STATISTICAL MODELLING OF RAINFALL DATA

1.0   INTRODUCTION

Hydrological data for water resources development and management should be consistent and homogeneous when they are used in frequency analyses or system simulations. A time series of hydrological data may exhibit jumps and trends owing to inconsistency and non-homogeneity. Inconsistency is a change in the amount of systematic error associated with the recording of data. It can arise from use of different instruments or methods of observations. Non-homogeneity is a change in the statistical properties of the time series. It is caused by both natural and man made processes. The man made changes include land use changes, impounding of waters and construction of diversion structures, etc.

A simple but efficient procedure for screening the hydrological data is to test the monthly, seasonal or annual time series for stationarity and for absence of trend. A time series of hydrological data is strictly stationary if its statistical properties e.g. mean and variance are not affected by the choice of the length and time period of data. Although stability of mean and variance indicates only a weak form of stationarity, this is enough to identify the non-stationarity of a time series. The tests for stability of variance and mean verify not only the stationarity of a time series but also its consistency and homogeneity. In the basic data screening procedure, these two tests are reinforced by a third one namely for absence of trend. If required, this basic screening procedure could be extended to include tests for absence of persistence using serial correlation and relative homogeneity and consistency using double mass analysis.

2.0   DATA SCREENING PROCEDURE

The data screening procedure consists of four principal steps. These are:

(i)   A rough screening of the data and computation to verify the totals for the hydrological year or season
(ii)   Plotting the totals according to a chosen time step say monthly, seasonal or annual and note any trend or discontinuities
(iii) Test the time series for absence of trend with Spearman's rank correlation method
(iv)   Apply the F test for the stability of the variance and the t test for the stability of the mean to split non-overlapping sub sets of the time series

The above steps form what was referred to earlier as the basic screening procedure. If need be the procedure can be extended to include two additional steps. These are:

(i)   Testing the time series for absence of persistence by computing the first serial correlation coefficient.
(ii)   Testing the time series for relative consistency and homogeneity with double mass analysis.

Together, the two sets of steps form the complete data screening procedure which is illustrated through a flow chart in Fig. 1.

2.1   Rough Screening of Data

The basic screening procedure begins with an initial rough screening of the data. For example in the case of rainfall this would facilitate a visual detection of whether the observations have been

consistently and wrongly entered on a wrong date, whether they show gross errors or whether they contain misplaced decimal points. Verifying the completeness of the data and checking the observer's arithmetic would be a useful exercise. Distinction should be made between data not available and nil(0) and missing data should be indicated clearly.
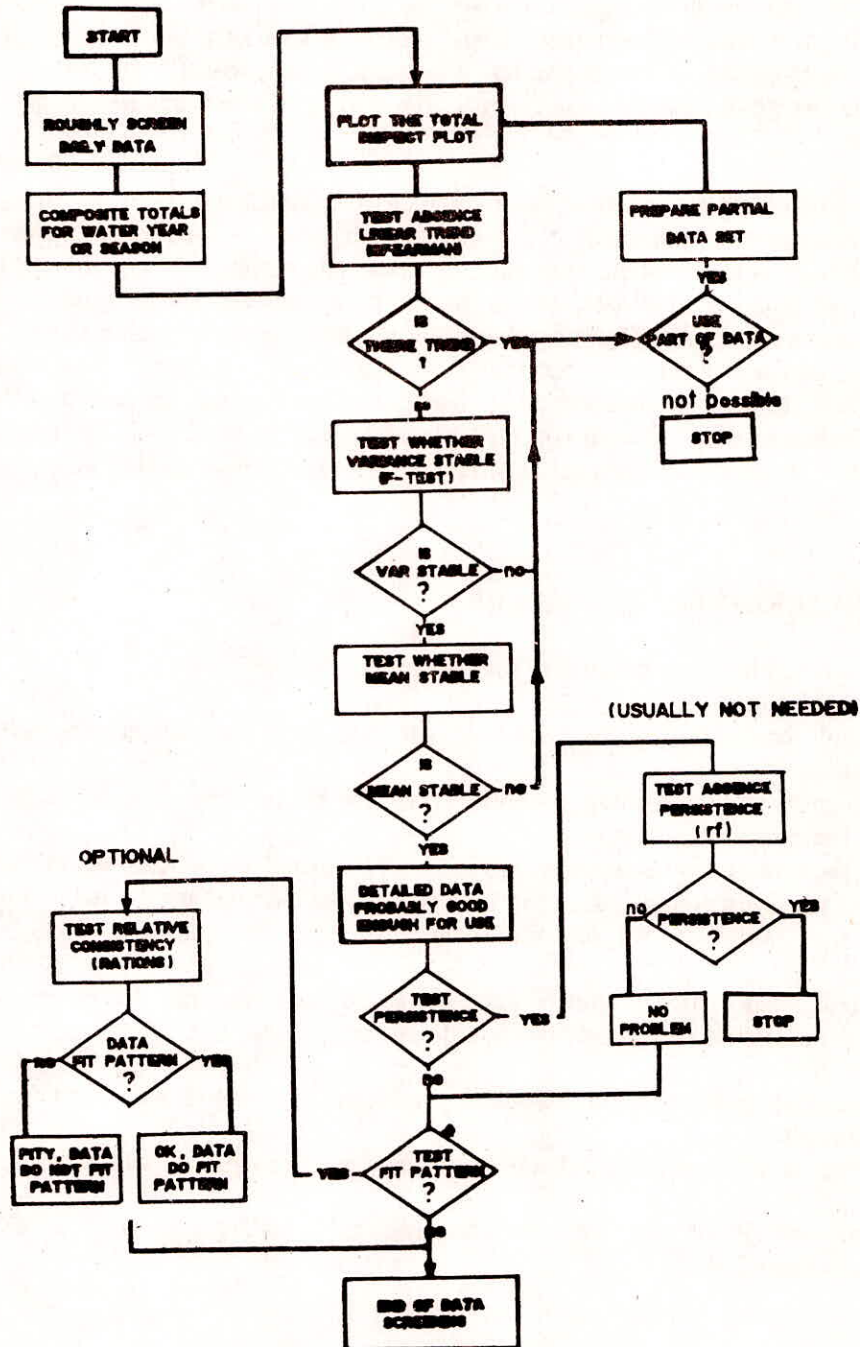

Fig. 1 : The Data Screening

2.2   Plotting the Data

After doing a rough screening of the data, the data are plotted on a linear or a semi-logarithmic paper. For this purpose the options available in the general purpose software GRAPHER can be used. In Fig. 2 the monsoon (Jun-Oct.) rainfall time series of a rainfall station Visakhapatnam in Andhra Pradesh are shown. It may be seen that the series does not show any obvious trend or discontinuities.
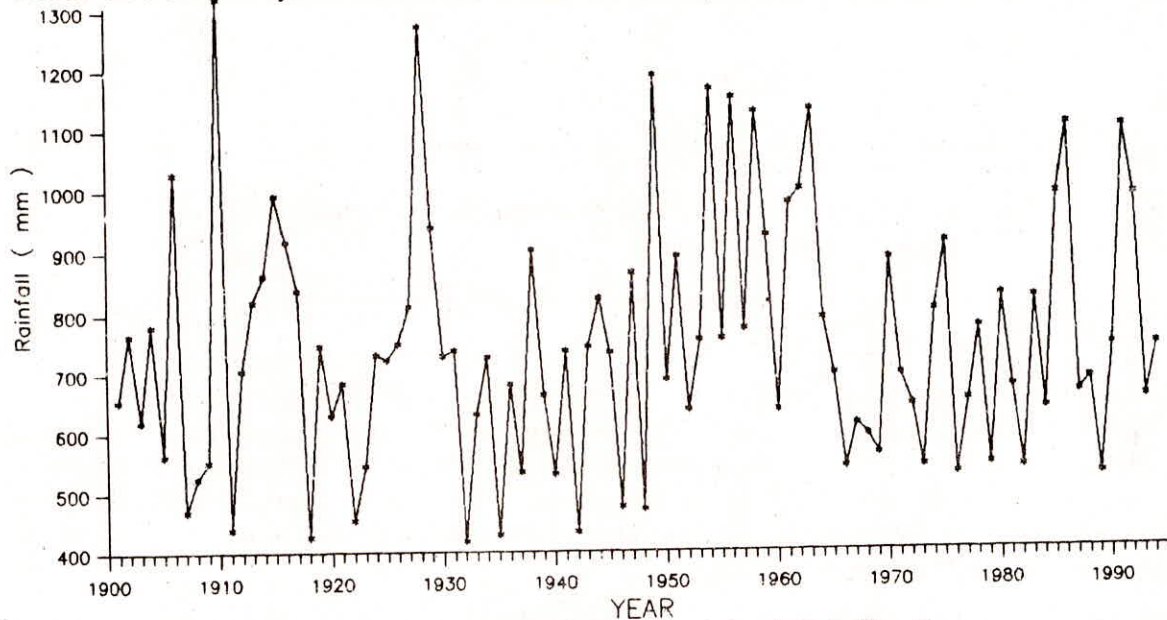

Fig. 2 : Time Series of Seasonal (Jun- Oct) Rainfall Totals (mm) at Visakhapatnam

2.3   Test for Absence of Trend

After plotting the time series, it must be ensured that there is no correlation between the order in which the data has been collected and the increase or decrease in magnitude of those data ; that is to say the occurrence of the events in the sequence is purely random, It is common practice to test the whole time series for absence of trend. Specific periods of the time series can also be chosen for examining the trend if there are reasons to suspect the possibility of existence of such a trend. However, it is not advisable to test for periods shorter than say 10 or 15 years.

A simple means of checking for the trend is by fitting a linear or a polynomial regression for the time series using for example seasonal or annual rainfall as the dependent variable and the year as the independent variable. The linear regression will be of the form as:

$$y = m x + C \qquad\qquad (1)$$

where x is the independent variable which number in the order of observation or the year. A positive value of m indicates a rising trend and a negative value would indicate a falling trend. A polynomial regression would have higher order terms and would indicate curvilinear trend. The options available in GRAPHER can be used for this purpose.

Another method is the Spearman's Rank correlation. It is simple and does not involve assumption of any underlying statistical distribution. Yet another advantage of the method is its nearly uniform power for linear and non-linear trends. The method is based on the Spearman's rank correlation coefficient R_{sp} which is defined as:

$$R_T = \frac{6 * \sum_{i=1}^{n} D_i^2}{n*(n^2-1)} \tag{2}$$

where, n is total number of data, D is the difference and i is the chronological order number. The difference in the rankings is computed with:

$$D_i = K_{xi} - K_{yi} \tag{3}$$

where, K_{xi} is the rank of variable x which is the chronological order of number of observations. The series of observations y_i is transformed to its rank equivalent K_{yi}, by assigning the chronological order number of an observation in the original series to the corresponding order number in the ranked series, y. If there are ties i.e. two or more ranked observations with the same value, the convention is to take K_x as the average rank. The significance of the Spearman rank correlation coefficient can be tested by computing the test statistic t as:

$$t_t = R_T \left[\frac{n-2}{1-R_T^2} \right] \tag{4}$$

where t_t has Student's t distribution with n-2 degrees of freedom. Generally the t value is tested for the 5 percent level of significance.

Examples : Let us apply the above method and find out if there is any trend in the time series of the seasonal rainfall totals of Visakhapatnam given in Fig. 2.

The t_t value corresponding to the Spearman's correlation coefficient is 1.00. Since 94 years of data is used, the degrees of freedom n - 2 is 92. The 5 % confidence limits for 92 degrees of freedom are (92, 2.5%) and (92, 97.5%) which are - 1.98 and +1.98 respectively. By putting - 1.98 < ? 1.00 ?< 1.98 it may be seen that the Spearman's rank correlation coefficient for the series in figure 2 is not significant indicating the absence of any trend.

Let us now apply the method to a non stationary time series of annual total rainfall at Station Pratapgarh in Rajasthan (Fig. 3).
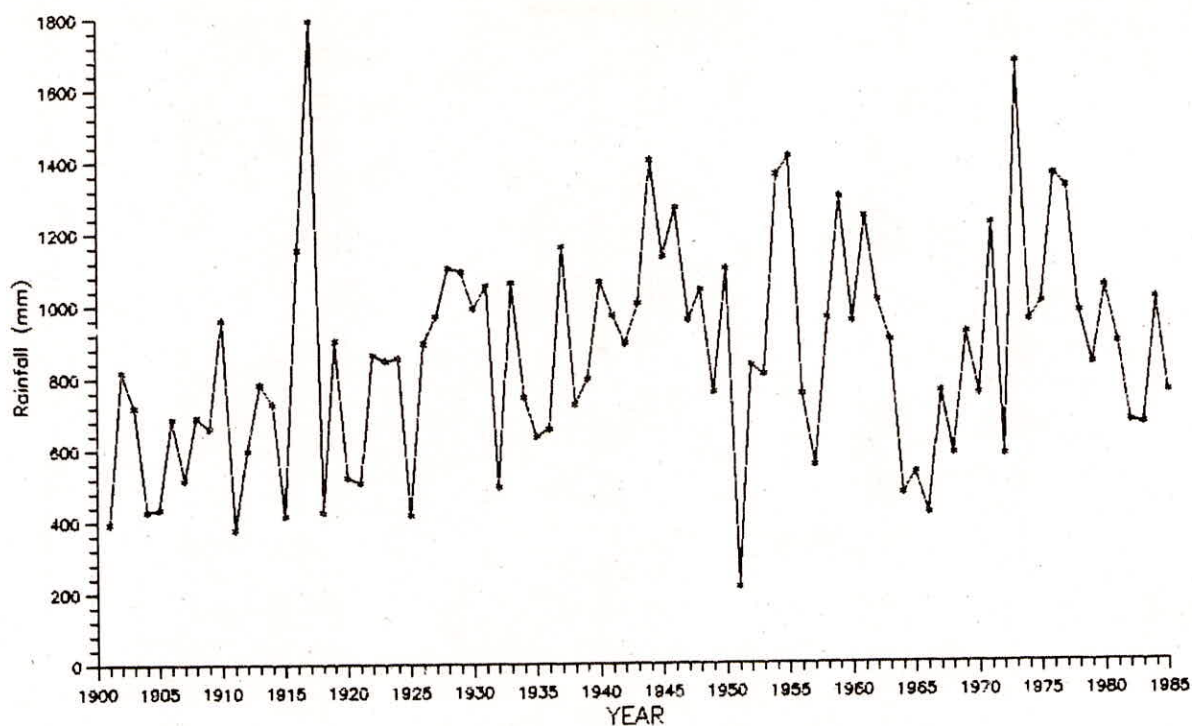
Fig. 3 : Time Series of Annual Rainfall Totals (mm) at Pratapgarh

The linear trend indicates a value of 3.42 indicating a rising trend. However, the application of the polynomial regression indicated the presence of curvilinear trend as shown in Fig. 3. The 't' value corresponding to Spearman's rank correlation is 2.993. The value of 't' are checked with the limits of the table 't' corresponding to 83 degrees (85-2) of freedom which are -1.99 and 1.99. Thus by putting -1.99 < ? 2.993 ? < 1.99 it may be seen that the value of the t corresponding to Spearman's correlation is significant. Similarly, the 't' value corresponding to the linear trend is 2.63 which is also significant at the 5% level of significance. The time series of Pratapgarh annual rainfall have, therefore, a positive trend.

The conclusions from the above tests can be further confirmed by carrying out the tests for stationarity i.e. the F test for the stability of the variance and the 't' test for the stability of the mean.

2.4   Tests for Stability of Variance and Mean

In addition to testing the time series for absence of trend, the series are also tested for stability of variance and mean. The stability of the variance is tested first. This is because if the variance is not stable the series could not be used for further analysis.

Table 1:

Percentile Points of the F-Distribution $F\{v_1, v_2, p\}$ for the 5-Per-Cent Level of Significance (Two-Tailed)

v_2	$p = P(F \le F_p)$	4	5	6	7	8	9	10	11	12	14	16
5	0.025	.107	.140	.169								
	0.975	7.39	7.15	6.98								
6	0.025		.143	.172	.195							
	0.975		5.99	5.82	5.70							
7	0.025			.176	.200	.221						
	0.975			5.12	4.99	4.90						
8	0.025				.204	.226	.244					
	0.975				4.53	4.43	4.36					
9	0.025					.230	.248	.265				
	0.975					4.10	4.03	3.96				
10	0.025						.252	.269	.284			
	0.975						3.78	3.72	3.66			
11	0.025							.273	.288	.301		
	0.975							3.53	3.47	3.43		
12	0.025								.292	.305	.328	
	0.975								3.32	3.28	3.21	
14	0.025									.312	.336	.355
	0.975									3.05	2.98	2.92

v_2	$p = P(F \le F_p)$	16	18	20	24	30	40	60	100	160	∞
16	0.025	.342	.362	.379							
	0.975	2.82	2.76	2.71							
18	0.025		.368	.385	.400						
	0.975		2.64	2.60	2.56						
20	0.025			.391	.406	.430					
	0.975			2.50	2.46	2.41					
24	0.025				.415	.441	.468				
	0.975				2.33	2.27	2.21				
30	0.025				.453	.482	.515				
	0.975				2.14	2.07	2.01				
40	0.025					.498	.533	.573			
	0.975					1.94	1.88	1.80			
60	0.025						.555	.600	.642		
	0.975						1.74	1.67	1.60		
100	0.025							.625	.674	.706	
	0.975							1.56	1.48	1.44	
160	0.025								.696	.733	
	0.975								1.42	1.36	
∞	0.025										1.00
	0.975										1.00

Test for Stability of Variance:

The test statistic for the F test is the ratio of the variances of two split, non-overlapping, sub-sets of the time series. Thus the test statistic reads:

$$F_t = \frac{\sigma_1^2}{\sigma_2^2} = \frac{S_1^2}{S_2^2} \tag{5}$$

where S^2 is variance. For computation of variance it is immaterial whether the population standard deviation (using n) or the sample standard deviation (using n-1) is used. The computed F_t is tested using the table values of F (Table 1) for the degrees of freedom ν_1 and ν_2 corresponding to n_1 and n_2 the number of values respectively in each sub-set.

Table 2 :   **Percentile Points of the T Distribution t { ν , p }
for the 5 % Level of Significance**

$p = p(t < = t_p)$	0.025	0.975
ν : 4	- 2.78	2.78
5	- 2.57	2.57
6	- 2.54	2.54
7	- 2.36	2.36
8	- 2.31	2.31
9	- 2.26	2.26
10	- 2.23	2.23
11	- 2.20	2.20
12	- 2.18	2.18
14	- 2.14	2.14
16	- 2.12	2.12
18	- 2.10	2.10
20	- 2.09	2.09
24	- 2.06	2.06
30	- 2.04	2.04
40	- 2.02	2.02
60	- 2.00	2.00
100	- 1.98	1.98
160	- 1.97	1.97
∞	- 1.96	1.96

Note : It is customary to take the next higher value if the required number of degrees of freedom are not listed in the table

Test for Stability of Mean:

The 't' test for the stability of the mean requires computing and then comparing the means of two or three sub-sets of the time series. The same sets used for testing the stability of the variance may be

used. The test statistic is:

$$t_t = \frac{\bar{x}_1 - \bar{x}_2}{\left[\dfrac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2} * \left(\dfrac{1}{n_1} + \dfrac{1}{n_2} \right) \right]^{0.5}} \tag{6}$$

where n is the number of data in each of the sub sets, x is the mean of the sub-set and S^2 is the variance of the sub set. In samples from a normal distribution $_t$ follows Student's t distribution. However, the condition of normality is not as stringent as for the F test. One can apply the t test to data of any frequency distribution but the length of the data in each of the subsets should be equal if the distribution is skewed i. e. not normal. The computed t_t is tested using the table values of t (Table 2) for the degrees of freedom ν_1 and ν_2 corresponding to n_1 and n_2 the number of values respectively in each sub-set.

Example : The tests described above for testing the stationarity of the series are illustrated by their application to the series of seasonal total rainfall at Visakhapatnam and Pratapgarh shown in Fig. 2&3.

In the case of the seasonal total rainfall series at Visakhapatnam the degrees of freedom corresponding to the two sub-sets ν_1 and ν_2 are 46 and 46 respectively. The lower and upper limits of F value from the table corresponding to 46 and 46 are 0.55 and 1.86 respectively. The computed F_t value for the observed seasonal rainfall series of Visakhapatnam is 1.008 which is within the limits and, therefore, the variance of the series is stable. Similarly the lower and upper limits of the table values of 't' are -1.99 and +1.99. The computed value of 't_t' which is - 1.683 is within the limits and the mean of the series is stable. The series, therefore, could be considered stationary.

The annual total rainfall series of Pratapgarh which was found to be having a trend through the test for absence of trend was further subjected to the tests of stability of variance and mean. The values of F_t and t_t are 0.835 and -2.389 respectively. The total number of values n of the Pratapgarh series is 85. Therefore the value of ν is 83 and those of ν_1 and ν_2 are 41 and 42 respectively and the lower and upper confidence limits for F corresponding these are 0.54 and 1.87. The computed value of F_t is within these limits and the variance of the series, therefore, could be considered as stable. However, the stability of the mean is still to be tested.

The lower and upper limits of the confidence limits for 't' from the table are -1.99 and + 1.99. The value of 't_t' which is - 2.389 is beyond these limits and the mean of the series is therefore not stable. As such the annual rainfall series of Pratapgarh cannot be considered as stationary.

2.5   Test for Absence of Persistence

The time series of yearly and seasonal totals are usually independent. However, there may be occasional cases where the series have some dependence on the previous year or season's value. For example rivers with large catchments have considerable carry over groundwater storage from one season to another season or from one year to another year. In such cases it is imperative to the series for the absence of persistence before they are used for any further analysis.

Serial Correlation:

The serial correlation coefficient can help to verify the independence of a time series. If a time series is completely random, the population auto correlation function will be zero for all lags other than zero. For lag zero the value of serial correlation will be 1.00 as all series are perfectly correlated with themselves. For the purpose of checking the independence of the series it is enough if the serial correlation is computed for the lag 1. The lag 1 serial correlation coefficient r_1 is given by :

$$r_1 = \frac{\sum\limits_{i=1}^{n-1}(X_i - \bar{X}) * (X_{i+1} - \bar{X})}{\sum\limits_{i=1}^{n-1}(X_i - \bar{X})} \tag{7}$$

where x is one observation and x is the following observation. $\bar{X}$ is the mean of the time series and n is the number of data. The series is considered to be independent if the values of lag 1 serial correlation coefficient are close to zero. However, in practice it is generally not possible and so if the values are within the acceptable levels corresponding to 5% level of significance the series is considered to be independent.

For example the lag 1 serial correlation in case of Visakhapatnam is 0.03 and the corresponding value for Pratapgarh is 0.19. Both these values are within the acceptable limits of -0.22 to +0.20 and the series can, therefore, be considered to be independent.

2.6   Relative Consistency and Homogeneity

Although the basic data screening procedure uses the statistics to test the individual time series of hydrological data, plotting the cumulative departure from the mean can be very helpful in testing such a time series for stability of mean. In Table 3 the cumulative departures from a stationary series X_1 and non stationary series X_2 are given.

Table 3 : Cumulative Departures from the Means of Two time Series

i	Time Series			
	I	I	II	II
	$\bar{X}1_i$	$\Sigma \bar{X}1_i - \bar{X}1$	$\bar{X}2_i$	$\Sigma X2_i - \bar{X}2$
0		0.00		0.00
1	1566	70.09	1387	85.64
2	1561	135.18	1450	234.27
3	1414	53.27	1584	516.91
4	1496	53.36	1423	638.55
5	1411	-31.55	1775	1112.18
6	1984	456.55	1187	997.82
7	1279	239.64	1225	921.45
8	1251	5.27	950	570.09
9	1799	297.82	1221	489.73
10	1330	131.91	987	175.36
11	1364	0.00	1126	0.00
	$\bar{X}1 = 1495.91$		$\bar{X}2 = 1301.36$	

By an examination of the cumulative departures it is easy to identify the breaks. For example in the above example there are no real differences in the time series I and the cumulative departures from the mean fluctuate randomly. Time series II on the other hand shows a positive trend in the first five years and a negative trend in the later years. A number of authors have proposed various procedures for locating the break points in plots of cumulative departures from the mean. The Computations involved are, however, very elaborate. The significance of the break, therefore, can be only ascertained by using the F test for stability of variance.

Double Mass Analysis:

A time series of hydrological data is relatively consistent if the periodic data are proportional to an appropriate time series. In other words, relative consistency means that the hydrological data at a certain observation station are generated by the same mechanism that generated similar rainfall or related data (runoff) at other stations. It is common practice to verify relative consistency with Double Mass analysis. The technique is described under a separate module and, therefore, not explained here.

3.0   SOFTWARE DATSCR

The easiest way to perform the data screening procedure is with the help of a dedicated computer programme. The computer program DATSCR developed by the International Institute of Land Reclamation and Improvement (ILRI Publ. No 49) is capable of running on computers with MS DOS. The description of DATSCR and details of its input data file are given in Appendix I.

4.0   CONCLUDING REMARKS

Reliable data are core of reliable hydrological studies. The extra effort required for screening of hydrological data before it is used is very negligible and is well worth the time as it will enhance the analyst's insight and understanding.

Most of the hydrological designers prefer long time series of hydrological data. However, the longer the time series the greater would be the chance that the series is neither stationary, consistent nor homogeneous. The later part of a time series, i.e. the recent data can represent better data set considering that the same conditions are likely to prevail for at least few years in future

It is customary to apply the screening procedure to time series of hydrological data over time periods of one year or season. It is however desirable to use the water year rather than the calendar year to avoid splitting of the wet season. It is some times better to carry out the analysis for individual seasons like wet (monsoon) season or dry (non-monsoon) season. Nevertheless the independence and acceptability of a time series depends on the level of aggregation and separation in time of the data points. For example successive hydrological events are not associated with the related weather systems is an essential pre-requisite for carrying out frequency analysis. Generally, time series of annual rainfall totals or flow volumes is regarded as statistically independent. Groundwater carry over and lake storage , however, can introduce persistence into a time series of flow volumes.

Rainfall records are extremely important. If they are consistent they are independent of the works of man, thus providing an index for evaluating changes in, for example stream flow.

**   **** **** ****

APPENDIX 1

HYDROLOGICAL DATA SCREENING 'DATSCR' (DOS VERSION)

To run the program (while being sure that COMMAND.COM is accessible), type: DATSCR <RETURN> at the DOS prompt while in the root directory. For making screen dumps via the <PRINT SCREEN> command, an appropriate version of GRAPHICS.COM should have been installed. Further information on the setup of disks and files is given.

The executable files DATSCR.EXE and FRONT.EXE are essential for the proper running of the package. DATED.EXE should be accessible when data files will have to be edited while running 'datscr'. DATED, however, can also be used independently.

<u>1. Using a single floppy drive:</u>

Normally the program will be run in a directory that contains the data to be analyzed while the programs and COMMAND.COM are in the root directory of the disk.When working in this setup the PATH and COMSPEC parameters should be set as follows:

         set PATH=A:\
         set COMSPEC=A:\COMMAND.COM

The computer's COMMAND.COM file should have been copied to the root directory of the disk. The actual setting of the two parameters can be checked by typing set<RETURN>.

<u>2. Using two floppy drives for 'datscr' only:</u>

Drive A: would normally contain the programs and the COMMAND.COM file in the root directory (or the programs would be in a separate directory 'dirname', c.f. section B.3) while the data are in directories on drive B:. When working in this manner the PATH and COMSPEC settings are:

         set PATH=A:\
         set COMSPEC=A:\COMMAND.COM
or: set PATH=A:\dirname\
         set COMSPEC=A:\COMMAND.COM

<u>3. Using a hard disk with or without a floppy drive:</u>

When the programs would be in a directory called 'dirname' on the hard disk C:, the COMMAND.COM file would usually be in the root directory of that disk. When using data files in any directory on drive A:, or in any directory of drive C:, the PATH and COMSPEC settings would be:

         set PATH= C:\; C:\dirname\
         set COMSPEC=C:\COMMAND.COM

Information on the programs and data file layout:

1. DATSCR.EXE

The current DOS version 2.1 of the program has been adapted for general usage. It therefore runs on machines with CGA, EGA, VGA and Hercules graphics adapters, under MS-DOS (tested on v3.3 and

higher) and at least one version of DC-DOS (DOSPLUS v2.1 albeit with a minor cosmetic flaw in inverse video in graphics mode). The program selects the highest available resolution automatically. The limitation to 156 data points is related to the capabilities of the above mentioned graphics adapters and generally available display screens.

2. DATED.EXE and DATSETUP.TXT

The data files for 'datscr' can be prepared in any editor or word processor that reads and writes ASCII files (also called plain text or DOS files). The layout of data files is fully described in the file DATSETUP.TXT on the distribution disk. Editors are useful when data lists can be imported from other files.

However, a dedicated editor 'dated', prepared by R Noorman of the International Institute for Hydraulic and Environmental Engineering (IHE) in Delft, the Netherlands, can be used independently from 'datscr' for data file preparation: type dated < RETURN > to run the program, and then either load a file for editing, or start filling the various fields with the required data, as in a spreadsheet. Alternatively, a file name can be appended to the command 'dated' (e.g. dated filename.dat < RETURN >) for loading an existing data file or opening a new file for saving.

The editor will automatically load the current file in use for data screening when called from within the program 'datscr'. Quitting the editor when called from 'datscr' will return the user to 'datscr' with the name of the original file as the default name for loading. A new version of the file can, however, always be saved in 'dated' (thus keeping the old version of the file available), and subsequently be loaded in 'datscr'. DATED does not automatically create backup files.

The length of the title field is limited to 57 characters. All observations are multiplied by the multiplication factor after having been read into 'datscr'; this factor thus offers the possibility to work with lists of data of which the mean (= multiplication factor) and fractions of the mean (= observations) have been published, or to convert observations in inches to values in millimetres, or to apply similar conversions.

The number of observations (line 3 of 'dated') determines the number of data that can be entered and the number of data that will be written to disk when saving the file. New slots for data entry are opened (and initialized to 0.0000) when a number greater than the current number is entered in line 3. The number of available data entry slots is diminished when the number in line 3 is reduced. Data in then obscured slots are, however, not immediately lost: they will reappear when the number of observations is increased again. These obscured data are, however, not written to the output file.

Data may be inserted at any point in the data list by pressing the INS key, or deleted by pressing DEL. The number of observations is automatically increased when using INS, but it is not automatically decreased when using DEL: slots set to 0.0000 will appear at the end of the list. Such slots can de deleted by adjusting the number of observations in line 3.

Data files that are being edited may be read from or written to any directory but only the xxxx.dat files in the currently selected directory are shown on the screen while running the program 'dated' (or 'datscr' for that matter).

3. FRONT.EXE

FRONT.EXE is the proprietary ILRI front end for DATSCR.EXE. It is loaded AFTER the graphics capabilities of the machine have been checked. The program 'front' will run on its own but DATSCR will call FRONT during execution. FRONT.EXE should therefore be present and accessible by setting the PATH parameter as required.

'dated.doc': on the layout of data files for 'datscr' (hydrological data screening)
(ILRI Publication No. 49)

The basic requirements for the layout of the data file are as follows:

1. Four label lines with an L (or l) in the first column shall precede data; comment lines may precede those label lines, but not follow such lines before the required number of data lines have been written.

2. The data lines following the four label lines shall contain:
  - after the 1st label line: the job title/identification (1 line)
  - after the 2nd label line: the number of observations (1 line)
  - after the 3rd label line: the multiplication factor (1 line)
  - after the 4th label line: the observations, one per line (n lines)

3. The list of observations shall be terminated with e or E (for end) in the first column

4. A <RETURN> (<ENTER>) shall be given after typing 'end'

Data lines start preferably with a blank ' ' in order to have portability on the screen when running 'datscr'; it is suggested to use apppropriate data formats (text, integer and real) although the program 'datscr' will convert reals to integers, and integers to real, where needed.

The text after the L in the label lines is ignored by the program; similarly any text after the 'e' of end is ignored.

Text after the 58th position of the title will not be shown on the screen during the running of the program and is lost when the data file is edited in 'dated'; a blank in the first column is useful for proper screen layout

All observations are multiplied by the multiplication factor after having been read into 'datscr'; this factor thus offers the possibility to work with the mean (= multiplication factor) and fractions of the mean (= observations), or to convert observations in inches to values in millimetres, or to apply similar conversions.

Here comment lines have a 'c' in the first column but this is not needed; an example of comment lines with a blank in the first column is given by before the label line for the multiplication factor in the following part of this file; but comment lines should of course not start with an L or l!

Comment lines are not only skipped by the program 'datscr' but also by the data editor 'dated' when loading files; this file can7 be edited in 'dated' but will then lose all comment lines in the process.

The lines after 'end of information' contain label lines, text and numbers that would result from editing this file in 'dated' (but the two comment lines would be absent and 'end of data' would be shortened to 'end'); on the other hand, the data editor would add -5, -10..... (at intervals of 5) after the observations to facilitate checking of the data input; such additions that follow the data are ignored by the program 'datscr' the superfluous zeros in data files prepared by 'dated' are the result of the write formats used in that program for input and output; they can be ignored; similarly can the blanks preceding the real numbers be ignored (but for the first one, see the remark in the second paragraph above!).

data file for 'datscr' (see file dated.doc for more information)

L job title, <= 58 characters...........................<

  MAY Maximum Temperature Series of VISAKHAPATNAM

L number of observations n

  44

L multiplication factor

          1.000

L obs 1.....n, one per line; terminate list with: end<ENTER> in 1st column

          32.60
          35.30
          36.70
          36.40
          33.30   - 5
          35.40
          36.30
          37.10
          36.40
          37.10   - 10
          36.90
          36.50
          35.80
          36.20
          36.80   - 15
          37.10
          36.60
          36.50
          34.40
          36.20   - 20
          34.00
          37.10
          36.80
          37.40
          36.90   - 25
          37.10
          35.10
          37.80
          36.20
          36.70   - 30
          36.30
          36.00
          36.80
          36.40
          37.40   - 35
          36.30
          35.10
          35.10
          36.00
          33.80   - 40
          36.20
          35.30
          34.10
          35.90
      end