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Flow Simulation through Multiple Leaky Aquifers

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ABSTRACT: A mathematical model for well flow in multiple leaky aquifer system with bottom aquifer partially penetrated was formulated with initial and boundary conditions. Numerical solution to the flow governing equation was then obtained by using central difference formula and Successive Over Relaxation (SOR) technique. Computer programmes were written in FORTRAN to predict the drawdown distribution caused by a well penetrating one or more aquifers. For validation of the developed mathematical model, a semi-circular sand tank model was constructed and pumping tests were conducted to generate the required discharge drawdown data at various depths of penetrations. The data obtained by conducting the experiments were converted for the field conditions using dimensional analysis. Comparison between the observed and the predicted drawdown values has been presented to indicate the errors (deviations) with respect to distance, discharge and penetration ratios for the purpose of validation. The errors in prediction of drawdown for the bottom, middle and top aquifers range from 1.11 to 16.11% with a mean average of 5.25% only.

Keywords: Drawdown, Partial Penetration, Multiple Leaky Aquifer.

INTRODUCTION

Most of the groundwater reservoirs are multi-aquifer systems separated by semi-confining layers, and these aquifers respond conjunctively to stresses imposed on any one of them. The fact that aquifers are part of a more complex geo-hydrologic system has long been recognized. DeGlee was apparently the first person to formulate a steady state leakage of water through less permeable layers into a aquifer (Gupta *et al.,* 1984).

The multiple leaky aquifer theory was studied by Papadopulos (1966), Neuman and Witherspoon (1969), Saleem (1973), Javandel and Witherspoon (1980) and later significant contributions were made by Hunt (1985), Cheng and Kwotsong (1989) and Cheng and Morohunfola (1993) using Eigenvalue and Laplace transform approach.

The numerical model described here is a finite difference two-dimensional groundwater flow (FD2DGW) model for analysing the flow through multiple leaky aquifer systems with varying number of layers, thicknesses and constants.

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DEVELOPMENT OF NUMERICAL MODEL

For the development of numerical model, the origin of the co-ordinate system is considered at the centre of the well in the top of the aquifer, with z-axis positive downward and r and z as the radial and vertical distances to any point in the aquifer. The grid size in the horizontal direction is Är and in the vertical direction is Äz,

$$\frac{\partial^2 s_i}{\partial r^2}+\frac{1}{r}\frac{\partial s_i}{\partial r}+\frac{\partial^2 s_i}{\partial z^2}-\frac{s_i}{B^2_i}=0 \qquad \ldots (1)$$

$$s_i(\infty, z)=0 \qquad \ldots (2)$$

$$3\frac{\partial s}{\partial z}(r,0)=\frac{\partial s}{\partial z}(r,b_n)=0 \qquad (3)$$

$$s_o(r,z)=s_{n+1}(r,z)=0 \;\ldots (4)$$

$$\frac{\lim}{r\to 0} l.r\frac{\partial s_i}{\partial r}=-\frac{Q}{2\pi K}0\le z\le l \qquad \ldots (5)$$

$$=0\; l\le z\le b_n \qquad \ldots (6)$$

Where s_i = drawdown in the ith aquifer,

B_i = leakage factor of the ith aquitard,

l = depth of penetration,

b_n = thickness of the ith aquifer, and

Q = rate of discharge.

NUMERICAL SOLUTION

Using the central difference formula the governing equation and the other equations are discretized. After discretization the governing equation can be written as given in equation (7) and (8).

After discretization,

$$\frac{1}{\Delta r^2}[s_{i+1,j}-2s_{i,j}+s_{i+1,j}]+\frac{1}{\Delta r\, i}\left[\frac{s_{i+1,j}-s_{i-1,j}}{2\Delta r}\right]+\frac{1}{\Delta z^2}[s_{i,j+1}-2s_{i,j}+s_{i,j+1}]-\frac{s_{i,j}}{B_i^2}=0 \quad \text{...(7)}$$

$$s_{i+1,j}\left(\frac{1}{\Delta r^2}+\frac{1}{2i\Delta r^2}\right)+s_{i-1,j}\left(\frac{1}{\Delta r^2}-\frac{1}{2i\Delta r^2}\right)$$
$$s_{i,j}\left(-\frac{2}{\Delta r^2}-\frac{2}{\Delta z^2}-\frac{1}{B_i^2}\right)+s_{i,j+1}\left(\frac{1}{\Delta z^2}\right)+s_{i,j-1}\left(\frac{1}{\Delta z^2}\right)=0 \quad \text{...(8)}$$

when $r=0,\ z\neq b$

$$\frac{s_{i+1,j}-s_{i-1,j}}{2\Delta r}=-\frac{Q}{2\pi K l i\Delta r}$$

$$\frac{1}{\Delta r^2}\left[s_{i+1,j}-2s_{i,j}+s_{i+1,j}+\frac{Q}{\pi K l i}\right]+\frac{1}{2i\Delta r^2}\left[-\frac{Q}{\pi K l i}\right]+\frac{1}{\Delta z^2}[s_{i,j+1}-2s_{i,j}+s_{i,j-1}]-\frac{s_{i,j}}{B_i^2}=0 \quad (9)$$

$$s_{i,j+1}\left(\frac{1}{\Delta z^2}\right)+s_{i,j-1}\left(\frac{1}{\Delta z^2}\right)+s_{i,j}\left(-\frac{2}{\Delta r^2}-\frac{2}{\Delta z^2}-\frac{1}{B_i^2}\right)+s_{i+1,j}\left(\frac{2}{\Delta r^2}\right)-\frac{Q}{2\pi K l i^2\Delta r^2}+\frac{Q}{\pi K l i\Delta r^2}=0 \quad \text{... (10)}$$

When $r\neq 0,\ z=b$

$$\frac{\partial s}{\partial z}=0$$

$$\frac{s_{i,j+1}-s_{i,j-1}}{2\Delta z}=0$$

$$s_{i,j+1}=s_{i,j-1}$$

$$\frac{1}{\Delta r^2}[s_{i+1,j}-2s_{i,j}+s_{i-1,j}]+\frac{1}{2i\Delta r^2}[s_{i+1,j}-s_{i-1,j}]+\frac{1}{\Delta z^2}[s_{i,j-1}-2s_{i,j}+s_{i,j-1}]-\frac{s_{i,j}}{B_i^2}=0 \quad \text{... (11)}$$

$$s_{i,j-1}\left(\frac{2}{\Delta z^2}\right)+s_{i,j}\left(-\frac{2}{\Delta r^2}-\frac{2}{\Delta z^2}-\frac{1}{B_i^2}\right)+s_{i+1,j}\left(\frac{1}{\Delta r^2}+\frac{1}{2i\Delta r^2}\right)+s_{i-1,j}\left(\frac{1}{\Delta r^2}-\frac{1}{2i\Delta r^2}\right)=0 \quad \text{... (12)}$$

When $r=0,\ z=b$

$$s_{i-1,j}=s_{i+1,j}+\frac{Q}{\pi K l i}$$

$$s_{i,j-1}=s_{i,j+1}$$

$$\frac{1}{\Delta r^2}\left[s_{i+1,j}-2s_{i,j}+s_{i+1,j}+\frac{Q}{\pi K l i}\right]+\frac{1}{2i\Delta r^2}\left[-\frac{Q}{\pi K l i}\right]+\frac{1}{\Delta z^2}[s_{i,j-1}-2s_{i,j}+s_{i,j-1}]-\frac{s_{i,j}}{B_i^2}=0 \quad \text{... (13)}$$

$$s_{i,j-1}\left(\frac{2}{\Delta z^2}\right)+s_{i,j}\left(-\frac{2}{\Delta r^2}-\frac{2}{\Delta z^2}-\frac{1}{B_i^2}\right)+s_{i+1,j}\left(\frac{2}{\Delta r^2}\right)-\frac{Q}{2\pi K l i^2\Delta r^2}+\frac{Q}{\pi K l i\Delta r^2}=0 \quad \text{...(14)}$$

When $r=0,\ z=0$

$$\frac{1}{\Delta r^2}\left[s_{i+1,j}-2s_{i,j}+s_{i+1,j}+\frac{Q}{\pi K l i}\right]+\frac{1}{2i\Delta r^2}\left[-\frac{Q}{\pi K l i}\right]+\frac{1}{\Delta z^2}[s_{i,j+1}-2s_{i,j}+s_{i,j+1}]-\frac{s_{i,j}}{B_i^2}=0 \quad \text{... (15)}$$

$$s_{i,j+1}\left(\frac{2}{\Delta z^2}\right)+s_{i,j}\left(-\frac{2}{\Delta r^2}-\frac{2}{\Delta z^2}-\frac{1}{B_i^2}\right)+s_{i+1,j}\left(\frac{2}{\Delta r^2}\right)-\frac{Q}{2\pi K l i^2\Delta r^2}+\frac{Q}{\pi K l i\Delta r^2}=0 \quad \text{... (16)}$$

MATERIALS AND METHODS

Due to spatial variation in the depth of aquifer system, its non homogeneous behaviour and the huge cost involved in the drilling of a large number of observation wells, a sand tank model was fabricated and pumping tests were conducted to study the flow phenomena of partially penetrating wells in multiple leaky aquifer system. The sand tank model setup

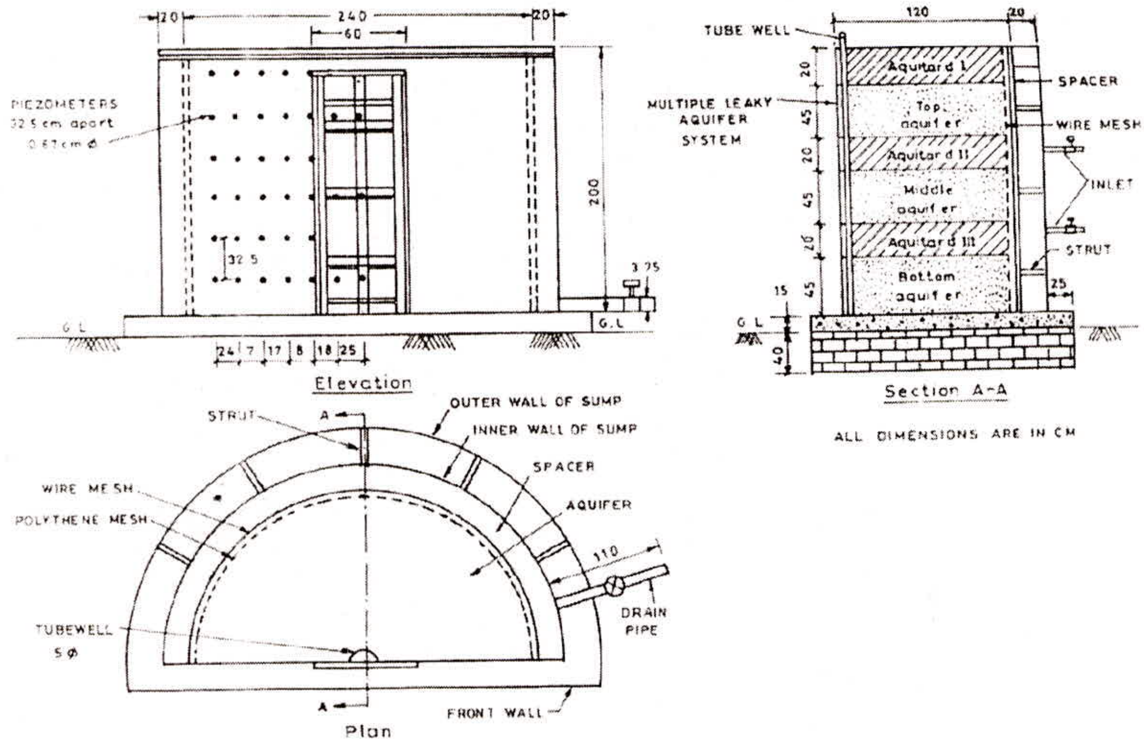

Fig. 1: Sand tank model

consists of a storage tank, constant head tank, sand tank model unit, drainage pit and a piezometer board assembly. The details of the sand tank model are shown in Figure 1 and schematic diagram of the water circulation system is shown in Figure 2. Pumping tests were conducted using three discharge rates of 1.0, 1.5 and 2.0 litre per minute and were replicated thrice. During testing drawdown readings were recorded at different radial distances. After completing testing for full penetration, the PVC well screen was pulled up by a height equal to 20% thickness of the bottom aquifer, thus achieving 80% penetration of the aquifer. Similarly 60, 40 and 20% penetrations were achieved. Testing was conducted at 100, 80, 60, 40 and 20 percent penetrations for all the three aquifers. The data obtained by conducting the pumping tests in the sand tank model were converted for the field condition, by dimensional analysis. Using Deglee's equation and observing the fact that the sand tank model could simulate only the part of the drawdown due to limitation of the radial distance, the prototype and model drawdown data at any radial distance was obtained by the following expression.

$$Sp = Sm\,.Lr/0.512$$

Where Sp = drawdown in the prototype
Sm = drawdown in the model
Lr = linear scale ratio

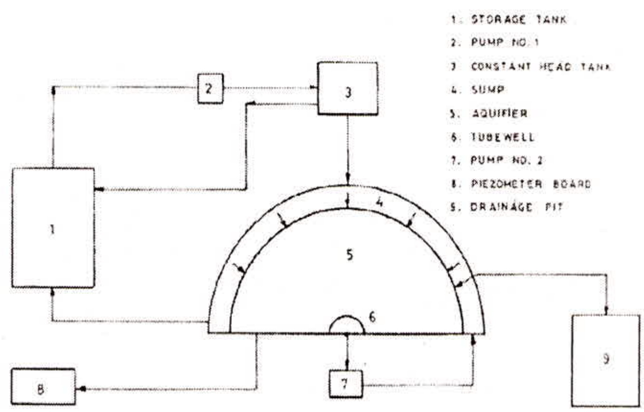

Fig. 2: Schematic diagram of the water circulation system in the sand tank model

RESULTS AND DISCUSSION

To validate the mathematical model when the top and middle aquifers are fully penetrating and the bottom aquifer is partially penetrating, the distance drawdown and the discharge drawdown relationships are considered. The relationships between the distance from the centre of the well and the observed and the predicted values of drawdown with the bottom aquifer partially penetrated and top and middle aquifers fully penetrated are presented in Figure 3(a), (b) and (c) for 0.8, 1.2 and 1.6 m^3/min discharges respectively. It can be seen from the figures that both the predicted and observed drawdown increase with the increase in discharge. However for a constant discharge at a given radial distance observed and predicted drawdown increase with the reduction in the penetration ratio. In Figure 3, relationships for 20, 60 and 100 percent penetration ratios are only presented. Relationships developed for other penetration ratios are found to have similar trends. It is also observed from the figure that for a given discharge rate and penetration ratio, both the predicted and observed drawdown decrease with the increase in distance from the centre of well.

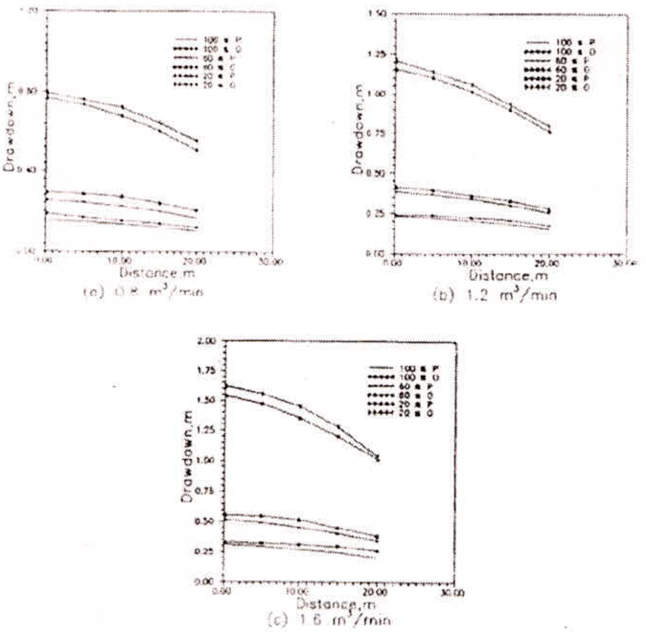

Fig. 3: Steady state predicted and observed drawdown *vs.* distance relationship for 20, 60 and 100% penetration of bottom aquifer of different discharges

Similar trends were observed for the middle and top aquifer except that the absolute values of corresponding drawdown were higher. This is due to the fact that thickness of the aquifer tapped becomes less when the middle and top aquifers are considered.

The discharge drawdown relationships for 20, 60 and 100% penetrations of bottom aquifer at 0.2, 10.0 and 20.0 m distances from the centre of the well are presented in Figure 4(a), (b) and (c) respectively. It can be concluded from Figure 4 that for a given radial distance and penetration ratio, both the observed and predicted drawdown values increase with the increase in discharge. However, for a given discharge rate as the penetration ratio decreases from 100 to 20% both the observed and predicted drawdowns increase. Discharge drawdown relationships at different radial distances and penetration ratios for the middle aquifer with the bottom aquifer non-penetrated were similar in nature except that the corresponding drawdown values were higher. Similar relationships were observed for different percent penetration ratios of the top aquifer also. In all cases the predicted drawdown curves are found to closely follow the observed drawdown curves. The differences are limited to within 10 to 15%.

Percentage errors for different discharges and penetration ratios of the bottom, middle and top aquifers are presented in Table 1. It is observed from the table that for a discharge of 0.8 m^3/min, the percentage errors in prediction of drawdown are 15.06, 8.03, 16.11, 10.57 and 4.90 for penetration ratios of 100, 80, 60, 40 and 20% respectively of the bottom aquifer.

Table 1: Errors in Prediction of Drawdown for Different Penetration Ratios of the Bottom, Middle and Top Aquifers

Aquifer	*Percent Penetration*	*Percentage Errors at Different Discharges, m^3/min*			*Average Error (%)*
		0.8	*1.2*	*1.6*	
Bottom	100	15.06	9.14	14.06	12.75
	80	8.03	13.81	13.48	11.77
	60	16.11	6.70	10.30	11.03
	40	10.57	7.25	2.71	6.84
	20	4.90	4.17	5.06	4.71
Middle	100	5.44	4.33	4.99	4.92
	80	5.94	2.63	3.18	3.91
	60	3.33	2.15	2.08	2.52
	40	2.31	2.69	2.70	2.56
	20	2.03	2.68	1.11	1.94
Top	100	8.84	2.43	2.53	4.6
	80	3.50	2.86	2.53	2.96
	60	4.32	2.53	2.06	2.97
	40	2.99	2.79	2.79	2.85
	20	2.54	2.66	2.90	2.70

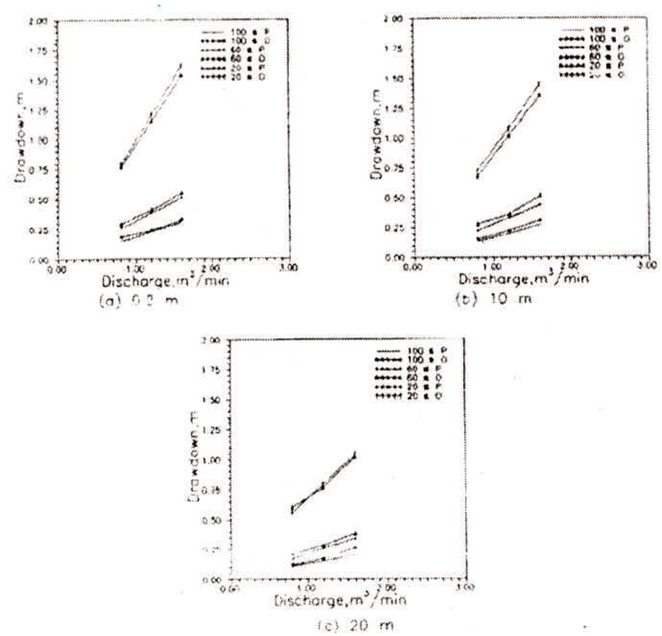

Fig. 4: Steady state predicted and observed drawdown vs discharge relationship for 20, 60 and 100% penetration of bottom aquifer at different radial distances

Similar errors ranging between 2.71 and 14.06% are found for other discharges and different penetration ratios. Although the errors are not following any particular trend, in general they are decreasing with the reduction in penetration ratio. The average percentage errors in prediction of drawdown for bottom, middle and top aquifers are 9.38, 3.17 and 3.21% respectively. Although in few cases the error exceed 10%, but in most of the cases they are limited within 5%. Therefore, the developed mathematical model can suitably applied for prediction of drawdown in multiple leaky aquifers under steady state condition.

CONCLUSIONS

The developed mathematical model predicts the drawdown precisely. In all cases the predicted drawdown curve closely follows the observed drawdown curve. The degree of errors for different layers of multiple leaky aquifer are as follows:

1. The errors in prediction of drawdown for 100, 80, 60, 40 and 20% penetration of bottom aquifer are 15.06, 8.03, 16.11, 10.57, and 4.9% respectively at 0.8 m^3/min discharge. However, the average errors in prediction of drawdown for all the three discharges for 100, 80, 60, 40 and 20% penetration of bottom aquifer are 12.75, 11.77, 11.03, 6.84 and 4.71% respectively.
2. The errors in prediction of drawdown for 100, 80, 60, 40 and 20% penetration of middle aquifer are 5.44, 5.94, 3.33, 2.31 and 2.03% respectively at 0.8 m^3/min discharge. However, the average errors in prediction of drawdown for all the three discharges for 100, 80, 60, 40 and 20% penetration of middle aquifer are 4.92, 3.91, 2.52, 2.56 and 1.94% respectively.
3. The errors in prediction of drawdown for 100, 80, 60, 40 and 20% penetration of top aquifer are 8.84, 3.50, 4.32, 2.99 and 2.54% respectively at 0.8 m^3/min discharge. However, the average errors in prediction of drawdown for all the three discharges for 100, 80, 60, 40 and 20% penetration of top aquifer are 4.6, 2.96, 2.97, 2.85 and 2.70% respectively.
4. The errors in prediction of drawdown for the bottom, middle and top aquifers ranges from 1.11 to 16.11% with a mean average of 5.25% only. Therefore, the developed mathematical model can be successfully used to predict drawdown in multiple leaky aquifers.

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APPENDIX

Notations

$Bi = (K_i b_i b_i')/ Ki'$	leakage factor, m
bi	thickness of the ith aquifer, m
bi'	thickness of the ith aquitard, m
b_n	thickness of the nth aquifer, m
Ki	hydraulic conductvity of ith aquifer, m/day
Ki'	hydraulic conductivity of the ith aquitard, m /day
l	depth of penetration, m
Q	discharge, m^3/day
r	horizontal distance, m
s	drawdown, m
Är	grid size in horizontal direction, m
Äz	grid size in vertical direction, m
z	vertical distance, m