TN-15

SEEPAGE FROM WATER BODIES

SATISH CHANDRA
DIRECTOR

STUDY GROUP

D N BHARGAVA
SATISH CHANDRA
G C MISHRA

NATIONAL INSTITUTE OF HYDROLOGY
JAL VIGYAN BHAVAN
ROORKEE-247667(UP)

INDIA

1985-86

CONTENTS

LIST OF SYMBOLS

a	a constant
B	width of the river or canal at the water surface
b	a constant, half river width
C_1, C_2, C_3	Constants
c	a constant
D	centre to centre distance between two parallel canals
D'_i	depth to impervious stratum measured from a high datum
E	initial saturated thickness
e	saturated thickness of aquifer below the river
H	maximum depth of water in the canal or river
$\bar{H}$	initial water table height before onset of recharge
h	total head
$h(n)$	total head under the river in the aquifer during time step n
h_c	capillary suction head
h_o	total head in the aquifer under the river
h_r, h_s	total head at the river boundary
K	coefficient of permeability
K_B	hydraulic conductivity of the semipermeable blanket
l	thickness of semipermeable layer
l_r	length of a river or canal reach
m	thickness of aquifer
p	pressure
q	recharge from unit length of line source
Q_r	return flow to a river reach
R	total no.of river reaches on either side of a reach influencing water table rise

$S_r(n)$	drawdown of the water table in the aquifer in the vicinity of a river measured from a high datum
T	transmissivity
t	time
w_p	wetted perimeter
w	recharge rate
x,y	cartesian co-ordinates
y_m	elevation of the semipermeable blanket
Φ	effective porosity
γ_w	unit weight of water
β	the angle the free surface makes with the horizontal
Γ_r	reach transmissivity
$\delta(.,.,.)$	discrete kernel coefficient for drawdown
$\sigma_r(n)$	drawdown of the water level in a river reach during n^{th} unit time period measured from a high datum.

LIST OF FIGURES

LIST OF TABLES

ABSTRACT

A mathematical model for seepage studies from two parallel canals has been developed and interference of two parallel canals is studied. One of the canals is situated on a high ridge and is not hydraulically connected with the aquifer. The other canal situated at a lower level has hydraulic connection with the aquifer. Its seepage rate is controlled by the water table position in the aquifer and is linearly dependent on the difference in the potentials at the canal and in the aquifer underneath. In the present study the temporal variation of seepage from the canal hydraulically connected with the aquifer has been quantified. The model predicts the time at which the seepage from the canal having hydraulic connection with the aquifer reduces to zero and quantifies the rate at which the canal receives water as a drain.

Recently there have been evidences that the exchange flow rate between a canal and a hydraulically connected aquifer can be very non-linear. In the present report seepage from a single canal when water table lies at a shallow depth has also been studied for a case in which the seepage rate has an exponential relation with the potential difference between the canal and the aquifer. The seepage rates corresponding to the linear and non linear relations have been compared.

1.0 INTRODUCTION

The assessment of seepage from canals and consequent recharge to aquifer are often required for the solution of problems of surface and ground water resources. Many research workers have investigated the steady free seepage from canals. A number of theoretically well established analysis are known after Kozeny (1931), Vedernikov (1934), Risenkampf (1940), Muskat (1940), Bouwer (1969). For homogeneous soil and idealized boundary conditions, the classical studied canbe used to predict steady seepage loss from canals. However, the soil and boundary conditions in nature do not conform always to the idealized soil and boundary conditions adopted in the various theoritical treatment of seepage study. Numerical methods and electrical analogue can handle the non homogeneous and anisotrophic nature of soils besides the various complex boundary conditions. Unless a canal is lined it is inevitable that seepage would occur from a canal. Impact of the seepage water on ground water regime and its distribution in the aquifer need equal attention as does the quantification of seepage.

The process of seepage from a canal starts as soon as water is conveyed in it. In the beginning the seepage rate undergoes rapid changes due to dispersion and swelling of soil particles after wetting and elimination of entrapped air by solution in the water. Few days after initiation of seepage, the seepage rate follows an exponential decay curve. It may be noted that seepage rate from a canal is not equal to the recharge rate at the water table at all time. When the wetting front position is some where between the canal bed and initial water table position, the recharge rate is zero. If the water content behind the wetting front is close to saturation recharge rate rises abruptly from zero to the prevailing seepage rate at the time the saturation front encounters the water table. The study of seepage prior to initiation of recharge to ground water is not the scope of the present report.

The computation of rise in water table due to recharge from water bodies has been dealt with by few authors. Hantush (1967) has derived an expression for rise in water table height due to recharge from a basin of finite length and width. If the dimension of length is increased to a very large value, the solution will correspond to rise in water table due to recharge from a canal. However, the solution involves numerical integration. Shestakov (1965) has tabulated special function for calculating water table rise due to recharge from a strip source using numerical method for integration. Glover (1974) has analysed the evolution of water table due to recharge from a line source, but has not taken the width of recharge body into consideration. Hantush and Glover have assumed that there is no hydraulic connection between the recharging water bodies and the aquifer. Morel Seytoux and Daly (1975) have developed stream aquifer interaction model in which the stream has hydraulic connection with the aquifer.

It has been often assumed for a stream (canal) which is hydraulically connected with the aquifer, that the exchange flow rate is linearly dependent on the potential difference between the aquifer and the stream (Aravin and Numerov, 1965, Herbert, 1970, Morel-Seytoux, 1975, Besbes et al.,1978, Flug et al.,1980). There have been evidences that the process can be very non linear (Dillon, 1983, 1984, Rushton and Redshaw, 1972). But as it is difficult to determine the exact non-linear relationship, the linear relationship is still in vogue.

In the present report using a linear relation and the basic solution of Glover and the discrete kernel approach of Morey Seytoux, seepage study from two parallel canals one of which has hydraulic connection with the aquifer, has been made. Also unsteady seepage from a canal having hydraulic connection with the aquifer has been determined for the case in which the seepage rate has an exponential relation with the potential difference between the canal and the aquifer underneath.

2.0 REVIEW

The literature on steady seepage from canal and its impact on ground water regime have been well documented (Bouwer, 1969; Glover, 1970; Harr, 1962; Kovacs 1981; Muscat, 1940; Schestakov, 1965). In the current report literature review has been made only for seepage from water body (canal or river) which has hydraulic connection with an aquifer.

Dillon and Liggett (1983) have analysed unsteady seepage from a strip source depicted in Fig.1, which is hydraulically connected with the underlying aquifer by solving the Laplace equation $\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}=0$ satisfying the following conditions:

i) $h = y$, and

$$\frac{\partial h}{\partial t} = \frac{K}{\Phi}\left[-\frac{1}{\cos\beta}\frac{\partial h}{\partial n} + \frac{w(t)}{K}\right]$$ along the unknown phreatic line,

ii) $\frac{\partial h}{\partial n} = \frac{q_b}{K}$ along the base of the aquifer,

iii) $\frac{\partial h}{\partial n} = 0$ for $x = 0$, and

iv) $h = h_s - B\frac{\partial h}{\partial n}$ on bed of the water body when $h > y_m$

in which

$$h = \frac{p}{\gamma_w} + y,$$

y = height above arbitrary datum chosen as the elevation of the aquifer basement,

K = coefficient of permeability of the aquifer material,

β = the angle the free surface makes with the horizontal,

n = outward normal direction to the boundary,

$y_m(x)$ = elevation of the semipermeable bottom of water body above the basement,

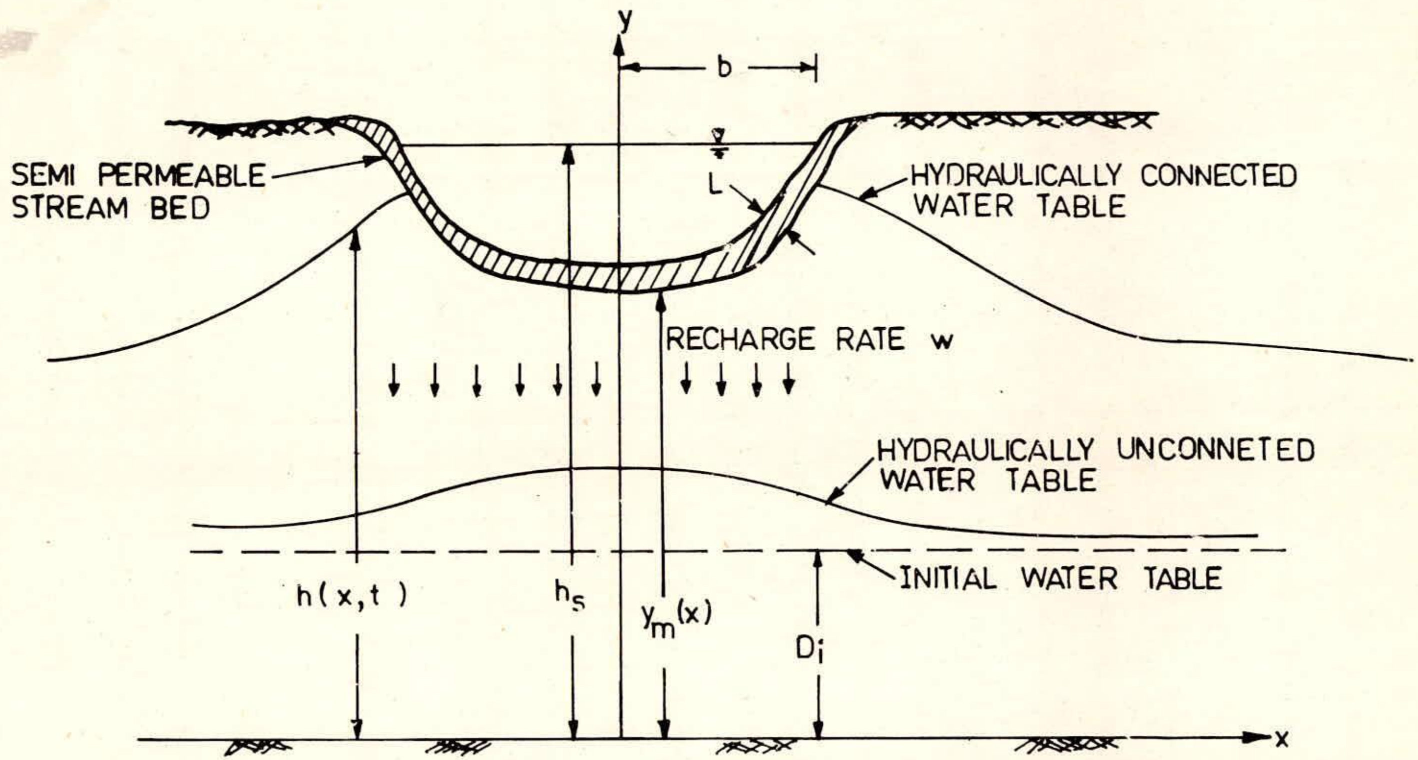

Fig.1 Stream aquifer system

$B = Kl/K_B$,

l = thickness of semipermeable bed comprising the boundary of the water body, and

K_B = hydraulic conductivity of the semipermeable blanket.

The recharge has been obtained by the expression

$$w/K = (h_s - h)/B \text{ for } (y_m + h_c) \leq h \leq y_m$$

$$w/K = (h_s - y_m - h_c)/B \text{ for } h < (y_m + h_c)$$

in which h_c is capillary suction head at the base of the blanket.

The Laplace equation with the above boundary conditions has been solved by boundary integral equation method.

Rushton and Tomlinson have presented typical non linear relationship between flow from an aquifer to the river as shown in Fig.2. The actual flow from a river into an aquifer is usually considerably less than the flow in the reverse direction for a similar head difference because of sediment in river bed.

According to Rushton and Redshaw (1979)[4] the non-linear relationship between flow to the aquifer and the potential difference between the aquifer and the river which appear to give a fair representation are as follows:

$$Q = C_1(h_o - h_r) + C_2[1 - e^{-C_3(h_o - h_r)}] \text{ for } h_o \geq h_r$$

and

$$Q = 0.3C_2 \ [e^{C_3(h_o - h_r)} - 1] \qquad \text{for } h_o \leq h_r$$

where C_1, C_2, C_3 are constants which depend on field condition.

Because of the difficulty in determining the actual nonlinear relationship, it is common practice to use a linear relationship. The constant of proportionality in the linear relationship has been designated as reach transmissivity or river resistance. Methods of calculating the hydraulic river resistance have been proposed by Aravin and Numerov

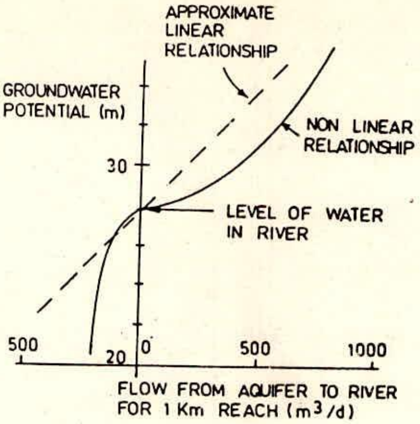

Fig.2 Typical relationship between flow from aquifer to river

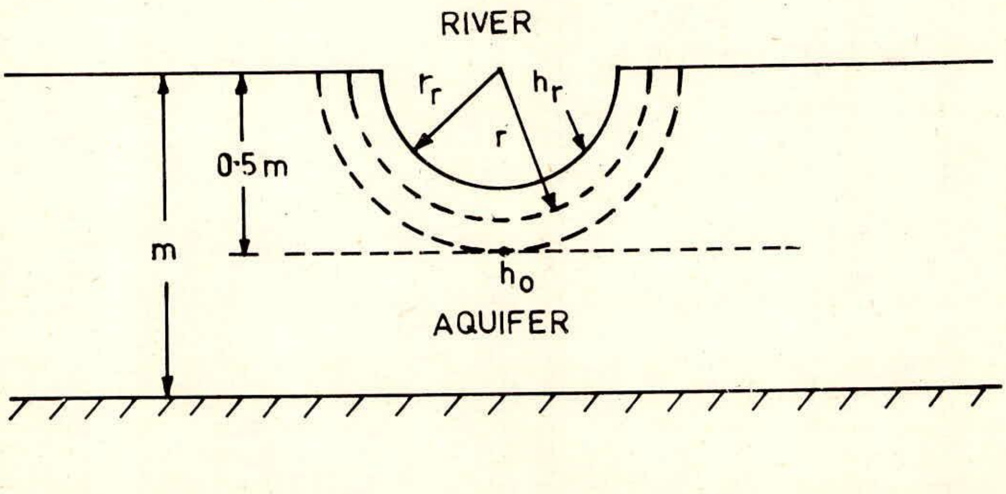

Fig.3 Representation of partially penetrating river

(1965), Herbert (1970), Streltsova (1974), Morel-Seytoux et al (1979). According to Herbert the linear relationship between the exchange flow rate and the potential difference between the river and the aquifer underneath is given by:

$$Q_r = \pi l_r K(h_r - h_o)/\log_e(.5m/r_r)$$

in which

l_r = length of river reach,

K = coefficient of permeability of the aquifer material,

h_r = potential at the river perimeter,

h_o = potential in the aquifer at a point under the river,

m = thickness of the aquifer as shown in figure 2,

r_r = radius of the semicircular river cross section.

Morel-Seytoux has postulated and verified the following relationship between return flow to a river and the potential difference between the river and in the aquifer in the vicinity of the river:

$$Q_r(n) = \Gamma_r[\sigma_r(n) - S_r(n)]$$

in which $Q_r(n)$ is the return flow to a river reach,

Γ_r = reach transmissivity,

$\sigma_r(n)$ = drawdown of the water level in the river reach during n^{th} time period measured from a high datum

$S_r(n)$ = drawdown of the water table in the aquifer measured from the same high datum in the vicinity of the river.

It may be noted that seepage rate from a canal is not the recharge rate at the water table at all time. With the wetting front position some where between the canal bed and initial water table position, and for initially dry soil the seepage rate varies in time but the recharge rate is constant and zero. If the water content behind the wetting front is close to saturation, recharge rates,rises abruptly from zero to the prevailing seepage rate at the time the saturation front encounters the water table (Abdulrazzak and Morel-Seytoux, 1983). The following expression for the time delay, t_d, for the recharge to reach the water table after the onset of seepage has been obtained by Dillon and Ligget by integrating Green and Ampt equation:

$$t_d = \frac{\Phi}{K} \left[y_m - D'_i - (h_s - h_c) \log_e \frac{h_s - h_c + y_m - D'_i}{h_s - h_c} \right]$$

where

Φ is the effective porosity, and

D'_i is the initial saturated thickness.

3.0 PROBLEM DEFINITION AND METHODOLOGY

3.1 Seepage from a Water Body, when the Flow Rate is Nonlinearly Dependent on Potential Difference

3.1.1 Statement of the problem

A river having hydraulic connection with the underlying aquifer is depicted in Fig.4. The water table lies at a shallow depth below the river bed. The recharge from the river to the aquifer is assumed to have the following non-linear relationship with the potential difference between the river and the aquifer that has been proposed by Rushton and Redshaw (1979):

$$Q_{1r}(n) = 0.3C_2 l_r \,[1 - \exp\{-C_3(h_r - h(n))\}] \quad \ldots(1)$$

The hydraulic head $h(n)$ in the aquifer during time period 'n' is governed by the recharges those take place from all reaches during time period 'n' and those took place from all reaches upto $(n-1)^{th}$ time period. It is required to find the recharge rate, $Q_{1r}(n)$, from a reach of length l_r at various time after the onset of recharge.

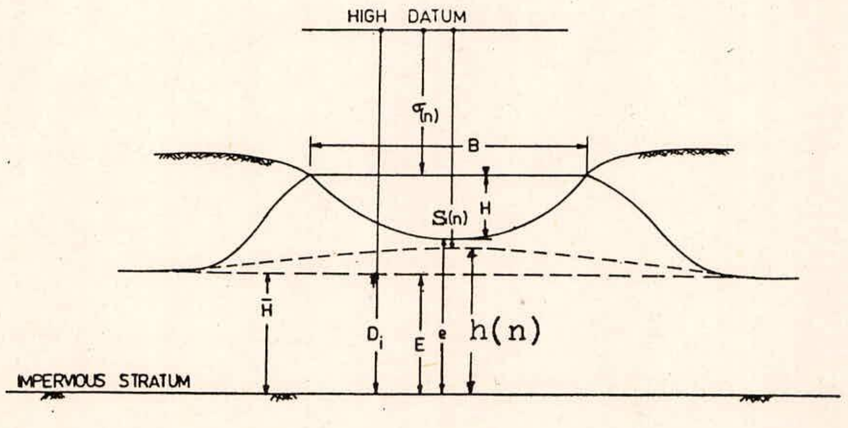

Fig.4 Schematic section of a river hydraulically connected to the aquifer

3.1.2 Methodology

The constants C_2 and C_3 which appear in equations (1) are evaluated in the following manner:

If the water table is at a large depth below the river bed i.e.when h(n) is small in comparison to h_r, the exponential term in equation (1) reduces to a small quantity. Neglecting the expo-exponential term for large values of $[h_r - h(n)]$

$$Q_{1r} = 0.3C_2 l_r \qquad ...(2)$$

According to Kozeny the seepage from a strip source of width B and a depth of water H is given by

$$Q_{1r} = K(B+2H)l_r \qquad ...(3)$$

where K is the coefficient of permeability of the aquifer material. Equating equations (2) and (3)

$$C_2 = K(B + 2H)/0.3 \qquad ...(4)$$

Substituting C_2 in equation (1)

$$Q_{1r}(n) = K\,(B + 2H)\,l_r\,[\,1\text{-exp}\,\{C_3(h_r - h(n)\}\,] \qquad ...(5)$$

For small difference between h_r and h(n) the higher order terms of the polynomial expansion of the exponential term appearing in equation (5) can be neglected and the recharge can be approximated to be

$$Q_{1r}(n) \simeq K(B+2H)l_r[1-1+C_3\{ h_r-h(n) \}]$$

or

$$Q_{1r}(n) \simeq K(B+2H)l_r C_3 [h_r-h(n)] \qquad ...(6)$$

Assuming a steady state condition and applying a simple potential theory Morel Seytoux et al.(1979) have derived the following relationship between return flow and the potential difference between river and the aquifer:

$$Q_{1r}(n) = (Tl_r/e) [(0.5w_p+e)/(5w_p+0.5e)][h_r-h(n)] \qquad ...(7)$$

in which e is the saturated thickness, T is the transmissivity equal to K e and w_p is the wetted perimeter of the river.

Though the derivation assumes a steady state of condition it has been applied to unsteady flow on the basis that a unsteady state can be approximated to be succession of steady state conditions. Comparing equations (6) and (7)

$$C_3 = 1/(B+2H) [(0.5w_p+e)/(5w_p+0.5e)] \qquad ...(8)$$

For a homogeneous aquifer, if the river stages in different reaches are not varying from each other, the recharge rates from river reaches during a particular time period are equal. Let N number of reaches on either sides of a particular reach influence the rise in water table. Let the time span be discretised by time steps of equal size and let during a particular time step the recharge rate be constant. With these assumptions the hydraulic head h(n) can be written as

$$h(n) = \bar{H} + \sum_{\gamma=1}^{n} Q_{r1}(\gamma)\,\partial(1,1,n-\gamma+1)+2\sum_{\gamma=1}^{n}\sum_{R=2}^{N} Q_{r1}(\gamma)\,\partial(1,R,n-\gamma+1) \qquad ...(9)$$

in which $\partial(.,.,.)$ are discrete kernel coefficients for drawdown and $\bar{H}$ is initial water table height before onset of recharge. Substituting h(n) in equation (5) and simplifying

$$1-Q_{1r}(n)/[K(B+2H)l_r] = \exp[-C_3\{D_i - \sigma_r(n) - \bar{H} - \sum_{\gamma=1}^{n} Q_{r1}(\gamma)\,\partial(1,1,n-\gamma+1)$$

$$-2\sum_{\gamma=1}^{n}\sum_{R=2}^{N} Q_{r1}(\gamma)\,\partial(1,R,n-\gamma+1)\}] \quad \ldots(10)$$

in which D_i is depth to impervious stratum measured from a high datum and $\sigma_r(n)$ is drawdown of the water level in the river measured from the same datum.

Taking logarithm of terms on either side

$$\log_e[1-Q_{1r}(n)/\{K(B+2H)l_r\}] = -C_3\{D_i - \sigma_r(n) - \bar{H}$$

$$-\sum_{\gamma=1}^{n} Q_{r1}(\gamma)\,\partial(1,1,n-\gamma+1)$$

$$-2\sum_{\gamma=1}^{n}\sum_{R=2}^{N} Q_{r1}(\gamma)\,\partial(1,R,n-\gamma+1)\} \quad \ldots(11)$$

splitting the summation into two parts

$$\log_e[1-Q_{1r}(n)/\{K(B+2H)l_r\}] - C_3Q_{1r}(n)\{\partial(1,1,1)+2\sum_{R=2}^{N}\partial(1,R,1)\}$$

$$= -C_3\{D_i - \sigma_r(n) - \bar{H} - \sum_{\gamma=1}^{n-1} Q_{r1}(\gamma)\,\partial(1,1,n-\gamma+1)$$

$$-2\sum_{\gamma=1}^{n-1}\sum_{R=2}^{N} Q_{r1}(\gamma)\,\partial(1,R,n-\gamma+1)\} \quad \ldots(12)$$

$Q_{1r}(n)$ can be solved in succession starting from time step 1 by an iteration procedure. The following simplification can be adopted without much loss of accuracy. $K(B+2H)L_r$ being the recharge rate when water table is at large depth, the ratio $Q_{1r}(n)/\{K(B+2H)L_r\}$ is less than 1. Expanding the logarithm term and neglecting higher order terms

$$-Q_{1r}(n)/[K(B+2H)L_r] - \frac{1}{2}[Q_{1r}(n)/\{K(B+2H)L_r\}]^2$$

$$- C_3Q_{1r}(n)[\partial(1,1,1) + 2\sum_{R=2}^{N}\partial(1,R,1)]$$

$$= -C_3[D_i - \sigma_r(n) - \bar{H} - \sum_{\gamma=1}^{n-1} Q_{r1}(\gamma)\,\partial(1,1,n-\gamma+1)$$

$$-2\sum_{\gamma=1}^{n-1}\sum_{R=2}^{N} Q_{r1}(\gamma)\,\partial(1,R,n-\gamma+1)] \quad \ldots(13)$$

Equation (13) is a quadratic equation in $Q_{1r}(n)$ and can be written in the form:

$$a\,Q_{rl}^{2}(n) \;+ b\,Q_{rl}(n) + c = 0 \qquad \ldots(14)$$

and $Q_{rl}\,(n)$ is given by

$$Q_{rl}(n) = [\,-b + \;(b^{2}-4ac)\;]/2a \qquad \ldots(15)$$

where

$$a = 0.5/\;[KL_{r}(B + 2H)\,]^{2}$$

$$b = 1/\;[K(B+2H)L_{r} \;+ C_{3}\,\partial\;(1,1,1)+2\sum_{R=2}^{N}\;\partial(1,R,1)\,]$$

$$c = -\,C_{3}[\;D_{i} - \sigma_{r}(n) - \bar{H} - \sum_{\gamma=1}^{n-1} Q_{rl}(\gamma)\,\partial(1,1,n-\gamma+1)$$

$$-2\sum_{\gamma=1}^{n-1}\;\sum_{R=2}^{N}\;Q_{rl}\,(\gamma)\,\partial(1,R,n-\gamma+1)\,]$$

3.2 Seepage from two Parallel Water Bodies one of which has Hydraulic Connection with the Aquifer

3.2.1 Statement of the problem

Two parallel canals have been constructed in a homogeneous and isotropic pervious medium of infinite aerial extent. The dimensions of the canals and the horizontal distance between centre to centre of the two canals are as shown in Fig.(5). One of the canals is situated on a high ridge and the other is at a much lower elevation. On account of large difference in the elevations of bed level of the ridge canal and the water table underneath, it is unconnected with the aquifer. Therefore, the seepage occurs at constant rate from ridge canal. The bed of the lower canal is near to the ground water table and is hydraulically connected to the aquifer. Its seepage rate will be controlled by the potential difference between the water levels in the canal and in the aquifer below the bed of the canal. The permeability of the

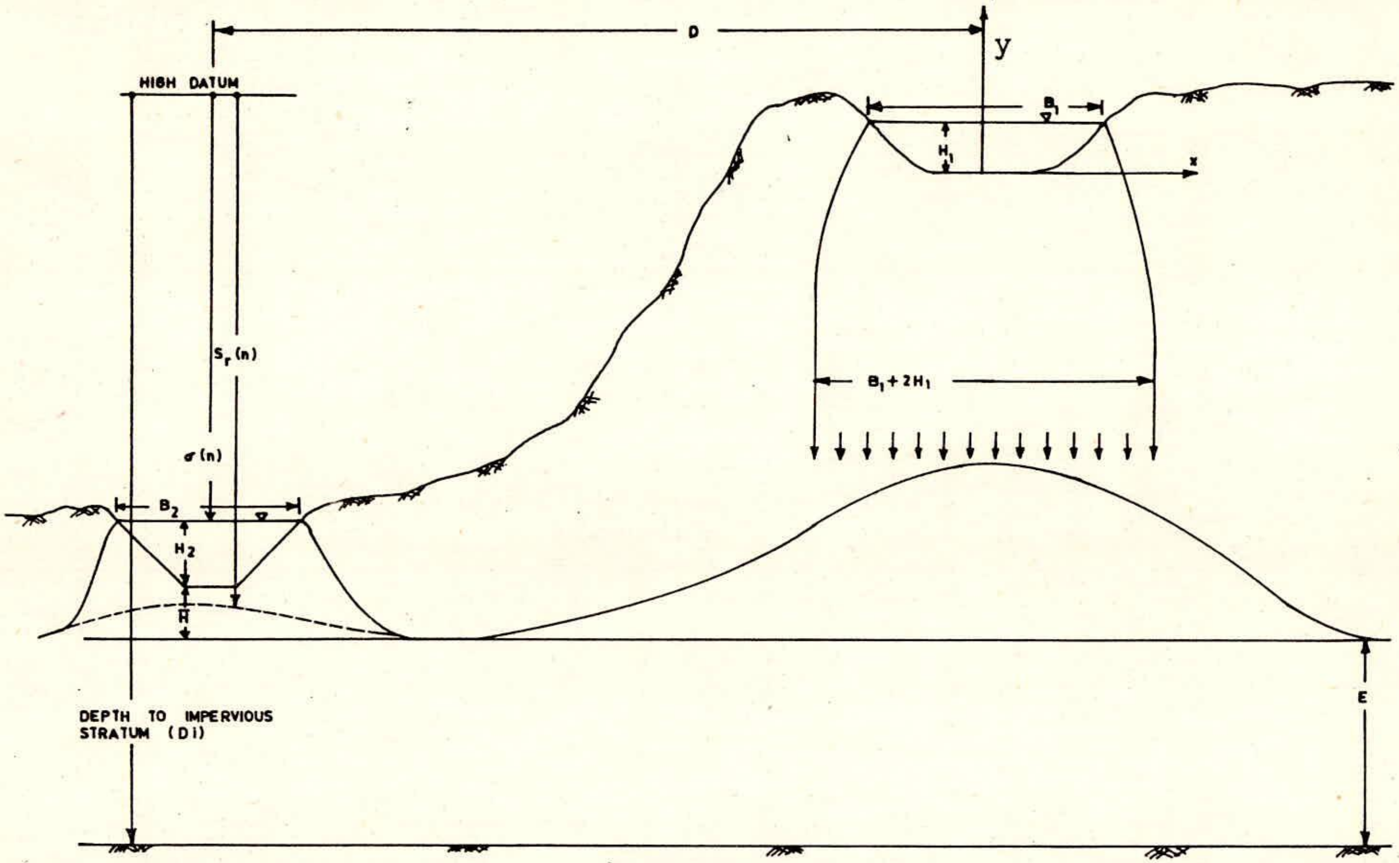

Fig.5 Schematic Section of two Parallel Canals

aquifer is K. The thickness of the saturated depth of aquifer is E and the storage coefficient is ϕ . It is required to determine the seepage from the lower canal and the temporal and spatial variation of water table rise.

3.2.2 Methodology

The assumptions made to carry out the analysis for evolution of water table by using Glover's basic equations and reach transmissivity-discrete kernel approach of Morel-Seytoux are as follows:

(i) Time interval in which the seeping water from ridge canal reaches the water table is small and can be neglected.

(ii) The hydraulic properties of the aquifer remain constant with respect to time and space.

(iii) The flow due to seepage from ridge canal is vertically downwards until it reaches the water table.

(iv) Dupuit's assumptions are valid.

(v) The time span is discretised by uniform time step. Within each time step the seepage from the lower canal is constant, but it varies from step to step.

A linear relationship which has been postulated and verified by various investigators (Morel Seytoux and Daly, 1975, Morel Seytoux, 1975, Morel Seytoux et al., 1979) is of the form $Q_r = \Gamma_r(\sigma_r - S_r)$, where Q_r is the return flow, σ_r is the drawdown to the water level in the stream measured from a high datum, S_r is the drawdown in the aquifer in the vicity of the reach measured from the same datum. The coefficient Γ_r has been designated as reach transmissivity which depends on the stream bed characteristics and shape of the stream cross section(Morel Seytoux, 1964, Bouwer, 1969).

Assuming a steady state condition and applying a simple potential theory for saturated flow, an expression of reach transmissivity has been derived by Morel-Seytoux et al.,(1979) which is given by

$$\Gamma_r = \frac{TL}{e} \frac{0.5w_p+e}{5w_p+0.5e},$$

where T is the transmissivity of the aquifer, L is the reach length, w_p is the wetted perimeter and 'e' is the saturated thickness under the river bed. Though the reach transmissivity has been derived on the assumption of steady state condition, it has been used to predict the return flow under transient condition supported by the argument that transient nature of a system can be represented as a continuous succession of steady states.

Making use of this approach for the case of lower canal, which is connected to the aquifer, the seepage from it is given by

$$Q_r(n) = -\Gamma_r[\sigma_r(n)-S_r(n)] \qquad ...(16)$$

where, $\sigma_r(n)$ and $S_r(n)$ are the depth to water surface in the canal and the depth to water table below the canal bed respectively, measured from the same high datum at the end of time step n, and Γ_r is the reach transmissivity.

The depth to water table below the lower canal bed $S_r(n)$, comprises two parts, $S_1(n)$ and $S_2(n)$, where, $S_1(n)$ is the rise on account of seepage from ridge canal and $S_2(n)$ is the rise due to its own seepage.

As stated above the ridge canal is unconnected with the aquifer. The evolution of rise of water table due to seepage from a canal can be ascertained by solving the following linearised one dimensional Boussinesq's equation for saturated flow

$$\frac{\partial^2 s}{\partial x^2} = \alpha \frac{\partial s}{\partial t} \qquad ...(17)$$

in which S is the rise in water table, $\alpha = T/\Phi$, T being the transmissivity of the aquifer and Φ is the storage coefficient. If the canal is assumed as a line source, the solution to the equation(17) has to satisfy the following boundary conditions:

At $x = 0$, $T \frac{\partial S}{\partial x} = -\frac{q}{2}$, and
at $x = \infty$, $S(\infty, t) = 0$

in which, q is recharge rate per unit length of the line source. If the ground water is at rest before the initiation of recharge, the initial condition to be satisfied is

$S(x,0)=0.$

Considering the canal to be a line source, the solution that satisfies one dimensional Boussinesq's equation and the above stated initial and boundary conditions has been given by Glover (1974), which is of the form

$$S(x,t) = \frac{q\sqrt{(\alpha t)}}{T\sqrt{\pi}} e^{-x^2/4\alpha t} - \frac{qx}{2T} + \frac{qx}{2T} \text{Erf}\left[\frac{x}{\sqrt{(4\alpha t)}}\right] \qquad \ldots(18)$$

In reality, however, a canal has a certain finite width which cannot be neglected. Therefore, it would be appropriate to treat it as a strip source instead of a line source. Since the water table is at large depth below the bed of ridge canal, the width of the strip can be taken approximately to be $(B_1 + 2H_1)$. Also, according to Kozney, the seepage rate per unit width of strip is K, as the water table lies at large depth. A strip source can be regarded to be consisting of a number of line sources. As the differential equation governing the flow is linear, the method of superposition can be used. Thus, the rise of water table due to seepage from a strip source can be obtained by integrating the rise of water table due to each line source. The solution to the equation (17) corresponding to seepage from a strip source has been given by Bhargava et al.,(1986) as below:

For $x \geq (0.5B_1+H_1)$ and $x \leq -(0.5B_1+H_1)$, the rise in water table is given by

$$S(x,t) = \frac{K\alpha t}{2T} \text{Erf}\left(\frac{x+0.5B_1+H_1}{\sqrt{4\alpha t}}\right) - \frac{K\alpha t}{2T} \text{Erf}\left(\frac{x-0.5B_1-H_1}{\sqrt{4\alpha t}}\right)$$

$$+\frac{K}{4T}(x+0.5B_1+H_1)^2 \text{Erf}\left(\frac{x+0.5B_1+H_1}{\sqrt{4\alpha t}}\right)$$

$$-\frac{K}{4T}(x-0.5B_1-H_1)^2 \text{Erf}\left(\frac{x-0.5B_1-H_1}{\sqrt{4\alpha t}}\right)$$

$$+\frac{K\sqrt{\alpha t}}{2T\sqrt{\pi}}(x+0.5B_1+H_1).e^{-(x+0.5B_1+H_1)^2/4\alpha t}$$

$$-\frac{K\sqrt{\alpha t}}{2T\sqrt{\pi}}(x-0.5B_1-H_1).e^{-(x-0.5B_1-H_1)^2/4\alpha t}$$

$$-\frac{K\sqrt{(x^2)}(B_1+2H_1)}{2T}$$

$$= F(x,t) - \frac{K\sqrt{(x^2)}(B_1+2H_1)}{2T} \qquad \ldots(19)$$

For $-(0.5B_1+H_1) \leq x \leq (0.5B_1+H_1)$, the rise is given by

$$S(x,t) = F(x,t) - \frac{K}{2T}[x^2 + (0.5B_1+H_1)^2] \qquad \ldots(20)$$

The rise of water table with time due to seepage from the ridge canal can be obtained from these solutions. The rise of water table under the lower canal due to seepage from the ridge canal at the end of time step 'n' is given by

$$S_1(n) = F(D,n) - \frac{KD(B_1+2H_1)}{2T} \qquad \ldots(21)$$

where D is the distance between centre to centre of the two canals.

The rise in water table due to seepage from the lower canal which is hydraulically connected with the aquifer can be predicted by the stream aquifer interaction model of Morel Seytoux and Daly. Let the length of the canal, from which the recharge causes appreciable rise in water table at a point, be divided into number of reaches. The rise in water table at the centre of the r^{th} reach at the end of n^{th} unit time step, $S_2(r,n)$ due to recharges which have taken place from all reaches upto n^{th} time step, is given by(Morel Seytoux and Daly,1975)

$$S_2(r,n) = \sum_{\gamma=1}^{n} \sum_{R=1}^{N} Q_R(\gamma)\ \partial(r,R,n-\gamma+1) \qquad \ldots(22)$$

in which

N = total number of reaches,

R = an index representing reaches,

$$\partial(r,R,I) = \frac{1}{4\pi T}\left[E_1\left(\frac{d_{rR}^2}{4\alpha I}\right) - E_1\left(\frac{d_{rR}^2}{4\alpha(I-1)}\right)\right],\ r \neq R, \qquad \ldots(23)$$

d_{rR} = distance from centre of the r^{th} reach to the centre of the R^{th} reach,

$$E_1(x) = \int_x^{\infty} \frac{e^{-u}}{u}\, du ,$$

$$\partial(r,r,I) = \frac{1}{\phi L_r B_2} \int_0^1 \text{Erf}\left(\frac{L_r}{4\sqrt{\alpha(I-Y)}}\right) \text{Erf}\left(\frac{B_2}{4\sqrt{\alpha(I-Y)}}\right) dY \qquad \ldots(24)$$

Let all the reaches be of equal length. Let the total number of reaches be 2N+1 and the central reach be the first reach. The rise in water table under the central reach can be obtained from equation (22) as given below:

$$S_2(n) = \sum_{\gamma=1}^{n-1} Q_1(\gamma)\ \partial(1,1,n-\gamma+1) + 2\sum_{\gamma=1}^{n}\sum_{R=2}^{N} Q_1(\gamma)\partial(1,R,n-\gamma+1) \qquad \ldots(25)$$

Spliting the temporal summation into two parts

$$S_2(n) = \sum_{\gamma=1}^{n-1} Q_1(\gamma)\,\partial(1,1,n-\gamma+1)+Q_1(n)\partial(1,1,1)$$

$$+ 2Q_1(n) \sum_{R=2}^{N} \partial(1,R,1) + 2 \sum_{\gamma=1}^{n-1} \sum_{R=2}^{N} Q_1(\gamma)\partial(1,R,n-\gamma+1) \qquad \ldots(26)$$

Referring to Fig.5, the relation between depth to water table from the high datum at the centre of the lower canal and the rise in water table underneath above the initial static stage can be written as:

$$S_r(n) = D_i - E - S_1(n) - S_2(n) \qquad \ldots(27)$$

Substituting for $S_r(n)$ in equation (16) and noting that $Q_r(n) = Q_1(n)$

$$Q_1(n)=-\Gamma_r[\sigma_r(n)-D_i-E+S_1(n)+S_2(n)] \qquad \ldots(28)$$

Substituting $S_1(n)$ and $S_2(n)$ from equations (21) and (25) respectively in equation (28) and rearranging

$$Q_1(n) = [\frac{1}{\Gamma_r} + \partial(1,1,1)+2\sum_{R=2}^{N}\partial(1,R,1)]^{-1}.[D_i-E-F(D,n)$$

$$+\frac{KD(B_1+2H_1)}{2T} - \sum_{\gamma=1}^{n-1} Q_1(\gamma)\,\partial(1,1,n-\gamma+1)$$

$$-2\sum_{\gamma=1}^{n-1}\sum_{R=2}^{N} Q_1(\gamma)\,\partial(1,R,n-\gamma+1)] \qquad \ldots(29)$$

In particular for the first time step the seepage rate is given by

$$Q_1(n) = \frac{D_i - E - F(D,1) + 0.5KD(B_1+2H_1)/T}{1/\Gamma_r + \partial(1,1,1)+2\sum_{R=2}^{N}\partial(1,R,1)} \qquad \ldots(30)$$

The seepage rate during any time step n can be found in succession starting from the first time step by using equation(29).

When the water table in the aquifer under the lower canal rises above the water level in the canal i.e.when $S_r(n) < \sigma_r(n)$, the lower canal will receive water and then onwards it will act as a drain. If the exchange flow rate is small in comparison to the discharge in the canal, it can be assumed that the change in water level in the canal is insignificant. Thus a constant head boundary condition can be assumed to prevail along the canal boundary after the canal receives water from the aquifer. Let the canal receives water after time step m. Since the head under the canal is constant for $n > m$,

$$S_1(n) + S_2(n) = \bar{H} + H_2 \qquad \ldots(31)$$

Substituting for $S_1(n)$ and $S_2(n)$ in above and solving for $Q_1(n)$, the rate at which water is entering into the canal is found to be

$$Q_1(n) = [\bar{H}+H_2-F(D,n)+0.5KD(B_1+2H_1)/T$$

$$- \sum_{\gamma=1}^{n-1} Q_1(\gamma)\partial(1,1,n-\gamma+1)$$

$$-2 \sum_{\gamma=1}^{n-1} \sum_{R=2}^{N} Q_1(\gamma)\,\partial(1,R,n-\gamma+1)]/$$

$$[\partial(1,1,1) + 2 \sum_{R=2}^{N} \partial(1,R,1)],\ n > m \qquad \ldots(32)$$

4.0 RESULTS AND DISCUSSION

4.1 Seepage from a Water Body when the Flow Rate is Nonlinearly Dependent on Potential Difference

Results pertaining to unsteady seepage from a canal hydraulically connected with the underlying aquifer have been presented in non- dimensional form in Fig.6 for B/H=20 and 10. The water table is assumed to be at a depth of 0.005E below the canal bed. As seen from figure the results decrease monotonnically with time. For B/H=10 when the time factor increases from $5x10^{-4}$ to $1x10^{-1}$, the nondimensional seepage rate decreases from $8.778x10^{-3}$ to $7.299x10^{-3}$. The corresponding results when the seepage rate is taken as linearly dependent on potential difference is presented in Table 1. It can be seen that there is no significant difference in the results pertaining to linear and nonlinear cases.

4.2 Seepage from two Parallel Water Bodies one of which has Hydraulic Connection with the Aquifer

Results pertaining to interference of two parallel canals have been presented in nondimensional form for different spacings and widths of the canals. The spacing of the canals has been varied from 80m to 240m. Both the canals are assumed to have equal width. The widths of the canals considered are 30m to 60m. The variations of seepage from the lower canal with time are shown in Fig.7 and 8.

At the beginning of the seepage during the early period, canal having larger width will loose at a higher rate than that of the canal with smaller width. As the seepage rate of the lower canal is governed by the potential difference between the canal and the aquifer underneath,

Table 1. Comparison of seepage rates from a single canal when the flow rate is taken linearly and non-linearly dependent on potential difference

Width of canal	Depth of water in canal	Nondimensional time $(\frac{Kt}{2\phi E})$	Nondimensional recharge $(\frac{Q_{1r}}{Klr.E})$ for linear variation (10^{-3})	Nondimensio-nal Recharge $(\frac{Q_{1r}}{Klr.E})$ for nonlinear variation (10^{-3})	Percentage difference $\frac{Col.4-Col5}{Col.4}$
1	2	3	4	5	6
30	3.0	$.5x10^{-6}$	10.130	8.915	12.00
		$.5x10^{-3}$	9.935	8.778	11.64
		$.1x10^{-2}$	9.844	8.714	11.48
		$.5x10^{-2}$	9.473	8.449	10.81
		$.1x10^{-1}$	9.210	8.259	10.32
		$.2x10^{-1}$	8.872	8.010	9.72
		$.3x10^{-1}$	8.647	7.842	9.31
		$.4x10^{-1}$	8.480	7.715	9.02
		$.5x10^{-1}$	8.348	7.615	8.78
		$.6x10^{-1}$	8.240	7.532	8.59
		$.7x10^{-1}$	8.148	7.461	8.43
		$.8x10^{-1}$	8.070	7.400	8.30
		$.9x10^{-1}$	8.000	7.346	8.18
		$1x10^{-1}$	7.940	7.298	8.08
60	3.0	$.5x10^{6}$	10.279	9.355	8.99
		$.5x10^{-3}$	10.089	9.208	8.73
		$.1x10^{-2}$	9.996	9.134	8.62
		$.5x10^{-2}$	9.613	8.833	8.11
		$.1x10^{-1}$	9.343	8.617	7.55
		$.2x10^{-1}$	8.995	8.337	7.32
		$.3x10^{-1}$	8.763	8.148	7.02
		$.4x10^{-1}$	8.591	8.007	6.80
		$.5x10^{-1}$	8.456	7.895	6.63
		$.6x10^{-1}$	8.345	7.803	6.50
		$.7x10^{-1}$	8.251	7.724	6.39
		$.8x10^{-1}$	8.171	7.657	6.29
		$.9x10^{-1}$	8.100	7.598	6.20
		$1x10^{-1}$	8.037	7.545	6.12

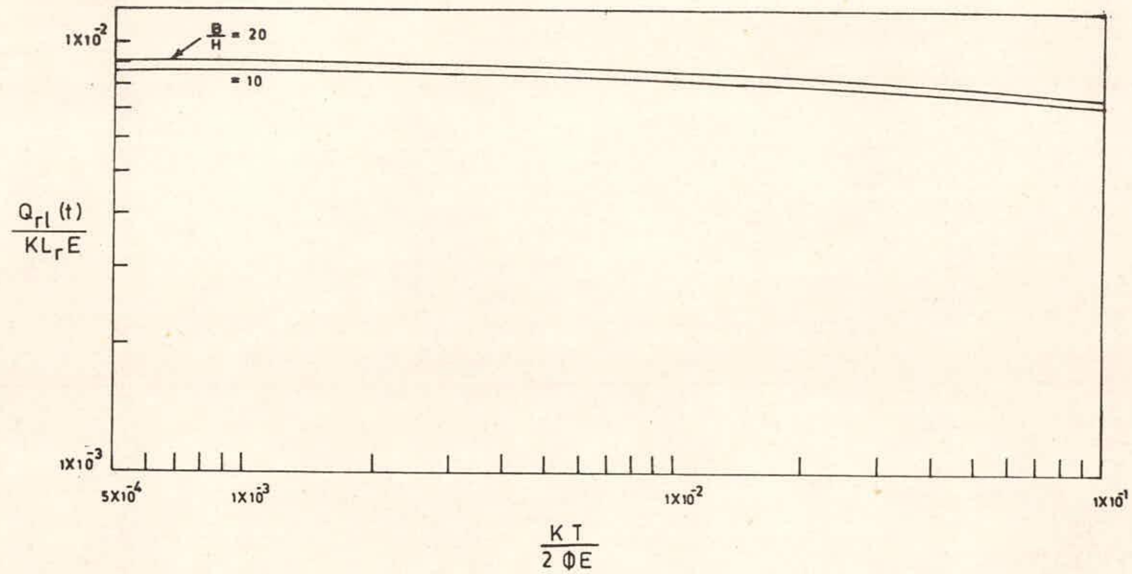

Fig.6 Variation of seepage from a canal when the flow rate is nonlinearly dependent on potential difference

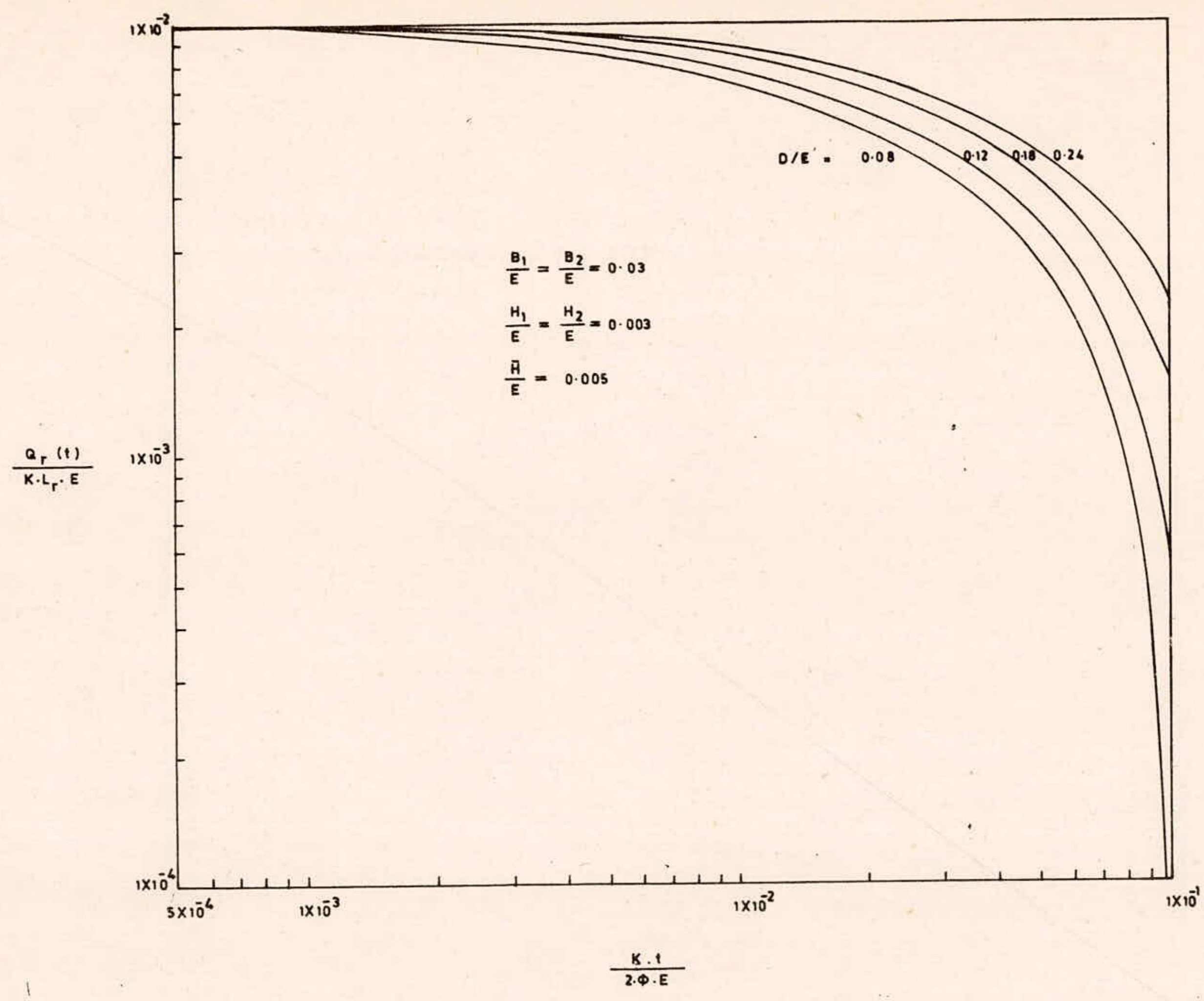

Fig.7 Variation of seepage rate with time for the lower canal for $B_1/E=B_2/E=0.03$

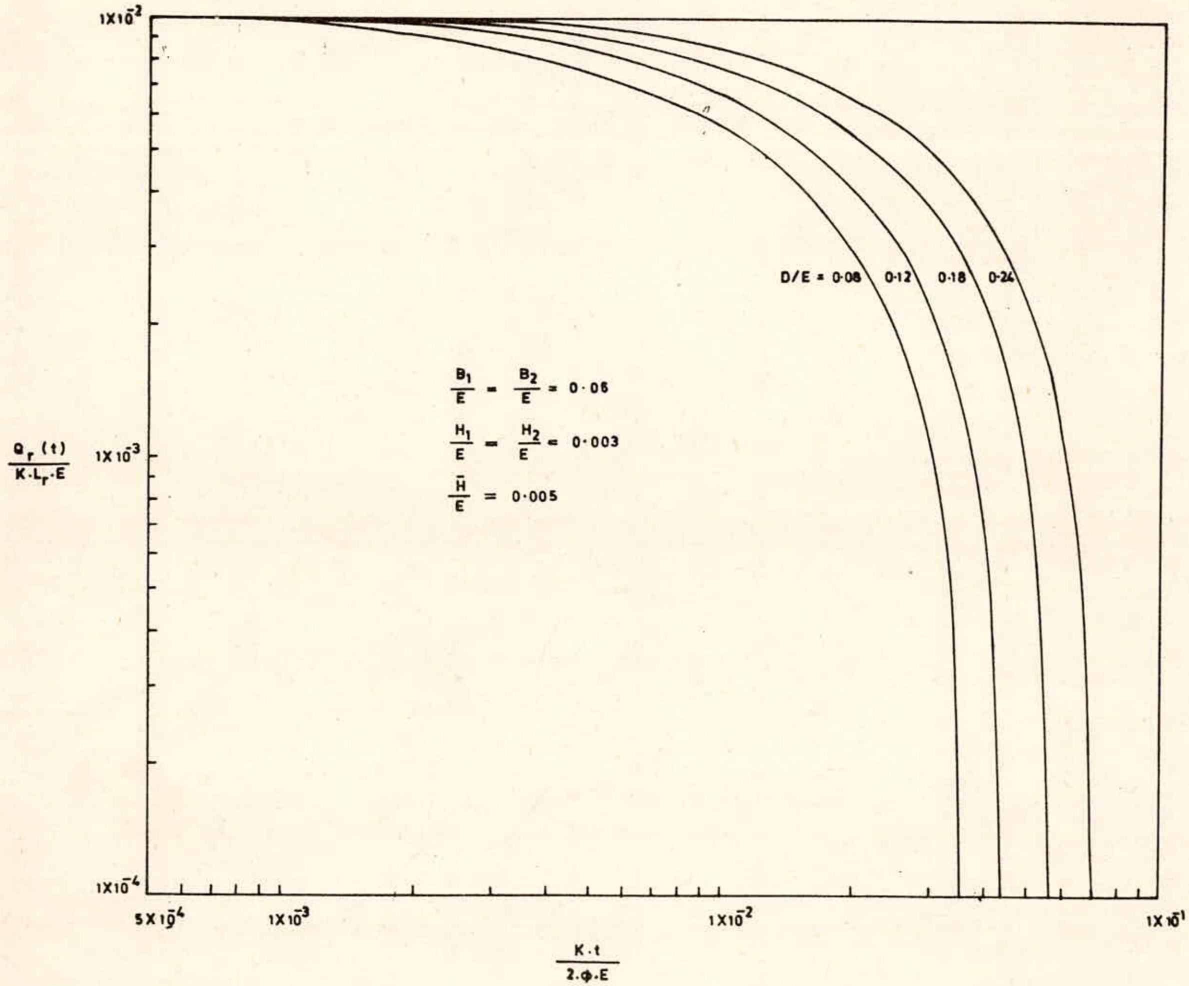

Fig.8 Variation of seepage rate with time for the lower canal for $B_1'/E = B_2/E = 0.06$

reduction in seepage is faster for canal with larger width due to rapid rise in water table. As seen from figures 7 and 8 the seepage rate from canal having 60m width is more than that of the canal having 30m width in the beginning of seepagge. At large time the seepage rate from canal having 30m width is more than the seepage rate of canal with 60m width. The variation of nondimensional seepage rate, $Q_r(t)/K L_r E$, with nondimensional time, $K.t/2\phi E$, has been presented in Fig.9 in a semilog plot. As seen from the figure the canal seepage reduces to zero and there after receives water from the aquifer. When the spacing is 80m for K=0.1m./day and ϕ =0.1, the time at which the seepage rate reduces to zero is 73 days. When the spacing is 120m., 180 and 240m. the corresponding times are 89, 114, 142 days respectively.

The evolution of water table has been presented for spacing between canals equal to 180m (D/E=0.18). The rise in water table under the lower canal at t=114 days is 8m and there after it remains constant due to imposition of boundary condition. The maximum rise in water table occurs under the ridge canal. The maximum rise of water table after 180 days is 14.3m which is equal to 1.43% of the saturated thickness.

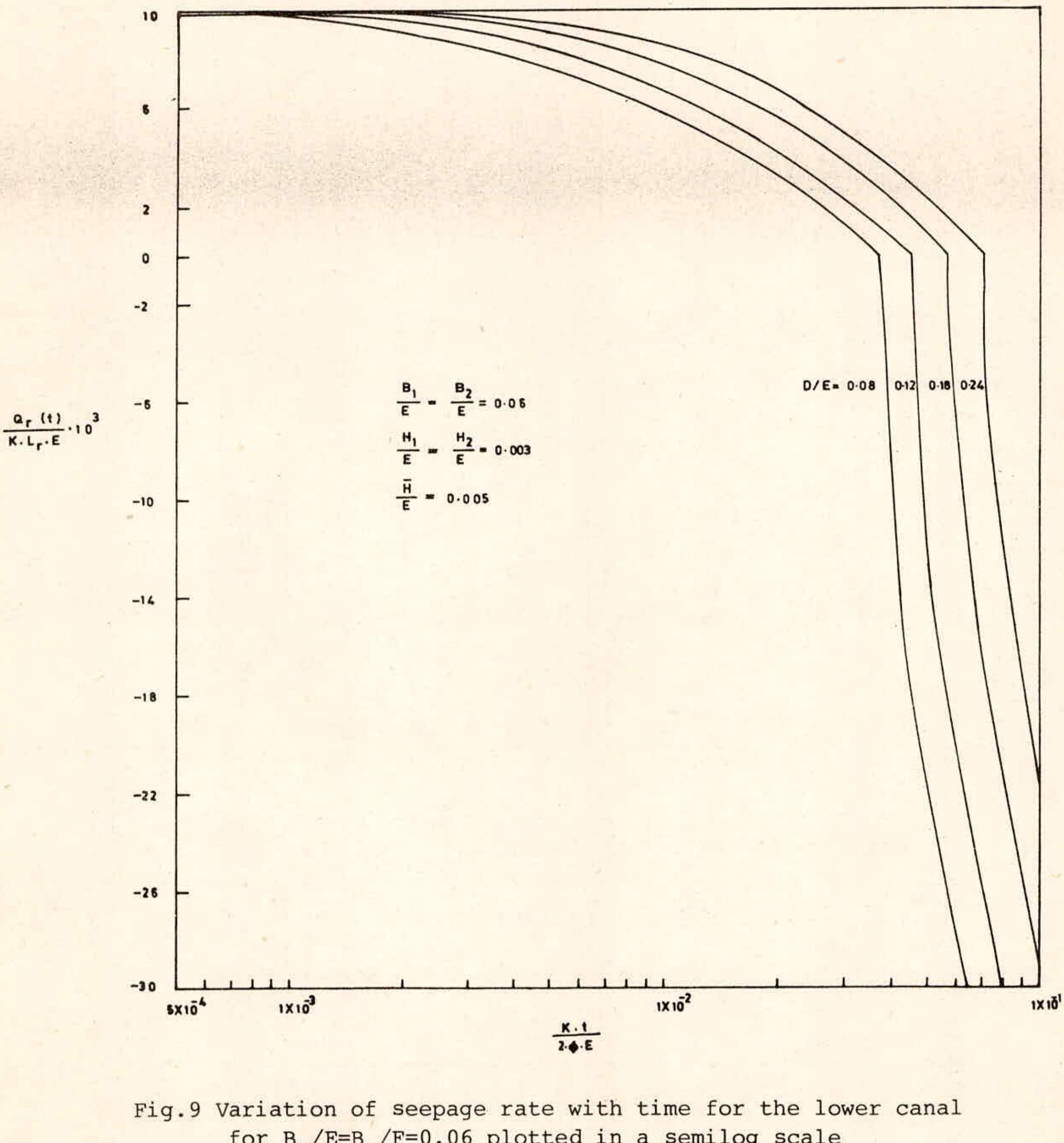

Fig.9 Variation of seepage rate with time for the lower canal for $B_1/E=B_2/F=0.06$ plotted in a semilog scale

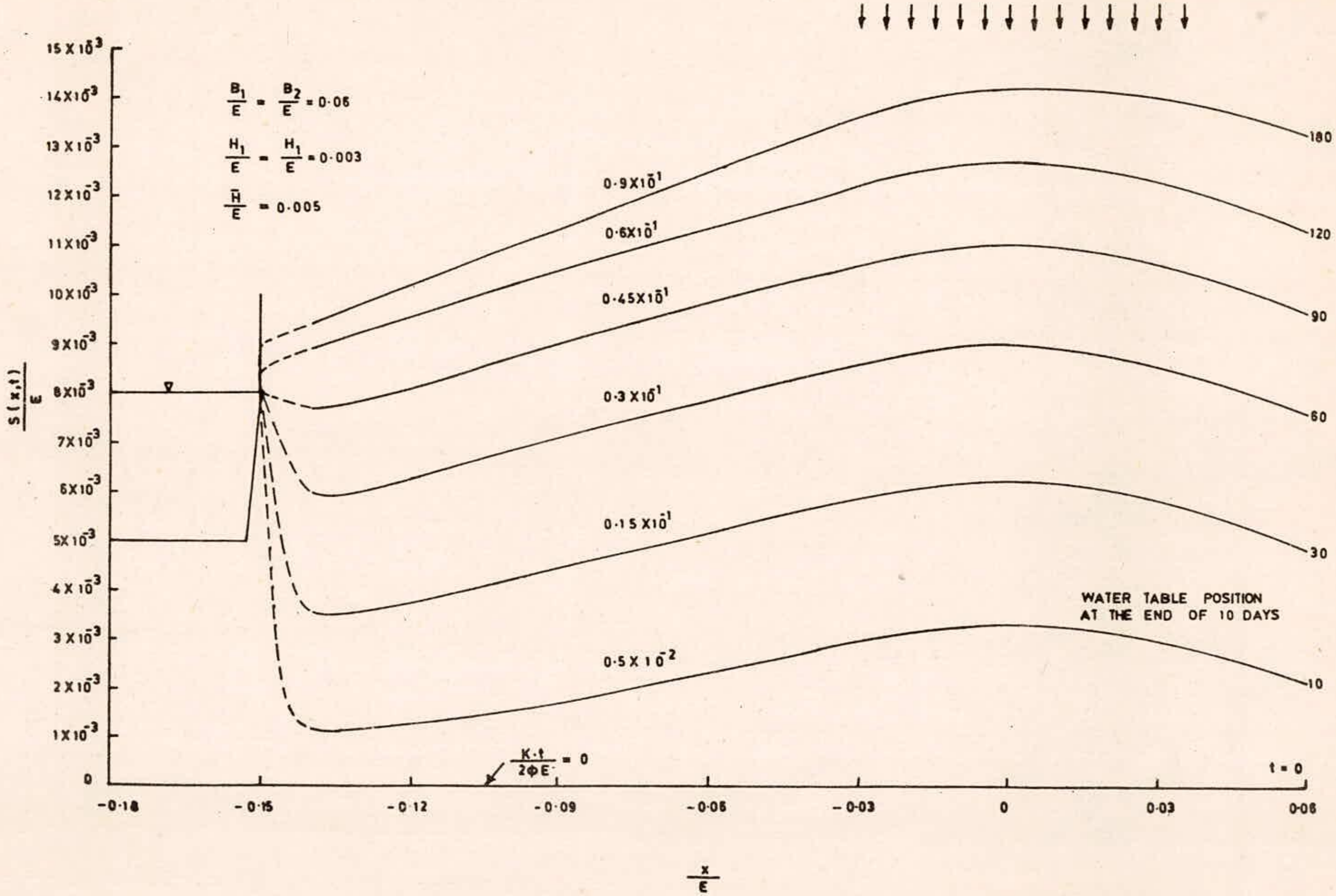

Fig.10 Evolution of water table for D/E=0.18

5.0 CONCLUSIONS

Unsteady seepage from a canal has been evaluated considering the flow rate to be nonlinearly dependent on the potential difference between the canal and the aquifer under the canal. The exponential relation between flow rate and potential difference proposed by Rushton and Redshaw has been used in the analysis. The constants which appear in the exponential relation have been evaluated analytically. It is seen that there is no significant difference in the results pertaining to linear and nonlinear cases.

A mathematical model has been developed to predict interference of two parallel canals. One of the canals is hydraulically connected with the aquifer. The temporal variation of seepage from the canal connected hydraulically with the aquifer has been quantified. The evolution of water table with time due to seepage from the canals has been determined. The model predicts the time at which the seepage from the canal, having hydraulic connection with the aquifer, reduces to zero. The model also quantifies the rate at which the canal receives water as a drain.

REFERENCES

1. Abdul Razzak, M.J., and H.J.Morel-Seytoux.1983.Recharge from an ephemeral stream following wetting front arrival to water table. Water Resources Research, Vol.19, No.1, pp.194-200.

2. Aravin, V.I., and S.N.Numerov.1965.Theory of Fluid Flow in Under formable Porous Media. Israel Program for Scientific Translation.

3. Besbes, M.,J.P.Delhomme and G.deMarsily.1978.Estimating recharge from ephemeral streams in arid regions. Water Resources Research, Vol.14, No.2, pp.281-290.

4. Bouwer, H.1969.Theory of seepage from open channels. Advances in Hydroscience, edited by V.T.Chow.5,pp.121-172, Academic Press.

5. Bhargava, D.N., G.C.Mishra and S.Chandra.1986.Seepage from two parallel canals. TN 27, NIH, Roorkee.

6. Dillon, P.J.1983.Boundary integral model of stream aquifer interaction. Hydrology and Water Resources Symposium, Institution of Engineers, Australia, Hobart.

7. Dillon, P.J.1984.Ephemeral stream aquifer interaction, Ph.D. Thesis, Dept. of Civil Engg.,University of Adelaide, Australia.

8. Dillon, P.J., and J.A.Liggett.1983.An ephemeral stream- Aquifer Interaction Model. Water Resources Research, Vol.19, No.3, pp. 621-626.

9. Flug, M.,G.V.Abi-Ghanem and L.Duckstein.1980.An event based model of recharge from an ephemeral stream. Water Resources Research, Vol.16, No.4, pp.685-690.

10. Glover, R.E.1974.Transient Ground Water Hydraulics. Water Resources Publication.

11. Hantush, M.S.1967.Growth and decay of ground water mounds in response to uniform percolation. Water Resources Research, Vol.3, pp.227-234.

12. Harr, M.E.1962.Ground Water and Seepage.McGraw Hill Book Company.

13. Herbert, R.1970.Modelling partially penetrating rivers on aquifers model. Ground Water, Vol.8, pp.29-36.

14. Kovacs, G.1981.Seepage Hydraulics, Elsevier scientific publishing company.

15. Kozeny, J.1931. Groundwasserbewegung bei freiem Spiegel, Fluss and Kanalversickerung, Wasserkraft und Wasserwirtschaft,no.3.

16. Morel-Seytoux, H.J.1975. Optimal legal conjunctive operation of surface and ground waters. proceedings, 2nd World Congress, IWRA, Vol.IV, pp.119-129.

20. Morel-Seytoux, H.J., and C.J.Daly.1975.A discrete kernel generator for stream aquifer studies. Water Resources Research, Vol.11, No.2, pp.253-260.

21. Morel-Seytoux, H.J.,T.Illangasekare, G.Peters.1979.Field verification of the concept of reach transmissivity. Proceedings of the Canberra Symposium, December, IAHS-AISH Publ.No.128,pp.355-359.

22. Muskat,M.1946.The Flow of Homogeneous Fluid through Porous Media. J.W.Edwards, An Arbor.Michigan.

23. Risenkampf, B.K.1940.Hydraulics of soil water, Seriyamat.Mekh, Vol.15.

24. Rushton, K.R., and S.C.Redshaw.1972. Seepage and Ground Water Flow. John Wiley and Sons.

25. Shestakov, V.M.1965. Theoretical principles of the evaluation of efflux, water level drop, and drainage (in Russian), IZd.M G U.

26. Streltsova, T.D.1974.Method of additional seepage resistances-theory and application. J.Hydraul.Div.,Am.Soc.Civil.,100,HY8, 1119-31.

27. Verdernikov, V.V.1934.Seepage from channels, Gosstroiizdat, Wasserkraft and Wasserwirtschaft, Nos.11, 12 and 13.