re-assess its hydrological safety by comparing the copula method results with those obtained from a hydrological lumped conceptual model.

INTRODUCTION

A large number of dams have been built in Italy in the first decades of the last century to supply the need of industrial, electric power, agricultural and drinking purposes. However, for that period data availability was limited, in terms of length of time series as well as of density on the territory. For this reason, the knowledge of the involved processes and the modelling techniques were affected from the scarcity of information. In the last years, the monitoring of numerous existing dams has allowed to collect a large amount of data, useful to develop sophisticated statistical methods addressed from extreme events that can be fundamental to review the hydrological safety of a reservoir.

Main part of the recent scientific literature has dealt with frequency analysis of flood peaks, which is the primary random variable in estimating the design flood of a dam (Stedinger *et al.*, 1993; Rao and Hamed, 2000). However this approach doesn't take into account the important role played by flood volume in the reservoir routing capacity of the dam, particularly in the definition of the spillway design flood, affecting the hydrological safety of the dam. Moreover, an univariate frequency analysis neglects the typically positive dependence structure between the two aforementioned random variables (flood peak and volume) and, for this reason, may lead to an overestimation or underestimation of the design flood, resulting, respectively, in a waste of money or in dam at high risk (Salvadori and De Michele, 2005). So, the complexity of flood processes makes a flood more likely to be a multivariate event that is characterized by a few correlated random variables, such as flood peak, volume and duration (Yue and Rasmussen, 2002). Multivariate models can offer improved understanding and modelling results regarding the hydrological safety of dams but also require considerably more data as well as sophisticated

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mathematical analysis. In order to overcome the drawbacks in using the univariate approach, many authors have proposed bivariate distribution for frequency analysis of flood peak and volume (Singh and Singh, 1991; Yue *et al.*, 1999; Yue *et al.*, 2001; Yue and Rasmussen, 2002; Shiau, 2003). All these methods were based on the assumption that the two variables have the same type of marginal probability distribution. However, in general this assumption is restrictive because the two afore-mentioned variables could follow different kinds of marginal probability distributions (Zhang and Singh, 2006). The use of copula approach to construct bivariate distributions can overcome this difficulty. In fact, copulas are functions that express the joint cumulative distribution function of two random variables in terms of their marginal probability distributions. The main advantage of using copulas is that they can separate the effect of dependence between the two variables from their marginal distributions (Shiau *et al.*, 2006). This allows to simplify greatly the calculations and may even yield analytical expressions for the isolines of the return periods.

The main purpose of this paper is to address the issues on hydrological safety of existing dams through a bivariate frequency analysis of flood peak and volume based on copula approach. To this end the results of the copula approach are compared with those obtained from a lumped rainfall-runoff model applied to the basin subtended by the dam. The proposed approaches have been applied to the Calcione dam, a small reservoir located in Central Italy, in order to reassess its hydrological safety by checking the routing capacity of the dam.

THEORETICAL BACKGROUND ON COPULAS AND BIVARIATE JOINT PROBABILITY

Copulas are functions that connect multivariate probability distribution of correlated variables to their one-dimensional marginal probability distributions (Nelsen, 1999), synthesizing the essential features of the existing dependence structure. Thus, the estimate of multivariate distributions is reduced to the selection of the marginal distributions that better fit the data set of correlated variables and to find the most suitable function that links them. As we are interested in determining the bivariate distribution of flood peak and volume, the following description refers to a bivariate case. Let X, Y be continuous random variables, their joint probability distribution function, $F_{X,Y}(x, y)$, is,

$$F_{X,Y}(x,y) = P(X \le x, Y \le y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f_{X,Y}(x',y')\, dx'dy' \quad \ldots (1)$$

where x is a value of X, y is a value of Y, P is the nonexceedance probability, $f_{X,Y}(x, y)$ is the joint probability density function. According to the Sklar's theorem (Sklar, 1959), $F_{X,Y}(x, y)$ can be estimated as,

$$F_{X,Y}(x,y) = C\left[F_X(x), F_Y(y)\right] = C(u,w) \quad \ldots (2)$$

where C is the copula function, $F_X(x)$, $F_Y(y)$ the marginal probability distributions of the variables X and Y, respectively, and u a specific value of $U = F_X(X)$ and w a specific value of $W = F_Y(Y)$.

Among the families of copulas proposed in literature, the Archimedean one has received much attention in hydrological analyses because it presents a large variety of copulas easy to construct (Nelsen, 1999). On this context, the one-parameter Archimedean copulas are chosen here to describe the joint probability distribution between flood peak and volume, that typically are positively correlated variables. The structure of a one-parameter Archimedean copula, C_δ, is given by,

$$C_\delta(u,v) = \phi^{-1}\{\phi(u) + \phi(w)\} \qquad 0 < u, w < 1 \ldots (3)$$

where ϕ is the copula generator, a convex decreasing function, with domain (0, 1] and range in $[0, \infty)$. It depends on the copula parameter, δ, that can be estimated through its relationship with Kendall's coefficient of correlation, τ, between X and Y. Different analytical expressions of ϕ produce different copulas applicable to hydrologic variables with positive or negative correlation. In particular, the copula families considered in this study are listed in Table 1 along with their main relationships.

Considering the expression of $\tau(\delta)$, we note that the Gumbel-Hougaard and the Cook-Johnson copulas can be applied only to bivariate data showing a positive dependence structure. On the contrary, the Ali-Mikhail-Haq copulas are suitable to construct joint probability distribution functions of both positively and negatively correlated variables.

Table 1: One-Parameter Archimedean Copula, C_δ, for the Families of Gumbel-Hougaard (G-H), Ali-Mikhail-Haq (A-M-H) and Cook-Johnson (C-J) (τ, Kendall's Correlation Coefficient, δ, Copula Parameter, ϕ, Generating Function)

Family	$\tau(\delta)$	δ *Constrain*	$\phi(t,\delta)$, $t = u, w$	$C_\delta(u,w)$
G-H	$1-\delta^{-1}$	$\delta \geq 1$	$(-\ln t)^\delta$	$\exp\left\{-\left[(-\ln u)^\delta + (-\ln w)^\delta\right]^{1/\delta}\right\}$
A-M-H	$\left(\frac{3\delta-2}{\delta}\right)-\frac{2}{3}\left(1-\frac{1}{\delta}\right)^2 \ln(1-\delta)$	$-1 \leq \delta \leq 1$	$\ln\frac{1-\delta\cdot(1-t)}{t}$	$\frac{uw}{1-\delta(1-u)(1-w)}$
C-J	$\frac{\delta}{\delta+2}$	$\delta \geq 0$	$t^{-\delta}-1$	$\left(u^{-\delta}+w^{-\delta}-1\right)^{-1/\delta}$

Copula Fitting Procedure

Given a sample of N bivariate observations (x_1, y_1), (x_2, y_2), …, (x_N, y_N), the procedure to construct the joint probability distribution function, $F_{X,Y}(x, y)$, of correlated variables X and Y is synthesized by the following steps:

1. estimate of the marginal distributions, $F_X(x)$ and $F_Y(y)$;
2. estimate of the dependence function, C_δ, between X and Y for each copula family;
3. identification of the most appropriate copula.

The first step can be carried out through the classical univariate frequency analysis. The second part of the procedure involves, first of all, the determination of the generating function, ϕ, through an estimate, τ_N, of Kendall's coefficient τ from observations as,

$$\tau_N = \binom{N}{2}^{-1} \sum_{i<j} sign\left[(x_i - x_j)(y_i - y_j)\right] \qquad \dots (4)$$

where the function sign is defined as sign = 1 if $(x_i-x_j)(y_i-y_j) > 0$, sign = −1 if $(x_i-x_j)(y_i-y_j)<0$, $i, j = 1, 2, \dots, N$. As it can be seen in Table 1, the value of τ_N defines the parameter δ and the function ϕ.

Finally, following Genest and Rivest (1993), the most suitable Archimedean copula to be used for obtaining the joint probability distribution function is identified through the following steps:

(a) define a new random variable $Z = Z(z)$ with probability distribution function $K(z)$, linked to copula generator ϕ as,

$$K(z) = z - \frac{\phi(z)}{\phi'(z)} \qquad \dots (5)$$

where ϕ' is the derivate of ϕ with respect to z;

(b) determine a non-parametric estimate of K, indicated as K_N, by computing:

- $z_i = N_i/(N-1)$, for $i = 1, 2, \dots, N$, where N_i represents the number of pairs (x_j, y_j) such that $x_j < x_i$ and $y_j < y_i$, for $j = 1, 2, \dots, N$ and $j \neq i$;
- $K_N(z_i)$, equal to the ratio of the number of observations z_j ($j = 1, 2, \dots, N$) with $z_j \leq z_i$ over the total number of observations;

(c) determine a parametric estimate of $K(z_i)$, by Eqn. 5, with z_i obtained in step b;

(d) for each Archimedean copula, plot $K_N(z_i)$ versus $K(z_i)$ to derive the copula that better fits observations;

(e) control the choice of the best copula using the AIC Akaike Information Criterion (Akaike, 1974), by means of Eqn. 6: specifically, the best copula is the one that has the minimum AIC value.

$$AIC = N \cdot \log\left\{\frac{1}{N-1}\sum_{i=1}^{N}\left[K(z_i) - K_N(z_i)\right]^2\right\} + 2 \qquad \dots (6)$$

Bivariate Return Period

In the bivariate analysis two joint return periods may be defined. The first one, T_{OR}, is defined as the return period in which either X or Y or both exceed given thresholds x or y, respectively,

$$T_{OR} = \frac{1}{P(X > x \text{ or } Y > y)} = \frac{1}{1-F_{X,Y}(x,y)} = \frac{1}{1-C_\delta(u,v)} \qquad \dots (7)$$

where $P(X > x$ or $Y > y)$ is the OR-exceedance probability of pair (x, y).

The second one is denoted by T_{AND} and is defined as the return period in which both X and Y are larger than fixed values x and y, respectively,

$$T_{AND} = \frac{1}{P(X > x \text{ and } Y > y)} = \frac{1}{1-F_X(x)-F_Y(y)+F_{X,Y}(x,y)} = \frac{1}{1-u-v+C_\delta(u,v)} \qquad \dots (8)$$

where $P(X > x$ and $Y > y)$ is the AND-exceedance probability of pair (x, y).

Since for Archimedean copulas $C_\delta(u, u) < u$ (De Michele *et al.*, 2005), the relationship among the corresponding return periods is,

$$T_{OR} \le \min(T_X, T_Y) \le \min(T_X, T_Y) \le T_{AND} \quad \dots (9)$$

where T_X and T_Y are the univariate return periods for X and Y, respectively. This relationship is extremely useful for a better understanding of the stochastic joint dynamics of the variables of interest. Moreover, it highlights the importance to choose which joint return period to use, as the same pair (x, y) may have greatly different return periods if considering T_{OR} or T_{AND}.

DESIGN FLOOD HYDROGRAPH

A lumped conceptual approach is employed in order to estimate the design flood. According to this approach, the basin is schematized as homogeneous in terms of soil hydraulic properties and rainfall spatial distribution, and the design hydrograph is given by,

$$q(t) = A\int_0^t \varepsilon(\tau)\, h(t-\tau)\, d\tau \quad \dots (10)$$

where t is time, $q(t)$ is the direct discharge at the basin outlet, A is the basin area, ε is the effective rainfall and h is the Geomorphologic Instantaneous Unit Hydrograph (GIUH). The GIUH depends on only one dynamic parameter, the basin lag-time L_B, defined as the time interval between the centroids of effective rainfall and direct runoff.

For effective rainfall estimate two different approaches are considered: the first one is based on bivariate frequency analysis by copula and the other on the design rainfall hyetograph abstracted by infiltration.

Copula Based Approach

For a fixed joint return period, the copula approach allows to select different couples of peak and flood volume (Q, V). Every pair has to be turned into a design hydrograph with a shape suitable to the geomorphologic characteristics of the basin. To this end, for each pair (Q, V) the flood hydrograph is derived through the convolution of a constant effective rainfall rate and the GIUH. Thus, the flood hydrograph $q(t)$ is given by,

$$q(t) = \begin{cases} \dfrac{V}{t_0}\displaystyle\int_0^t h(t-\tau)\, d\tau & 0 \le t \le t_0 \\ \dfrac{V}{t_0}\displaystyle\int_0^{t_0} h(t-\tau)\, d\tau & t_0 \le t \end{cases} \quad \dots (11)$$

where t_0 is the duration of effective rainfall, determined by assuming max $q(t) = Q$.

Rainfall-Runoff Model Approach

For a fixed return period the design rainfall hyetograph is obtained with the same duration of the GIUH and is derived by the alternating block method using an areal rainfall depth-duration curve (Chow *et al.*, 1988). The effective rainfall is estimated considering infiltration as the main loss and simulating the process of infiltration with a parametric local infiltration model (Flammini *et al.*, 2004). This model is a new formulation of the semi-analytical model developed by Corradini *et al.* (1997) for the estimate of rainfall infiltration into a homogeneous soil under complex rainfall patterns, including successive infiltration/redistribution cycles. Assuming an initial soil water content, θ_i, invariant with depth, z, and the dynamic wetting profile, $\theta(z)$, of distorted rectangle shape, the time evolution of surface water content, θ_0, is described by,

$$\frac{d\theta_0}{dt} = \frac{(\theta_0-\theta_i)\beta(\theta_0)}{I'\left[(\theta_0-\theta_i)\dfrac{d\beta(\theta_0)}{d\theta_0}+\beta(\theta_0)\right]} \left[\left[r-\varepsilon(t)\right]-K_0-\frac{(\theta_0-\theta_i)G(\theta_i,\theta_0)\beta(\theta_0)pK_0}{I'}\right] \quad \dots (12)$$

where r is the rainfall rate, K_0 is the surface hydraulic conductivity, I' is the cumulative dynamic infiltration depth, p and β are parameters linked to profile shape, $G(\theta_i, \theta_0)$ is the net capillary drive at the wetting front depending on the soil water matric potential, Ψ, and hydraulic conductivity, K, according to,

$$G(\theta_i,\theta_0) = \frac{1}{K_0}\int_{\Psi(\theta_i)}^{\Psi(\theta_0)} K\, d\Psi \quad \dots (13)$$

Eqn. 12 may be solved numerically during each of infiltration process phases (infiltration-saturation-distribution-reinfiltration), expressing the soil hydraulic properties by functional forms (Corradini *et al.*, 1997) which incorporate the well-known pore size distribution index, λ, the air entry potential, Ψ_b, the residual soil water content, θ_r, and the water content at natural saturation, θ_s, as well as two empirical coefficients, c and d. The new model version incorporates parameterizations of a few hydraulic properties through empirical parameters and allows the estimation of time evolution for infiltration rate and surface water content through simple explicit relations. In particular, the quantity G is parameterized according to,

$$G(\theta_i,\theta_0)=\frac{K_s}{K_0}\left(\frac{(\theta_0-\theta_r)}{(\theta_0-\theta_r)}\right)^{3+\frac{2}{\lambda}}$$
$$\left\{\Psi_b\left[\frac{(\theta_s-\theta_i)^{1+\frac{4}{5}\lambda}}{(\theta_0-\theta_i)^{1+\lambda}(\theta_s-\theta_0)^{\frac{2}{5c}}}\right]^{\frac{1}{c}-\lambda}+d\right\}\frac{(\theta_s-\theta_r)^{\frac{2}{c}+\frac{\lambda}{4}}}{(\theta_s-\theta_0)^{\frac{1}{2c}}} \quad \text{... (14)}$$

where K_s is the saturation value of K. Eqn. 12, rewritten for time to ponding, t_p, and for the post-ponding stage produces,

$$\int_0^{t_p}\left(r(t)-K_i\right)dt=\frac{(\theta_s-\theta_i)G(\theta_i,\theta_s)K_s\beta(\theta_s)p}{\left(r(t_p)-K_s\right)} \quad \text{... (15a)}$$

$$f_c=K_s+\frac{(\theta_s-\theta_i)G(\theta_i,\theta_s)K_s\beta(\theta_s)p}{I'} \quad \text{... (15b)}$$

where f_c is the infiltration capacity for $t > t_p$ and G is given by Eqn. 14. In the infiltration model adopted here, Eqn. 12 is also parameterized in order to derive θ_0 as a function of time during the unsaturated stages.

Lastly, during a reinfiltration stage due to an intense rainfall, starting at a time t_r, a new time to ponding could be reached; it may be computed by considering t_r as the new initial time. Then, during the new post-ponding stage the infiltration capacity is assumed to be, at each time, the maximum of the two values derived assuming as initial time the start of the complex rainfall event and the beginning of the last reinfiltration period. The parameterizations here described lead to represent any stage during successive infiltration/redistribution cycles by eliminating computations of complex integrals and the numerical solution of the ordinary differential equation (Eqn. 12). In particular, during the stages of saturated soil surface, the effective rainfall rate, ε, is derived as,

$$\varepsilon=r-f_c \quad \text{... (16)}$$

DESIGN HYDROGRAPH RESERVOIR ROUTING

In order to re-assess the hydrological safety of a dam, a simulation of the reservoir behaviour can be carried out considering flood events related to a fixed return period that should represent the expected design life of the dam. In particular, operating the routing of design hydrographs allows to check adequacy of the dam spillway. The simulation of flood routing requires the definition of the initial reservoir level, chosen according to cautious criterion. The mathematical model for flood routing is based on the continuity equation expressed in the form,

$$q_a-q_s=\frac{dV}{dt} \quad \text{... (17)}$$

where q_a is the inflow to the reservoir, q_s is the overflow spillway discharge, the unique outlet considered, and V represents the water volume stored in the reservoir. Eqn. 17 can be solved numerically through the Runge-Kutta algorithm (Castorani and Moramarco, 1995) when the initial volume is known. The dam spillway can be considered adequate if during the routing of design hydrograph the reservoir level does not overcome the maximum water level.

CASE STUDY

Dam Basin Description and Data Used

The procedures described in the previous sections have been applied to the Calcione dam, a small reservoir located in Central Italy, with a subtended drainage area of 20.4 km^2. The maximum water storage of the reservoir is about $4.0 \cdot 10^6$ m^3, while the maximum regulation level, the maximum water level and the crest level are 362.50, 364.00 and 366.00 m a.s.l., respectively. In the catchment a hydro-pluviometric network has been operating since 1983, consisting of one rain gauge station and one hydrometric station both situated at the dam. Hourly data of the level reservoir along with information on the bottom outlet discharge were available from 1983 to 1997 (except for 1988) and from 2004 to 2006. Through Eqn. 17 the more salient flood hydrographs entering the lake were determined. In particular, 29 flood events have been estimated and the maximum annual flood, characterized by both the maximum flood peak and the maximum flood volume, has been selected. The principal characteristics of these events are reported in Table 2.

Application of the Copula Method

The application of copula method to the random variables flood peak and volume implicates: 1) the determination of marginal probability distributions that better fit the empirical distributions of the two variables; 2) the identification of the most suitable Archimedean copula to describe the dependence structure function.

Table 2: Main Characteristics of Maximum Annual Floods Occurred in the Basin of the Calcione Dam: Peak Flow, Q_{TOT}, Direct Peak Flow, Q, Total Runoff Volume, V_{TOT}, Direct Runoff Volume, V, Total Areal Rainfall Depth, R_m, Storm Duration, D, and Maximum Rainfall Rate, r_{max}

Date	Q_{TOT} (m^3/s)	Q (m^3/s)	V_{TOT} $(10^5\ m^3)$	V $(10^5\ m^3)$	R_m *(mm)*	D *(h)*	r_{max} *(mm/h)*
Feb 1983	18.5	17.7	8.98	7.79	59.2	24	9.8
Feb 1984	4.6	4.5	2.90	2.53	32.8	23	5.4
Mar 1985	8.5	7.7	3.27	2.59	29.0	15	7.8
Jan 1986	10.2	10.1	6.42	5.29	65.8	22	7.0
Dec 1987	25.9	25.2	5.33	4.37	44.0	12	10.0
Jun 1989	10.4	10.2	2.97	2.60	30.0	19	7.0
Dec 1990	3.8	3.7	2.32	1.66	56.2	26	6.2
Feb 1991	10.3	8.1	2.67	1.41	21.2	11	6.4
Oct 1992	46.8	46.7	5.93	5.33	52.6	8	24.4
Oct 1993	65.1	65.0	6.27	5.87	30.0	12	15.6
Nov 1994	1.7	1.6	0.47	0.34	18.2	11	8.6
Dec 1995	13.6	13.5	4.31	3.69	54.8	21	9.2
Nov 1996	5.3	5.0	2.16	1.66	56.0	18	10.0
Jun 1997	22.7	22.2	5.78	4.74	93.8	13	20.0
Oct 2004	85.0	84.6	12.22	11.00	146.2	9	39.4
Nov 2005	26.3	25.2	7.22	5.91	67.0	19	9.0
Jan 2006	38.7	38.6	16.59	15.18	83.0	27	12.0

Marginal Distributions of Flood Peak and Volume

The Gumbel, Lognormal and TCEV (Rossi *et al.*, 1984) distributions have been considered for univariate frequency analysis of annual maximum direct flood peak, Q, and annual maximum direct runoff volume, V. On the basis of the Kolmogorov-Smirnov test, all the three theoretical distributions resulted to be consistent with the data. Therefore, the choice of the distribution was based on the comparison of the three probability distributions versus the sample empirical distributions shown in Figure 1. As it can be seen, the TCEV distribution can be considered as the most suitable probability distribution function to fit observed values of both flood peak and volume for the catchement of the Calcione dam. In fact, this distribution guarantees the better reproduction of the highest observed values. Fixing a return period (T_Q and T_V) for both variables equal to 1,000 years, which is the design return period for the dam in accordance with the directives followed in Italy, the fitted TCEV distributions furnished a flood peak and volume value of 204 m^3/s and 3.1 10^6 m^3, respectively.

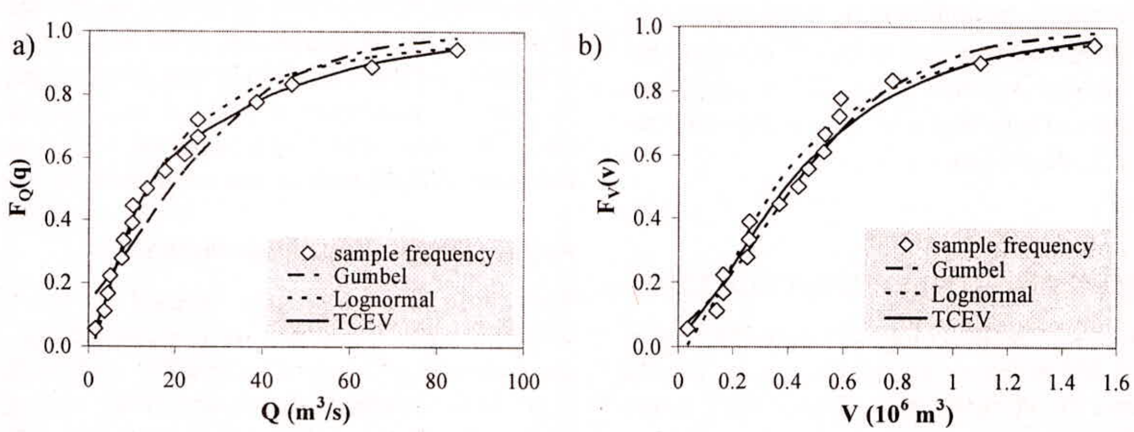

Fig. 1: Lognormal, Gumbel and TCEV probability distributions *versus* sample empirical probability distribution of (a) flood peak, b) flood volume

Joint Probability Distribution of Flood Peak and Volume

The couples of observed Q and V for the Calcione dam were found well correlated, with a Kendall's coefficient equal to 0.691. Their bivariate probability distribution, $F_{Q,V}(q, v)$ has been estimated by the previously described Archimedean copulas. Among them the Cook-Johnson (C-J) copula was found to be the most suitable to represent the bivariate distribution of the correlated flood peak and volume for the study catchment. In fact, it is characterized by the lowest value of the AIC parameter as well as by the minimum deviation between K and K_N, as shown in Figure 2. The selected joint probability distribution function is shown in Figure 3.

Joint Return Period of Design Flood Peak and Volume

The marginal return periods, T_Q and T_V, and joint return periods T_{OR} and T_{AND}, of variables flood peak and volume for the catchment of the Calcione dam have been calculated. In Figure 4 T_{OR} and T_{AND} are plotted versus T_Q and T_V, while in Figure 5 observed flood peak and volume couples are displayed with reference to different return periods estimated by the copula approach. The analysis of Figure 5 can provide univariate and bivariate risk assessment of flood events. In fact, univariate analysis reveals that the flood event occurred on January 2006, characterized by a peak flow value of 38.6 m^3/s and volume of 15.18 10^5 m^3, presents an univariate peak flow return period of 4.5 years and an univariate flood volume return period of 25.5 years, while its T_{OR} is 4.3 years and its T_{AND} is 35.1 years. For the flood event occurred on October 2004, with peak flow equal to 84.6 m^3/s associated to a volume of 11.00 10^5 m^3, the values of the afore-mentioned T_Q, T_V, T_{OR} and T_{AND} are, respectively, equal to 19.0, 9.7, 7.5 and 45.5 years. These observations mean that, if flood control planning focuses on flood peak only, the second event would be considered quite relevant. On the contrary, by assuming also the volume as safety factor, the first event would be relevant too. Then, the analysis of the bivariate behaviour of flood peak and volume can give an instrument to correctly evaluate the relevance of flood events. In fact, the two above-mentioned events are characterized by a similar joint return period.

Design Hydrographs for the Calcione Dam

Both the approaches considered in this study for design hydrograph estimation require the determination of the GIUH for the catchment subtended by the dam. This has been derived from the dimensionless instantaneous unit hydrograph estimated for a geographical area of Central Italy including the catchment itself (Corradini *et al.*, 1995; Melone *et al.*, 2002). The basin lag-time was computed by the rainfall-runoff model applied for simulating six observed flood events. In particular, L_B was found equal to 2.9 hours.

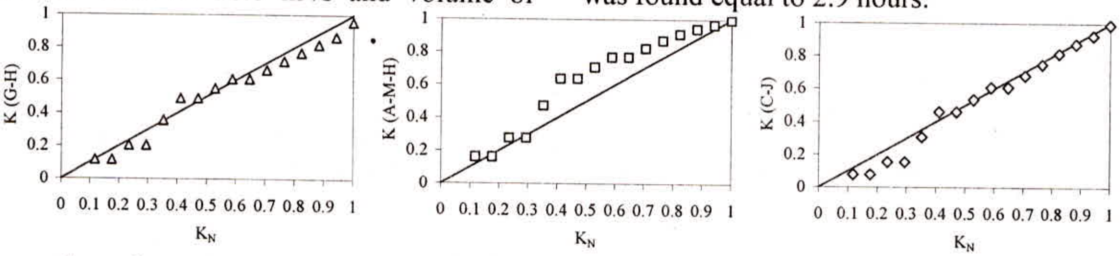

Fig. 2: Comparison of nonparametric $K_N(z)$ and parametric $K(z)$ for Gumbel-Hougaard (G-H), Ali-Mikhail-Haq (A-M-H), Frank (F) and Cook-Johnson (C-J) copulas referring to correlated variables flood peak and volume

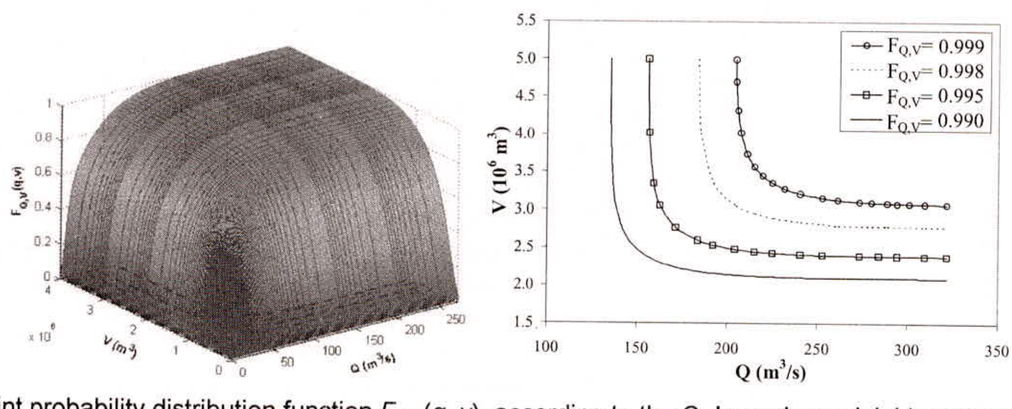

Fig. 3: a) Joint probability distribution function $F_{Q,V}(q, v)$, according to the C-J copula model, b) contours of $F_{Q,V}(q, v)$

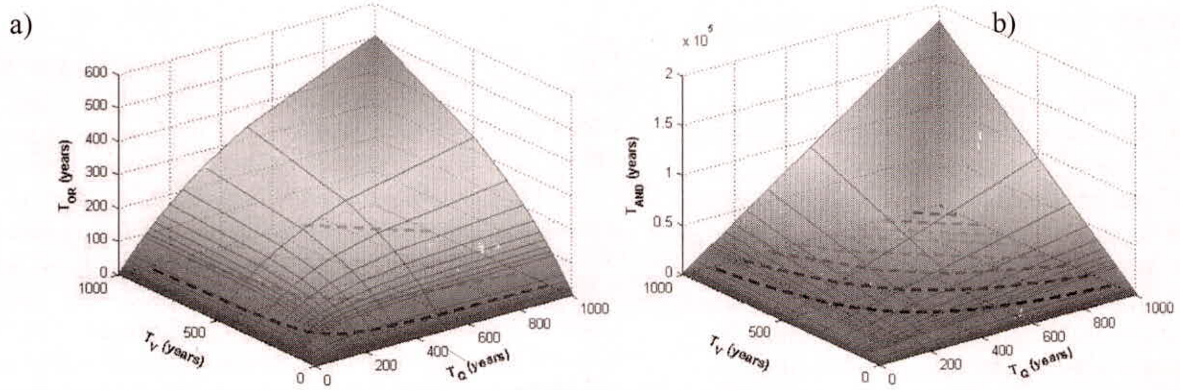

Fig. 4: Joint return period of flood peak and volume for the basin of the Calcione dam, according to the C-J copula model related to a) OR-case, b) AND-case, versus marginal return period

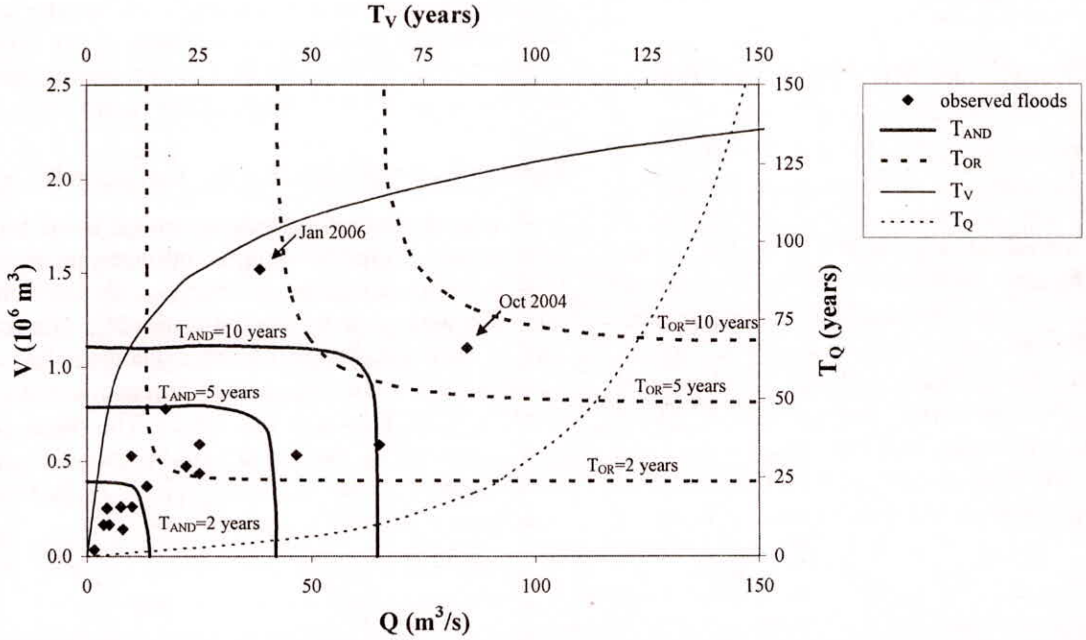

Fig. 5: Contours of joint return periods T_{OR} and T_{AND}, and marginal return periods T_Q and T_V of flood peak and volume. Observed annual maximum peak and volume are also shown

Copula Approach

For a fixed joint return period, infinitive couples of flood peak and volume (Q, V) can be determined through the copula approach. Thus, an appropriate selection of representative couples is necessary. In particular, considering a 0.999-joint nonexceedance probability curve and a T_{OR} = 1,000 years for cautious reasons, three couples were selected: one couple is characterized by intermediate values of peak and volume, whereas the other refer to the asymptotic values assumed by the two variables (Figure 6a). The corresponding hydrographs are shown in Figure 6b.

Rainfall-Runoff Model

The rainfall-runoff model requires the determination of both the design rainfall hyetograph and the model parameters, that are the basin lag-time and the main parameter involved in estimating the losses, θ_i. These parameters have been estimated for the catchment of the Calcione dam through a procedure of calibration. The other parameters involved were assigned on the basis of previous analyses on the soil hydraulic properties in the Upper River Tiver basin (K_s = 0.4 mm/h, θ_s = 0.41, θ_r = 0.08) and subsequently tested through the calibration procedure. At the purpose, the rainfall-runoff events used are reported in Table 3 together with the calibration procedure results. In particular, the calibration allowed to check that for salient flood events the lag-time can be assumed invariant and equal to 2.9 hours, whereas the antecedent soil conditions can be represented by a medium-high initial water content.

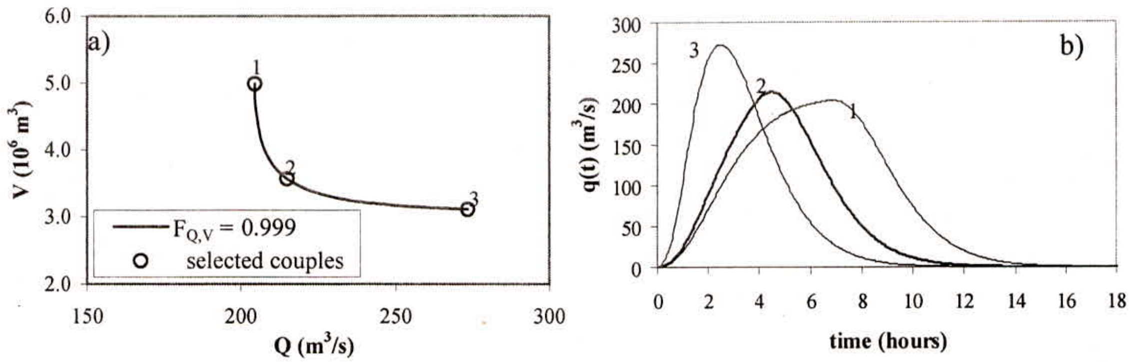

Fig. 6: a) Selected design couples of flood peak and volume for a joint probability distribution function $F_{Q,V}(q, v) = 0.999$, b) flood hydrographs derived from the selected couples

Table 3: Rainfall-Runoff Model Calibration Results (θ_i, initial soil water content, ε_Q, ε_t and ε_V, error on peak flow, time to peak and volume, respectively)

Flood Event	θ_i	ε_Q *(%)*	ε_t *(%)*	ε_V *(%)*
Jan 1986	0.297	27.7	0.0	2.0
Feb 1991	0.318	2.5	–10.0	1.4
Dec 1987	0.365	9.0	0.0	–0.1
Nov 2005 (1)	0.272	4.7	0.0	1.6
Nov 2005 (2)	0.298	–1.0	0.0	0.2
Jan 2006	0.409	–3.4	–11.1	–12.6

For a fixed return period of 1,000 years, the design rainfall hyetograph has been calculated using the rainfall depth-duration curve obtained from the regionalization procedure referred to the area of Compartimento Bologna-Pisa-Roma (CNR-GNDCI, 2000), that was found in good accordance with local statistical analyses. Considering a rainfall hyetograph of alternating block shape and saturated initial soil conditions, the design hydrograph estimated by the rainfall-runoff model was characterized by a peak flow and flood volume of 193 m^3/s and 3.4 10^6 m^3, respectively. This design hydrograph was compared with the results of the copula-based bivariate analysis. As it can be seen in Figure 7, the values of flood peak and volume derived from the rainfall-runoff approach are quite close to the ones related to a joint return period T_{OR} equal to 1,000 years and greatly more relevant than the ones corresponding to a joint return period T_{AND} equal to 1,000 years. In particular, the design hydrograph derived from the rainfall-runoff model is characterized by a T_{OR} equal to 500 years and a T_{AND} equal to 183,000 years.

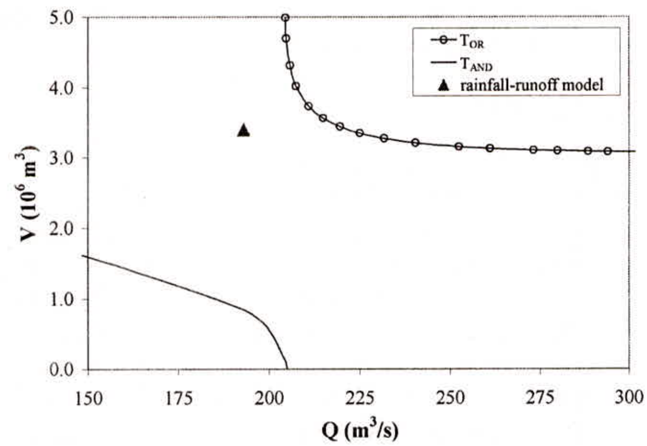

Fig. 7: Design flood estimated by the rainfall-runoff model in relation with the results of the copula-based bivariate analysis (T_{OR}, T_{AND}), for a return period of 1,000 years

Reservoir Routing of Estimated Design Hydrographs

In order to re-assess the hydrological safety of the Calcione dam, a simulation of the reservoir behaviour has been carried out considering the four flood events previously derived (three by the copula method and one by the rainfall-runoff model) and assuming as initial reservoir level the maximum regulation one. This choice has been suggested by cautious reasons as the reservoir levels observed before each maximum annual flood never overcame the maximum regulation level. Figure 8 shows the simulation results for the design flood events estimated by the copula approach and by the rainfall-runoff model. As it can be seen, the maximum water level was never overcome during the simulated flood events and this assess the hydrological safety of the Calcione dam in terms of adequacy of dam spillway.

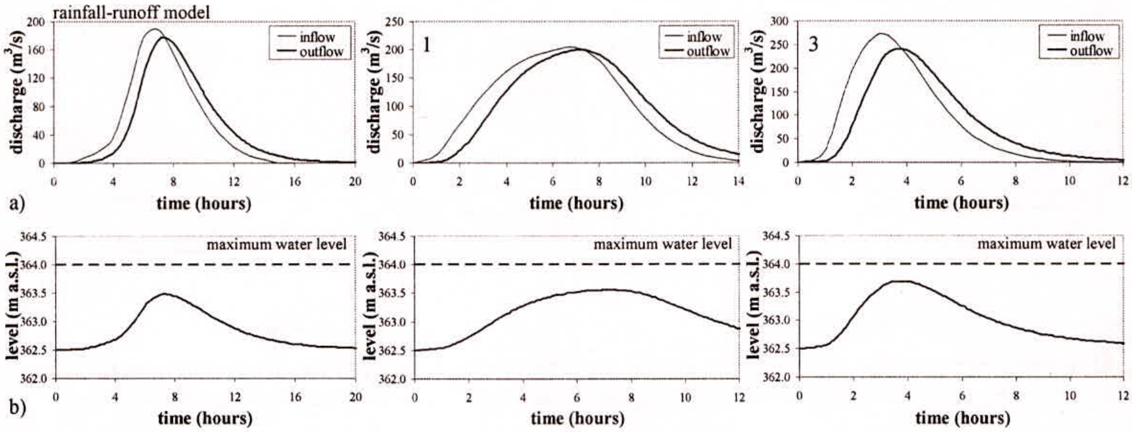

Fig. 8: Results of reservoir routing simulation for different design flood events (rainfall-runoff model one, events number 1 and 3 of Figure 6) with a return period of 1,000 years: a) inflow and outflow hydrographs, b) water level

CONCLUSIONS

The bivariate frequency analysis based on one-parameter Archimedean copulas allowed to estimate the joint probability distribution of annual maximum flood peak and volume. These quantities are fundamental in order to assess the hydrological safety of dams in terms of flood design estimation and analysis of the reservoir routing capacity through the spillway. Moreover, the copula approach can be useful to verify the design hydrograph estimated by a lumped rainfall-runoff model, calibrated with few observed flood events.

On the basis of the results obtained for the Calcione dam the following conclusions can be drawn: (1) the univariate analysis provided 1,000 years flood peak and volume comparable with the corresponding ones from the rainfall-runoff model; (2) the bivariate analysis led to significantly different evaluation of design flood with regard to the chosen return period, related to OR or AND conditions of critical peak and volume occurrence; (3) for a return period of 1,000 years, the values of flood peak and volume derived from the rainfall-runoff model were found close to the ones given by the OR-condition of the copula method and much greater than the ones obtained under AND-condition; (4) the hydrological safety in terms of dam spillway has been assessed by operating the reservoir routing considering as inflows the design events obtained from the rainfall-runoff model and the copula method. The use of both the approaches investigated here can be considered a valuable method for checking the adequacy of dam spillway mainly for the existing dams with short time series of flood peak and volume.

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