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Genetic Algorithm Based Unit Hydrograph

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ABSTRACT: Unit hydrograph approach is one of the commonly used techniques employed for the determination of flood hydrographs. For this purpose various distribution functions have been utilized in the past. The suitability of the distribution functions are depend on the accurate estimation of their parameter. Different parameter estimation techniques are common in practice viz., method of moments, least squares, maximum likelihood, etc. However, these methods have their own limitations that induce the errors in estimation of flood hydrograph. Contrary to this, the Genetic Algorithm (GA) is used as an optimization tool which minimizes the global error in the estimation of the model parameters. In this study, the two-parameter Gamma and two-parameter Beta distribution functions were utilized to derive the unit hydrograph and the GA was used for the optimization of the parameters. The suitability of two-parameter Beta distribution function was also explored in the derivation of unit hydrograph. The methodology was found to work well in the reproduction of unit hydrograph as well as the direct runoff hydrographs.

INTRODUCTION

Generally, the watershed's rainfall-runoff process is nonlinear and complex in nature and involves number of variables in the modeling. The input-output mathematical models based on the linear theory of hydrologic system attempt to establish a causal linkage between the input and output without detailed description of physical process under investigation and are widely accepted theory in hydrologic modeling. The Unit Hydrograph (UH) concept is one such theory and perhaps the most popular modeling tool for the computation of flood hydrograph. The unit hydrograph or unit pulse response function is an important tool for system identification in hydrologic analysis and can be used as mathematical description of a linear system (Sherman, 1932; Dooge, 1959, 1973; Singh, 1988; etc.). Since, the shape of Probability Density Function (PDF) is similar to the conventional Unit Hydrograph (UH), therefore, many attempts have been made in the past to use the PDFs of Gamma and three parameter Beta distribution function for the derivation of UH (Gray, 1961; Croley, 1980; Haktanir and Sezen, 1990; Singh, 2000; Yue *et al.*, 2002; Bhunya *et al.*, 2003; etc.). However, the application of two-parameter Beta distribution function still remains unexplored for the derivation of a UH. Such application of PDFs for the derivation of UH requires the fitting of their parameters. For this purpose, several approaches are available for example, least square, method of moment, maximum likelihood, etc. (Singh, 1988). However, these methods of fitting the parameters gives errors in matching the resulting direct runoff hydrographs, particularly in the peak flood magnitude. Contrary to this, the Genetic Algorithm (GA) enhanced the optimization which minimizes the global error in the estimation of the model parameters (Goldberg, 1989). In the recent years, many applications of GA have been reported in the hydrologic modeling (Wardlaw and Sharif, 1999; Kuo *et al.*, 2000; Jain *et al.*, 2005; Rabuñal *et al.*, 2007; etc.). Looking into these facts, following objectives have been set for the present study: (i) parameter estimation of two-parameter Gamma and two-parameter Beta distribution functions using GA, and (ii) derivation of UH and resulting Direct Runoff Hydrograph (DRH).

DISTRIBUTION FUNCTIONS

In the present study, PDFs of two-parameter Gamma and two-parameter Beta distribution functions have been used for the derivation of UH.

Two-parameter Gamma Distribution

Nash (1959) and Dooge (1959) proposed the mathematical derivation of instantaneous unit hydrograph (i.e. unit impulse response function) in the form of

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Gamma function utilizing the concept of n-linear reservoirs of equal storage coefficient K as follows,

$$h(t)=\frac{1}{K\,\Gamma(n)}\left(\frac{t}{K}\right)^{n-1}\exp(-t/K) \quad \ldots (1)$$

In equation (1), n and K define the shape of the IUH, $h(t)$ is the flow rate $[T^{-1}]$, and $\Gamma(n)$ is the Gamma function of n. The dimension of K is similar to the time $[T]$. For the deriving UH of desired duration, step response function is derived which is analogous to the S-hydrograph in surface hydrology. The step response is computed from the following equation,

$$g(t)=\int_0^t h(t)dt=\int_0^t \frac{1}{K\,\Gamma(n)}\left(\frac{t}{K}\right)^{n-1}\exp(-t/K)dt$$
$$=\frac{1}{\Gamma(n)}\int_0^{t/K}\left(\frac{t}{K}\right)^{n-1}\exp\left(-\frac{t}{K}\right)d\left(\frac{t}{K}\right) \quad \ldots (2)$$

where, $g(t)$ is the step response function used to compute the UH of desired duration. The Δt Hour-UH is computed from the following relationship,

$$u(\Delta t,t)=\frac{10.0\,A}{3.6\,\Delta t}\times\{g(t)-g(t-1)\} \quad \ldots (3)$$

where, $u(\Delta t,t)$ is the UH of Δt duration, A is the drainage area (km^2). Following discrete form of convolution expression is used for the computation of Direct Runoff Hydrograph (DRH),

$$Q(t)=\sum_{j=1}^{t}u(\Delta t,t-j+1)\cdot r_e(j) \quad \ldots (4)$$

In above relationship (Eqn. 4), r_e is the excess rainfall depth (cm) and $Q(t)$ is the ordinates of DRH at time step of t.

Two-parameter Beta Distribution

The *pdf* of two-parameter Beta distribution function is given as follows,

$$f(t)=\frac{1}{B(a,b)}t^{a-1}\cdot(1-t)^{b-1},\quad (0\le t\le 1) \quad \ldots (5)$$

where, $B(\cdot)$ is the Beta function, and a and b are the parameters which define the shape of the UH. The value of t must lie in the interval of [0, 1]. The feasibility of Eqn. (5) in the derivation of UH is given in Appendix. The Beta function is defined as follows,

$$B(a,b)=\int_0^1 t^{a-1}(1-t)^{b-1}dt \quad \ldots (6)$$

Similar to Eqn. (2), the Beta cumulative distribution function is given by the following relationship,

$$F(t\,|\,a,b)=\frac{1}{B(a,b)}\int_0^t t^{a-1}(1-t)^{b-1}dt \quad \ldots (7)$$

Equation (7) gives the step response function base on the two parameters. The UH and DRH are computed using the Eqns. (3) and (4). Since applicability of Eqn. (7) is within the range of $0\le t\le 1$, therefore, the computation is performed by normalizing t by dividing it with base time $(=t_B/t_r)$. In which $t_r=$ is duration of excess rainfall.

OPTIMIZATION USING GENETIC ALGORITHM (GA)

The parameters of the used distribution function were carried out using the Genetic Algorithm (GA) which needs to define an objective function. The objective function used for the optimization of parameters is given as follows,

$$\min f(\cdot)=\sum_{i=1}^{t_B}\{w(i)\times|\hat{Q}(i)-Q(i)|\} \quad \ldots (8)$$

where, $f(\cdot)$ is the function of the parameters, $\hat{Q}(i)$ and $Q(i)$ are the computed and observed direct runoff, respectively, and $w(i)$ is defined as follows,

$$w(i)=Q(i)/\sum_{i=1}^{t_B}Q(i) \quad \ldots (9)$$

To perform the optimization using GA, the program was developed on MATLAB platform.

RESULTS AND DISCUSSION

To test the applicability of the proposed methodology following cases are considered.

Case-I: Applicability of the methodology on the data having UH derived from other methods (Haktanir and Sazen, 1990).

Case-II: Applicability in the computation of DRH for the published data of Gaur and Mathur (2003).

In totality, six storm events have been considered to test the methodology. Out of six events, two events were taken from the data of Anatolia river basin reported in Haktanir and Sazen (1990) (Case-I) and four events were picked up from the four Indian watersheds reported by Gaur and Mathur (2003) (Case-II). The brief description of these watersheds is given in Table 1.

Case I

To start with, the methodology was first applied on to the storm event data of two sub-basins of Anatolia river basin namely Hamam and Inderesi watersheds. The performance is evaluated by using the statistical criteria viz. Coefficient of efficiency, CE (Nash and Sutcliff, 1970) and mean absolute percent error, MAPE. The CE is expressed as follows,

$$CE = 1 - \left\{ \sum_{t=1}^{n} \left[\hat{Q}(t) - Q(t) \right]^2 \Big/ \sum_{t=1}^{n} \left[Q(t) - \bar{Q} \right]^2 \right\} \quad \dots (10)$$

where $\hat{Q}(t)$ and Q(t) are the computed and observed UH ordinates, respectively; $\bar{Q}$ is the mean UH; and n is the total number of ordinates.

The Mean Absolute Percent Error (MAPE) is defined as follows,

$$MAPE = \frac{1}{n}\left(100 \times \sum_{i=1}^{n} ABS(\hat{Q}_i - Q_i) \right) \quad \dots (11)$$

Using GA, the parameters of the Gamma distribution function (Eqns. 1 through 3, and Eqns. 8 and 9) have been optimized and given in Table 2. The derived UH for the selected events are shown in Figure 1. The results obtained using the GA has also been compared with the results reported by Haktanir and Sazen (1990). Besides the visual comparison of the results, performance evaluation of the approaches has been worked out using the statistical measures. The estimated values of statistical measures viz. CE and MAPE are given in Table 2. The estimated values of CE and MAPE obtained from the results reported by Haktanir and Sazen (1990) is given in the parenthesis of Table 2.

Table 1: Description of Watersheds and Event Information

S. No.	Watershed Description	(A) Haktanir and Sazen (1990)		(B) Gaur and Mathur (2003)			
		Hamam, Goksu River	Inderesi, Inderesi Creek	W-8	Jhandoo Nala	B-319	B-719
(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)
1.	Darainage Area (km^2)	4300.0	141.5	0.012	0.177	0.82	14.0
2.	Slope (%)	0.83	1.18	1.90	48.8	9.2	6.8
3.	Length of Main Channel (m)	16.1×10^4	23.5×10^3	177.6	900.0	1650.0	7200.0
4.	Event Date	–	–	23/08/1995	08/08/1991	05/08/1964	11/08/1965
5.	Excess Rainfall Duration (min)	480.0	120.0	20.0	10.0	50.0	60.0
6.	Time to Peak (min)	1440.0	360.0	15.0	20.0	50.0	150.0
7.	Peak Flow Rate (m^3/s)	417.0	54.0	15.0	6.83	16.03	10.03
8.	Base Time (hr)	100.0	28.0	1.42	1.167	2.67	6.5

Table 2: Parameters of the Gamma and Two-parameter Beta Distribution Functions along with the Estimated Values of CE and MAPE

Case	Watershed	Two-parameter Gamma				Two-parameter Beta			
		n	K	CE	MAPE	a	b	CE	MAPE
(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)
Case-I	Hamam	3.8119 (5.6)	6.3882 (5.233)	0.9227 (0.8817)	1772.8 (3108.6)	3.2478	10.794	0.9313	2107.5
	Inderesi	4.0991 (5.4)	1.6015 (1.358)	0.9888 (0.9782)	132.16 (199.53)	3.2722	11.403	0.981	167.23
Case-II	W-8	1.01	0.2602	0.9450	0.2265	0.7883	3.8089	0.9346	0.2302
	Jhandoo	3.4042	0.0975	0.9582	1.512	2.3618	6.5872	0.9361	1.925
	B-319	3.9815	0.1529	0.891	25.66	2.7915	9.5666	0.893	25.96
	B-719	5.4986	0.4750	0.981	138.98	3.0148	4.7285	0.979	101.54

Visual comparison (Figure 1) as well as the statistical criteria (Table 2) shows that the results obtained from the GA gives better agreement with observed UH as compared to the Haktanir and Sazen (1990) especially in the estimation of peak flow rates.

In a similar lines, the UH were computed using the two-parameter Beta distribution function (i.e. Eqns. 5 through 9, and 3). The results are presented in Figure 2. Also, same statistical criteria were used to evaluate the model performance. The estimated values of CE and MAPE (Table 2) give the evidence to the applicability of the two-parameter Beta distribution function. Figure 2 shows that the two-parameter Beta distribution function is very much capable in reproducing the peak flow rate as well as the shape of the UH.

Case-II

In case-II, the applicability of two-parameter Gamma as well as the two-parameter Beta distribution functions fitted with GA is tested in the determination of DRH for the watersheds considered under case-II. The DRH was computed by convoluting the excess rainfall onto the derived UH (i.e. Eqn. 4). The results computed through the Gamma as well as the Beta distribution function are depicted in Figure 3, and. statistical criteria viz:, CE and MAPE is estimated for the performance evaluation. The estimated values of

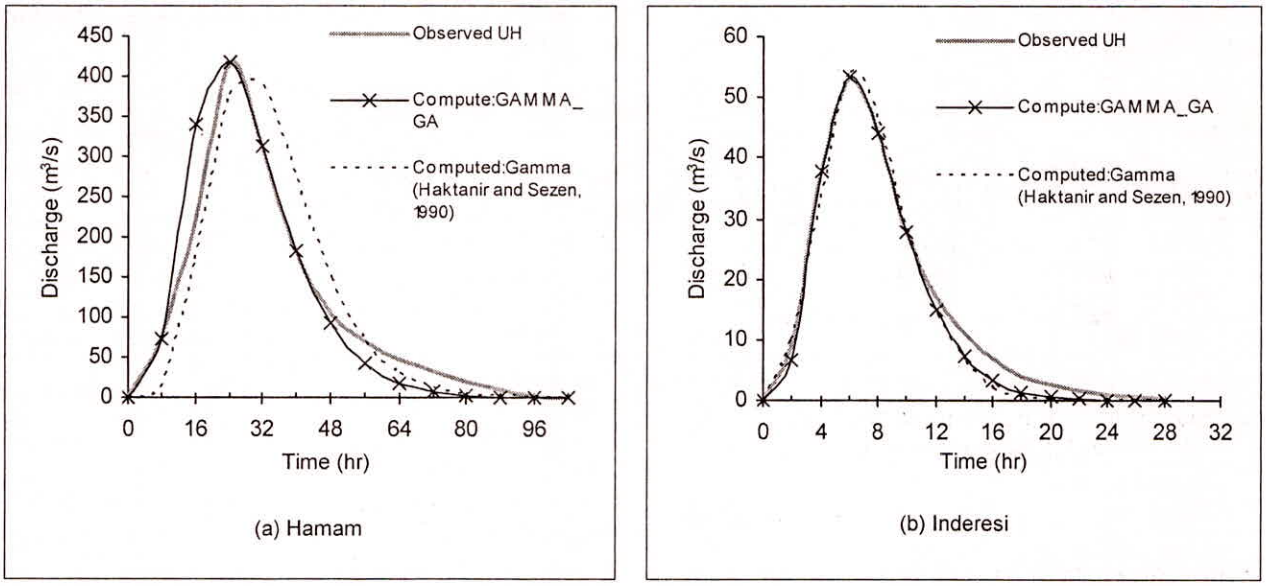

Fig. 1: Comparison of observed and computed UH from Gamma function for two watersheds (Case-I)

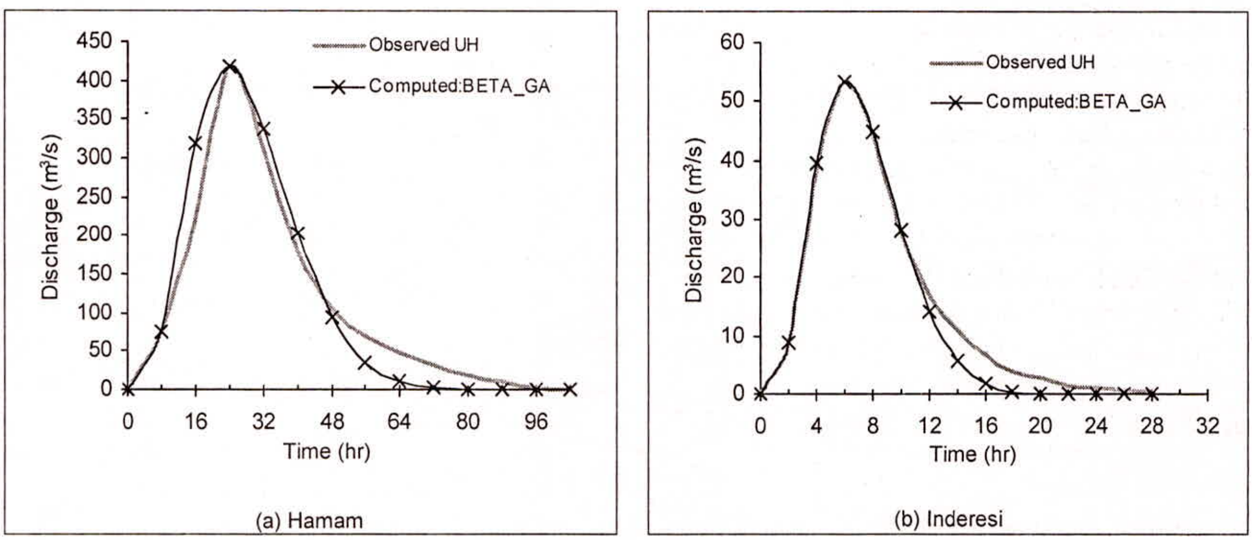

Fig. 2: Comparison of observed and computed UH from two-parameter Beta distribution function for two watersheds (Case-I)

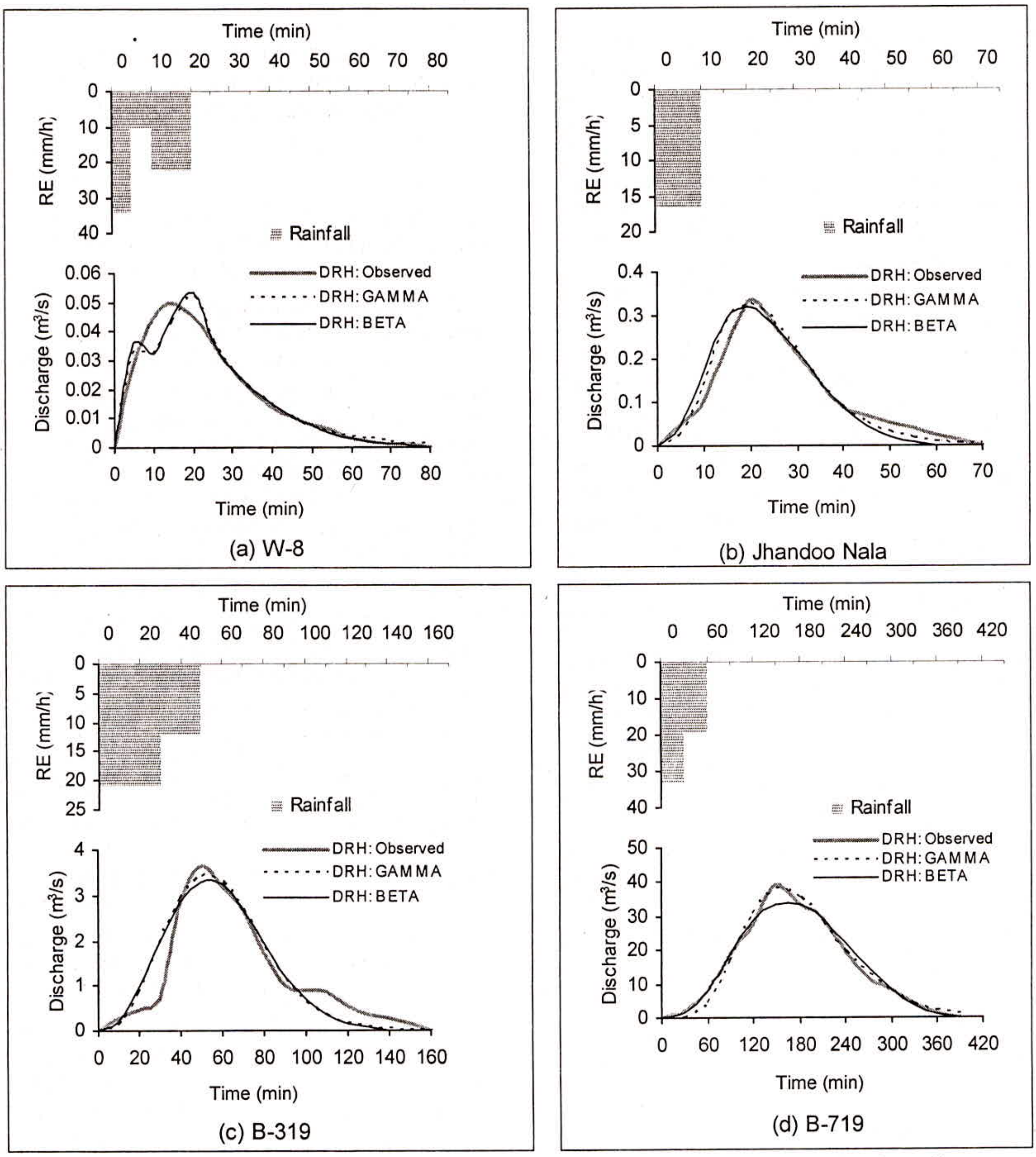

Fig. 3: Comparison of observed and computed DRHs for the watersheds under Case-II

these criteria are mentioned in Table 2 (i.e. Case-II). It is evident from the visual comparison (Figure 3) as well as from the statistical measures (Table 2), both the distribution functions give better agreement to the observed DRH.

CONCLUSIONS

The following conclusion can be drawn from this study:

1. The parameter estimation using the GA gives better agreement in fitting the UH than other commonly used parameter estimation methods.
2. The fitted two-parameter Beta distribution function gives better agreement with the observed UH, thus shows its potential in deriving the UH.
3. DRH computed by employing the two-parameter Beta distribution function are close to the observed DRH, which again prove its capability in deriving the UH and therefore, can be a useful substitute to the commonly used Gamma function model.
4. The visual comparisons as well as statistical measures clarify the overall advantage in the estimation of parameters using GA.

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