LINEAR PERTURBATION MODEL

1.0 INTRODUCTION

River flow forecasting from the knowledge of the occurrence of the rainfall in the catchment is one of the frequently used technique by the hydrologists. This is used both for real time operation or for planning purposes. This is mainly because of the availability of rainfall data and the lag time that is available for preparedness.
Input rainfall are converted to corresponding runoff through a model. Such models are constructed and calibrated based on the observed series of rainfall and runoff. The structure of the model may differ from one another. The complexity included in these models vary from model to model.

1.1 Model Requirements

In hydrological models the physical processes occuring in the catchment are described to the extent possible to achive the objectives. They are having definite advantage over purely empirical formulae as the parameters used in the model can be assigned a definite physical interpretation.
A models should:

1) Represent as closely as possible the actual processes occurring in the catchment;
2) It should transform the input accurately into the output;
3) It should be consistent through different samples of the data;
4) It should be versatile i.e. provide accurate and consistent results when subjected to diverse applications.

1.2 Model Development

The model development consists of three stages viz i) Mathematical structure ii) Calibration and iii) Validation. Mathematical structure depends on the conceptual view taken by the modeller for the specific problem. This depends on the data availability, the accuracy/details needed, and the time available. Calibration of the model is estimation of parameters for a particular catchment that can best reproduce the outputs. These estimated parameters are used and validation is carried out. The available data are aportioned into two slots viz. one for calibration and the other for validation. It is advantageous to use larger data for calibration. The distinction of the calibration and verification periods is to test the consisitency of the model. Obviously they can be finally combine to recalibrate the model. Adative calibration is the use of data as soon as they are available. That is restimation of the parameters for every new observation. It is difficult to comphrehend that hydrologic system changes in such an interval of time. Hence recalibration may not be of any real advantage except an attraction.

1.3 Efficiency Criteria

A model is judged on its performance in achieving its objectives i.e. reproducing the observed ones. Model accuracy is quantitatively expressed as

$$F = \Sigma (y_{oi} - y_c)^2 \qquad 1.0$$

Here F is a measure of residual error reflecting the ability of the model to reproduce the observed phenomena. This expresses the model accuracy and can be used to compare application of different models on to a single catchment. Defining an initial variance, models can be compared based on a dimensionless number. Just as the use of coefficient of determination in linear regression efficiency is defined by Nash et al (1970).

Initial variance:

$$F_0 = \Sigma (y_{oi} - y_{av})^2 \qquad 2.0$$

$$y_{av} = (l/n) \sum_{i=1}^{n} y_{oi}$$

n is the number of values of y in the data set.

The criteria now

$$R^2 = (F_0 - F)/F_0 \qquad 3.0$$

This criteria is identical to the coefficient of determination and varies between zero to one. It is usual to use the initial variance as F_0 for the verification of the model. Initial variance is preferred because it expresses a condition of zero model i.e. the average (of the observed values during calibration period) is the model prediction. Hence R^2 can take negative values in the verification period.

Similar criteria may also be used to express relative accuracies of different models wherein F_0 is replaced by that pertaining to one of the model. It is not appropriate to use average as a base model when the flow data shows large variations especially where rainfall is concentrated in a season. Garrick et al. (1978) proposed seasonal model as the base model.

1.4 Gainfactor

The ratio of total output to the total input is known as gainfactor (G) which is non dimensional.

$$G = \Sigma y_i / \Sigma x_i$$

In case of the input rainfall and the output discharge the gainfactor will normally be less than one.

2.0 SEASONAL MODEL

This model only needs the observed flow data. This does not relate any input to the output. The data pertaining to calibration period is used for developing this model which is given below:

$$y_d = (1/n) \sum_{r=1}^{n} y_{d,r} \qquad 4.0$$

where $y_{d,r}$ is the observed discharge on date d, of the year r, and n is the number of years of record used for calibration. This model takes the observed date-wise average is the best estimate. These average values take seasonal variations into account.

Now

$$R^2 = (F_d - F)/F_d \qquad 5.0$$

where F_d is found from eq. 2.0 by replacing y_{av} by y_d.
Garricks suggestion is that one should judge the model performance with the best extimates which could be obtained without any model. As n becomes large, the estimate of Y_d is expected to have a smooth variation. Seasonal Character of the runoff can be seen in many catchments due to seasonal variation of the potential evaporation. It is found that one of the main cause of non-linearity present in the the rainfall-runoff relation is this seasonality.

3.0 LINEAR MODEL

Ever since the application of linear Systems in hydrology by Sherman in 1936, this kind of analysis has been in use as an important tool in hydrlogical studies.

Important characteristics of these models are

1) Principle of superimposition

If $X_3 = X_1 + x_2$, then

$$y_3 = mX_3 = mX_1 + mX_2 = y_1 + y_2$$

2) Principles of proportionality

If $y_1 = mX_1$ and if $X_c = CX_1$ where C is a constant, then

$$y_c = mX_c = CmX_1 = Cy_1$$

Discrete form of the linear model for relating input x to output y is

$$y_i = X_i h_1 + X_{i-1} h_2 + X_{i-2} h_3 + \ldots + X_{i-m+1} h_m + e_i$$

$$= \sum_{j=1}^{m} X_{i-j+1} h_j + e_i$$

Here h_j refers discrete values of the response function. They represent the idendified linear system and are computed from the observed X_i and y_i. The length of this function depends upon the system itself.

3.1 Memory Length

Effect of the rainfall can be felt even after its duration. The effect of early part of the rainfall may continue even after the rain but cesses to contribute significantly. Thereafter only the next part of rain continues to contribute. The time interval between the start of the rainfall and ending of appreciable effect (compared to its original contribution) in the stream discharge is known as the memory length. The value of m in the above equation represnt the memory length. For a solution this should be known prior hand. The best estimate is obtained by trial and error method.

In a matrix notation this can be written as:

$$\begin{bmatrix} y_i \\ y_2 \\ . \\ . \\ . \\ \hdashline y_m \\ y_{m+1} \\ . \\ . \\ . \\ y_n \end{bmatrix} = \begin{bmatrix} x_1 & 0 & . & . & . & 0 \\ x_2 & x_1 & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \hdashline x_m & x_{m-1} & . & . & . & x_1 \\ x_{m+1} & x_m & . & . & . & x_2 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ x_n & x_{n-1} & . & . & . & x_{n-m+1} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ . \\ . \\ . \\ h_m \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ . \\ . \\ . \\ \hdashline e_m \\ . \\ . \\ . \\ . \\ e_n \end{bmatrix}$$

It is assumed that the input and the output start from X_1 and y_1 .

$$\{y_{n,1}\} = [X_{n,m}] \{H_{m,1}\} + E_{n,1}$$

From matrix algebra the {H} can be found

$$\{H\} = [X^T X]^{-1} X^T y$$

A solution based on Recursive Least Sqaures for this case is attractive as it can avoid the matrix inversion. Details are not included in this notes.

4.0 PERTURBATIONS

Fourior series is used to smooth the output of seasonal model. The departures of rainfall (input) and the discharge (output) from their respective smoothed seasonal values are known as perturbations. In Linear Perturbation model these departures are assumed to be linearly related Nash et al. (1980).

The Steps used in the model are:

1) The input data is divided into two parts one for calibration and the other for validation. Larger portion of the data is preferably used for calibration purpose. For example if 7 years data are available ; 5 years data are used for calibration and 2 years for validation.

2) Seasonal mean rainfalls and seasonal mean discharge were calculated for the period of calibration.

3) Fourior series is used and a smooth function is fit upon each of the above values. Four to five harmonics will generally be enough for this modelling.

4) Departures are calculated by subtracting above values from rainfall and discharge.

5) Linear model is then applied onto departures.

6) The estimated discahrge series are calculated at by adding the seasonal mean discharge to the estimated departures.

7) After validation the entire data can be used for parameter estimation.

5.0 CASE STUDIES

Kandasamy et al. (1992) applied LPM upon few catchments in Kerala. They found that LPM produces better results when compared to other linear models including Tank model. They recommended LPM for application to small mountainous catchments subjected to seasonal raifall. James et al. (1992), applied LPM upon 5 catchments and concluded that the model is acceptable for estimating the runoff values for larger river basins of the Western Ghats region. They noticed a uniform reduction in efficiency with decrease in the size of the basin.

The models, viz. LM, and LPM have been applied on the data pertaining to 6 catchments as shown in Table 1. A brief description of the catchments used are given below:

5.1 Catchment Balephikola

Saptakosi river in the eastern region of Nepal is the biggest river in Nepal and has three main affluent. The river Balephikola used in this study forms a part of the above system, Balephikola drains out Dorje Lakpa range and joins Kosi at Balephi Dobhan. The discharge of Balephikola becomes low in March and reaches the peak from June to August. Snowmelt contributes to the flow for some period of a year.

TABLE 1 : CATCHMENT USED

Sl. No.	Name of the Catchment	Area Sq. Km.	Length of Record (days)
1.	Iruvanjipuzha	206	2556
2.	Koodathai	117	2556
3.	Kuthrapuzha	220	3652
4.	Balephikhola	630	1826
5.	Ranganadi	1730	2922
6.	Kulsi	2147	2498

5.2 Catchment Chaliyar

The Chaliyar river starts from Elambalari hills at an altitude of 2067 m in Kerela. The total length of the river before it falls into Arabian sea at Beypore is 169 Km. There are 13 gauging stations in the total catchment area of 2923 sq. km. The Iruvanjipuzha, Koodathai and Kuthrapuzha are sub basins of this River Basin.

5.3 Catchment Kulsi

This forms a sub-basin of the River Brahmaputra and lies partly in the State of Mehalaya and partly in the State of Assam. The catchment is bounded on the South by the West Khasi Hill ranges and on the North by the Brahmaputra. The river originates at an elevation of about 1850 m. The total catchment area is 2147 Sq.Km. The catchment consists of Khasi Hill range in the upper part, Middle reserve forest area and lower alluvial plain. The famous Chandubi lake is situated in the reserve forest area.

5.4 Catchment Ranganadi

The River Ranganadi starts from Lower Subansri District of Arunachal Pradesh. Flowing through the State of Arunachal Pradesh, Ranganadi joins the River Subansri before its confluence with Brahmaputra. The catchment area upto Yazali is 1730 sq.km. Average annual rainfall is 1232 mm.

5.5 Model Performance

An evaluation of these models in terms of efficiency criteria is given in Table 2. The LPM produced consistently better results than other two. Improvement produced by LPM model over LM, suggest that even if one approximately removes the possible non-linearity present in the rainfall-runoff data and relate the rest linearly the model could produce reasonably good result. Removal of seasonal non-linearity makes LPM producing better relation.

TABLE 2 MODEL EFFICIENCY ON DIFFERENT CATCHMENTS

Sl. No.	Name of the Catchment	Efficiency LM	% LPM
1.	Iruvanjipuzha	64	91
2.	Koodathai	70	73
3.	Kuthrapuzha	55	61
4.	Balephikhola	77	90
5.	Ranganadi		42
6.	Kulsi		76

The hydrological diagram of Balephikhola and the pulse response function derived by linear model are shown in Fig.1 and 2 respectively. The results of Balephikhola indicates that LPM could work admirably even with snowmelt component in the flow. The hydrological diagram and the response function derived by Linear Perturbation model for the case of Iruvanjipuzha are shown in Fig.3 and 4.

A typical comparison of the computed and observed discharge for the case of the river Kulsi is shown in Fig.5. It may be mentioned that a long memory of 40 days is used in this case as shown in Fig.6. The poor predictions and long memory are indicative of inadequacy of LPM. The case is similar with the river Ranganadi.

6.0 ANALYSIS & DISCUSSIONS

The efficiency produced by LPM and the catchment area are plotted as shown in Fig.7. The results of James et al.(1992) and others are also shown in this plot. It is clear that the efficiency of the model does not depend upon the size of the catchment even for catchments of a single region (Western Ghats) analysed. There is no noticeable trend in the plot.

LPM model have been extensively used for relating rainfall and discharge on daily intervals. However it has been proved that the model is capable of relating them in monthly intervals also. There were few occasions in which hourly scale (for flood routing) have been used and found to produce satisfactory results.

The poor performance in some cases especially for the catchments of North East may be due to the effect of soil moisture (the long memory is indicative of this). A soil moisture accounting model or a non-linear model may produce better results.

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Appendix - I

FOURIER SERIES

Hydrologic data such as discharge of a river, water level can be expressed through Fourier series approximately. If 2N measurements are available at equal interval x. The general form of the equation is

$$F = a_0 + \sum_{k=1}^{M} (a_k \text{Cos}kx + b_k \text{Sin}kx)$$

Here M is the number of desired harmonic components used in the representation. The appoximate function will more closely represent the data used as M increases. However M > N will not have any addition of accuracy. i.e. The hormonic component of the highest frequency that can be included in the analysis in Fourier series in discrete form has a wave length equal to $2x=2L/(2N-1)$, which is the finest possible resolution. Here L is the total length of the data. Depending on the application one can choose the number of components. If smooth function is only the objective 5 to 10 % of N is sufficient for example Seasonal model can be well represented by 10 to 15 components.

Any interval can be changed to the range 0 to 2π. The interval is then divided into 2N equally spaced parts. The coefficients a_k are found by least Square Techniques.

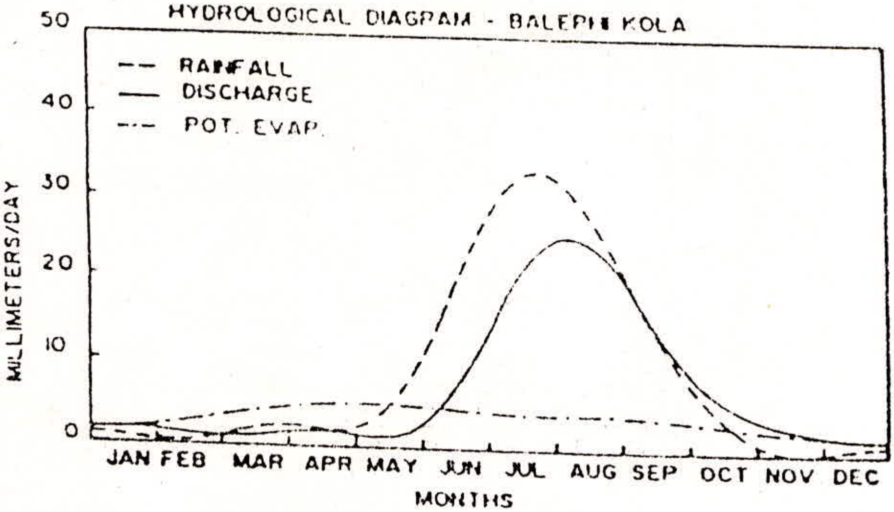

FIG. 1 SMOOTHED SEASONAL MEAN RAINFALL AND DISCHARGE

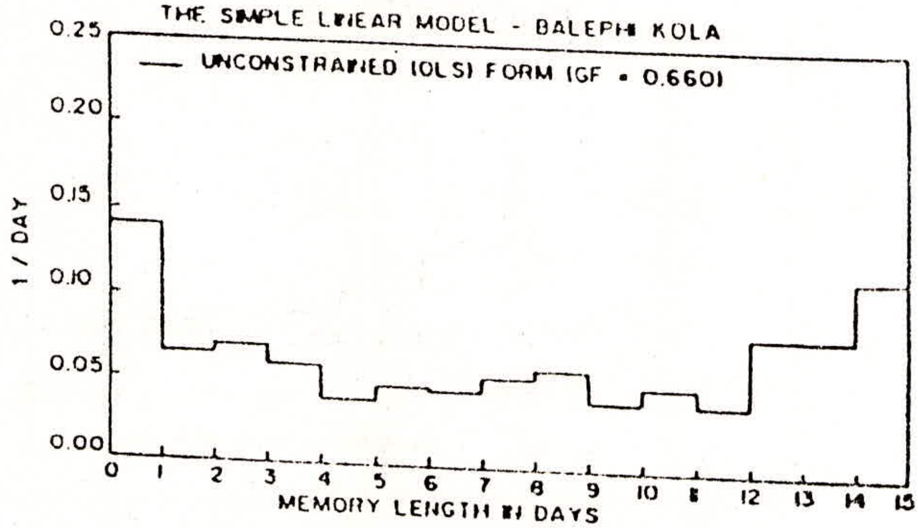

FIG. 2 PULSE RESPONSES OF THE LM, DERIVED BY OLS.

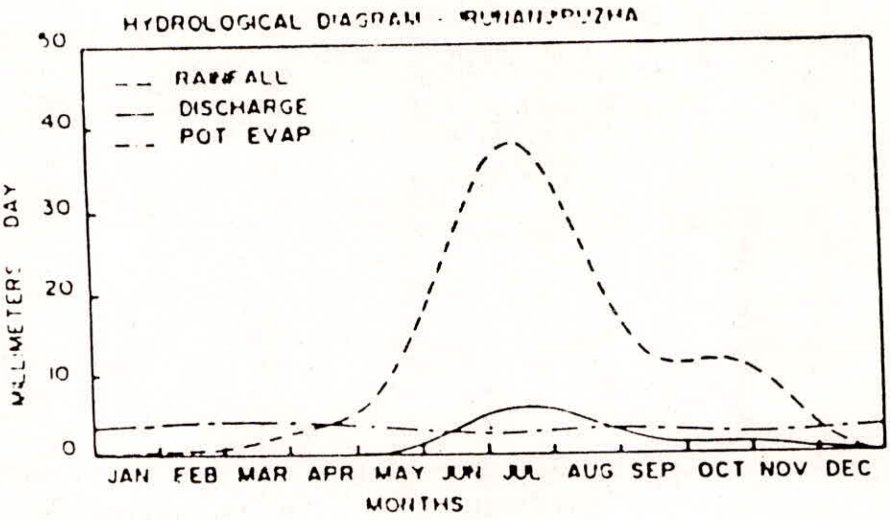

FIG. 3 SMOOTHED SEASONAL MEAN RAINFALL AND DISCHARGE

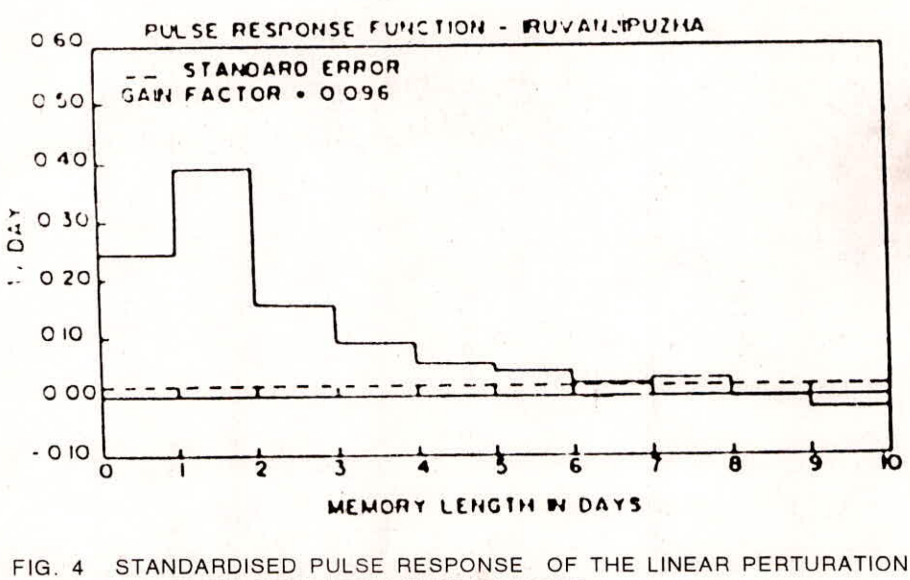

FIG. 4 STANDARDISED PULSE RESPONSE OF THE LINEAR PERTURATION MODEL BY ORDINARY LEAST SQUARES

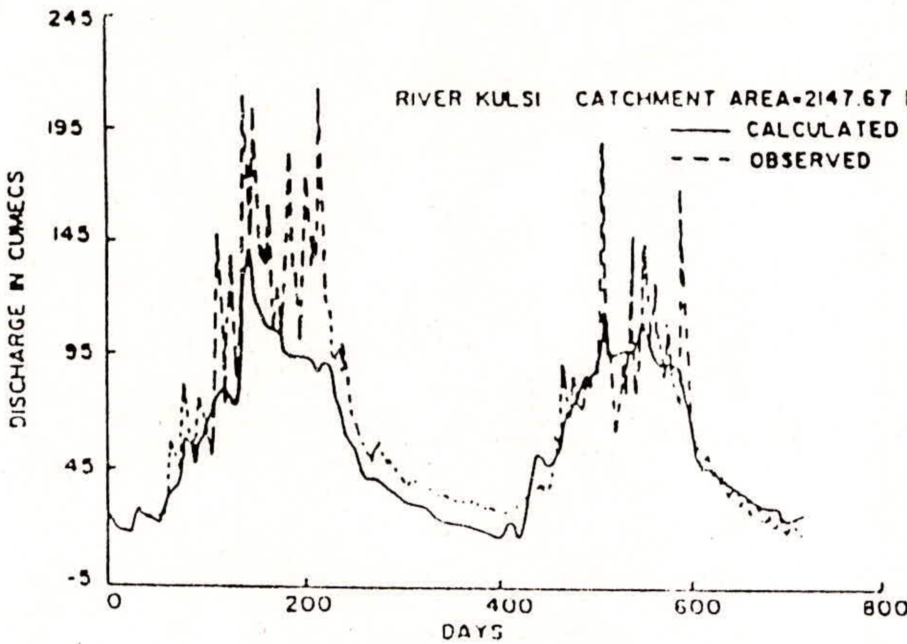

FIG. 5 COMPARISON OF OBSERVED HYDROGRAPH WITH CALCULATED ONE BY LPM.

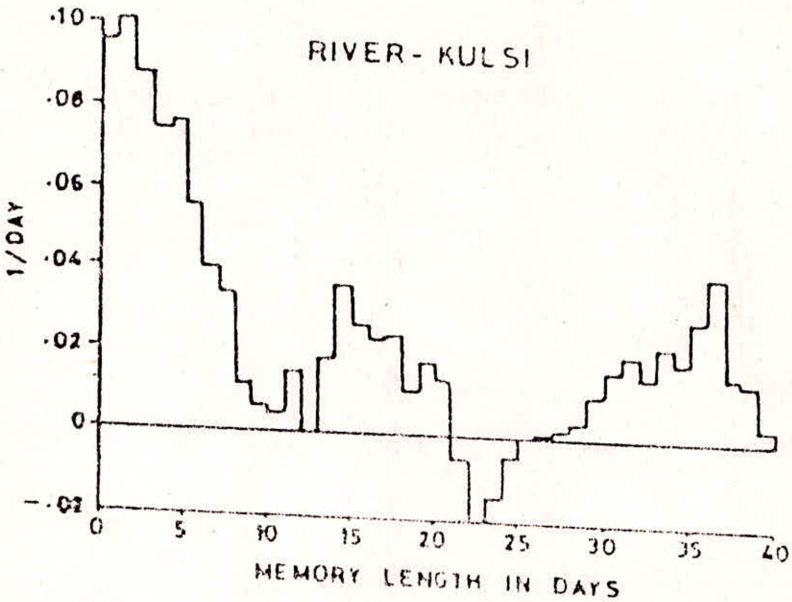

FIG. 6 STANDARDISED PULSE RESPONSE OF LPM

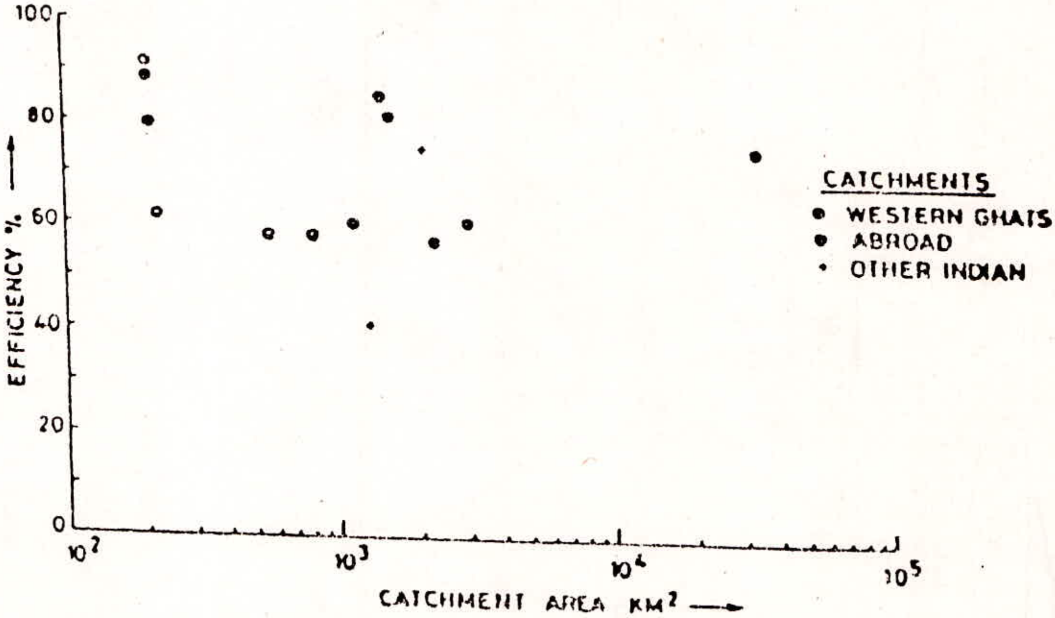

FIG. 7 EFFICIENCY OF LPM FOR DIFFERENT CATCHMENT AREA