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A Parsimonious Trigonometric Model of Reservoir Operating Rules

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ABSTRACT: With the ever increasing demand for scarce water resources, it is imperative that water resource utilization is optimized in the short and the long-term. A major component of this endeavour is the optimization of reservoir system operation which until recently had been dominated by Linear Programming (LP) and Dynamic Programming (DP). An alternative to LP and DP is direct simulation-optimization within which the operating rule curves can be defined by decision variables and therefore obtained directly. The simulation—optimization approach which is now easily enabled by the use of evolutionary optimization techniques, is easier to understand and enables the incorporation of required system performance more easily and comprehensively than LP or DP. One of the main challenges experienced in applying simulation-optimization approaches is high computation intensity and reducing computation intensity whilst maintaining effectiveness leads to greater acceptability of the approach.

This paper investigates the applicability of simple trigonometric functions as a means of parsimoniously defining operating rule curves and thereby reducing the number of decision variables in reservoir system optimization. The procedure is applied to a system of two reservoirs whose total yield needs to be maximized while meeting multiple reliability constraints of supply and reservoir storage state. The results obtained do not indicate the need for trigonometric functions and suggest that an even simpler model that defines rule curves as straight horizontal lines with no monthly variation is adequate. While this may be the case for the system studied here, it is proposed that the trigonometric function or its variants be studied with more complex systems or other objectives. The shuffled complex evolution (SCE-UA) method is applied in the optimization and is found to be effective and efficient through the use of multiple randomly initialized runs.

INTRODUCTION

As global water demand continues to spiral while the availability of the resource becomes more uncertain with increased pollution and climate change/ variability, the need to utilize water resources as efficiently as possible in short term operation and long term planning cannot be overemphasized.

Most large water resource systems typically include one or more reservoirs that need to be operated efficiently in order to maximize benefits and minimize losses. The operation of reservoirs typically includes rules that specify when and how much water needs to be released depending on several variables including the storage state of the individual reservoir and that of the complete system, the demand/s and the period (season, month, 10 day period etc.) of the year. Inflow forecasts, if adequately reliable can be included in real time/short term operation. For the system is to meet the water demands and storage state based uses (e.g. recreation) as adequately as possible, then the rule curves need to be optimized applying an objective function that maximizes overall benefit.

Reservoir operation optimization is still and active area of research (Wardlaw and Sharif, 1999, Chang *et al.*, 2005a, b, Prakash and Shanthi, 2006, Shiau and Lee, 2005, Consoli *et al.*, 2007, Shih and Re Velle, 1995, Chen *et al.*, 2007, Suiadee and Tingsanchali, 2007, Ndiritu, 2005, Ndiritu, 2003) and the Genetic Algorithm (GA) seems the predominant optimization approach in recent studies. This observation is in agreement with the suggestions for increased application of the GA in reservoir operation (Labadie 2004). The GA and other evolutionary techniques enable the inclusion of as much detail as needs to be included as they are easily amenable to a simulation-optimization (s-O) approach unlike linear programming

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or dynamic programming. The need to run a large number of simulations in applying evolutionary techniques is a drawback of the S-O approach and may also limit the level of detail that can be practically included. The long computation times (varying from 100 minutes to 8 hours) observed in the optimization of a system of two reservoirs (Ndiritu, 2005) inspired the current study. Each monthly rule curve value had been set as an independent value to be optimized and this lead to an optimization problem with a large number of parameters (varying from 77 to 158). Multiple randomly initialized runs obtained reasonably close objective functions but poor parameter identification indicating that the problem was over-parameterized.

The objective of the current study is to investigate the applicability of using simple trigonometric functions to define rule curves as a means to eliminate over-parameterization and thereby reduce computation time and obtain better parameter identification. In addition, this study also serves to demonstrate the application of the shuffled complex evolution (SCE-UA) method (Duan *et al.*, 1992) as an alternative to the popular GA in reservoir system optimization. Although the SCE-UA has been shown to be highly effective and efficient in catchment model calibration, only a single application in reservoir system optimization has been found (Cui and Kuczera, 2003). A simulation-optimization approach is applied to the system of two reservoirs used in an earlier study (Ndiritu, 2005). The objective is determine the monthly operating rule curves that maximize total system yield subject to multiple reliability constraints to supply and reservoir storage states.

SYSTEM SIMULATION

The main water balance components included in the analysis are shown on Figure 1. The upper dam, Rust de Winter has a catchment area of 1 145 km^2 and the incremental area to the downstream reservoir Mkombo is 2 578 km^2. On the basis of simulated monthly streamflow sequences supplied by the South African Department of Water Affairs and Forestry (DWAF) for the period 1920–1996, the mean annual runoff to Rust de Winter is 19.8 mm while the incremental area to Mkombo has a mean annual runoff of 3.9 mm. The historic mean point rainfalls for the same period at Rust de Winter and Mkombo are 605 and 243 mm respectively. Rust de winter and Mkombo have live storage capacities of 27.1 Mm^3 and 205 Mm^3 respectively and these were assumed in the current analysis. The South African DWAF also provided the monthly average Symon's pan evaporation depths. These were factored by 0.85 to obtain reservoir evaporation rates. Surface area-storage volume relationships for computation of net evaporation losses were obtained and modelled using second order polynomials. On Figure 1 and for the rest of this paper, Rust de Winter is denoted as Reservoir 1 and Mkombo as Reservoir 2.

Assuming the two reservoirs to be initially half-full, mass balance was carried out at a monthly time step as described by Eqn. (1).

$$S_{i,j+1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} S_{i,j} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} Q_{i,j} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} NEV_{i,j} + \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix} RF_{i,j} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} AS_{i,j} + \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix} SP_{i,j} \quad \ldots (1)$$

where:

$S_{i,j}$ is the vector of the initial storage volumes at the beginning of month *j* of year *i*

$Q_{i,j}$ is the vector of the incremental inflows in month *j* of year *i*

$NEV_{i,j}$ is the vector of net evaporation losses in month *j* of year *i*

$RF_{i,j}$ is the vector of regulated flows from the upper to the lower reservoir assuming neither transmission losses nor abstractions

$AS_{i,j}$ is the vector of the direct diversions to supply

$SP_{i,j}$ is the vector of spill volumes.

The net evaporation losses were obtained as,

$$nev_{i,j,k} = 0.5\left[a_{i,j,k} + a_{i,j+1,k}\right] \times \left[0.85ev_{j,k} - ra_{i,j,k}\right] \quad \ldots (2)$$

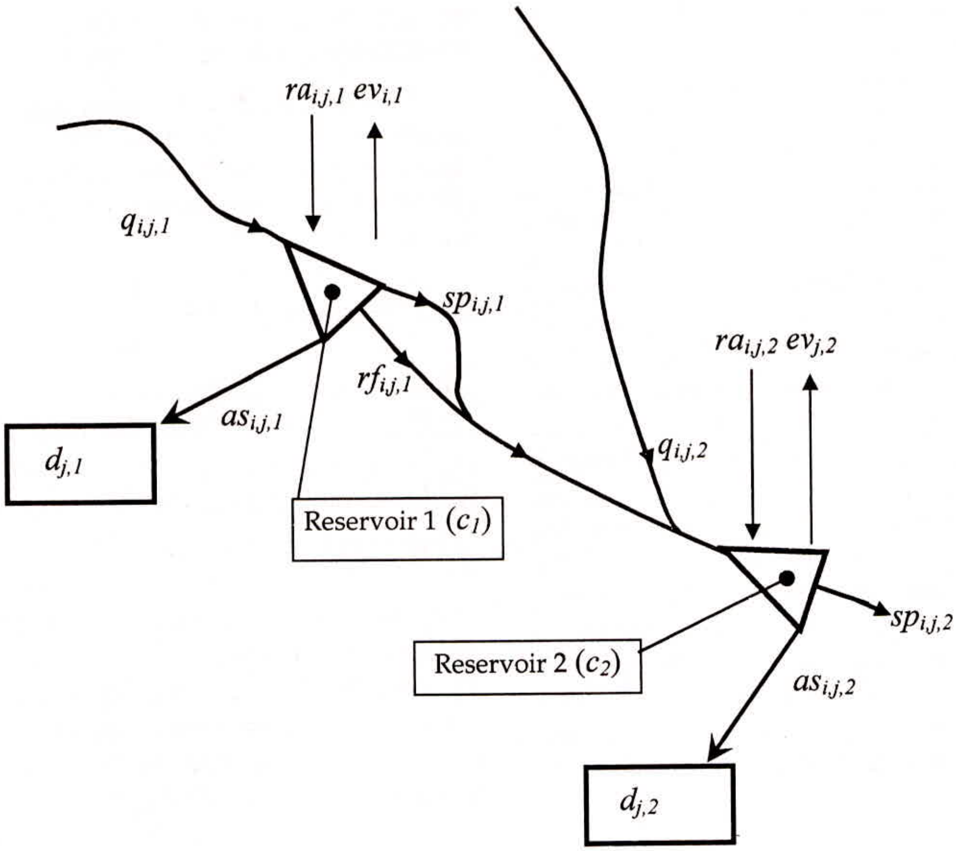
where:

$a_{i,j,k}$ is the surface area of reservoir *k* at the beginning of month *j* of year *i*

$ev_{j,k}$ is the average Symon's pan evaporation rate for reservoir *k* in month *j*

$ra_{i,j,k}$ is the point rainfall at reservoir *k* in month *j* of year *i*.

The direct diversion to supply $as_{i,j,k}$ and the regulated flow from Reservoir 1 to 2, $rf_{i,j,1}$ depend on the storage state of the individual reservoirs, the total storage state of the system, the monthly demand and the month of the year. Operating rule curves therefore needed to be defined for each reservoir and for total



$as_{i,j,k}$	direct diversion from reservoir k in month j of year i
c_k	capacity of reservoir k
$d_{j,k}$	demand for reservoir k in month j
$ev_{j,k}$	average Symon's pan evaporation for reservoir k in month j
$q_{i,j,k}$	inflow to reservoir k in month j of year i
$ra_{i,j,k}$	point rainfall at reservoir k in month j of year i
$rf_{i,j,k}$	regulated flow from reservoir k in month j of year i
$sp_{i,j,k}$	spill from reservoir k in month j of year i

Fig. 1: Main water balance components in system simulation

storage. Each reservoir was assumed to supply at four levels of restriction (100, 80, 50 and 30%) and three rule curves therefore needed to be defined for each. It had been found (Ndiritu, 2005) that it was sufficient to obtain the rule curves for total storage as a weighted linear average of the rule curves of the individual reservoirs and this finding was adopted in the current analysis. The direct diversions to supply from each reservoir and the controlled release from Reservoir 1 to 2 were obtained as follows:

- If the overall storage and individual reservoirs are in the same supply zone, that level of direct diversion is provided with no controlled release from Reservoir 1 to 2.
- If the upper reservoir is in a lower zone than that of total storage, the diversion to supply from Reservoir 1 is based on its zone.
- If the lower reservoir is in a lower zone than that of the total storage and the upper reservoir is in the zone of the total storage or a higher one, then a controlled release is made from Reservoir 1 to Reservoir 2. This release equals the extra demand that Reservoir 2 would fail to supply for being in a lower zone than the total storage. Once the controlled release is made, Reservoir 2 provides a direct diversion corresponding to the zone of total storage.
- If no controlled release is made and Reservoir 2 is in a lower zone than the total storage, Reservoir 2 provides a direct diversion based on its supply zone.
- If a reservoir is in a zone higher than that of the total storage, the direct diversion is based on the supply zone of total storage.

Quantitative details of this procedure are available in the earlier study of the system (Ndiritu, 2005).

To reflect the reality of supply to multiple users, It was assumed that reservoir 1 would be supplying municipal demand while reservoir 2 would be supplying irrigation demand at the levels of reliability presented in Table 1. As expected, these reflect a higher reliability requirement for municipal water supply. Assuming the reservoirs would also be used for some storage state based utilization (e.g. some form of recreation), the reliability of levels of storage that would need to be maintained to enable an adequate level of utilization were also included and are presented in Table 1. The reliability constraints were defined as the maximum proportion of time out of the total simulation period that a particular event is allowed to happen. The event could be a certain restriction level to supply or a storage state lower than a set value. These reliability levels formed the constrained the amount of yield that the system is capable of supplying and are therefore termed as reliability constraints. Note that the reliability of 1.0 for a storage state of 20% is realistic for a historic analysis (as in this study) but would not be for more comprehensive stochastic analysis applying synthetically generated sequences.

RULE CURVE DEFINITION USING TRIGONOMETRIC FUNCTIONS

The trigonometric function needed to be defined in a manner that would allow the rule curves to take a large variety of shapes and thereby enable a robust search. A function that was considered adequate for the purpose was defined as,

$$rl_{j,l,k} = tp_k + wp_k(3-l) + ap_k\left[\sin\left(2\pi\left(\frac{j}{12} - lp_k\right)\right)\right] \quad \ldots (3)$$

where $rl_{j,l,k}$ is the rule curve value for month j (1, 2, ..., 12) and level l (l = 1, 2, 3) and reservoir k (k = 1, 2), tp_k, wp_k, ap_k, and lp_k is the translation, width, amplitude and lag parameter for reservoir k.

The search range for all the four parameters except wp_k was [0–1] while that for wp_k was [0–0.5]. If the computed rule curve exceeded 1.0 or was negative, the solution was rendered infeasible by reducing the objective function to a negligible value (1×10^{-10}). Although the trigonometric function is not as flexible as a free definition of all the rule curve values, it was found reasonably versatile as Figure 2 illustrates. This

Table 1: Reliability Constraints of Water Supply and Reservoir Storage States

Storage Zone	*Level of Supply (%)*	*Reliability*		*Storage State*	*Reliability*	
		Reservoir 1	*Reservoir 2*		*Reservoir 1*	*Reservoir 2*
1	100	0.95	0.8	0.8	0.1	0.1
2	80	0.98	0.8	0.6	0.5	0.5
3	50	0.99	0.9	0.2	1.0	1.0
4	30	0.995	0.95			

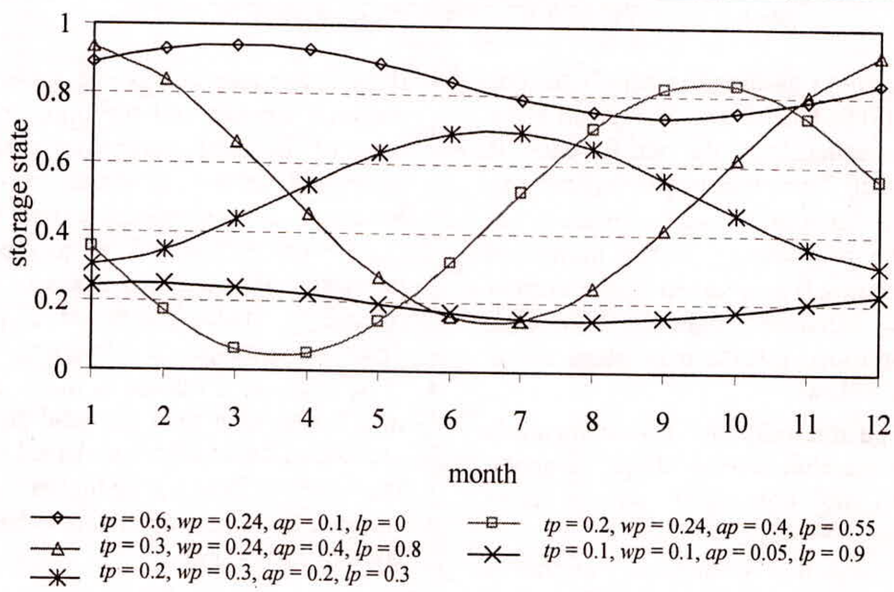


Fig. 2: The versatility of the simple trigonometric function

Figure shows the rule curves that would be obtained for various combinations of the four parameter sets for l = 2 (the second rule curve). Compared with a free specification of all the rule curve values, the trigonometric function uses 4 parameters to define the 3 rule curves of a single reservoir while a free specification would need 36. In order to test the need for the trigonometric function, a case in which the amplitude parameter (ap_k) was set to a very low value was carried out as well. This case was equivalent to a rule curves that apply only the translation (tp_k) and width parameter (wp_k) as deactivating the amplitude parameter deactivates the lag parameter (lp_k).

SYSTEM OPTIMIZATION

The optimization involved determining the rule curves that maximized the total yield (Y) from the system.The objective function was therefore defined as,

$$\text{Maximize} \quad Y = \frac{\sum_{i=1}^{N}\sum_{j=1}^{12}\sum_{k=1}^{2} as_{i,j,k}}{TR} \qquad \dots (4)$$

$$as_{i,j,k} = \frac{pr_{l,k}}{100} d_{j,k} \qquad \dots (5)$$

Where N is the total number of years of analysis, $as_{i,j,k}$ is the actual supply in month j of year i for reservoir k, $pr_{l,k}$ is the supply percentage for storage zone l for reservoir k, (Table 1) $d_{j,k}$ is the water demand for month j from reservoir k and TR is the total runoff in the simulation period. The reliability constraints (Table 1) were imposed by reducing the objective function to an extremely low value (1×10^{-10}) in case of any violation.

Each reservoir required 4 parameters to define the three rule curves and the rule curve values for total storage ($trl_{j,l}$) was defined as linear functions of those of the individual reservoirs as described in Eqn. 6 and the weight (we) was optimized.

$$trl_{j,l} = we \times rl_{j,l,1} + (1 - we) \times rl_{j,l,2} \qquad \dots (6)$$

The demands $d_{j,k}$ (j = 1, 2...., 12, k = 1, 2) were obtained as fixed percentages of the annual demands d_k and the annual demands were themselves optimized. A constant distribution was assumed for reservoir 1 as municipal demand typically show little monthly variation while the distribution for reservoir 2 was highly varied, typical of the seasonal variation of irrigation demand. In total there were therefore 11 parameters to optimize.

The optimization was carried out by linking the simulation model to a shuffled complex evolution (SCE-UA) optimizer (Duan *et al.*, 1992). The SCE-UA is based on the following 4 concepts: i) combination of deterministic and probabilistic approaches ii) systematic evolution of a complex of points spanning the parameter space in the direction of global improvement iii) competitive evolution and iv) complex shuffling. A population of solutions is generated and divided into a number of complexes. Each complex evolves independently using the downhill simplex method for a set number of evolutions. The complexes are then shuffled thereby enabling exchange of information among them. If convergence is not reached, a new set of evolutions for each complex is carried out. A more detailed explanation of the method is provided by Duan *et al.* (1992, 1994). The SCE-UA has been used extensively in catchment model calibration and has been found to be effective and efficient. The SCE-UA optimization parameters applied here were based on the recommendations of the developers of the method (Duan *et al.*, 1994). To verify the effectiveness of optimization and parameter identification, 5 randomly initialised optimization runs were made for each case.

RESULTS AND DISCUSSION

Table 2 presents the objective function and number of model simulations obtained for the two cases (with the complete trigonometric model and with the amplitude parameter deactivated). All 10 optimization runs give almost similar objective functions with the lowest value greater than 95% of the largest one indicating effective optimization by the SCE-UA. The optimization when the amplitude parameter is inactivated however takes a considerably lesser number of model simulations. Table 3 presents the optimized parameters for all the runs of the two cases while Figure 3a displays the parameter values of the complete model (with amplitude activated). Figure 3a reveals a satisfactory level of consistency of parameter values from different runs suggesting the trigonometric model is adequately parsimonious for the problem. Figure 3b illustrates the high cross correlation between the optimized annual demands d_1 and d_2 which is again is an indication of effective optimization. This Figure also demonstrates the role that multiple randomly initialised optimizations can play in identifying efficient alternative solutions to water resource utilization. Figure 4 presents the rule curves obtained from the individual runs giving the best objective functions (run3 for with amplitude activated and run 1 with amplitude deactivated). The curves indicate that the optimum solution includes a significant amplitude parameter although this activation does not provide superior operating rule curves since the optimized yields with amplitude deactivated are just as good (Table 2).

Table 2: Optimized Yields and Number of Model Simulations

run	*Amplitude Activated*		*Amplitude Deactivated*	
	n.sim	*Yield (Y)*	*n.sim*	*Yield (Y)*
1	21439	0.2728	12992	0.2819
2	17299	0.279	13873	0.2699
3	19761	0.2833	14027	0.2785
4	14681	0.2778	12784	0.2808
5	17610	0.2789	12884	0.2773

n.sim—number of simulations

Table 3: Optimised Parameter Values

Parameter	*Units*	*Amplitude Activated*				
		run 1	*run 2*	*run 3*	*run 4*	*run 5*
tp_1	–	0.0168	0.0187	0.0221	0.0087	0.0099
wp_1	–	0.0757	0.0545	0.0401	0.0416	0.0451
ap_1	–	0.0154	0.0099	0.0215	0.0029	0.0046
lp_1	–	0.0589	0.0329	0.2371	0.5529	0.6064
tp_2	–	0.1804	0.3578	0.4756	0.3629	0.2998
wp_2	–	0.0961	0.0777	0.0659	0.0937	0.1237
ap_2	–	0.0647	0.0679	0.015	0.0666	0.0376
lp_2	–	0.6035	0.4973	0.444	0.2629	0.1028
d_1	Mm^3/year	6.6649	5.3761	4.2582	4.9773	5.2671
d_2	Mm^3/year	2.4315	3.9392	5.2101	4.282	4.0257
we	–	0.9738	0.9417	0.939	0.9457	0.9563
		Amplitude Deactivated				
tp_1		0.0046	0.0058	0.0059	0.0025	0.0062
wp_1		0.0273	0.0764	0.0645	0.0285	0.0567
tp_2		0.5151	0.0701	0.2447	0.4555	0.2568
wp_2		0.0244	0.1054	0.1057	0.1029	0.136
d_1	Mm^3/year	3.9654	6.8197	6.016	3.9607	5.6081
d_2	Mm^3/year	5.4321	2.1757	3.2873	5.4001	3.6454
we		0.9409	0.7434	0.9116	0.9396	0.9258

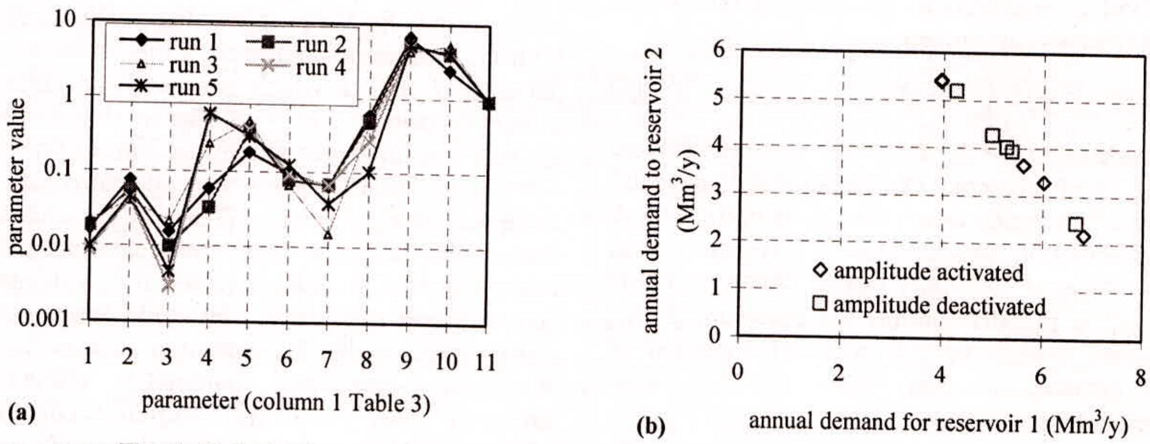


Fig. 3: Optimized parameter values and correlation of optimized annual demands

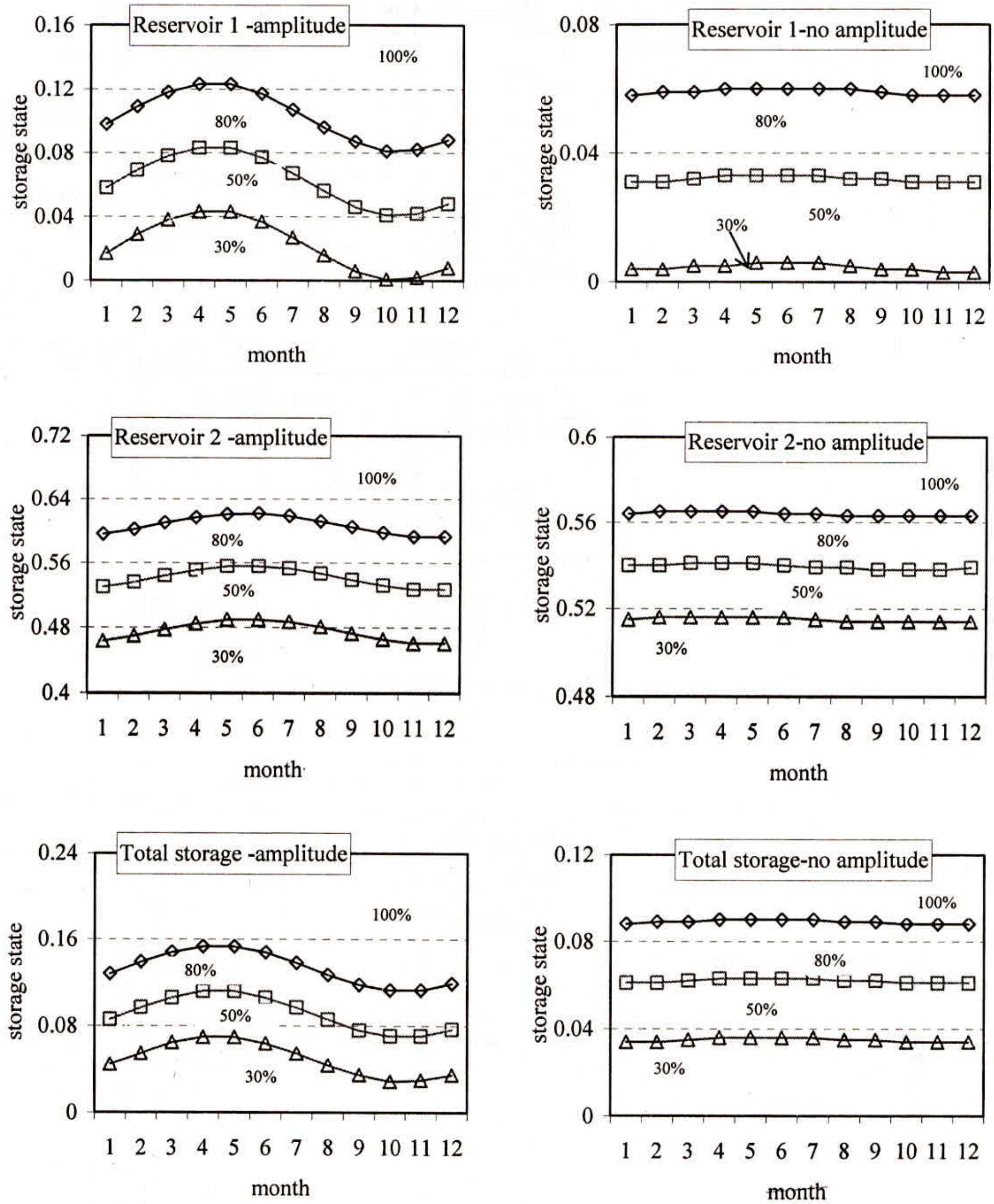


Fig. 4: Optimized rule curves with amplitude activated and deactivated

The results therefore suggest there is no need for a trigonometric function. To investigate possible further application of the trigonometric function, an analysis with very restrictive reservoir storage constraints that vary on a monthly basis (Figure 5) replacing those defined in Table 1 was carried out. The results of this analysis presented in Table 4 however indicated that activating amplitude actually obtained poorer results than the simpler rule curve model that includes a translation and width parameter only. The computation time for 5 randomly initialized runs for all cases was less than 10 minutes—much less than the computation times in a previous analysis of the system (Ndiritu, 2005).

It was observed for all the optimization was the redundancy of most of the reliability constraints. For the first scenario with the simple and less restrictive storage state constraints, the only active reliability constraint was the requirement to supply 80% of the demand at a reliability of at least 98%. For the second scenario with very restrictive monthly varying storage state constraints, the active constraints were the 3 storage state constraints for months 1, 2 and 3 for reservoir 2. Figure 6 compares the reliability

Table 4: Optimization Results with Monthly Varying Storage State Restrictions

Run	*Amplitude Activated*		*Amplitude Deactivated*	
	n.sim	*Yield (Y)*	*n.sim*	*Yield (Y)*
1	9890	0.0347	7788	0.0377
2	7941	0.0366	7224	0.0376
3	11467	0.0376	7918	0.0377
4	3910	0.0333	8456	0.0377
5	2990	0.0315	7243	0.0377

n.sim—number of simulations

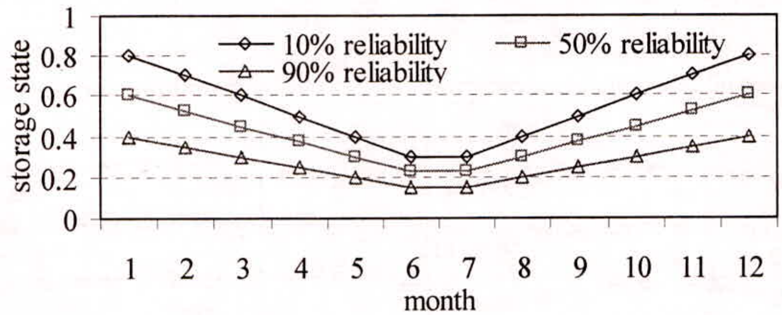


Fig. 5: Monthly varying reservoir storage state constraints

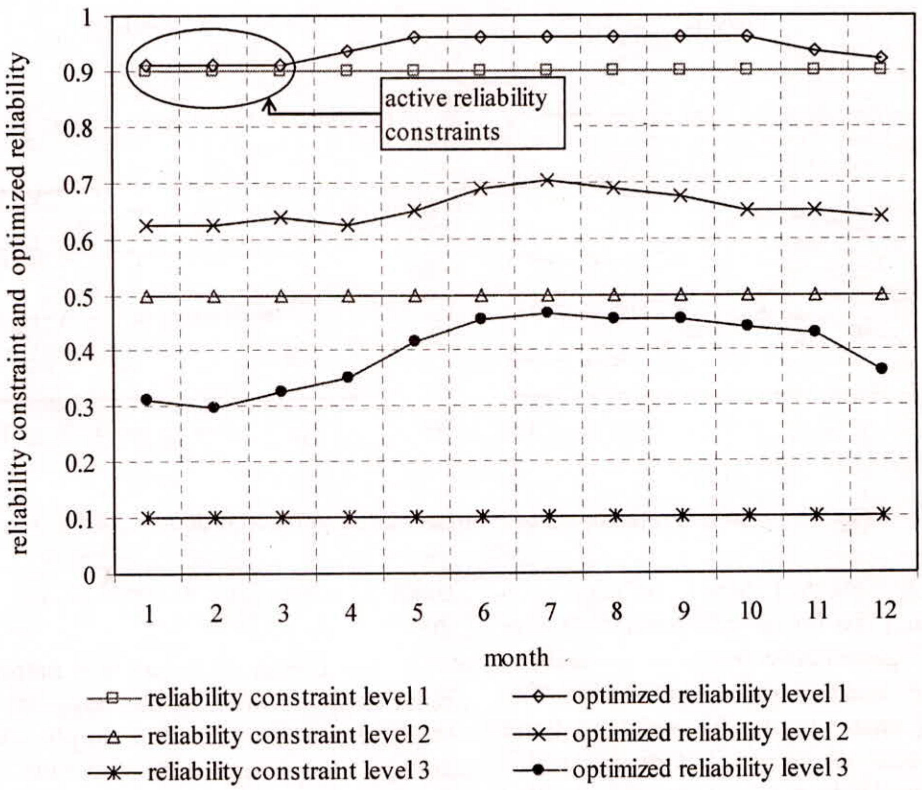


Fig. 6: Illustration of active and redudant reliability constraints

constraints with the optimized reliabilities for reservoir 2 highlighting the 3 active constraints. This type of analysis could provide useful information for practical decision making as it could provide ideas about how the system redundancy may be reduced to achieve a higher level of resource utilization.

CONCLUSIONS AND RECOMMENDATIONS

A simple trigonometric function for parsimonious modelling of reservoir operating rule curves was formulated and applied in the maximization of the total system yield obtainable from a system of two reservoirs using a simulation-optimization approach. Yield maximization was subjected to multiple reliability constraints of demand and reservoir storage state and optimization was carried out using the shuffled complex evolution (SCE-UA) method. The analysis found that for the specific problem, there is no need for the use of trigonometric functions as simpler rule curves that do not vary on a monthly basis obtained similar yields using considerably lower numbers of model simulations.

It is recommended that additional analysis using other reservoir operating problems be carried out before dismissing the applicability of the simple trigonometric function. While the analysis here suggests the trigonometric function is more complex than necessary, for more complex systems it could turn out that the trigonometric function (or its variant) could be applied for the initial stages of optimization to help reduce computation time. It may also be useful to compare the rule curves applied in this analysis with the general form of curve that defines each rule curve value independently.

To verify the effectiveness of optimization and parameter identification, 5 randomly initialised optimization runs were carried out for all scenarios. The objective function values from different runs found close indicating the optimizations were all effective. For optimization methods that do not ensure the location of the global optimum such as the GA, the SCE-UA and other evolutionary techniques, it is suggested that multiple randomly initialised runs need to be included as a valuable means of verifying the adequacy of optimization. The effective and efficient performance of the SCE-UA optimization in this analysis encourages its wider application in reservoir system optimization.

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