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APPLICATION OF FINITE ELEMENT METHOD TO SOME FLOW PROBLEMS

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S. Mohan

INDIAN NATIONAL COMMITTEE ON HYDROLOGY
(Committee Constituted by Ministry of Water Resources, Govt. of India)

INCOH SECRETARIAT
NATIONAL INSTITUTE OF HYDROLOGY
ROORKEE - 247 667, INDIA

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PREAMBLE

Water is the most essential natural resource for life next to air and is likely to become a critical scarce resource in many regions of the world. The availability of water resources in India shows a great deal of spatial and temporal variability. The population in the country is steadily growing and is expected to approach 160 crores by 2050. The per capital food availability is at present low and needs to be increased. This rate of growth in food grain production can be achieved through extension of irrigated areas and by increasing the grain yield per unit area assuming that there may not be any significant increase in net sown area. It has been established that productivity of irrigated areas is atleast double, if not more than, that of unirrigated areas in respect of wheat and rice crops. This calls for better water management in the projects to bring more area under irrigation, reduce the cost/ha and thereby increase production. The growth process, the increase in population and the expansion of economic activities inevitably, lead to increasing demands for water for diverse purposes.

The Indian National Committee on Hydrology is the apex body on hydrology constituted by the Government of India with the responsibility of coordinating the various activities concerning hydrology in the country. The committee is also effectively participating in the activities of UNESCO and is the National Committee for International Hydrology Programme (IHP) of UNESCO. In pursuance of its objective of preparing and periodically updating the state-of-art in hydrology in the world in general and India in particular, the committee invites exerts in the country to prepare these reports on important areas of hydrology. Realising the importance of irrigation water management, the committee considered it appropriate to get prepared a state of art in this important area.

This state-of-art report analyses and reviews the present practices of water management being followed in projects of the country. The report also attempts to cover various technical, social, economic and organisational aspects related to the command area management and suggests possible action for improving the water management.

The Indian National Committee on Hydrology with the assistance of its erstwhile Panel on Surface Water has identified this important topic "Application of Finite Element Method to Some Flow Problems" for preparation of this state-of-art report and the report has been prepared by Dr. B Vasudeva Rao of IIT, Mumbai. The guidance, assistance and review etc. provided by the Panel are worth mentioning.

It is hoped that this state-of-art report would serve as a useful reference material to practising engineers, researchers, field engineers, planners and implementation authorities, who are involved in correct estimation and optimal utilisation of the water resources of the country.

(S.M. SETH)
Executive Member, INCOH
& Director, NIH
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APPLICATION OF FINITE ELEMENT METHOD TO SOME FLOW PROBLEMS

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INTRODUCTION:

During the past few decades, a significant number of research papers have appeared which apply the finite element technique to the solution of wide variety of problems in Hydraulic Engineering and Water Resources. These papers in general point out that the stability and accuracy of the Finite Element models are comparable to those found in Finite Difference models. In some cases where complex geometries are involved, Finite Element models score well over Finite Difference models. The advent of computers also contributed to the development of Finite Element models to solve wide variety of engineering problems. Further improvement over FEM in recent years is the application of Boundary Integral method to some engineering problems and it was claimed by Brebbia (1988) that it offers an excellent alternative to Finite Element for the solution of some practical problems. In BEM only the boundary is discretised and the solution is obtained in terms of boundary nodes. The values of field variables are evaluated using the fundamental solution associated with the governing equation. The size of the matrix formed is small compared to the FEM. However, the discussion will be restricted to the FEM only in this report. The report deals with the finite element formulation of some of the problems encountered in Hyd. Engg./Water Resources. These are mainly: (a) Water Distribution Networks, (b) Water Hammer Problems, (c) Dispersion of pollutants in aquifers, (d) Ground Water management, (e) Two-Dimensional Stream Flow modelling, (f) Surface Runoff modelling.

The most popular method used in the structural mechanics to formulate the element matrices is the variational technique. As reported by Narasimhan et al (1982), Finlayson and Scriven have compared the variational technique with the weighted integration technique and concluded that it is easier to formulate the finite element equations using the Galerkin's technique. At present, the Galerkin's method is the most widely used method to solve the flow problems using the Finite Element method. In all the formulations reported here, Galerkin's method has been used. A key task in the application of the finite element method is that of setting up of a matrix of algebraic equations to be solved simultaneously for the various unknowns. Matrix solution technique can be broadly divided into two classes: (i) direct and (ii) iterative solvers. The difficulty with direct solvers when applied to large problems is that of computer memory. These matrices formed may be sparse and storing and operating on zero elements will waste considerable amount of computer time. As opposed to direct solvers, iterative methods have the advantage of storing and operating only on non-zero quantities so that the storage requirements are always minimal. These iterative schemes may take more time to converge, but the computer memory requirements are minimum. One of the attractive features of the FEM is the ease with which the geometric information to define the discretised problem can be provided as input. Now this method has come to be recognized as a powerful tool of analysis of flow problems. The compact storage schemes for solution of matrices by direct method is as follows.

COMPACT STORAGE SCHEME TO STORE THE GLOBAL MATRIX: With the increase in the number of nodes of the problem, the matrix size will increase inordinately, but most of the elements of the matrix are zero and only about 10 percent of the elements are nonzero elements. If conventional schemes are used, it will lead to increase in storage capacity requirements and also the computational time. The following compact storage scheme has been used here to store mostly the nonzero elements of the matrix so that the problem can be solved fast with minimum storage requirements.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} & a_{47} & a_{48} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} & a_{57} & a_{58} \\ 0 & 0 & a_{63} & a_{64} & a_{65} & a_{66} & a_{67} & a_{68} \\ 0 & 0 & 0 & a_{74} & a_{75} & a_{76} & a_{77} & a_{78} \\ 0 & 0 & 0 & a_{84} & a_{85} & a_{86} & a_{87} & a_{88} \end{bmatrix} \quad (1)$$

$$[AU] = [a_{11};\ a_{12}, a_{22};\ a_{13}, a_{23}, a_{33}; a_{14}, a_{24}, a_{34}, a_{44};\ a_{15}, a_{25}, a_{35}, a_{45}, a_{55}; a_{36}, a_{46}, a_{56}, a_{66};\ a_{47}, a_{57}, a_{67}, a_{77}; a_{48}, a_{58}, a_{68}, a_{78}, a_{88}] \quad (2)$$

$$[AL] = [1;\ a_{21}, 1;\ a_{31}, a_{32}, 1;\ a_{41}, a_{42}, a_{43}, 1; a_{51}, a_{52}, a_{53}, a_{54}, 1;\ a_{63}, a_{64}, a_{65}, 1;\ a_{74}, a_{75}, a_{76}, 1; a_{84}, a_{85}, a_{86}, a_{87}, 1] \quad (3)$$

The diagonal pointer array is [LIMIT] = [1,3,6,10,15,19,23,28], which will point out the diagonal elements in both the AU and AL arrays. The compact storage schemes for matrices save considerable amount of computational time, some times may be upto 90 % for very large matrices. In finite element method, for one iteration, the computational time approximately is:

(a) Numerical integration, formulation of element matrices and global assembly ... ... 80% of total
(b) Decomposition of global matrix into lower and upper triangular matrices ... ... 16% of total
(c) Back substitution and forward elimination .. 4% of total.

Incorporation of Dirichlet type Boundary Condition into Global Matrix:

One of the essential boundary condition to be incorporated in to the matrix before t can be solved is the Dirichlet's condition. Consider the following system of equations in matrix form as given by Eqn. 4.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{Bmatrix} \tag{4}$$

If $x_3 = V$ is to be incorporated, then the matrix is to be rewritten as given by Eqn. 5.

$$\begin{bmatrix} a_{11} & a_{12} & 0 & a_{14} \\ a_{21} & a_{22} & 0 & a_{24} \\ 0 & 0 & 1 & 0 \\ a_{41} & a_{42} & 0 & a_{44} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} b_1 - a_{13}V \\ b_2 - a_{23}V \\ V \\ b_4 - a_{43}V \end{Bmatrix} \tag{5}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & 10^{30} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ V(10^{30}) \\ b_4 \end{Bmatrix} \tag{6}$$

It is better to use the method described in Eqn. 5 when ever there is an iterative scheme is involved, though the method described in Eqn. 6 is simpler to adopt on computers.

1.0 ANALYSIS OF FLOW NETWORKS:

The subject of water distribution networks is of considerable interest to the hydraulic engineers as well as environmental engineers since long. Many publications have appeared about the analysis of flow networks in general and design of pipe distribution systems in particular. to mention a few - Shamir Uri (1974) discussed the aspects of analysis and design exhaustively; Shaake and Lai (1986), Liang (1971), Jacoby (1968) discussed the designing aspects with mathematical programming techniques. The purpose of this chapter is to present the analysis of flow networks by using FEM (Vasudeva Rao 1987), some times known as linear method. In order to appreciate the advantage of this method over the other two popular methods, it is appropriate to review briefly the other two methods, namely (i) Hardy-Cross method and (ii) Newton-Raphson method. The analysis of flow networks is based on the analogy drawn from the Kirchoff's laws applicable electrical networks. These are: (i) the algebraic sum of the flows into or out of any node should be zero, (ii) the algebraic sum of the pressure head losses around any closed loop should be zero.

1.1 Hardy-Cross method for Flow Networks:

Considering a small network of four nodes and five elements (pipes) as shown in Fig. 1 for illustration purposes, the nodal continuity equations can be written in matrix form as:

$$[A1]\{Q\} = \{C\} \quad where$$

$$[A1] = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & -1 & -1 \end{bmatrix} \qquad (7)$$

$\{Q\}^T = \{Q_1,Q_2,Q_3,Q_4,Q_5\}$ and $\{C\}^T = \{-C_1,-C_2,-C_3,-C_4\}$
Here, Q_i,i=1,5 are the discharges and C_i,i=1,4 are the consumptions at the nodal points. As per the notation used, inputs will be negative. The loop equations in matrix form can be written as:

$$[B]\{HF\} = \{0\} \quad where$$

$$[B] = \begin{bmatrix} -1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & -1 \end{bmatrix} \qquad (8)$$

and $\{HF\}_T = \{hf_1,hf_2,hf_3,hf_4,hf_5\}$.
At high Reynolds numbers, the relationship between the pressure head loss and discharge in any pipe is nonlinear. According to Darcy-Weisbach formula, the relationship is of the form $Q = K.hf^{0.5}$ where K is the conveyance factor of the pipe, $K^2 = 12.1D^5/fL$, where Q is the discharge in cubic meters per second, f is the friction factor, D and L are the diameter and length of the pipe in meters, hf is the pressure head loss in pipe due to friction. Before the Eqns. 1 and 2 can be solved for discharges, it is to be noted that the matrix in Eqn. 1 is of rank 3. The Eqn. 2 when expressed in terms of discharges is nonlinear, hence can be solved by iterative procedure only. Out of the 4 rows in Eqn. 1, any three rows can be taken into account. Retaining the first three rows in Eqn. 1, the modified matrix is of the form given by Eqn. 9.

$$\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{Bmatrix} = - \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \end{Bmatrix} - \begin{Bmatrix} Q_5 \\ 0 \\ -Q_4 \end{Bmatrix} \qquad (9)$$

Assuming Q^4 and Q_5 to start with, the other values Q_1, Q_2 and Q_3 can be evaluated by solving the matrix. The values thus generated will satisfy the continuity equations, but they may not satisfy the loop equations. Hence corrections are to be applied to the discharge values till they satisfy the loop equations. It can be shown that the correction to the discharges in loop 1, δQ_1 can be written as given by Eqn. 10.

$$\delta Q_1 = \frac{\sum hf_i}{2\sum K_i|Q_i|}; \qquad for\ i=1,2,5 \qquad (10)$$

The corrections to the discharges in loop 2, δQ_2 is as given by Eqn. 11.

$$\delta Q_2 = \frac{\sum hf_i}{2\sum K_i |Q_i|}; \quad for\ i=2,3,5 \tag{11}$$

Finally $Q_i = Q_i + \delta Q_1$ for i=1,2 and 5;
$Q_i = Q_i + \delta Q_2$ for i=2,3 and 5:

The solution procedure by Hardy-Cross method is as follows:

1. Assume Q_i,i=N,NE and let the iteration counter m=1
2. Compute Q_i, i=1,N-1
3. Compute δQ_j, j=1,NL
4. Apply corrections $Q_i = Q_i + \delta Q_j$ for all the values of i, i being the element of loop set j.
5. If ABS(δQ_j) $\leq$ 0.00001 GOTO step 7
6. Put m=m+1 and goto step 3
7. Compute the pressure heads at all the nodes and print the results, that is the values of discharges and pressure heads.

This procedure was tested on three different networks and was found to be satisfactory in terms of convergence. The matrix in this case is unsymmetric, hence for large networks when stored in full form it requires considerable amount of computer memory. the presence of constant head reservoirs will complicate the problem.

1.2 Newton-Raphson Method for Flow Networks:

This method is essentially a pressure head correction method wherein the corrections are applied to the pressure heads at the nodes from iteration to iteration. The loop equations are not needed to be solved in this method which are automatically satisfied if the nodal continuity equations are satisfied. When the nodal continuity equations are expressed in terms of pressure heads at the nodes, a set of nonlinear equations are obtained which are of the form $F_i = F_i(H_1,H_2,H_3,H_4)$ for i=1,4. These are:

$$F_1 = -K_1(H_1-H_2)^{1/2} - K_3(H_1-H_3)^{1/2} - K_5(H_1-H_4)^{1/2} + C_1 = 0 \tag{12}$$

$$F_2 = K_1(H_1-H_2)^{1/2} - K_2(H_2-H_4)^{1/2} - C_2 = 0 \tag{13}$$

$$F_3 = K_3(H_1-H_3)^{1/2} - K_4(H_3-H_4)^{1/2} - C_3 = 0 \tag{14}$$

$$F_4 = K_2(H_2-H_4)^{1/2} + K_4(H_3-H_4)^{1/2} + K_5(H_1-H_4)^{1/2} - C_4 = 0 \tag{15}$$

These set of nonlinear equations given by Eqns. 12 to 15 can be solved by linearising them into Jacobian form using Taylor series expansion. The linearised equations in matrix form will be as given by Eqn. 16.

$$\begin{bmatrix} \frac{\partial F_1}{\partial H_1} & \frac{\partial F_1}{\partial H_2} & \frac{\partial F_1}{\partial H_3} & \frac{\partial F_1}{\partial H_4} \\ \frac{\partial F_2}{\partial H_1} & \frac{\partial F_2}{\partial H_2} & \frac{\partial F_2}{\partial H_3} & \frac{\partial F_2}{\partial H_4} \\ \frac{\partial F_3}{\partial H_1} & \frac{\partial F_3}{\partial H_2} & \frac{\partial F_3}{\partial H_3} & \frac{\partial F_3}{\partial H_4} \\ \frac{\partial F_4}{\partial H_1} & \frac{\partial F_4}{\partial H_2} & \frac{\partial F_4}{\partial H_3} & \frac{\partial f_4}{\partial H_4} \end{bmatrix} \begin{Bmatrix} \delta H_1 \\ \delta H_2 \\ \delta H_3 \\ \delta H_4 \end{Bmatrix} = \begin{Bmatrix} -F_1 \\ -F_2 \\ -F_3 \\ -F_4 \end{Bmatrix} \tag{16}$$

The rank of the matrix in Equation 16 is 3, hence an additional equation is needed to solve these Eqns. uniquely. This additional equation can be in the form of fixed head at any node of the network. the matrix in Eqn. 16 should be suitably modified suitably to incorporate this condition before this can be solved for pressure head corrections $\{\delta H\}$, then $H_j = H_j + \delta H_j$ for j=1,NN where NN is the number of nodes. This procedure is to be repeated till the values of nodal continuity equations. F_j for j=1,NN approach a value close to zero. The matrix formed is symmetric in this case, hence compact storage schemes can be used for matrix. The convergence depends upon the starting values of pressure heads, if the solution converges, the loop equations are automatically satisfied. The presence of constant head reservoirs can be taken care of easily.

1.3 Finite Element Method for Flow Networks:

The pipe discharge versus pressure head relationship can be written as $Q = T_e\, hf$, where $T_e = [12.1\ D^5/(fL\ |hf|)]^{1/2}$, T_e being the transmissivity of the pipe element. For a typical element shown in Fig. 1.2, the flow entering the element at its ends in terms of the pressure heads can be written as:

$$\begin{Bmatrix} Q_1^{(e)} \\ Q_2^{(e)} \end{Bmatrix} = \begin{bmatrix} T_e & -T_e \\ -T_e & T_e \end{bmatrix} \begin{Bmatrix} H_i \\ H_j \end{Bmatrix} \tag{17}$$

For the network configuration shown, the individual element Eqns. are:

$$\begin{Bmatrix} Q_1^{(1)} \\ Q_2^{(1)} \end{Bmatrix} = \begin{bmatrix} T_1 & -T_1 \\ -T_1 & T_1 \end{bmatrix} \begin{Bmatrix} H_1 \\ H_2 \end{Bmatrix} \tag{18}$$

$$\begin{Bmatrix} Q_2^{(2)} \\ Q_4^{(2)} \end{Bmatrix} = \begin{bmatrix} T_2 & -T_2 \\ -T_2 & T_2 \end{bmatrix} \begin{Bmatrix} H_2 \\ H_4 \end{Bmatrix} \tag{19}$$

$$\begin{Bmatrix} Q_1^{(3)} \\ Q_3^{(3)} \end{Bmatrix} = \begin{bmatrix} T_3 & -T_3 \\ -T_3 & T_3 \end{bmatrix} \begin{Bmatrix} H_1 \\ H_3 \end{Bmatrix} \tag{20}$$

$$\begin{Bmatrix} Q_3^{(4)} \\ Q_4^{(4)} \end{Bmatrix} = \begin{bmatrix} T_4 & -T_4 \\ -T_4 & T_4 \end{bmatrix} \begin{Bmatrix} H_3 \\ H_4 \end{Bmatrix} \tag{21}$$

$$\begin{Bmatrix} Q_1^{(5)} \\ Q_4^{(5)} \end{Bmatrix} = \begin{bmatrix} T_5 & -T_5 \\ -T_5 & T_5 \end{bmatrix} \begin{Bmatrix} H_1 \\ H_4 \end{Bmatrix} \tag{22}$$

The global assembly of these element Eqns. 18 to 22 is as given by Eqn. 23.

$$\begin{bmatrix} (T_1+T_3+T_5) & -T_1 & -T_3 & -T_5 \\ -T_1 & (T_1+T_2) & 0 & -T_2 \\ -T_3 & 0 & (T_3+T_4) & -T_4 \\ -T_5 & -T_2 & -T_4 & (T_2+T_4+T_5) \end{bmatrix} \begin{Bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{Bmatrix} = \begin{Bmatrix} Q_1^{(1)}+Q_1^{(3)}+Q_1^{(1)} \\ Q_2^{(1)}+Q_2^{(2)} \\ Q_3^{(3)}+Q_3^{(4)} \\ Q_4^{(2)}+Q_4^{(4)}+Q_4^{(5)} \end{Bmatrix} \tag{23}$$

or $[A]\{H\} = \{Q^{(e)}\}$. It can be seen that matrix formed is symmetric and the right hand side of the Eqn. 23 can be simplified still further. From the nodal continuity equations and Fig. 1.3, the right hand side of Eqn. 23 can simplified as shown in Eqn. 24.

$$\begin{Bmatrix} Q_1^{(1)}+Q_1^{(3)}+Q_1^{(5)} \\ Q_2^{(1)}+Q_2^{(2)} \\ Q_3^{(3)}+Q_3^{(4)} \\ Q_4^{(2)}+Q_4^{(4)}+Q_4^{(5)} \end{Bmatrix} = \begin{Bmatrix} Q_1+Q_3+Q_5 \\ -Q_1+Q_2 \\ -Q_3+Q_4 \\ -Q_2-Q_4-Q_5 \end{Bmatrix} = \begin{Bmatrix} C_1 \\ -C_2 \\ -C_3 \\ -C_4 \end{Bmatrix} \tag{24}$$

The right hand side of the equation 24 can now be written as $\{-C_1,-C_2,-C_3,-C_4\}^T$, where the input at any node should be treated as negative. In Finite Element formulation also the rank of the matrix formed is NN-1, where NN is the number of node points. Hence an additional equation in the form of boundary condition must be incorporated into the matrix equations

This additional equation can be in the form of fixed pressure head at any of the node points. If the pressure head at node 1 is HFIX, the matrix should now be modified as given by Eqn. 25.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & (T_1+T_2) & 0 & -T_2 \\ 0 & 0 & (T_3+T_4) & -T_4 \\ 0 & -T_2 & -T_4 & (T_2+T_4+T_5) \end{bmatrix} \begin{Bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{Bmatrix} = \begin{Bmatrix} HFIX \\ -C_2+T_1(HFIX) \\ -C_3+T_3(HFIX) \\ -C_4+T_4(HFIX) \end{Bmatrix} \quad (25)$$

One complication with the element transmissivity is that it is a function of the pressure head difference at the nodes connecting the element. As these are unknown initially, the transmissivity values are to be evaluated with assumed values of pressure heads. If by chance the pressure head difference $hf^{(e)}$ is zero, then it should be treated that transmissivity, T_e is equal to the conveyance factor K_e for the element. The algorithm by FEM for flow networks proceeds as follows:

1. Evaluate the conveyance factor for all the elements of the network using the discharge versus pressure head difference equation. Read the starting values of the pressure heads at all the nodes. Set the iteration counter m=0.

2. let m=m+1

3. Compute the transmissivity T_e. If $hf^e = 0$ then $T^e = K_e$.

4. Assemble and form the global matrix and prescribe the proper boundary conditions and solve for the pressure heads.

5. If $ABS(H_j^{m+1} - H_j^m) \leq 10^{-6}$ then goto step 7.

6. Put $H_j^m = H_j^{m+1}$ for j=1,N and goto step 2.

7. Print the results and stop.

1.4 Numerical Example:

The following data has been assumed for the network shown in Fig. 1.1. The Darcy-Weisbach friction factor is 0.02. The diameters of the pipes are 100,120,120,100,150 mm and the lengths are 100, 120, 150, 90, 170 m respectively. The consumptions at the nodes are -60, 15, 20 and 25 lit/sec, the negative value at the node 1 indicates the input at node 1. The starting values of the pressure heads at all the nodes is assumed to be 20 m. The pressure head at node 1 is fixed at 20 m. The solution by FEM converged in less than 10 iterations. The solution is {H} = {20, 16.942, 16.978, 16.957} meters. The same values have been obtained by using the Hardy-Cross and Newton-Raphson methods. Three different networks have been studied by these three methods. The first network consists of 6 nodes and 9 pipes with one fixed head node. The second network contains 36 nodes and 38 pipes with two constant head reservoirs. The third network consists of 158 pipes and 148 nodes with one constant head reservoir. In all these cases the final solution by different methods converged to the same values. In FEM, the final solution for all the networks converged in less than 10 iterations with starting values of pressure heads being at a particular value for each network for all the nodes. Convergence is faster in FEM compared to the other two methods. The time of computation is also very less compared to the other two methods. Moreover, FEM has been applied subsequently to about 20 different networks and it was observed that convergence is independent of the starting values.

Table No. 1.1

Comparative statement of the three different methods for solving the flow networks.

	Hardy-Cross method	Newton-Raphson method	Finite element method
1.	Iterative procedure based on loop oriented discharge correction method; $Q_i=Q_i+\delta Q_j$ for all pipes in loop j	Iterative procedure based on pressure head correction method; $H_j=H_j+\delta H_j$ for all the nodes j in the network	Iterative procedure to evaluate the pressure heads directly.
2.	Complete data regarding the node to pipe connections and loop connections is needed	Node-pipe connection data is to be given in proper order.	Only the nodes connection the elements are to be given
3.	Will lead to unsymmetric matrix	Will lead to symmetric matrix. Nodal continuity Eqns. for all nodes are to be evaluated for each iteration which is a major time consuming job.	Will lead to symmetric matrix. Nodal continuity Eqns. need not be evaluated for each iteration.
4.	Data preparation is laborious especially for large networks. Direction of flow is to be taken into account in the data preparation.	Same as Hardy-Cross method.	Data preparation is simple. Direction of flow in the input data need not be taken into account.
5.	Final solution is in the form of discharges only. Pressure heads at the nodes are to be evaluated separately	Final solution is in the form of pressure head corrections. Pressure heads and Discharges are to be evaluated at each iteration to compute the nodal balance Eqns..	Final solution is the form of pressure heads directly. Discharges are to be evaluated at the end of the solution if needed.
6.	Convergence is slow, depends upon the initial starting values	Convergence is slow, depends upon the initial starting values	Faster convergence, Solution is independent of the starting values.

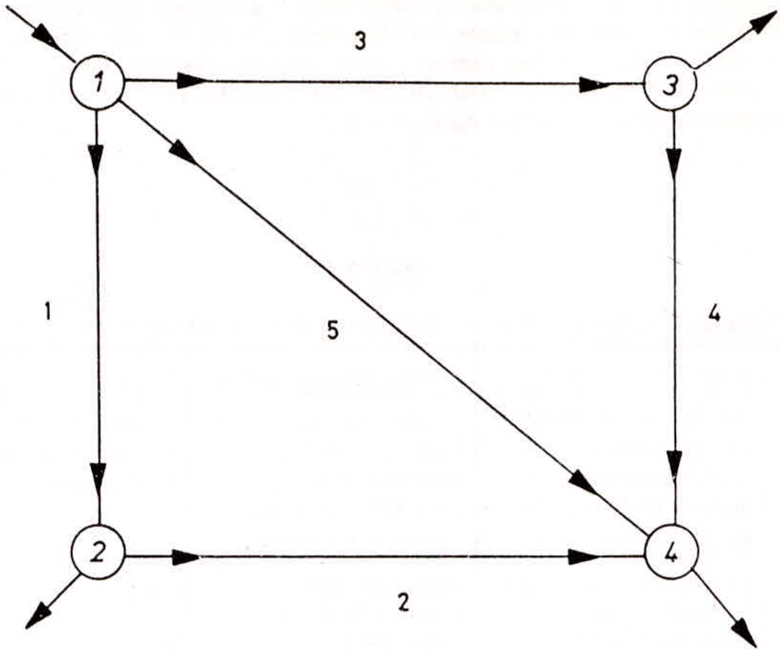


FIG. 1.1 FLOW NETWORK

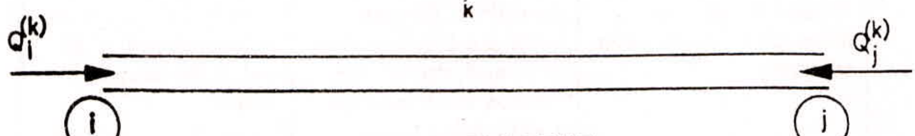


FIG. 1.2 PIPE ELEMENT

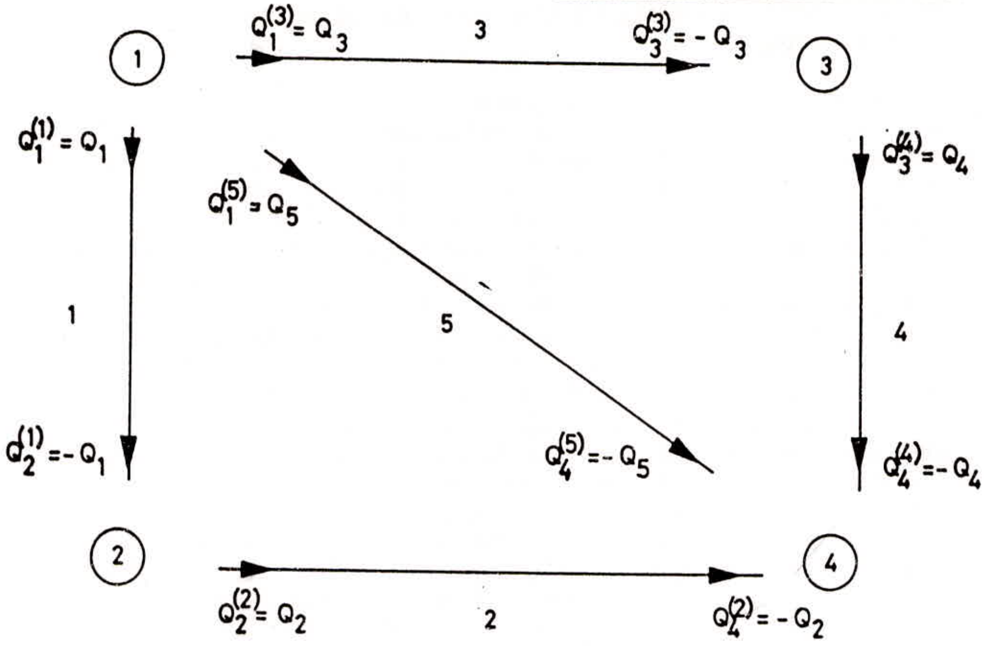
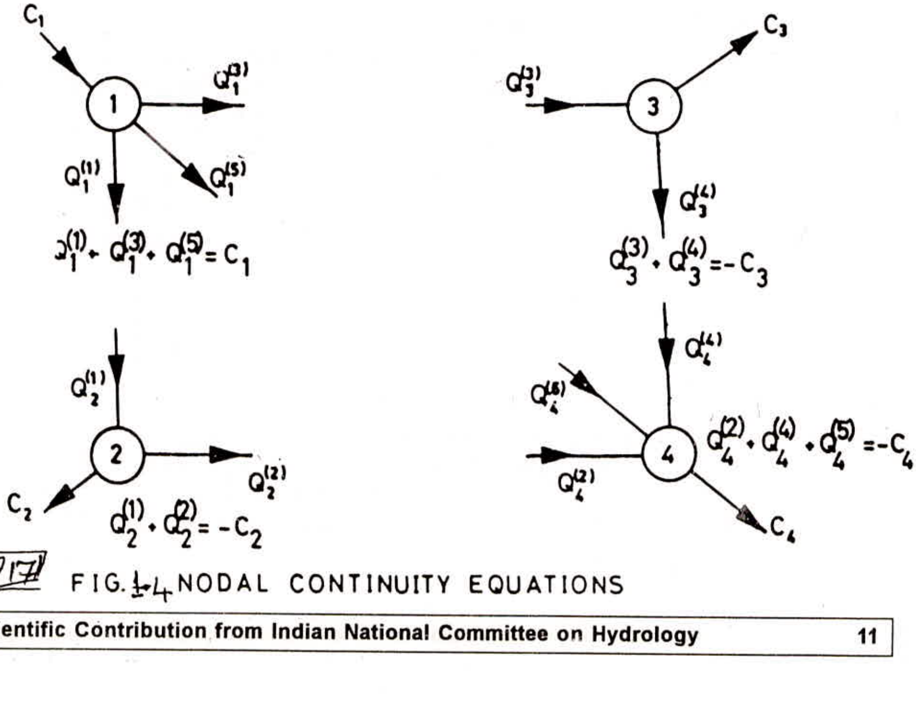


FIG. 1.3 DISCRETISED FLOW NETWORK



A17 FIG. 1.4 NODAL CONTINUITY EQUATIONS

2.0 FLUID TRANSIENTS FOLLOWING A VALVE CLOSURE IN A PIPE LINE OR WATER HAMMER:

In the case of closed conduits flowing under pressure, hydraulic transients commonly known as water hammer occur when there is either a retardation of flow due to closure of a valve or an acceleration due to the opening of a valve. This may sometimes cause the damage to the pipe line, valve and other pipe fittings along with the pumps and turbines etc. in the conduit system. Increase in pressure due to water hammer is not serious in case the valve is operated graduated gradually. The pressure rise can be evaluated by surge methods by treating the liquid as incompressible and the pipe wall as rigid. But for the case of sudden closure (or opening) of the valve, there is a sudden reduction of flow which causes an increase in the head on the upstream side of the valve. A high pressure wave propagates upstream from the valve at a speed equal to the sonic wave speed for the given fluid medium. This pressure wave reduces the velocity of flow. On the downstream side of the valve, there is a sudden decrease in the pressure and a wave of this reduced pressure travels in the downstream direction at the sonic speed. This also reduces the velocity. If the closure is quite rapid and the normal pressure is sufficiently low, this may cause cavitation and produce high pressure wave downstream on the collapse of the cavity. The transient flow phenomenon in pipes has been studied by a number of investigators. The derivations of the basic governing differential equations were presented by Rich (1963), Wylie et al. (1978), Chaudhry (1979) etc. According to Wylie et al., the method of characteristics is the widely accepted method for the solution of the water hammer problems. Watt et al. (1980) described an experimental rig constructed to measure the transient response of a water pipeline following a rapid valve closure. The aim of this chapter is to present the Finite element formulation and solution of the basic governing equations of the water hammer problem and compare the results obtained by this method with those obtained by the method of characteristics (Paygude D.G., B.Vasudeva Rao and S.G.Joshi (1985)).

2.1 Governing Equations:

In the analysis of fluid transients, two basic principles of mechanics namely (a) the principle of the law of conservation of mass and (b) the principle of conservation of linear momentum are applied to an elemental segment of fluid in a pipe, which give rise to the following two partial differential equations as given by Eqns. 26 and 27.

$$L_1(V,H) = \frac{\partial H}{\partial t} + \frac{C_p^2}{g}\frac{\partial V}{\partial x} + V\frac{\partial H}{\partial x} - V \, Sin\, \varphi = 0 \qquad (26)$$

$$L_2(V,H) = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial H}{\partial x} + \frac{f\,V\,|V|}{2D} = 0 \qquad (27)$$

in which $V=V(x,t)$ is the mean velocity of flow through the pipe, $H=H(x,t)$ is the piezometric head, x is the horizontal distance measured along the pipe, t is the time elapsed since start of the valve closing operation, g is the acceleration due to gravitation, f is the Darcy-Weisbach friction factor, D is the diameter of the pipe, C_p is the celerity of the pressure wave and θ is the angle of inclination of the pipe to the horizontal.

In the method of characteristics, the solution to the problem is somewhat indirect in the sense that the governing partial differential equations are converted into total differential equations known as characteristic equations and then to finite difference form before the solution can be obtained. The Finite Element method deals with the governing equations directly and converts them into element equations which are to be assembled and the boundary conditions are to be substituted before the solution can be obtained.

2.2 Element Matrices:

A trial solution of the following form is assumed: $V = V(x,t) = \sum N_i(x)\, V_i(t)$ and $H = H(x,t) = \sum N_i(x)\, H_i(t)$
where $N_i(x)$, $i=1,2,\ldots NN$; (NN is the number of nodes) are the interpolation functions satisfying the boundary conditions over the domain. The functions $V(x,t)$ and $H(x,t)$ are the exact solutions of the governing equations only if the residuals $L_1(V,H)$ and $L_2(V,H)$ are equal to zero. As the trial solutions are not exact, the resulting error of the residual over the domain can only be minimized to have a satisfactory solution. This is possible by making the trial functions orthogonal to the residual (Galerkin's criterion), which can be written as: $\int N_i(x)L_1(V,H)\,dx = 0$ and $\int N_i(x)L_2(V,H)\,dx = 0$. For NN interpolation functions, there are NN undetermined coefficients V_i and H_i and NN orthogonality conditions are to be satisfied.

$$\int N_i(x,t)\; L_1\left[\sum N_j(x)\; V_j(t),\;\; \sum N_j(x)\; H_j(t)\right]\; dx \;=\; 0 \quad (28)$$

Putting the Galerkin's criterion (Eqns. 28 and 29) into the governing differential Eqns. 26 and 27, the resulting equations are as given by Eqns. 30 and 31.

$$\int N_i(x,t)\; L_2\left[\sum N_j(x)\; V_j(t),\;\; \sum N_j(x)\; H_j(t)\right]\; dx \;=\; 0 \quad (29)$$

$$\int N_i\,[\,\frac{\partial}{\partial t}(\sum N_j\, V_j) + \sum N_j\, V_j \frac{\partial}{\partial x}(\sum N_j V_j) + g\frac{\partial}{\partial x}(\sum N_j H_j) + \frac{f}{2D}\sum N_j V_j (\sum N_j |V_j|)\,]\; dx \;=\; 0 \quad (30)$$

$$\int N_i\,[\,\frac{\partial}{\partial t}(\sum N_j\, H_j) + \frac{C_p^2}{g}\frac{\partial}{\partial x}(\sum N_j\, V_j) + (\sum N_j V_j)\frac{\partial}{\partial x}(\sum N_j H_j) - Sin\varphi (\sum N_j V_j)\,]\; dx \;=\; 0 \quad (31)$$

Integration of each term in Eqns. 30 and 31 can be done by assuming suitable interpolation functions. It is to be noted that these Eqns. 30 and 31 are based on FEM in space domain only and Finite Difference scheme in time domain. The field variables V and H can be expressed as, $H = (H_{t+\Delta t} + H_t)/2$ and $V = (V_{t+\Delta t} + V_t)/2$ and the derivatives of the field variables H and V can be expressed as $(\partial H/\partial t) = (H_{t+\Delta t} - H_t)/\Delta t$ and $(\partial V/\partial t) = (V_{t+\Delta t} - V_t)/\Delta t$. Putting these into Eqns. 30 and 31 and simplifying, the element equations are as given by Eqns. 32 and 33.

Eqns. 32 and 33 can be combined together to form a combined element matrix which is as

$$\begin{bmatrix} \frac{L}{3\Delta t}-\frac{(2V_1+V_2)}{12} & \frac{L}{6\Delta t}+\frac{(2V_1+V_2)}{12} \\ \frac{L}{6\Delta t}-\frac{(V_1+2V_2)}{12} & \frac{L}{3}\Delta t+\frac{(V_1+2V_2)}{12} \end{bmatrix}^t \begin{Bmatrix} H_1 \\ H_2 \end{Bmatrix}^{t+\Delta t} + \frac{C_p^2}{4g}\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} V_1 \\ V_2 \end{Bmatrix}^{t+\Delta t}$$

$$= \begin{bmatrix} \frac{L}{3\Delta t}+\frac{(2V_1+V_2)}{12} & \frac{L}{6\Delta t}-\frac{(2V_1+V_2)}{12} \\ \frac{L}{6\Delta t}+\frac{(V_1+2V_2)}{12} & \frac{L}{3\Delta t}-\frac{(V_1+2V_2)}{12} \end{bmatrix}^t \begin{Bmatrix} H_1 \\ H_2 \end{Bmatrix}^{t} \qquad (32)$$

$$+\frac{C_p^2}{4g}\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}\begin{Bmatrix} V_1 \\ V_2 \end{Bmatrix}^{t} + \frac{LSin\varphi}{6}\begin{Bmatrix} 2V_1+V_2 \\ V_1+2V_2 \end{Bmatrix}$$

$$\begin{bmatrix} \frac{L}{3\Delta t}-\frac{(2V_1+V_2)}{12} & \frac{L}{6\Delta t}+\frac{(2V_1+V_2)}{12} \\ \frac{L}{6\Delta t}-\frac{(V_1+2V_2)}{12} & \frac{L}{3}\Delta t+\frac{(V_1+2V_2)}{12} \end{bmatrix}^t \begin{Bmatrix} V_1 \\ V_2 \end{Bmatrix}^{t+\Delta t} + \frac{g}{4}\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} H_1 \\ H_2 \end{Bmatrix}^{t+\Delta t}$$

$$= \begin{bmatrix} \frac{L}{3\Delta t}+\frac{(2V_1+V_2)}{12} & \frac{L}{6\Delta t}-\frac{(2V_1+V_2)}{12} \\ \frac{L}{6\Delta t}+\frac{(V_1+2V_2)}{12} & \frac{L}{3\Delta t}-\frac{(V_1+2V_2)}{12} \end{bmatrix}^t \begin{Bmatrix} V_1 \\ V_2 \end{Bmatrix}^{t} + \frac{g}{4}\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}\begin{Bmatrix} H_1 \\ H_2 \end{Bmatrix}^{t} \qquad (33)$$

$$+\frac{fL}{24D}\begin{Bmatrix} 3V_1|V_1|+V_1|V_2|+|V_1|V_2+V_2|V_2| \\ V_1|V_1|+V_1|V_2|+|V_1|V_2+3V_2|V_2| \end{Bmatrix}$$

given by Eqn. 34.

2.3 Boundary Conditions:

The element equations can be assembled to form a global matrix which should be solved for the field variables in the domain at time step t+Δt, when all the field variables at time step t are known. The global matrix formed as it is, is singular and cannot be solved unless proper boundary conditions are applied to it. When the domain is divided into NE elements, there are NE+1 nodes and hence there are 2(NE+1) unknowns in the problem as there is a two degree freedom at each node. But the element equations provide for only 2(NE) independent equations. Hence two more additional equations are needed to determine all the unknowns and they are provided by the boundary conditions. Constant head in the reservoir becomes the upstream boundary condition. At the downstream end, the outlet discharge calculated in terms of the valve data and the head at the outlet can be taken as the downstream boundary condition.

2.4 Numerical Example:

This finite element formulation has been applied to a case of a horizontal pipe of uniform diameter discharging water from constant level reservoir into the atmosphere as shown in Fig. 2.1. a solution to this problem in the form of method of characteristics is

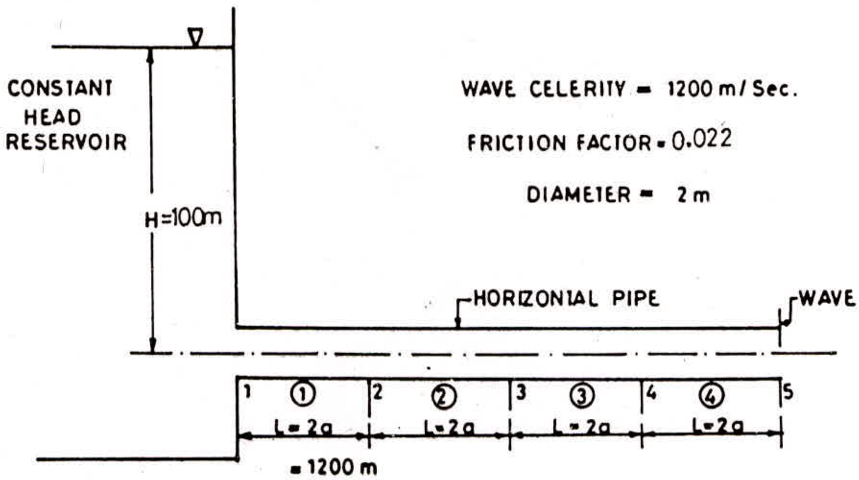


FIG. 2.1 DEFINITION SKETCH OF THE PROBLEM

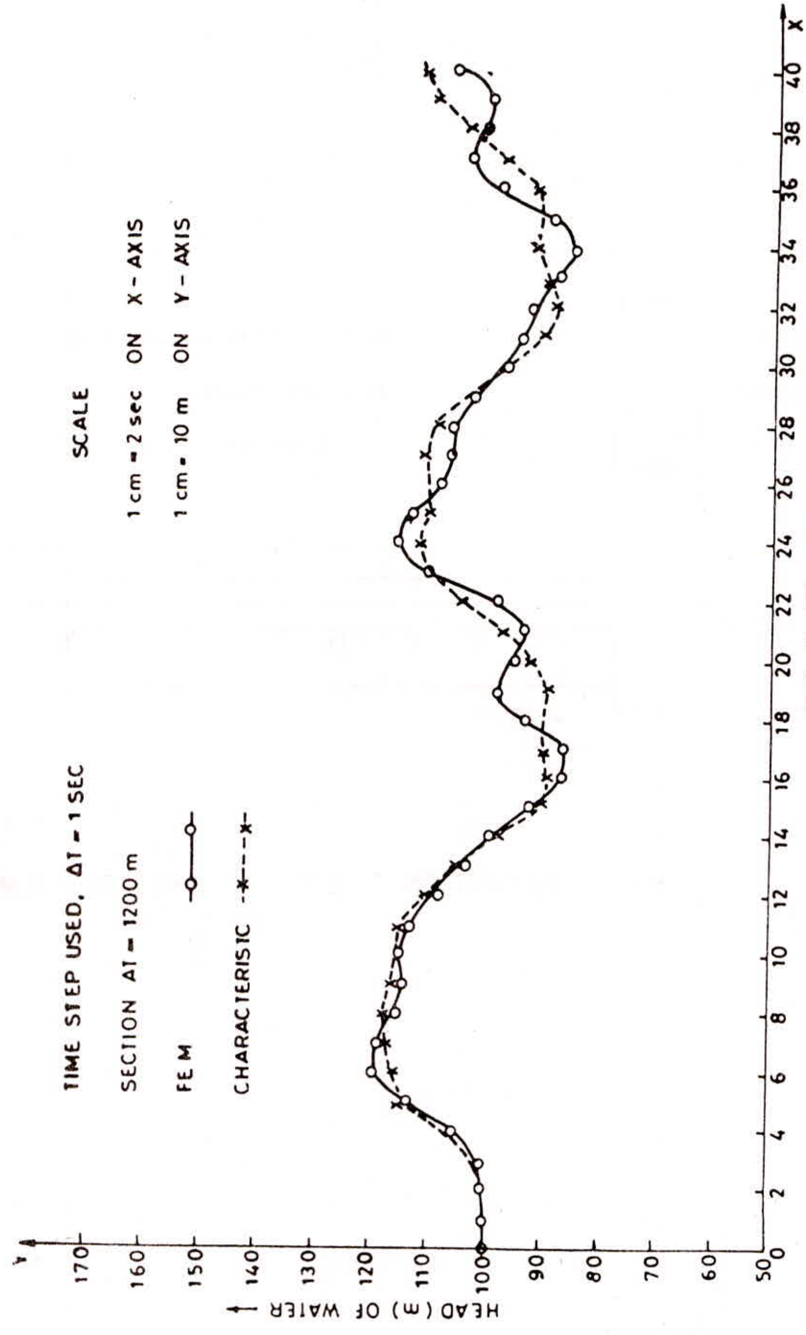


FIG. 2.2 PRESSURE HEAD VARIATIONS WITH TIME AT SECTION X = 1200 m

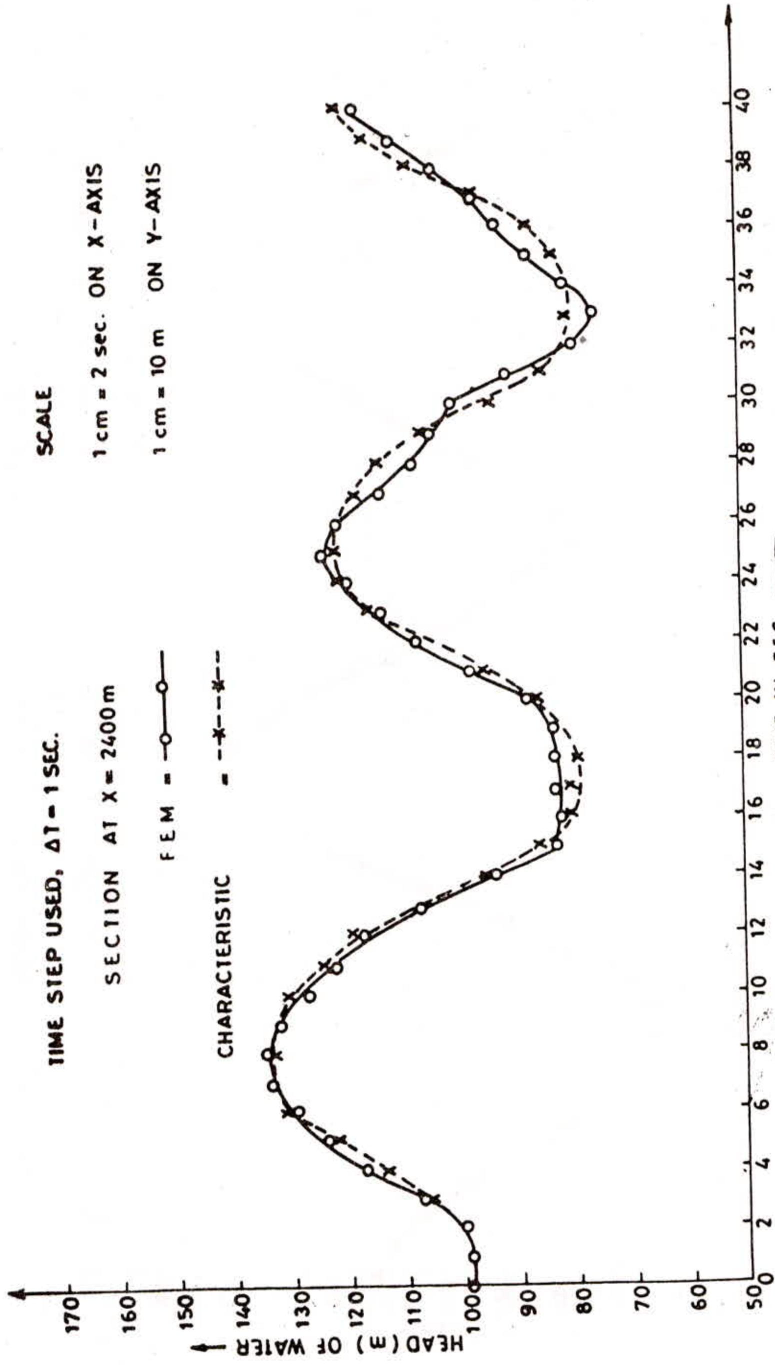


FIG.2.3 PRESSURE HEAD VARIATIONS WITH TIME AT SECTION X – 2400 m

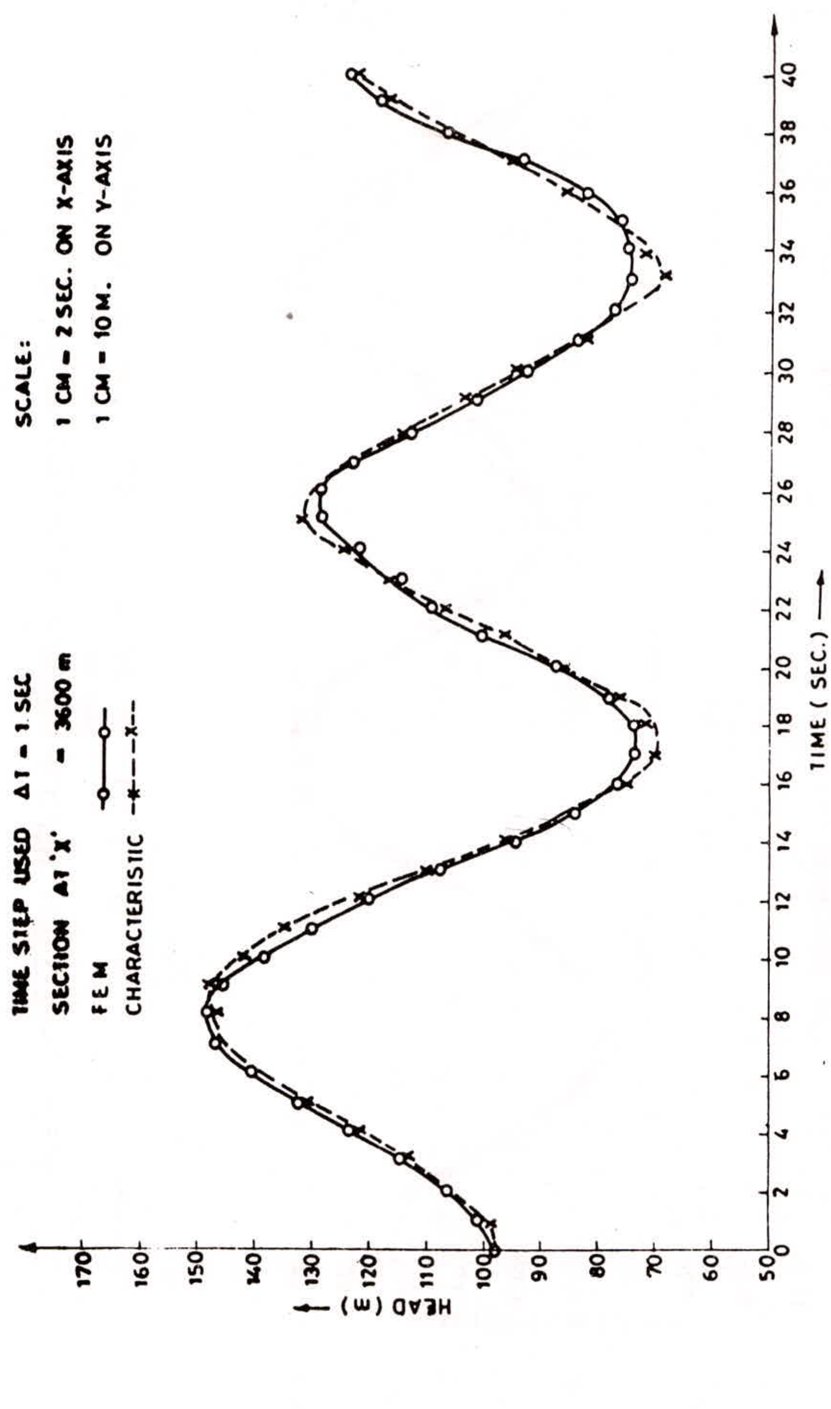


FIG.2·4 PRESSURE HEAD VARIATIONS WITH TIME AT X = 3600 m

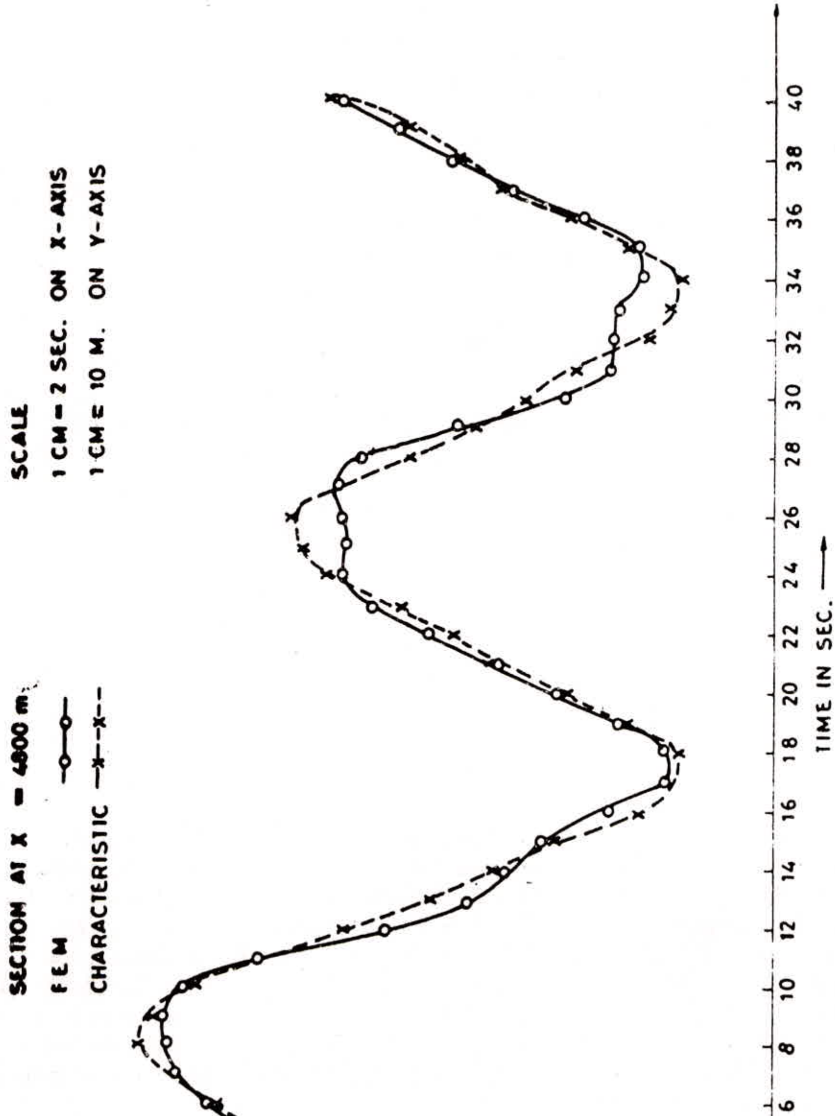


FIG.2.5 PRESSURE HEAD VARIATIONS WITH TIME AT X = 4800 m

$$\begin{bmatrix} \frac{L}{3\Delta t}-\frac{(2V_1+V_2)}{12} & \frac{L}{6\Delta t}+\frac{(2V_1+V_2)}{12} & -\frac{g}{4} & \frac{g}{4} \\ \frac{L}{6\Delta t}-\frac{(V_1+2V_2)}{12} & \frac{L}{3}\Delta t+\frac{(V_1+2V_2)}{12} & -\frac{g}{4} & \frac{g}{4} \\ -\frac{C_P^2}{4g} & \frac{C_P^2}{4g} & \frac{L}{3\Delta t}-\frac{(2V_1+V_2)}{12} & \frac{L}{6\Delta t}+\frac{(2V_1+V_2)}{12} \\ -\frac{C_P^2}{4g} & \frac{C_P^2}{4g} & \frac{L}{6\Delta t}-\frac{(V_1+2V_2)}{12} & \end{bmatrix}^{t} \begin{Bmatrix} V_1 \\ V_2 \\ H_1 \\ H_2 \end{Bmatrix}^{t+\Delta t}$$

$$= \begin{bmatrix} \frac{L}{3\Delta t}+\frac{(2V_1+V_2)}{12} & \frac{L}{6\Delta t}-\frac{(2V_1+V_2)}{12} & \frac{g}{4} & -\frac{g}{4} \\ \frac{C_P^2}{4g} & -\frac{C_P^2}{4g} & \frac{L}{6\Delta t}+\frac{(V_1+2V_2)}{12} & \frac{L}{3\Delta t}-\frac{(V_1+2V_2)}{12} \\ \frac{C_P^2}{4g} & -\frac{\frac{C_P}{2}}{4g} & \frac{L}{6\Delta t}+\frac{(V_1+2V_2)}{12} & \frac{L}{3\Delta t}-\frac{(V_1+2V_2)}{12} \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \\ H_1 \\ H_2 \end{Bmatrix}$$

$$\begin{Bmatrix} \frac{-fL}{24D}(3V_1|V_1|+V_1|V_2|+|V_1|V_2+V_2|V_2|) \\ \frac{-fL}{24D}(V_1|V_1|+V_1|V_2|+|V_1|V_2+3V_2|V_2|) \\ \frac{LSin\varphi(2V_1+V_2)}{6} \\ \frac{Lsin\varphi(V_1+2V_2)}{6} \end{Bmatrix} \quad (34)$$

available. A valve is provided at the discharging end of the pipe. Closure of the valve is gradual. The length and diameter of the pipe 4800 m and 2 m respectively. The constant head in the reservoir is 100 m. The entire length of the pipe is divided into four elements of equal length. The system has a valve opening $C_DA = 0.06$ sq met. At intervals of 5 seconds, C_DA takes the values 0.03, 0.01,0.003, 0.001, 0.0005, 0.0002, 0.0000 and remains closed. The transients of the system for about 40 seconds after the valve starts to close, have been worked out. The pressure transients occurring at different sections of the pipe have been worked out and plotted for different time intervals. The results are compared with those obtained by the method of characteristics as shown in Figs. 2.2 to 2.5. The comparison shows that these two results agree closely with each other.

3.0 DISPERSION OF POLLUTANTS IN POROUS MEDIA FLOW - ONE DIMENSIONAL APPROACH:

With the rapid growth in urbanization and industrialization, the possibilities of the contamination of both the surface water and ground water sources are rapidly increasing. The ground water sources which were being considered as relatively free from contamination are gradually becoming degraded as a result of transport of soluble chemicals via the rainwater percolating into the subsoil and ultimately meeting the ground water. Disposal of solid wastes and sewage on land, agricultural activities, disposal of liquid wastes originated from industries and disposal of radioactive waste are the major sources of ground water contamination. Remedial measures are absolutely essential to prevent this contamination. If not detected in time, it may result in damage of the aquifers beyond repair.

Various contaminants polluting the ground water sources are in the form of soluble chemicals or dissolved substances called solute. There are other forms of solutes namely the natural constituents of minerals and artificial tracers. The transport of these solutes occur through an aquifer by convection and hydrodynamic dispersion. Convection or advection is the process by which the solutes are transported by the bulk motion of the flowing ground water. Hydrodynamic dispersion involves mechanical or hydraulic dispersion and molecular dispersion. As a result of convection and dispersion, the solute is transported by the ground water with varying degree of concentration throughout the domain of flow. The differential equation describing the physical processes of convection and hydrodynamic dispersion known as Convective-Dispersion Equation is basically obtained from the statement of principle of conservation of mass which is: NET RATE OF CHANGE OF MASS OF SOLUTE WITHIN AN ELEMENTAL VOLUME = FLUX OF SOLUTE OUT OF ELEMENT - FLUX OF SOLUTE INTO THE ELEMENT ± LOSS OR GAIN OF SOLUTE MASS DUE TO INTERNAL REACTIONS. Convection and hydrodynamic dispersion are physical processes controlling the influx and efflux of the solute, whereas adsorption and radioactive decay are chemical or biochemical processes that cause the loss or gain of the solute mass.

Looking into the need for the control of the ground water quality, it is necessary for a ground water hydrologist to study the engineering aspects of the problem of contaminant transport such as distribution of solute concentration in the aquifer, estimation of the level of contamination and planning of suitable remedial measures to prevent further contamination. From this point of view, it is necessary to solve the differential equation describing the process of solute transport, that is, the solution of the dispersion equation for the case of saturated, homogeneous isotropic porous media. Because the straight forward analytical solution for the most general form of this equation is not possible, a numerical solution using the F.E.M. has been attempted and the digital model is developed for the case of one-dimensional form. As the analytical solution is available for this case, the results from the digital model have been compared with the analytical solution. To simplify the problem, adsorption and radioactive decay were not considered in this formulation.

The problem of contaminant transport has been studied extensively during the last three decades. In the earlier part of this period, the investigations were mainly concerned with the theoretical development and the experimental work, whereas in the later part, attention seems to be focused mainly on the numerical solution of the problem using numerical techniques. Bear (1961), (1972) and Scheidegger (1961) presented the general theory of dispersion in the porous media flow. Bachamat and Bear (1964) described general equations governing the hydrodynamic dispersion of a homogeneous fluid in a homogeneous and isotropic medium. Dagan (1967) solved the one-dimensional dispersion equation using perturbation technique. Guymon (1970) gave an equivalent variational principle to the governing partial differential equation of one-dimensional diffusion-convection and developed a F.E. solution requiring approximation in space domain only. The analytical solutions were developed by Marino (1974) for two longitudinal dispersion problems in saturated porous media.

The basic differential equation describing the process of solute transport in saturated, homogeneous isotropic porous media and accounting for the effects of convection, hydrodynamic dispersion adsorption and radioactive decay can be written (Bachamat 1964, Freeze 1974) as given by Eqn. 35.

$$n\frac{\partial C}{\partial t} + (1-n)\frac{\partial S}{\partial t} = \frac{\partial}{\partial x_i}\left[n\, D_{ij}\frac{\partial C}{\partial x_j} - u_i\, C\right] - \lambda\,[n\,C + (1-n)\,S] + q\,C^{*} \qquad (35)$$

The flow through the porous medium is assumed to be steady and obeys Darcy's law. Assuming linear equilibrium adsorption, the concentration in the solid phase can be expressed as S = k C, where k is the constant of proportionality between concentrations in solid and liquid phases, C is the concentration of the dispersing mass in liquid phase and S is the concentration of the dispersing mass in solid phase.

$$R\frac{\partial C}{\partial t} - \frac{\partial}{\partial x_i}\left[D_{ij}\frac{\partial C}{\partial x_j} - \frac{u_i}{n}\,C\right] - \lambda\,R\,C - \frac{q}{n}\,C^{*} = 0 \qquad (36)$$

In Eqn. 36, R is the retardation factor R = [1 + k(1-n)/n]. For the case of one-dimensional dispersion, Eqn. 36 takes the form given by Eqn. 37.

$$L(C) = R\frac{\partial C}{\partial t} - D_{xx}\frac{\partial^2 C}{\partial x^2} + \frac{1}{n}\frac{\partial}{\partial x}(u_x\,C) + \lambda\,R\,C - \frac{q}{n}\,C^{*} = 0 \qquad (37)$$

3.1 Formulation of Element Matrices:

In this method, a trial solution of the following form is assumed: $C = C(x,t) = \sum N_i(x)$. $C_i(t)$ where $N_i(x)$, i=1,2,...NN; (NN is the number of nodes) are the interpolation functions satisfying the boundary conditions over the domain. The function C(x,t) is the exact solution of the governing equation only if the residual L(C) is equal to zero. As the trial solution is not exact, the resulting error of the residual over the domain can only be minimized to have a satisfactory solution. This is possible by making the trial functions orthogonal to the residual over the domain. That is, $\int N_i(x)\, L(C)\, dx = 0$. For NN interpolation functions, there are NN undetermined coefficients C_i and NN orthogonality conditions are to be satisfied. Therefore,

$$\int N_i(x)\; L\left[\sum N_j(x)\; C_j(t)\right] dx = 0 \qquad (38)$$

Putting Eqn. 38 in Eqn. 37, the resulting equation is as given by Eqn. 39.

$$\int [R\, N_i \frac{\partial}{\partial t} (\sum N_j\, C_j) + D_{xx} \frac{\partial N_i}{\partial x} \frac{\partial}{\partial x} (\sum N_j C_j) - \frac{u_x}{n} \frac{\partial N_i}{\partial x} (\sum N_j C_j) + \lambda R N_i \sum N_j C_j]\, dx = D_{xx} \frac{\partial C}{\partial x} N_i + \int \frac{q}{n} C^{*} N_i\, dx \quad (39)$$

The field variable C can be expressed as, $C = (C_{t+\Delta t} + C_t)/2$ and the derivative of the field variable C can be expressed as $(\partial C/\partial t) = (C_{t+\Delta t} - C_t)/\Delta t$. Now Eqn. 39 can be written as: $[P]\{\partial C/\partial t\} + [K]\{C\} = \{F\}$, where $p_{ij} = \int N_i\, N_j\, dx$ and $k_{ij} = \int [D_{xx}\, (dN_i/dx)(dN_j/dx) - (u_x/n)(dN_i/dx)N_j + \lambda R N_i N_j]\, dx$, and $f_i = [D_{xx}(\partial C/\partial x) - (u_x/n)]N_i + \int (q/n)C^{*}N_i\, dx$. Putting these into Eqn. 39 and simplifying, the matrix form of the equations is as given by Eqn. 40.

$$\left(\frac{1}{\Delta t}[P] + \frac{1}{2}[K]\right)\{C\}_{t+\Delta t} = \left(\frac{1}{\Delta t}[P] - \frac{1}{2}[K]\right)\{C\}_t + \{F\} \quad (40)$$

The Eqn. 40 represents the general assembly of the set of Finite Element Equation. Solution of Eqn. 40 after incorporating the initial and boundary conditions yields the unknown values of concentration C at the nodal points at any time instance when the values at the earlier time instance are known.

3.2 Initial and Boundary Conditions:

For the dispersion problem, the initial concentration distribution over the entire domain under consideration is assumed to be zero, that is, $C(x,0) = 0$ for $x > 0$. The following forms of upstream boundary conditions given by Marino [8] are assumed in the present problem:

Case I: $C(0,t) = C_o$, $t > 0$

Case II: $C(0,t) = C_o\, \mathrm{Exp}(\gamma t)$, $t > 0$

Case III: $C(0,t) = C_o\, [1 - \exp(-\gamma t)]$, $t > 0$

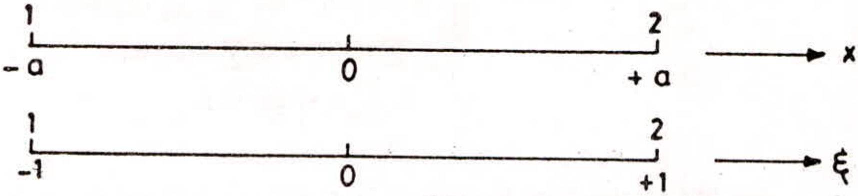
The set of simultaneous equations whose coefficients were stored in compact form solved by Gauss-Seidel iterative scheme. This method is particularly suitable to large unsymmetric matrices when stored in compact form using row oriented method.

3.3 Numerical Example:

Marino (1974) has given the following form of analytical solution to the one-dimensional dispersion equation for the above cases of upstream boundary conditions.

$$CASE\ I: \quad C(x,t) = \frac{1}{2} C_o \left\{ erfc\left(\frac{(x-ut)}{2\sqrt{D_t}}\right) + Exp\left(\frac{ux}{D}\right) erfc\left(\frac{(x+ut)}{2\sqrt{D_t}}\right) \right\} \quad (41)$$

In Eqns. 41, 42 and 43 $\psi = (u^2+4D\gamma)^{1/2}$ and $\varphi = (u^2 - 4D\gamma)^{1/2}$. In this formulation, the adsorption and radioactive decay are not taken into consideration. The domain chosen for study is 600 m long has been descretised using a simple line element with linear interpolation



- LINE ELEMENT

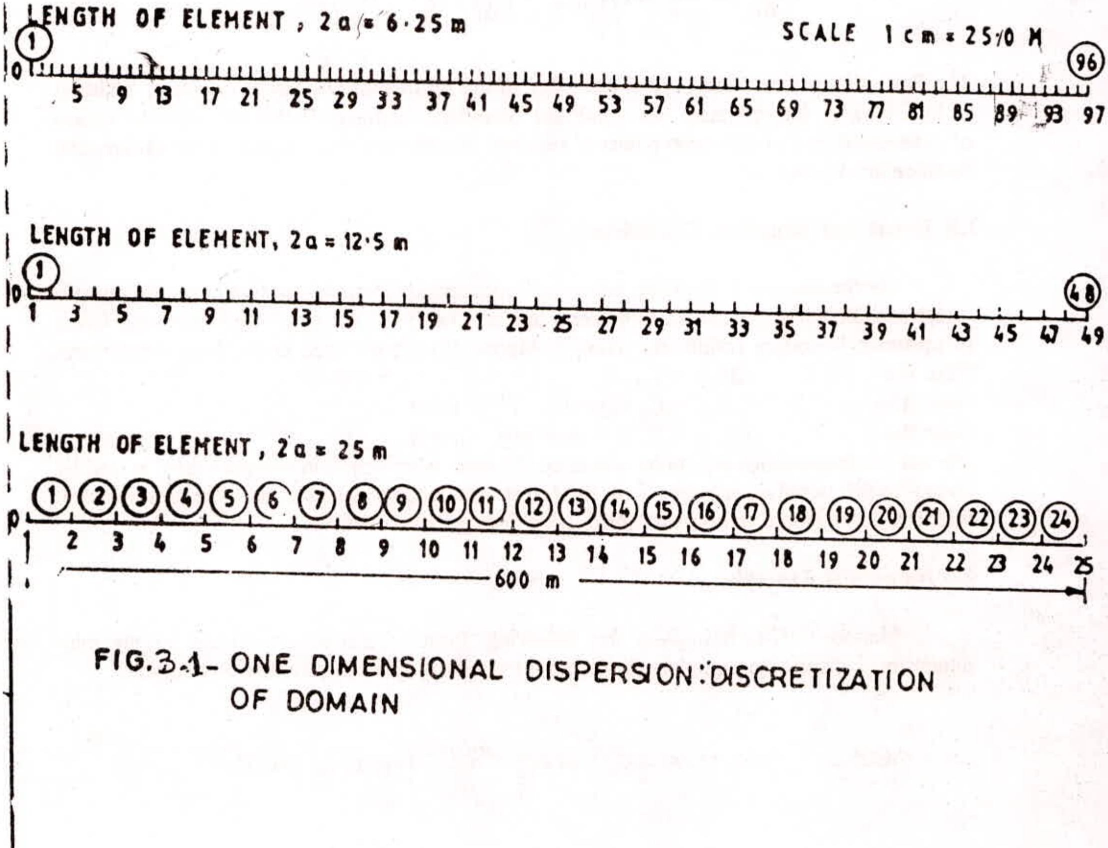


FIG.3.1- ONE DIMENSIONAL DISPERSION: DISCRETIZATION OF DOMAIN

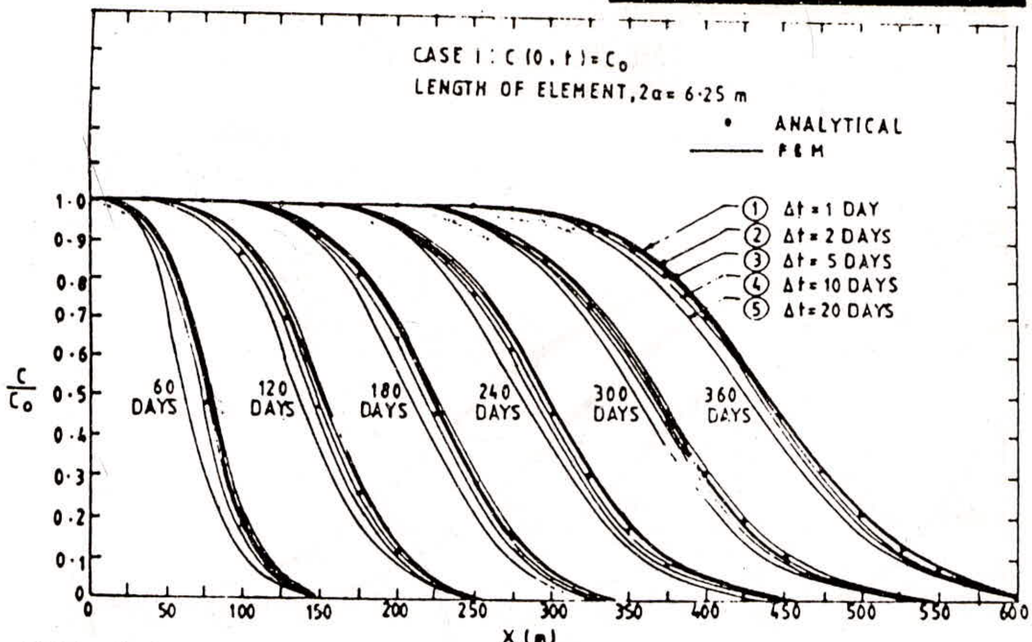


FIG.3-2 ONE - DIMENSIONAL DISPERSION : COMPARISON OF FEM AND ANALYTICAL RESULTS (TIME STEP VARIATION)

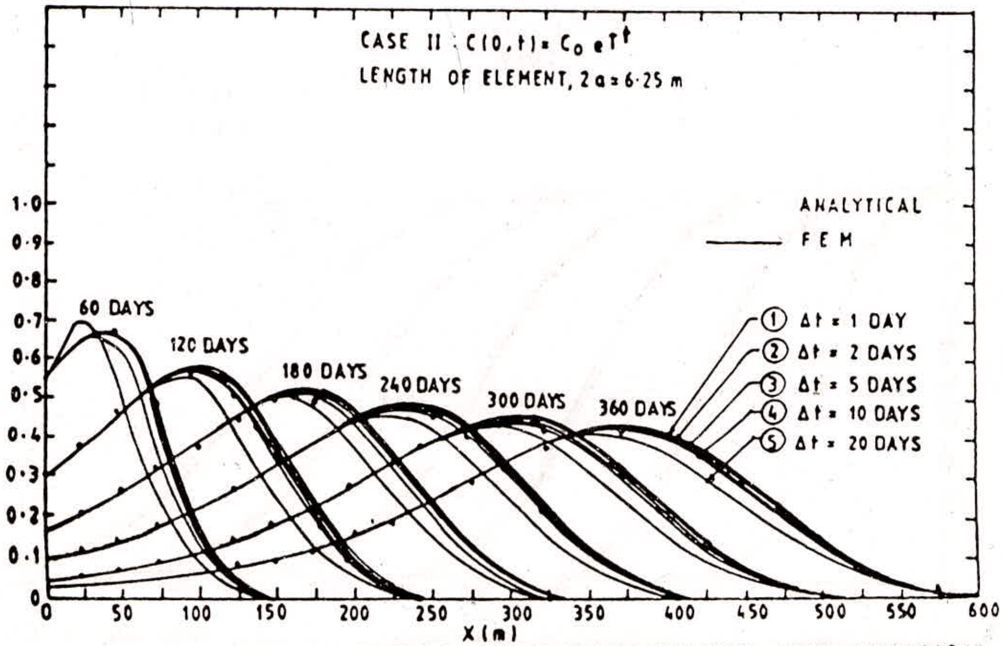


FIG 3-3 ONE-DIMENSIONAL DISPERSION : COMPARISON OF FEM AND ANALYTICAL RESULTS (TIME STEP VARIATION)

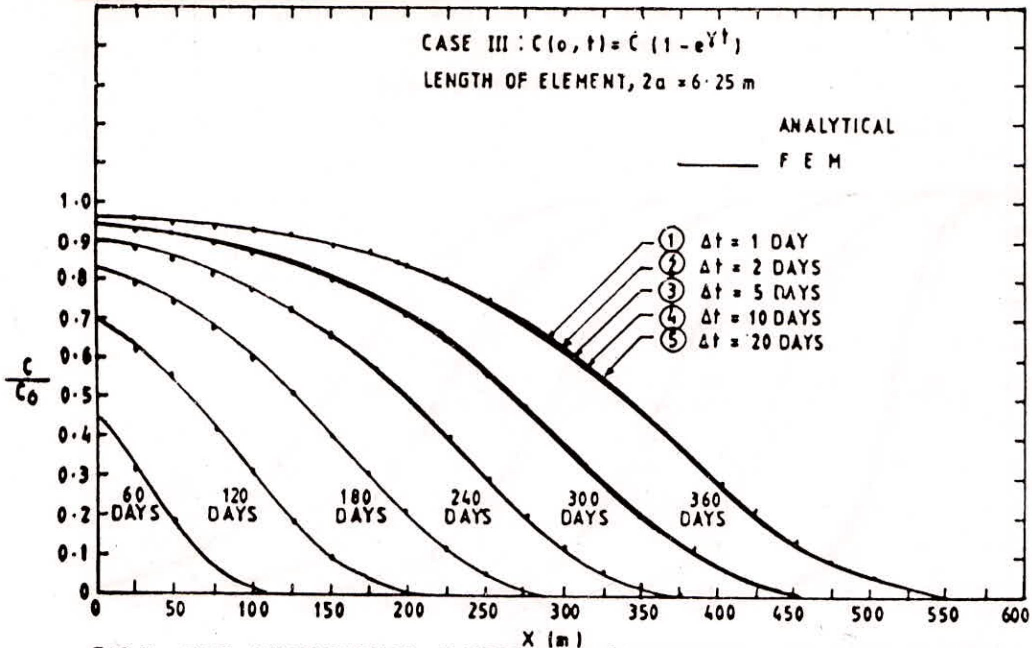


FIG.3.4 ONE-DIMENSIONAL DISPERSION: COMPARISON OF F E M AND ANALYTICAL RESULTS (TIME STEP VARIATION)

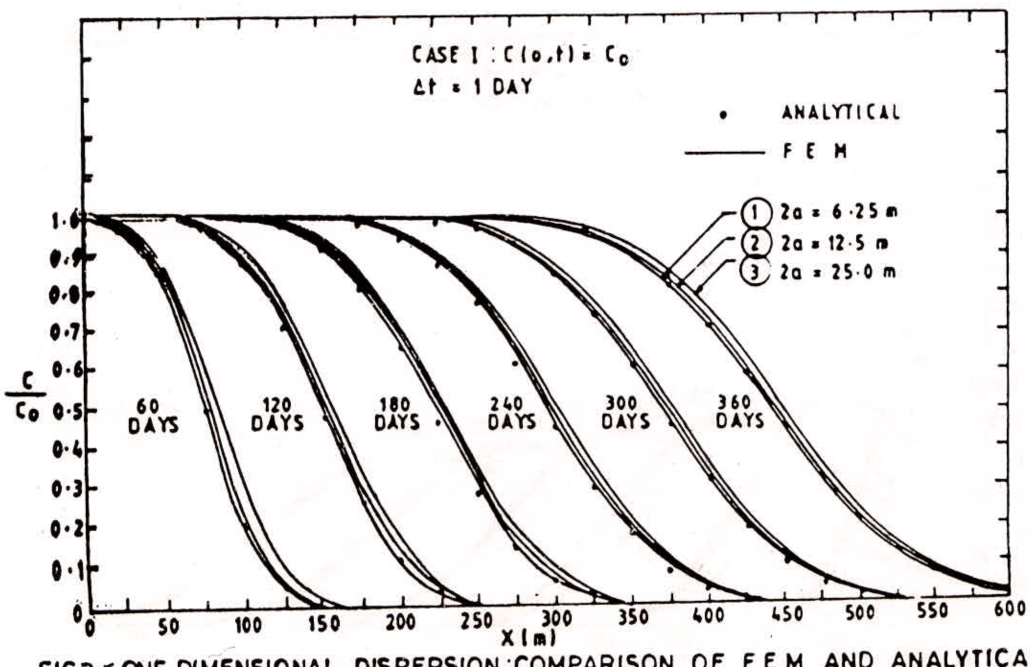


FIG.3.5 ONE DIMENSIONAL DISPERSION: COMPARISON OF F E M AND ANALYTICAL RESULTS (ELEMENT SIZE VARIATION)

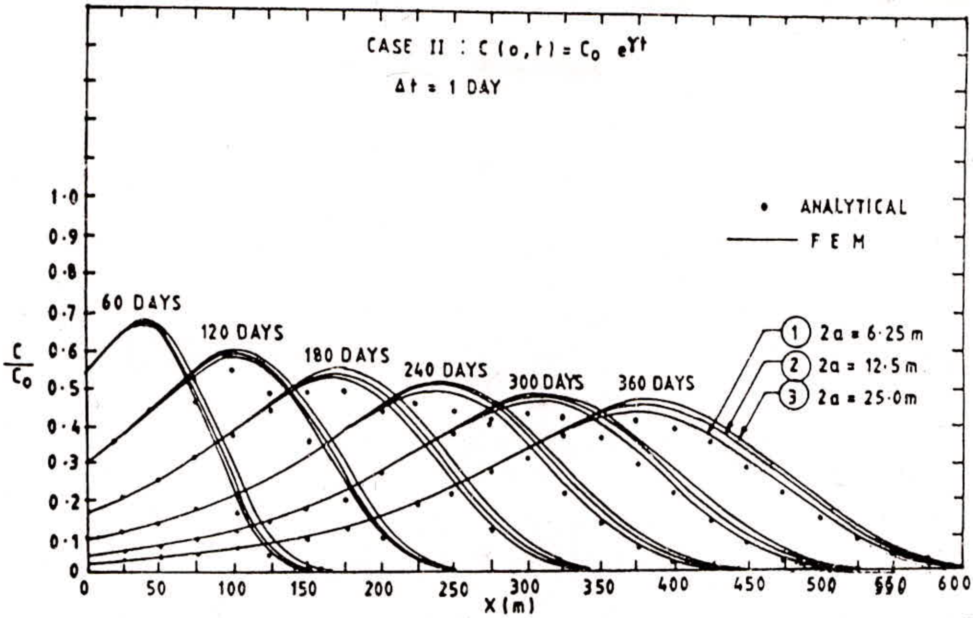


FIG.3.6 ONE-DIMENSIONAL DISPERSION : COMPARISON OF FEM AND ANALYTICA RESULTS (ELEMENT SIZE VARIATION)

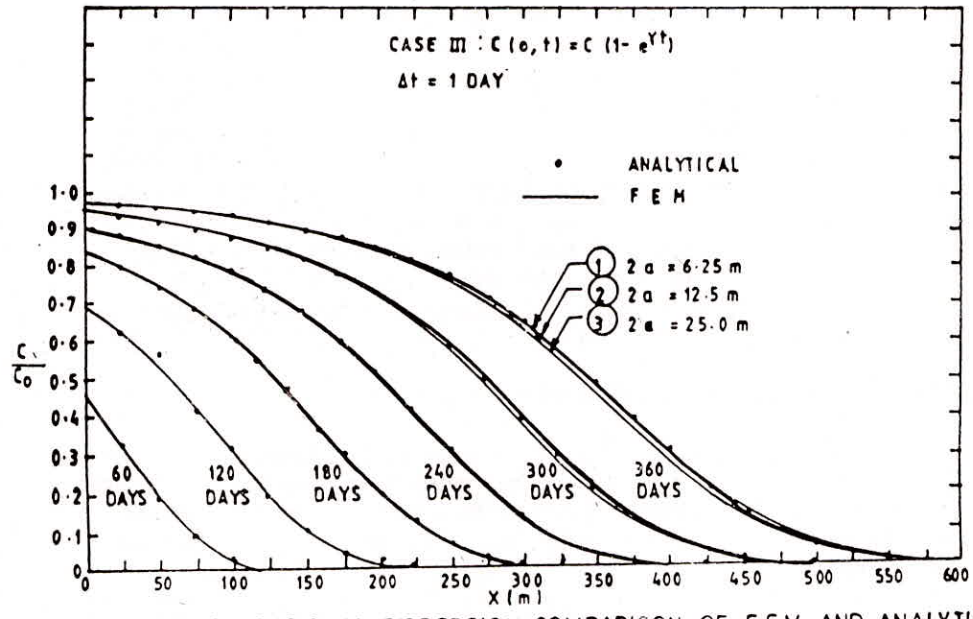


FIG.3.7 ONE-DIMENSIONAL DISPERSION : COMPARISON OF FEM AND ANALYTIC RESULTS (ELEMENT SIZE VARIATION)

$$CASE\ II:\quad C(x,t) = \frac{1}{2}C_o \exp(\gamma t)\left\{Exp\left[\frac{x(u-\psi)}{2D}\right]erfc\left(\frac{(x-\psi t)}{2\sqrt{D_t}}\right)\right\} + \tag{42}$$

$$\frac{1}{2}C_o \exp(\gamma t)\left\{Exp\left[\frac{x(u+\psi)}{2D}\right]erfc\left(\frac{(x-\psi t)}{2\sqrt{D_t}}\right)\right\}$$

$$CASE\ III:\quad C(x,t) = \frac{1}{2}C_o\left\{erfc\left(\frac{(x-ut)}{2\sqrt{D_t}}\right) + Exp\left(\frac{ux}{D}\right)erfc\left(\frac{(x+ut)}{2\sqrt{D_t}}\right)\right\} -$$

$$\frac{1}{2}C_o \exp(-\gamma t)\left\{Exp\left(\frac{x(u-\varphi)}{2D}\right) erf\left(\frac{x-\varphi t}{2\sqrt{D_t}}\right)\right\} -$$

$$\frac{1}{C_o}\exp(-\gamma t)\left\{\left(\frac{x(u+\varphi)}{2D}\right)erfc\left(\frac{x+\varphi t}{2\sqrt{D_t}}\right)\right\} \tag{43}$$

function. The results have been obtained for all the possible combinations of the element size and time step in order to study the variation of FEM results in terms of dimensionless parameters Courant Number and Peclet number.

Figs 3.2 to 3.4 show the graphs of relative concentration against distance for a typical element length of 6.25 m and for various time steps and Figs. 3.5 to 3.7 show the similar graphs for a typical time step of 1 day and for various element sizes in respect of the three boundary conditions stated above in cases I to III. The results of the analytical solution have been superimposed on these graphs. From the graphs, it can be observed that the FEM results compare well with those of the analytical solution. For oscillation free numerical solution with regard to temporal and spatial distribution respectively, the criteria for Courant number and Peclet number in respect of one-dimensional dispersion are specified and given by Daus and Frind (1978) are: (i) Courant number, $CN_x = u_x\Delta t/\Delta x \leq 1.0$ and (ii) Peclet number, $PN_x = u_x\Delta x/D_{xx} \leq 2.0$. In these studies, the Courant number ranges from 0.05 to 3.9 and Peclet number ranges from 1 to 4. Even for this wider range of numbers, the FEM results are found to be stable and oscillation free. The FEM results closely follow the analytical solution results when the Courant number is of the order 0.5.

4.0 SIMULATION OF MULTI-AQUIFER BASINS BY FEM:

In a multiaquifer basin, the aquifers are separated by semi-impervious layers or aquitards which transmit water from the adjoining aquifers to the pumped aquifer when there is head difference between the pumped aquifer and the adjoining aquifer. While analyzing the multiaquifer system, the contribution in the form of leakage from the adjoining aquifers should be taken into account.

Amongst the earlier investigators may be mentioned the names of Jacob and Hantush (1955), who developed type curves for radial flow towards a well in a leaky aquifer system.

Pinder and Bredehoeft (1968) developed a finite difference model for two-dimensional flow in an artesian aquifer taking into account leakage from the adjoining aquifer through the intervening semi-pervious layer, but neglecting contribution from storage of the aquitard and assuming zero drawdown in the adjoining aquifer. Neumann and Witherspoon (1969) developed an analytical solution for the problem of flow to a well in an artesian aquifer, receiving leakage from the upper confined aquifer separated by an aquitard and drawdown in the unpumped aquifer. The solutions are applicable to homogeneous and isotropic aquifers which are confined and separated by an aquitard. Bredehoeft and Pinder (1970) gave a finite difference model for two aquifer system, the upper aquifer being water table aquifer and lower pumped one an artesian aquifer. Both the drawdown in the water table aquifer and contribution from the storage of the aquitard taken into account. The model was tested by comparing the solution with theoretical results obtained from Hantush's (1960) modified leakage theory, in which drawdown in the water table aquifer is assumed to be zero, Pinder and Frind (1972) gave a finite element model for a two-dimensional flow in a leaky aquifer considering zero drawdown in the unpumped aquifer and neglecting contribution from aquitard storage. Yuang and Sonnenfeld (1974) presented a numerical method based on three-dimensional finite elements for determining the unsteady drawdown around an artesian well. Gray and Pinder (1974) applied the finite element method to the unsteady flow in a homogeneous isotropic infinite confined aquifer using time as the third dimension. Huang and Wu (1975) presented a three-dimensional finite element model for unsteady in a confined aquifer, pumped by an artesian well. It can be used for determining the drawdown in the aquifer by specifying the discharge or recharge at various nodes. They applied it to a hypothetical aquifer. Gupta and Tanzi (1976) developed a finite element model for three dimensional ground water flow for multiaquifer basin (Sutter basin of California in USA) for steady state condition using mixed isoparametric elements. Fuzinawa (1977) developed an integrated finite element model for two confined aquifer systems with one dimensional finite element analysis of the aquitard, considering the entire aquitard thickness as one element for the leakage flux. Frind and Verge (1978) developed finite element model for a three-dimensional ground water flow problem, based on general saturated-unsaturated continuity equation. The model was tested by comparing the results of one-dimensional and two-dimensional solutions. Chorely and Frind (1978) developed a quasi-three-dimensional finite element model for multiaquifer system. For determining the leakage flux from the aquitard one-dimensional finite elements were used. The model was tested by comparing the results obtained from the analytical solution of Neumann and Witherspoon (1969) for radial flow in pumped and unpumped aquifers having identical properties. The results were also compared with those obtained from the finite element leaky aquifer model neglecting storage of aquitard and drawdown in the unpumped aquifer.

The development of a digital model for simulating the behaviour of multiaquifer basin in response to hydrologic stresses in the form of pumping or recharge through wells in artesian and/or water-table aquifers, taking into account contribution from storage of aquitard and drawdown in the unpumped aquifer has been presented here. This model was applied to a multiaquifer basin Mehsana district of Gujarat state.

4.1 Governing Equations:

The continuity equation for two-dimensional horizontal flow in a leaky aquifer can be written as given by Eqn. 44.

$$S\frac{\partial h}{\partial t} - \nabla_{xy}.\{[T]\,[\nabla_{xy}h]\} + Q(x,y) + q_{z|z2} - q_{z|z1} = 0 \qquad (44)$$

where z_1 and z_2 are the elevations of the lower and upper confining layers, $q_z|_{z1}$ and $q_z|_{z2}$ represent the leakage fluxes from the lower and upper aquifers respectively, S is the storage coefficient and T is the transmissivity of the aquifer, h is the piezometric head and Q is the strength of a source or sink function defined as given by Eqn. 45,

$$Q = \sum_{k=1}^{NW} Q_w(x_k, y_k)\ \delta(x-x_k, y-y_k) \qquad (45)$$

where Q_w is the volumetric discharge from the aquifer, δ is the Dirac delta function and NW the number of wells. If the principal components of the transmissivity tensor are collinear with the coordinate axes, the cross derivatives vanish. Putting $q_z|_{z2} = q_{z2}$ and $q_z|_{z1} = q_{z1}$, the expanded form of the Eqn. 45 will be as given by Eqn. 46.

$$\frac{\partial}{\partial x}\left(T_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_{yy}\frac{\partial h}{\partial y}\right) - S\frac{\partial h}{\partial t} - Q - q_{z2} + q_{z1} = 0 \qquad (46)$$

For three layered system consisting of water table aquifer, aquitard and artesian aquifer Eqn. 46 will be reduced to (after putting q_{z1}=0 and q_{z2}=q_L) the one as given by Eqn. 47.

$$\frac{\partial}{\partial x}\left(T_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_{yy}\frac{\partial h}{\partial y}\right) - S\frac{\partial h}{\partial t} - Q - q_L = 0 \qquad (47)$$

The continuity equation for the water table aquifer is similar to Eqn. 47 except that transmissivity is a function of hydraulic head in case of water table aquifer. It will be of the form as given by Eqn. 48.

$$\frac{\partial}{\partial x}\left(K_{xx}b'(h_w)\frac{\partial h_w}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy}b'(h_w)\frac{\partial h_w}{\partial y}\right) - S_y\frac{\partial h_w}{\partial t} - Q_w - q_L = 0 \qquad (48)$$

In Eqn. 48, K is the hydraulic conductivity, $b'(h_w)$ is the saturated thickness, h_w is the hydraulic head, S_y is the specific yield and Q_w is the strength of source or sink function for the water table aquifer. The vertical flow in the aquitard is governed by the equation 49.

$$\frac{\partial^2 h^*}{\partial z^2} = \frac{S_s^*}{K^*}\frac{\partial h^*}{\partial t} \qquad (49)$$

In Eqn. 49, h^* is the hydraulic head in the aquitard, K^* is the hydraulic conductivity and S^*_s is the specific storage of the aquitard. The boundary conditions are of the usual Dirichlet and Neumann type, these are:(i) $h = h_o$ on Γ_1 and (ii) $-T(\partial h/\partial n) = q_o$ on Γ_2 for artesian aquifer and (iii) $h_w = h_{wo}$ on Γ_1 and (iv) $-K_w b'(\partial h_w/\partial n) = q_{wo}$ on Γ_2 for water table aquifer, where q_o and q_{wo} normal fluxes entering the boundaries, $\Gamma_1 + \Gamma_2 = \Gamma$ is the areal boundary of the domain and h_o and h_{wo} are constant piezometric heads and water levels in the artesian and water table aquifers respectively.

The solution to Eqn. 49 for vertical one-dimensional flow through the aquitard is obtained using the method of Laplace Transforms (Patel and Rao, 1982). The solution for the case of initial piezometric surface coinciding with the initial water table surface is of the form as given by Eqn. 50.

$$h'(z,t) = h_o - \Delta h + \frac{\Delta h z}{b'} + \sum_{n=1}^{\infty}\left(\frac{2\Delta h}{n\pi}\right) Sin\left(\frac{n\pi z}{b'}\right) \exp\left(\frac{-n^2\pi^2\alpha t}{b'^2}\right) \quad (50)$$

In Eqn. 50, h_o is the initial piezometric head, α is the ratio (K'/S_s') and Δh is the change in head at the lower boundary of the aquitard in time t. The leakage flux q_L entering the artesian aquifer at the lower boundary of the aquitard (z=0) is given by $q_L = -(K' \partial h'/\partial z) = -(K' \Delta h/b')\{1 + 2 \leq \exp[-(n^2\pi^2\alpha t/b'^2)]\}$, n varies from 1 to ∞. The change in head at the lower boundary of the aquitard Δh in time t is nothing but the drawdown $s = h_w - h$ in time t. Assuming head change to be applied at one-half the elapsed time, the leakage flux in terms of drawdown and leakage coefficient is given by Eqn. 51.

$$q_L = \frac{K's}{b'}\left[1 + 2\sum_{n=1}^{\infty} \exp\left(\frac{-n^2\pi^2 K' t}{2S_s' b'^2}\right)\right] = \frac{K' s\theta}{b'} \quad (51)$$

$$\text{where} \quad \theta = 1 + 2\sum_{n=1}^{\infty} \exp\left(\frac{-n^2\pi^2 K' t}{2S_s' b'^2}\right)$$

After substituting Eqn. 51 into Eqns. 47 and 48 for q_L, the problem reduces to that of two-dimensional flow in a leaky aquifer. A nondimensional plot of leakage flux versus time is given in Fig. 4.1. It may be seen from the graph that the contribution from the aquitard storage takes place only upto non-dimensional time approximately equal to 0.2. Leakage from the water table aquifer begins only after this time. Hence, if the duration of continuous pumping is less than this time, the analysis should be confined to the artesian aquifer and the aquitard only.

4.2 Element Matrices:

A trial solution of the form $h = h(x,y,t) = \sum N_i(x,y)\, H_i(t)$ is to be assumed if this problem is to be solved by using FEM, where $N_i(x,y)$, i=1,2,...NN; (NN is the number of nodes) are the interpolation functions satisfying the boundary conditions over the domain, $H_i(t)$ are the undetermined time dependent coefficients, which are shown to be solution to Eqn. 47 at the nodal points in the solution domain. For the artesian aquifer, the standard procedure yields a finite element equation which can be written in matrix form as given by Eqn. 52. In the same manner, the finite element equation for the water table aquifer can be

$$[A]\{H\} + [B]\left\{\frac{dH}{dt}\right\} = \{F\}$$

$$\text{where} \quad a_{ij} = \iint\left(T_{xx}\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x} + T_{yy}\frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y} + \frac{K'}{b'}\theta N_i N_j\right) dxdy \quad (52)$$

$$\text{and} \quad b_{ij} = \iint S\, N_i N_j \, dx\, dy$$

$$\text{and} \quad f_i = \iint N_i\left(\frac{K'}{b}*\theta h_w - Q\right) dx\, dy + \oint q_n N_i \, ds$$

obtained in the matrix form as given by Eqn. 53.

$$[A_w]\{H_w\} + [B_w]\left\{\frac{dH_w}{dt}\right\} = \{F_w\}$$

$$\text{where} \quad a_{wij} = \iint\left(K_{xx}b'\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x} + K_{yy}b'\frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y} + \frac{K^*}{b^*}\theta N_i N_j\right)dxdy \quad (53)$$

$$\text{and} \quad b_{wij} = \iint S_y N_i N_j \, dx\, dy$$

$$\text{and} \quad f_{wi} = \iint N_i\left(\frac{K^*}{b}*\theta h_w - Q_w\right)dx\, dy + \oint q_n N_i \, ds$$

When the field variable H was expressed as, $H = (H_{t+\Delta t} + H_t)/2$ and the derivative of the field variable H was expressed as $(\partial H/\partial t) = (H_{t+\Delta t} - H_t)/\Delta t$, the Eqns. 52 and 53 can be written as Eqn. 54.

$$\left(\frac{1}{\Delta t}[B] + \frac{1}{2}[A]\right)\{H\}_{t+\Delta t} = \left(\frac{1}{\Delta t}[B] - \frac{1}{2}[A]\right)\{H\}_t + \{F\}$$
$$\text{and} \quad (54)$$
$$\left(\frac{1}{\Delta t}[B_w] + \frac{1}{2}[A_w]\right)\{H_w\}_{t+\Delta t} = \left(\frac{1}{\Delta t}[B_w] - \frac{1}{2}[A_w]\right)\{H_w\}_t + \{F_w\}$$

The solution of Eqn. 54 after incorporating the initial and boundary conditions yields the unknown values of piezometric levels H and water table aquifer levels H_w at the nodal points at any time instance when the values at the earlier time instance are known. Integration of the parameters over the elements was performed by using the Gaussian quadrature scheme. The above algorithm was used to handle a three layered medium. It can also be used for multiaquifer systems with slight modifications. Compact storage schemes to store the global matrix have been used to optimize the main memory. The matrix formed is symmetric and only the upper half only needs to be stored.

4.3 Case Study:

Mehsana district of Gujarat state, India is a draught prone area and depends mainly for its irrigation requirements on ground water, which is developed through tube wells. Water levels have been depleted greatly (more than 70 m in some parts) and are found to be lowering at the rate of 1.5 m per year on an average. This has caused an alarming situation demanding the scientific study of the problem and taking remedial measures in the form of taking up schemes of artificial recharge and controlling over-development of ground water, if necessary. Two major aquifer units have been identified within the explored depth of 600 m. The upper unit is phreatic, which consists of relatively coarse-grained sediments. The lower unit comprises of few hundred meters of alternatively sandy and argillaceous beds forming semiconfined and confined systems. On account of lowering of water table levels due to extensive exploitation for irritation and leakage in the lower aquifer, most of the open wells are dry and thrown out of use. At present the irrigation water is mostly supplied by the tube wells. The area selected for study includes that portion of the alluvial tract of Mehsana district, for which sufficient data for determining aquifer parameters and observed water levels are available. The boundaries are selected such that north-eastern and south-eastern boundaries are normal to the direction of flow and thus are equipotential lines, whereas the other two boundaries are parallel to the direction of flow and are streamlines. The project area is a rectangle of size 90 km x 60 km, which is divided into 216 square elements, each of size 5 km x 5 km. The number of nodes are 247, as shown in Fig. 4.4.

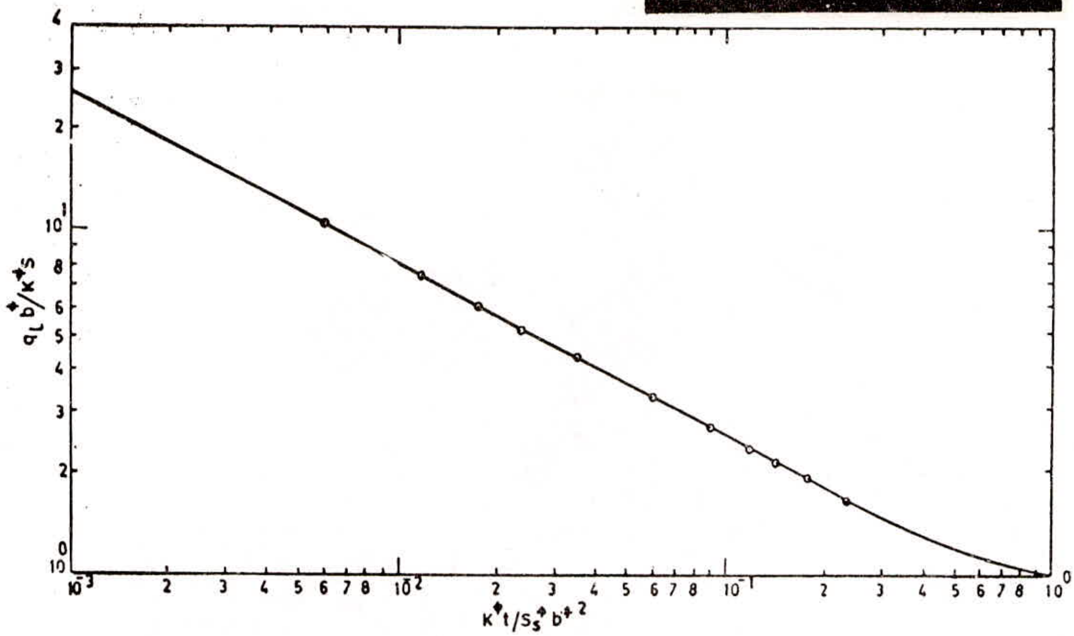
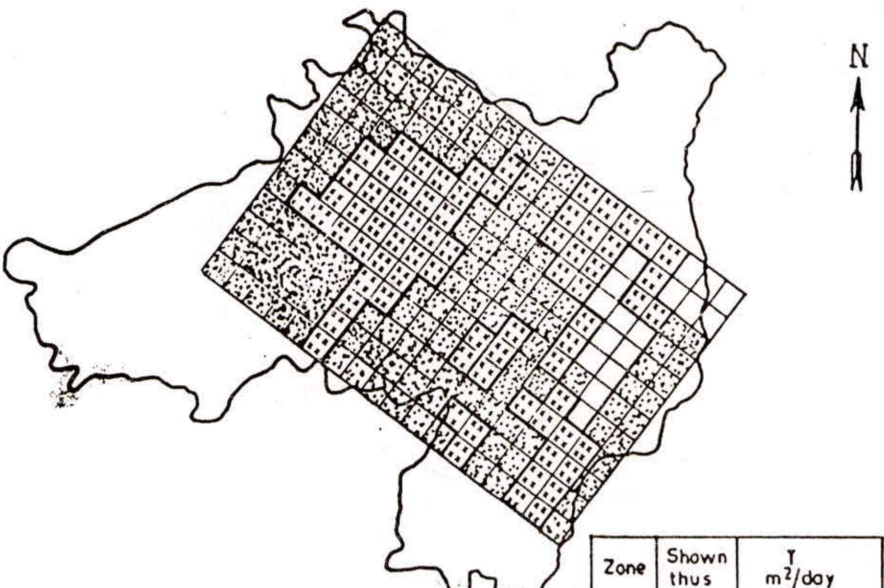


FIG. 4.1 DIMENSIONLESS LEAKAGE FLUX vs. DIMENSIONLESS TIME



Zone	Shown thus	T m²/day	S
A		2.97×10^2	0.00015
B		9.30×10^2	0.00030
C		2.97×10^3	0.001

FIG. 4.2 PROJECT AREA SHOWING ZONES ACCORDING TO 'S' AND 'T' DISTRIBUTION

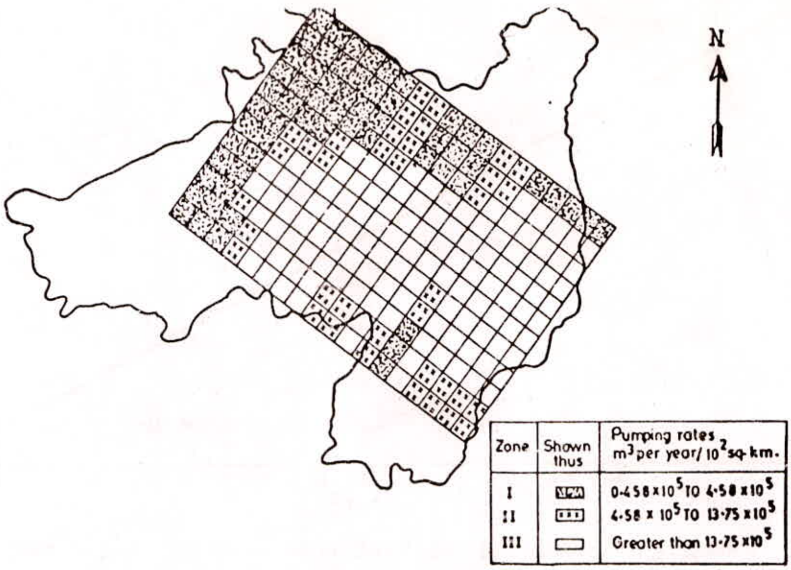


FIG. 4.3 PROJECT AREA SHOWING ZONES ACCORDING TO PUMPING RATES

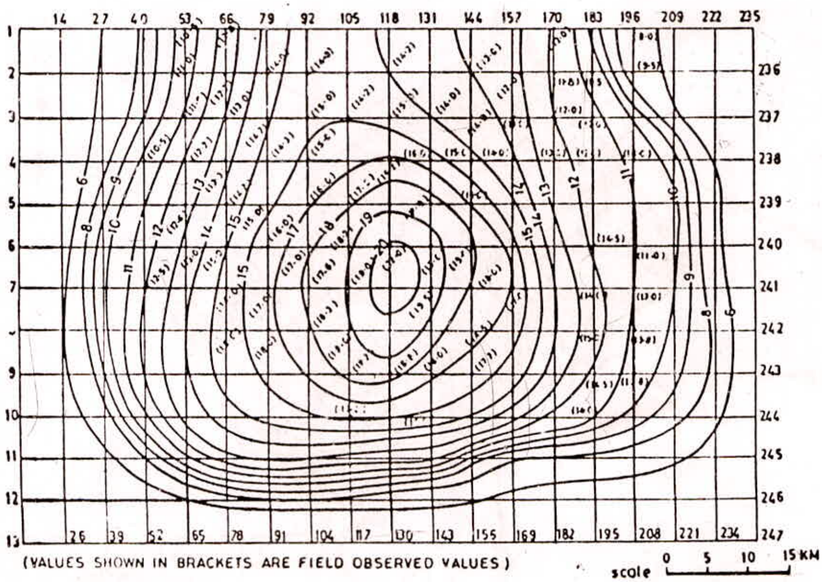


FIG. 4.4 ARTESIAN AQUIFER - 13 YEARS RIGIONAL DRAWDOWNS CALIBRATION RUN.

The data of observed drawdowns in piezometric levels and water table levels and pumping rates for the period from 1959 to 1971 and values of storage coefficients, transmissivities for artesian and water table aquifers was obtained from Gujarat Water Resources Development Corporation, Gujarat State and Gujarat Engineering research Institute, Baroda. The entire project area was divided into three zones, each zone having same values of transmissivity and storage coefficient as given in Table No. 4.1. For the purpose of distribution of withdrawal rates the area was divided again into three zones, each zone having the same rate of withdrawal as given in Table No. 4.2. The division of area into above zones is as shown in Figs. 4.2 and 4.3.

Table No. 4.1
Zones according to distribution of T and S

Zone	Average T	Average S
A	297 m^2/day	0.00015
B	930 m^2/day	0.00030
C	2970 m^2/day	0.001

Table No. 4.2
Zones according to distribution of withdrawal rates

Zone	Average withdrawal rate
I	45800 m^3 to 458000 m^3 per year per 100 sq.km.
II	458000 m^3 to 1375000 m^3 per year per 100 sq.km.
III	More than 1375000 m^3 per year per 100 sq.km.

The northeast boundary of the study area is located adjoining the recharge area. Hence this boundary is treated as flux boundary. The inflow flux was computed as 2.5 cumecs per day per meter length of the boundary. This inflow is divided equally into two parts, one entering the phreatic aquifer and the other entering the artesian aquifer as both the aquifers have the common outcrop area. An average value of transmissivity of 160 m^2/day was taken for the water table aquifer (from UNDP project report). The model was calibrated using the above data of two aquifer system of Mehsana district. The model was run continuously to compute drawdowns of both the aquifers for thirteen years from 1959 to 1971. The values of leakage coefficient and storage coefficient for the aquitard were adjusted by trial till a reasonable match was made between the computed values and the observed values of drawdown, the final values of which are shown in Fig. 4.4. Except the northeast recharge boundary the other three boundaries are treated as Dirichlet boundaries. It was observed that at 56% of the nodes the error is less than 5%, at 14% of the nodes the error is between 5% & 10% that is at 70% of the nodes the results may be considered as within reasonable limits in studies of such large regional problem. At 30% of nodes the deviation exceeds 10%.

5.0 TWO DIMENSIONAL STREAM FLOW MODELLING:

Many problems of hydraulic engineering require information concerning surface water elevations and magnitudes of velocities in the x- and y- directions in the two-dimensional horizontal domain. Typical cases involve bays or creeks, estuaries, harbours and wide rivers. Recent advent in computers made it possible to develop mathematical models to predict the flow phenomenon in such cases. Because of the relative ease and economy of the computations on one hand and the ever increasing demand for reliable information on the other hand, mathematical modelling has become a useful tool in the field of hydraulic engineering. The modelling can be carried out either by using Finite Difference Method (FDM) or by Finite Element Method (FEM). Each method has its own merits and demerits. If the domain can be descretised as rectangular elements, then FDM with ADI (Alternating Direction Implicit method) is advantageous. In this case, there is no need to form and store the global matrix before solving for the field variable. But if the domain is irregular, FEM scores well over the other methods. Here, the discussion will be limited to the F.E. method only.

5.1 The Governing Equations:

The depth averaged hydrodynamic Eqns. used in the formulation of the finite element model are based on the following assumptions: (a) The medium is homogeneous and incompressible, (b) Pressure distribution is hydrostatic, (c) All the shear stresses except at the boundary are negligible, (d) Friction losses in unsteady flow are equivalent to those of steady uniform flow and (e) The channel bed is fixed. The equations are as given by Eqns. 55, 56 and 57.

$$\frac{\partial H}{\partial t} + \frac{\partial}{\partial x}(uh) + \frac{\partial}{\partial y}(vh) = 0 \qquad (55)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + g\frac{\partial H}{\partial x} + \frac{u|u|}{C^2 h} = 0 \qquad (56)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + g\frac{\partial H}{\partial y} + \frac{v|v|}{C^2 h} = 0 \qquad (57)$$

where u=u(x,y,t); v=v(x,y,t) are the velocities in the x- and y- directions respectively, g is the acceleration due to gravity, C is the Chezy's coefficient, H=H(x,y,t) is the water surface elevation at any point with respect to some datum, h is the depth of flow at any point in the stream such that H=h+z where z is the bed level at any node, t is the time elapsed. To solve these differential equations, finite element scheme in space domain and finite difference scheme in time domain were used.

5.2 Finite Element Formulation:

To obtain the finite element matrices, it is necessary to write the different unknowns by a set of trial functions, $H = \Sigma N_i(x,y) H_i(t)$, $u = \Sigma N_i(x,y)U_i(t)$, $v = \Sigma N_i(x,y) V_i(t)$, i=1,..,NN, where H_i, U_i, V_i are the values of water surface elevation, velocity components in the x-, and y-directions which are functions of time. These trial functions can also be written as: H =[N]{H}, U =[N]{U} and V =[N]{V}, where $[N]=[N_1\ N_2\ N_3]$ is the array of trial functions or interpolation functions, {H}, {U} and {V} are the arrays of the values of the field variables, H, U and V at the nodes. One can use Galerkin's procedure to obtain the element equations, where the error is made orthogonal to the trial function. Let L_1 be the equation 55, L_2 be the equation 56 and L_3 be the equation 57. Then according to Galerkin's criterion, $\int N^T L_1\, dA = 0$, $\int N^T L_2\, dA = 0$ and $\int N^T L_3 dA=0$. Let us denote $N_x = \partial N/\partial x$, $N_y=\partial N/\partial y$, then the governing equations can be written as given by Eqns. 58, 59 and 60.

$$\left[\iint N^T N\, dA\right]\{\frac{\partial H}{\partial t}\} + \left[\iint N^T h_m\, N_x\, dA\right]\{U\} + \left[\iint N^T h_m\, N_y\, dA\right]\{V\} = 0 \quad (58)$$

$$\left[\iint N^T N\, dA\right]\{\frac{\partial U}{\partial t}\} + \left[\iint N^T\, U_m\, N_x\, dA\right]\{U\} + \left[\iint N^T\, V_m\, N_y\, dA\right]\{U\} + \left[g\iint N^T N_x\, dA\right]\{H\} + \left[\frac{g|V|}{C^2\, h}\iint N^T N\, dA\right]\{U\} = 0 \quad (59)$$

$$\left[\iint N^T N\, dA\right]\{\frac{\partial V}{\partial t}\} + \left[\iint N^T U_m\, N_x\, dA\right]\{V\} + \left[\iint N^T V_m\, N_y\, dA\right]\{V\} + \left[g\iint N^T N_y\, dA\right]\{H\} + \left[\frac{g|V|}{C^2\, h}\iint N^T N\, dA\right]\{V\} = 0 \quad (60)$$

Let $\{\partial H/\partial t\} = (\{H\}_{t+\Delta t} - \{H\}_t)/\Delta t$, $\{\partial U/\partial t\} = (\{U\}_{t+\Delta t} - \{U\}_t)/\Delta t$, and $\{\partial V/\partial t\} = (\{V\}_{t+\Delta t} - \{V\}_t)/\Delta t$ and also let $H = H_{t+\Delta t}$, $U = U_{t+\Delta t}$, $V = V_{t+\Delta t}$, then the Eqns. 58 to 60 can be written as as given by Eqns 61 to 63.

$$\left[\frac{1}{\Delta t}\iint N^T N\, dA\right]\{H\} + \left[\iint N^T\, h_m\, N_x\, dA\right]\{U\} + \left[\iint N^T\, h_m\, N_y\, dA\right]\{V\} = \left[\frac{1}{\Delta t}\iint N^T N\, dA\right]\{H\}_t \quad (61)$$

$$\left[g\iint N^T\, N_x\, dA\right]\{H\} + \left[\left(\frac{1}{\Delta t} + \frac{g|V|}{C^2\, h}\right)\iint N^T\, N\, dA\right]\{U\} + \left[\iint (N^T\, U_m\, N_x + N^T\, V_m\, N_y)\, dA\right]\{U\} = \left[\frac{1}{\Delta t}\iint N^T\, N\, dA\right]\{U\}_t \quad (62)$$

$$\left[g\iint N^T\, N_y\, dA\right]\{H\} + \left[\left(\frac{1}{\Delta t} + \frac{g|V|}{C^2\, h}\right)\iint N^T\, N\, dA\right]\{V\} + \left[\iint (N^T\, U_m\, N_x + N^T\, V_m\, N_y)\, dA\right]\{V\} = \left[\frac{1}{\Delta t}\iint N^T\, N\, dA\right]\{V\}_t \quad (63)$$

In matrix form they can be written as given by Eqn. 64.

$$\begin{bmatrix} \frac{1}{\Delta t}\iint N^T N\, dA & \iint N^T h_m N_x dA & \iint N^T h_m N_y dA \\ g\iint N^T N_x dA & a_{22} & 0 \\ g\iint N^T N_y dA & 0 & a_{33} \end{bmatrix} \begin{Bmatrix} H \\ U \\ V \end{Bmatrix} = \begin{Bmatrix} \frac{1}{\Delta t}\iint N^T N dA \, \{H\}_t \\ \frac{1}{\Delta t}\iint N^T N dA \, \{U\}_t \\ \frac{1}{\Delta t}\iint N^T N dA \, \{V\}_t \end{Bmatrix} \qquad (64)$$

$$\textit{where} \quad a_{22} = a_{33} = \left(\frac{1}{\Delta t} + \frac{g|V|}{C^2\, h_m}\right)\iint N^T N\, dA + \iint (N^T u_m N_x + N^T v_m N_y)\, dA \qquad (65)$$

In Eqns. 64 and 65, Δt is the time step chosen, the variables with subscript t are at the previous time step which are known. The variables at the time step $t+\Delta t$ are to be evaluated using the matrix Eqns. 64. Proper boundary conditions must be introduced before these Eqns. 64 are solved for each time step. These Eqns. work well for problems involving large surface areas relative to its depths. Also, a cold start is needed for these equations, where we assume that the field variable U=V=0 at t=0 and H=constant every where in the domain. When this finite element formulation was tried on a two-dimensional stream flow case, the results were not satisfactory. Hence the following new formulation was tried.

5.3 New Finite Element Formulation for Stream Flow

The main assumption involved in the alternative formulation is that the order of magnitude of the convective terms in the momentum equation is small compared to the other terms and that the velocity in the x- and y- directions can be written using the Chezy's formula as $U = C\, h_m^{0.5}\, S_{fx}^{0.5}$ and $V = C\, h_m^{0.5}\, S_{fy}^{0.5}$, where C is the Chezy's coefficient, h_m is the mean depth of water over the element, S_{fx} is the gradient of the water surface elevation in the x-direction, $S_{fx} = -\partial H/\partial x$, S_{fy} is the gradient of the water surface elevation in the x-direction, $S_{fy} = -\partial H/\partial y$. The equations for U and V can be linearized and can be rewritten as given by Eqn. 66.

$$U = -\frac{C\sqrt{h_m}}{\sqrt{|\partial H/\partial x|}}\frac{\partial H}{\partial x} = -K_1\frac{\partial H}{\partial x} \;;and$$
$$V = -\frac{C\sqrt{h_m}}{\sqrt{|\partial H/\partial y|}}\frac{\partial H}{\partial y} = -K_2\frac{\partial H}{\partial y} \qquad \mathbf{(66)}$$

Putting Eqns. 66 into Eqn 55, the resulting equations are:

$$\frac{\partial H}{\partial t} - \frac{\partial}{\partial x}\left(K_1\, h_m\, \frac{\partial H}{\partial x}\right) - \frac{\partial}{\partial y}\left(K_2\, h_m\, \frac{\partial H}{\partial y}\right) = 0 \qquad \mathbf{(67)}$$

In Eqn. 68
, $K_x = C\, h^{1.5}/(|\partial H/\partial x|)^{0.5}$ and $K_y = C\, h^{1.5}/(|\partial H/\partial y|)^{0.5}$

$$L_4 = \frac{\partial H}{\partial t} - \frac{\partial}{\partial x}\left(K_x \frac{\partial H}{\partial x}\right) - \frac{\partial}{\partial y}\left(K_y \frac{\partial H}{\partial y}\right) = 0 \qquad (68)$$

Putting Galerkin's criterion, $\int N^T L_4 \, dA = 0$ into Eqn. 68, we have,

$$\iint N^T \left[\frac{\partial H}{\partial t} - \frac{\partial}{\partial x}\left(K_x \frac{\partial H}{\partial x}\right) - \frac{\partial}{\partial y}\left(K_y \frac{\partial H}{\partial y}\right)\right] dA = 0 \qquad (69)$$

In Eqn. 69, the field variable $H = H(x,y,t)$ can be written as $H = [N]\{H\}$ where $N_i = N_i(x,y)$ is the array of the nodal values of the interpolation function and $H_i = H_i(t)$ is the array of the nodal values of the field variable.

Partially integrating the second and third term of equation 69, the result is as given by Eqn. 70.

$$\iint N^T\left[\frac{\partial}{\partial x}\left(K_x\frac{\partial H}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y\frac{\partial H}{\partial y}\right)\right] dx\, dy = \int N^T\left[K_x\frac{\partial H}{\partial x}l_x + K_y\frac{\partial H}{\partial y}l_y\right] dS - \iint\left[K_x\frac{\partial N^T}{\partial x}\frac{\partial H}{\partial x} + K_y\frac{\partial N^T}{\partial y}\frac{\partial H}{\partial y}\right] dx\, dy \qquad (70)$$

The first integral term on the right hand side of Eqn. 70 represents the flow into or out of the domain, +ve for inflow and -ve for outflow. Combining Eqns. 69 and 70, Eqn. 71 can be obtained.

$$\left[\iint N^T N \, dA\right]\left\{\frac{\partial H}{\partial t}\right\} + \left[\iint\left(K_x\frac{\partial N^T}{\partial x}\frac{\partial N}{\partial x} + K_y\frac{\partial N^T}{\partial y}\frac{\partial N}{\partial y}\right) dA\right]\{H\} = \oint N^T Q \, dS \qquad (71)$$

Let $\partial H/\partial t = (\{H\}_{t+\Delta t} - \{H\}_t)/\Delta t$, then Eqn. 71 can be rewritten as:

$$\left[\frac{1}{\Delta t}\iint N^T N \, dA\right]\{H\}_{t+\Delta t} + \left[\iint\left(K_x\frac{\partial N^T}{\partial x}\frac{\partial N}{\partial x} + K_y\frac{\partial N^T}{\partial y}\frac{\partial N}{\partial y}\right) dA\right]\{H\}_{t+\Delta t} = \left[\frac{1}{\Delta t}\iint N^T N \, dA\right]\{H\}_t + \oint N^T Q \, dS \qquad (72)$$

The last term of Eqn. 72 represents the discharge into or out of the domain, hence it is a flux boundary condition which is to be supplied before the equations can be solved. The boundary conditions play vital role in the solution. Here the field variable is H and the other parameters U and V can be evaluated from $\partial H/\partial x$ and $\partial H/\partial y$. There are two types of boundary conditions: (i) Dirichlet type or field variable is specified at some nodes, (ii) Neumann type or flux condition, that is discharge is specified at some other specified nodes. The finite element matrix formed as it is, is of the rank NN - 1 where NN is the number of nodes and hence atleast one value of field variable is to be specified without which the solution cannot be obtained. It is important to note that these two boundary conditions cannot be applied simultaneously at one particular node, if applied simultaneously, the flux condition will be nullified. In the problems solved, the field variable H is specified on the upstream side and the discharge is specified on the downstream side.

When Manning's formula is used in place of Chezy's, the values of K_x and K_y will be as given by Eqn. 73.

$$K_x = \frac{1}{n}\frac{h_m^{5/3}}{\sqrt{\left|\frac{\partial H}{\partial x}\right|}}\frac{\partial H}{\partial x}; and \quad K_y = \frac{1}{n}\frac{h_m^{5/3}}{\sqrt{\left|\frac{\partial H}{\partial y}\right|}}\frac{\partial H}{\partial y} \qquad (73)$$

In this study, the triangular elements with linear interpolation function were used. Here, the field variables $H = H(x,y,t)$, $u = u(x,y,t)$, and $v = v(x,y,t)$ are written as: $H = N_1 H_1 + N_2 H_2 + N_3 H_3$; $u = N_1 u_1 + N_2 u_2 + N_3 u_3$ and $v = N_1 v_1 + N_2 v_2 + N_3 v_3$; where $N_i = N_i(x,y)$, $H_i = H_i(t)$, $u_i = u_i(t)$ and $v_i = v_i(t)$. Here N_i can be written as $N_i = (a_i + b_i x + c_i y)/2A$, where A is the area of the element, and $a_1 = x_2y_3 - x_3y_2$, $a_2 = x_3y_1 - x_1y_3$, and $a_3 = x_1y_2 - x_2y_1$; $2A = a_1 + a_2 + a_3$, $b_1 = y_2 - y_3$, $b_2 = y_3 - y_1$, $b_3 = y_1 - y_2$, $c_1 = x_3 - x_2$, $c_2 = x_1 - x_3$, $c_3 = x_2 - x_1$. Also $N = \{N_1\ N_2\ N_3\}$, $\{H\}^T = \{H_1\ H_2\ H_3\}$. Now, $\partial H/\partial x = \{b_1\ b_2\ b_3\}\{H\}/2A$ and $\partial H/\partial y = \{c_1\ c_2\ c_3\}\{H\}/2A$.
For triangular elements:

$$\iint N^T N\, dA = \left(\frac{A}{12}\right)\begin{bmatrix}2 & 1 & 1\\ 1 & 2 & 1\\ 1 & 1 & 2\end{bmatrix} \qquad (74)$$

$$\iint \left[K_x \frac{\partial N^T}{\partial x}\frac{\partial N}{\partial x} + K_y \frac{\partial N^T}{\partial y}\frac{\partial N}{\partial y}\right] dA =$$

$$\frac{K_x}{4A}\begin{bmatrix}b_1b_1 & b_1b_2 & b_1b_3\\ b_2b_1 & b_2b_2 & b_2b_3\\ b_3b_1 & b_3b_2 & b_3b_3\end{bmatrix} + \frac{K_y}{4A}\begin{bmatrix}c_1c_1 & c_1c_2 & c_1c_3\\ c_2c_1 & c_2c_2 & c_2c_3\\ c_3c_1 & c_3c_2 & c_3c_3\end{bmatrix} \qquad (75)$$

Because the problem is nonlinear, especially with respect to K_x and K_y, the problem is to be solved iteratively till the convergence is achieved.

5.4 Program Highlights:

1. Read the initial data, element nodes, x-, y- and z- coordinates and other boundary values to be prescribed.

2. Generate the element parameters, column heights, diagonal pointer array for compact storage scheme and store these values in a separate file. These are to be evaluated

only if the element mesh is changed.

3. Formulate the element matrix and assemble them into global matrix in compact form.

4. If depth < 0 at an time of computation, then depth = 0.

5. Prescribe proper boundary conditions.

6. Solve for the field variable, H, that is the water level.

7. Check for convergence, if abs[$(H_{k+1} - H_k)/H_k$] < 0.0001, then stop.

8. Let $H_k = H_{k+1}$ and goto step 3.

5.5 Case Study: Comparison of Experimental and Computed Results: This new simplified formulation has been applied (Vasudeva Rao 1994) to three particular cases: two cases of meandering channels and one channel junction. Some of the results are as shown in Table No. 5.1 and 5.2.

Table No. 5.1

S.No.	Total Disch. Q,m^3/s	Chezy's Coeff. C	Discharge prescribed at nodes	Water Levels in meters upstream obs	upstream comp	downstream obs	downstream comp
1	147	41.5	{10,5(25),12}	89.25	89.25	88.05	88.05
2	540	41.5	{15,5(100),40}	91.48	91.48	90.57	90.22
3	700	41.5	{25,5(130),25}	92.18	92.18	91.15	91.38
4	800	41.5	{50,5(140),50}	92.46	92.46	91.45	91.3
5	850	41.5	{75,5(150),75}	92.58	92.58	91.6	91.63
6	900	41.5	{75,5(160),75}	92.72	92.72	91.75	91.57
7	1000	41.5	{100,5(160),100}	92.95	92.95	92.05	91.94

Number of elements used = 369 and number of node points = 222.

Table No. 5.2

S.No.	Total Disch. Q,m^3/s	Manning's Coeff. C	Discharge prescribed at nodes	Water Levels in meters upstream obs	upstream comp	downstream obs	downstream comp
1	147	0.022	{10,5(25),12}	89.25	89.25	88.05	88.23
2	540	0.022	{15,5(100),40}	91.48	91.48	90.57	90.26
3	700	0.022	{25,5(130),25}	92.18	92.18	91.15	91.43
4	800	0.022	{50,5(140),50}	92.46	92.46	91.45	91.74
5	850	0.022	{75,5(150),75}	92.58	92.58	91.6	91.66
6	900	0.022	{75,5(160),75}	92.72	92.72	91.75	91.78
7	1000	0.022	{100,5(160),100}	92.95	92.95	92.05	91.74

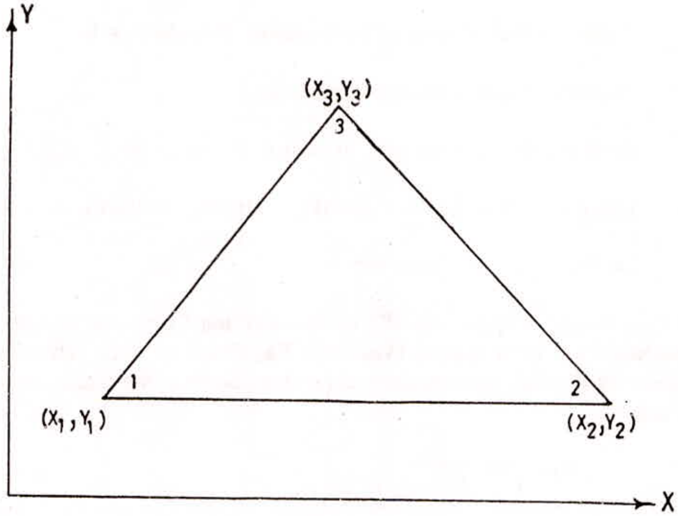


FIG.5.1 TRIANGULAR ELEMENT

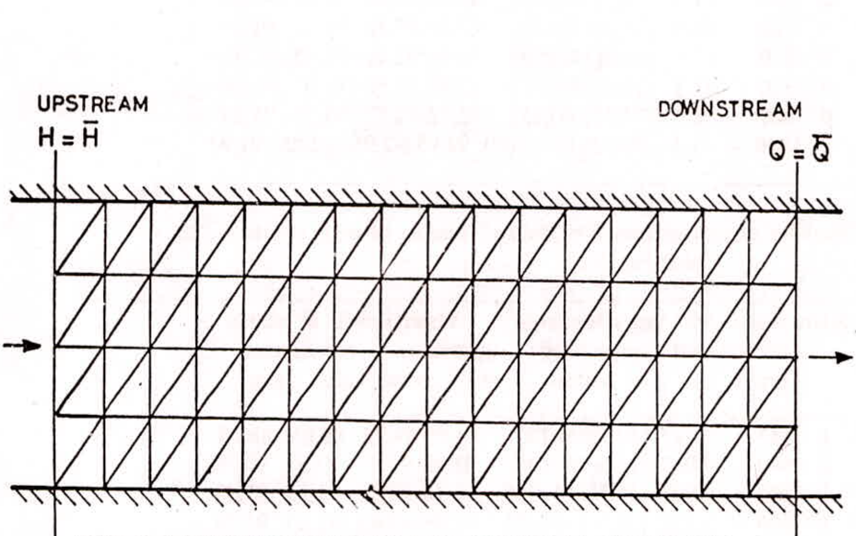


FIG.5.2 DISCRETISATION OF STREAM SEGMENT

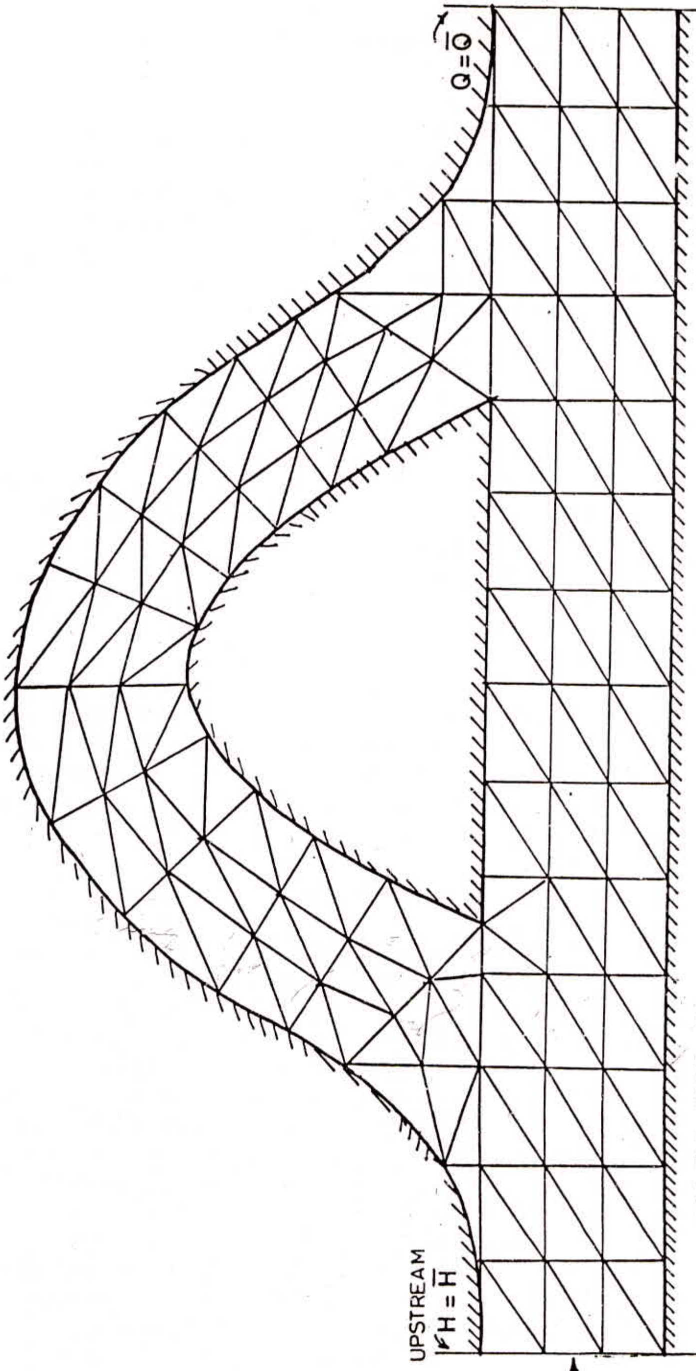


FIG.5·3 DISCRETISATION OF BRANCHING STREAM

6.0 SURFACE RUNOFF MODELLING OF WATERSHEDS FORMULATION AND APPLICATION TO REAL CATCHMENTS

The formulation of a mathematical model to estimate surface runoff from watersheds using the overland flow and stream flow Eqns. has been presented here. Using the net-rainfall as input into the overland flow Eqns., the surface runoff is calculated which will become input into the stream flow Eqns. for each time step. The Diffusion Wave Model (DWM) is solved using Finite Element Method in space domain and by using Finite difference in time domain. This model has been used to simulate storm hydrographs from two real catchments and compared with the observed hydrographs. Sensitivity of the solution with respect to the changes in some parameters involved in this model is presented.

For many river basins, stream flow data is either scanty or not available at all. This adversely affects the planning and design of water resource projects. Runoff estimate by statistical correlations is available in some cases, but the distribution of runoff with respect to time (hydrograph) cannot be obtained with these correlations. The hydrograph for any given storm is important in a river basin for flood forecasting, warning and control. Any model based on natural physical laws would give the reliable estimates of peaks and hydrograph for a given storm. The nonlinearity of these physical laws and complexity of the solution procedure have discouraged many of the investigators to attempt any solution. The advent of computers in recent times and the development of efficient numerical schemes have paved the way to obtain some solutions based on these physical laws.

The common practice to solve the gradually varied unsteady flow Eqns. is to employ the Method of Characteristics (MOC) and the Finite Difference Method (FDM). The Finite Element Method (FEM) is relatively recent approach for solving these Eqns. (Taylor et al., 1974; Cooley and Moin, 1976; and Jayawardena and White, 1977 and 1979). Only a few investigators have attempted to solve the complete form of these Eqns. using FEM (Cooley and Moin, 1976; Keuning, 1976; King, 1976; and Nwaogazie and Tyagi, 1984).

Limitations of computer resources led to the approximations such as Kinematic Wave Model (KWM) and Diffusion Wave Model (DWM). The KWM which is popular, assumes that inertia and pressure terms in the momentum equation are negligible compared to the friction and gravity terms. However, some available examples in the literature indicate that performance of KWM is poor compared to other methods (Hromadka et al. 1986a; Akan and Yen, 1981; Katopodes and Schamber, 1983; Weinmann and Laurenson, 1979). The DWM assumes that the inertia terms negligible compared to the pressure, friction and gravity terms. Henderson (1966); Cunge et al. (1980) have shown that the inertia terms are generally small compared to the other terms. Ponce et al. (1978) have shown that the diffusion wave model describes the flood wave subsidence better than the KWM. Hromadka et al. (1986b) have shown that DWM provides a considerable improvement over the often used KWM. Hence DWM has been used in the present formulation.

Any complex mathematical model when applied to a real catchment may have to be greatly simplified because of the heterogeneous nature of the hydrologic parameters of the catchment. The efficacy of any model can be evaluated to some extent by comparing the computed hydrograph with the observed hydrograph of a catchment. The model developed here requires the information of rainfall intensity on a continuous basis, which is not normally available for all the catchments. But, in view of the requirement of such data for the present study, considerable effort has been made to collect the rainfall and runoff on hourly basis for some catchments. The literature on runoff simulation using mathematical models from real catchments of the sizes of few hundred square kilometres is rather rare (Refsgaard et al., 1992; and Jain et al., 1992). In this paper, the formulation of a mathematical model, application of the same to real catchments and sensitivity of the solution with respect to change in some parameters involved in the model are described. This model can be used to simulate the hydrographs of ungauged basins, for which the observed hydrographs are not available.

6.1 Formulation of the Mathematical Model:

The model presented here has two parts: (i) the computation of overland flow for a given rainfall excess, (ii) the computation of stages and flow rates at different points along the stream with the computed overland flow as input.

6.1.1 Computation of overland flow:

Though some investigators have attempted to evaluate the overland flow from the St. Venant Eqns., the mathematical rigor provided by the numerical solution is not merited in light of its high cost, stability and convergence problems and uncertainty concerning friction losses and other phenomena being modelled (Bennett, 1974). The mass balance equation on the other hand does not pose many of the uncertainties mentioned here. Therefore, an equation is developed (Vasudeva Rao and Panakala Rao, 1988) here from mass balance criterion for evaluating the overland flow. The mass balance equation may be written in the form: INFLOW - OUTFLOW = STORAGE INCREMENT, that is,

$$P - q \,.\, L = \Delta V / \Delta t \tag{76}$$

where P is the inflow into the catchment which is mainly in the form of excess precipitation, q is the overland flow from the catchment segment per unit length of the stream in the form of cross flow into the stream, ΔV is the change in detention storage over the overland, Δt is the time step chosen and L is the length of the stream segment intercepting the overland flow. The Eqn. 76 can be rewritten as given by Eqn. 77.

$$0.5(P_{t+\Delta t} + P_t)A_c - 0.5(q_{t+\Delta t} + q_t)L = A_c(d_{t+\Delta t} - d_t)/\Delta t \tag{77}$$

In Eqn. 77, d is the depth of overland flow, A_c is the area of the watershed segment and variables with subscripts t and $t+\Delta t$ are at the beginning and end of the time step respectively.

Assuming that the slope of the overland plane S, in the case of overland flow, is equal to the friction slope S_f, the overland flow q can be written as given by Eqn. 78.

$$q = d \,.\, d^{2/3} \,.\, S^{1/2} / n \tag{78}$$

In Eqn. 78, n is the Manning's roughness coefficient applicable to overland flow. If P is expressed in mm/hour, d in mm, L in meters and A_c in square meters, the Eqn. 78 can be written as given by Eqn. 79. For a given initial depth d_t, the depth $d_{t+\Delta t}$ can be obtained by solving Eqn. 79 iteratively, which can be used in Eqn. 78 to obtain overland flow q.

$$K1 \ . \ d_{t+\Delta t}^{5/3} + 100 \ d_{t+\Delta t} = K2$$
$$where \quad K_1 = L \ . \ S^{1/2} \ \Delta t \ / \ (2 \ n \ A_c) \tag{79}$$
$$and \quad K_2 = 100 \ d_t + \Delta t(P_t + P_{t+\Delta t})/72 - K_1 d_t^{5/3}$$

6.1.2 Computation of stream flow:

There are many forms of differential equation sets to describe a large spectrum of unsteady flows (Lai, 1986). The one-dimensional gradually varied unsteady flow equations used here are of the form given by Eqns. 80 and 81.

$$(a) Continuity Equation: \quad \frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} - q = 0 \tag{80}$$

$$(b) Momentum Equation: \quad \frac{\partial Q}{\partial t} + \frac{\partial (Q^2/A)}{\partial x} + g\,A\left(\frac{\partial H}{\partial x} + S_f\right) = 0 \tag{81}$$

In Eqns. 80 and 81, A is the flow area, Q is the flow rate, H is the stage or water surface elevation, S_f is the friction slope, g is the acceleration due to gravity and x, t are the spacial and temporal coordinates. S_f can be approximated using Manning's equation (Akan and Yen, 1981) as $Q = A\ R^{2/3}\ S_f^{1/2}/n$, where R is the hydraulic radius, n is the Manning's roughness coefficient applicable to channel flow. The channel bend losses and expansion losses can be accounted for by varying the value of n.

In the DWM, the local and convective acceleration terms in the momentum equation are neglected (Akan and Yen, 1981), thus Eqn. 81 is simplified as

$$S_f = -\left(\frac{\partial H}{\partial x}\right) \tag{82}$$

$$Q = K\left(\frac{\partial H}{\partial x}\right) \quad where \quad K = -\frac{1}{n} A.R^{2/3} \Big/ \left|\frac{\partial H}{\partial x}\right|^{1/2} \tag{83}$$

Combining Eqns. 80 and 82 gives a diffusive type of relationship (Hromadka et al., 1986b) which is as given by Eqn. 84.

$$\frac{\partial A}{\partial t} - \frac{\partial}{\partial x}\left(K\frac{\partial H}{\partial x}\right) - q = 0 \tag{84}$$

For a constant channel width, W, Eqn. 84 reduces to Eqn. 85.

$$W \frac{\partial H}{\partial t} - \frac{\partial}{\partial x}\left(K \frac{\partial H}{\partial x}\right) - q = 0 \qquad (85)$$

6.2 Finite Element form of the Channel Flow Equation:

The FEM formulation of Eqn. 85 for channel flow developed by the authors has already been reported in brief (Vasudeva Rao and Panakala Rao, 1988). Over the channel domain, the stage H can be approximated as $H = \Sigma N_i H_i$, i=1,NN where NN is the number of node points and N_i are the interpolating functions. The application of Galerkin's criterion to the Eqn. 85 can be written as (Panakala Rao, 1990):

$$\int_0^L [N]^T [N] \, dx \left\{\frac{\partial H}{\partial t}\right\} - \frac{K}{W} [N]^T \left[\frac{\partial H}{\partial x}\right]_{x=0}^{x=L} + \frac{K}{W} \int_0^L \frac{\partial [N]^T}{\partial x} \frac{\partial [N]}{\partial x} dx \; \{ H \} - \frac{q}{W} \int_0^L [N]^T dx = 0 \qquad (86)$$

Here the field variable H is approximated as $H = (H_{t+\Delta t} + H_t)/2$ and the time derivative of the field variable is $\partial H/\partial t = (H_{t+\Delta t} - H_t)/\Delta t$. The terms in Eqn. 86 are integrated over the length of the element to form the element matrix, which are assembled to form a global matrix.

6.3 Solution Procedure: The discretization of overland flow regions is based on drainage pattern, topography, soil characteristics and rainfall variation etc. All the overland flow which is joining the stream at one place/stretch can be demarcated as one region. The depth of flow is assumed to be the same over the whole element of the overland region. The channel is discretized keeping the following points in view, (i) the element stretch is straight, (ii) channel bed slope is unique, (iii) no abrupt change occurs in any parameter of the channel within the element.

If any tributary catchment area exceeds 20% of the total catchment area, then it is considered as a separate stream with its own discretization as described earlier and joins the main stream. If its area is less than 20 %, then it is considered as an overland flow region contributing to the main stream to restrict the overland regions and corresponding channel segments and also the computational time.

The success of this model depends on the evaluation of the excess rainfall from the catchment area. The FESHM model (Ross et al., 1979) uses parameters like soil texture, depth of 'A' horizon of soil, soil hydrology group and land use cover for calculating the infiltration rate and excess rainfall. In the absence of any such type of data for the catchments considered, Φ-index method, i.e., the average rainfall above which the rainfall volume is equal to the runoff volume, has been used. The Φ - index is derived from the rainfall hyetograph with the knowledge of the resulting runoff volume and the base flow. The Φ - index method accounts for the total abstraction and enables runoff magnitudes to be estimated for a given rainfall hyetograph. If there is no observed base flow or runoff volume, then a value for minimum base flow and loss rate as recommended by Central water Commission (CWC) report (1973 and 1982) for the specified region has been used.

The roughness coefficient normally varies with flow depth, however, this variation was not accounted for in this model. The flow depth is assumed to be zero as an initial condition. The boundary conditions imposed in the model before obtaining the solution for each time step are: (i) upstream discharge and (ii) downstream depth either normal depth or critical depth depending on the end condition of the channel (Freeze, 1978). The input parameters into this model are: (i) number of rain gauge stations and their Influence Area Factors (IAF), (ii) duration of rainfall for each rain gauge station for a given storm at all the stations, (iii) discharge from the catchment and its duration, (iv) number of overland flow regions, their areas, slopes, roughness values and lengths of stream segments intercepting the overland flow, (v) number of channel elements including major tributaries, their lengths and roughness values, bed levels and bed widths at each node of the channel element and (vi) time step Δt and the maximum duration t_{max} for which the hydrograph is to be simulated.

The sequence of steps in the solution procedure is as follows:

1. Compute the excess rainfall for each region. Set t=0

2. Compute the overland flow for each region.

3. Formulate the element matrices for all the channel segments and assemble into global form and prescribe the proper boundary conditions.

4. Compute the stage values {H} at all the node points at the end of the time step and the discharge in each channel segment. Set $t=t+\Delta t$.

5. If $t > t_{max}$ then goto step 7.

6. Set $\{H\}_t = \{H\}_{t+\Delta t}$, print {H} values and goto step 2.

7. Stop.

6.4 Application to Real Catchments:

This model has been applied to two natural catchments (i) Pimpalgaon Joge and (ii) Bhatsa in the state of Maharastra, India. The data needed for this model from these catchments has been extracted from the Top-sheets (1:50,000) of survey of India. Some trial runs were made for simulation using the same time step for overland flow and channel flow. During these runs, it was found that the solution was stable when the time step Δt was varied from 5 sec to 900 sec. For Pimpalgoan Joge catchment, each simulation run took 278.6 sec for a time step Δt=5 sec, 6.94 sec for a time step Δt=300 sec and 3.35 sec for a time step Δt=900 sec of CPU time on CYBER 180/840 system for Storm-1 (discussed subsequently). A time step of 5 minutes was finally chosen when it was found that it has given near average value of peak flow. Results are numerically unstable for time step Δt above 900 sec.

6.4.1 Pimpalgaon Joge catchment:

The Pimpalgaon Joge catchment, of area 102.2 square kilometres is on the leeward side of Western Ghats forming the watershed of Arr river, consists of eight Self Recording Rain Gauge stations (SRRG). The main Arr river has a tributary called Dudhwar river has a catchment area which is nearly 30 % of the total catchment area, hence considered as tributary to Arr. The main river including its tributary has been divided into 12 channel elements and 26 overland flow regions. Thiessen polygons have been drawn for the rain gauge stations as shown in Fig. 6.1 and influence area factors have been calculated. An average base flow of 0.05 m^3/sec/sq.km is recommended by Central Water Commission report (1982) for this catchment region, this works out to 5 m^3/sec for this catchment. The loss rate has been calculated as 0.34 mm/hour by the Φ - index method. The Manning's roughness coefficient for the channel is taken as 0.03 as it is a Ghat region (Chow, 1959), the overland roughness coefficient is taken as 0.039.

Two storms have been identified which have the rainfall and corresponding outflow hydrographs. The Storm-1 is of 18 hour duration with a peak flow of 853.26 m^3/s and Storm-2 is of 36 hour duration with a peak flow of 429.92 m^3/s.

With the given catchment parameters, the outflow hydrograph was simulated for Storm-1 and the results are as shown in Fig. 6.2. It can be seen that the simulated hydrograph agrees well with the observed hydrograph, the peak flow being 3.1% less and time to peak being 25% early (1 hour). With the calibrated values of loss rate and overland flow and roughness values used for the first storm, outflow hydrograph has been simulated for the Storm-2 and the results are as shown in Fig. 6.3. Despite small differences, the simulation of peaks and troughs is reasonably good with the peak flow being 4.5% less and time to peak being 22% early (4 hours).

In another attempt to simulate outflow hydrograph for Storm-1, a minimum loss rate of 2 mm/hour as suggested by CWC report (1982) was used instead of computed loss rate of 0.34 mm/hour, overland roughness of 0.03, the other parameters being the same as earlier. The simulated hydrograph is shown in Fig. 6.2 as ungauged simulation, because the loss rate is adopted from the CWC reports. This ungauged simulation gave a peak flow which is 5.8% less and a time to peak which is 25% earlier compared to the observed hydrograph. This may be treated as a fair agreement. The same simulation was repeated for Storm-2 also and the resulting hydrograph is as shown in Fig. 6.3 as ungauged simulation. It can be seen that the simulation of peaks and troughs is reasonably good with peak flow being 13.8% less and time to peak being 22.2% early. Despite the fact that 488% increase in loss rate and a 23% decrease in overland roughness, the peak flows decreased by 2.7% and 9.7% respectively for Storm-1 and storm-2, without any change in time to peak compared to the earlier simulation.

6.4.2 Bhatsa Catchment:

Bhatsa catchment of 398.86 sq.km area is on the western slopes of Western Ghats covering the watershed of Bhatsa river. Here also, two storms were identified for which both the rainfall and corresponding outflow hydrographs are available. These are referred to as Storm-3 with a measured peak flow of 1592.4 m^3/sec and of 39 hours duration and Storm-4 with a measured peak of 1858.95 m^3/sec and of 29 hours duration. For Storm-3, rainfall data from 10 SRRG records is available while for Storm-4, rainfall data from 5 SRRG records is available. The main river Bhatsa has a tributary Chapinai river whose catchment area is nearly 50% of the total catchment area, hence it is considered as a channel in the computations. The main river and its tributary were divided into 23 channel segments and 48 overland flow regions. The IAF values have been calculated using Thiessen polygons separately for the two storms. The location of 10 SRRG stations, Thiessen polygons and discretization are as shown in Fig. 6.4.

As per the recommendations of the CWC report (1973), the base flow for this catchment has been taken as 0.22 m^3/sec/sq.km which works out to 90 m^3/sec. The loss rate has been calculated as 0.35 mm/hour by the Φ - index method. The Manning's roughness for channel is taken as 0.03 and the overland roughness is taken as 0.036. For Storm-3, the simulated hydrograph is shown in Fig. 6.5. It can be seen from this figure that the simulation of peaks and troughs is reasonably good. The simulated peak flow is about 7% less and time to peak is about 20% earlier (2 hours) as compared to the observed hydrograph.

With the same values of loss rate and overland and channel roughness values used for Storm-3, hydrograph for Storm-4 was simulated and the results are as shown in Fig. 6.6. In this case also, the simulation of peaks and troughs is reasonably good compared to the observed one. The peak flow is about 0.4% less and the time to peak is 16.7% earlier (2 hours) compared to the observed.

In another attempt to simulate outflow hydrograph for Storm-3, a minimum loss rate of 2 mm/hour as suggested by CWC report (1973) was used instead of computed loss rate of 0.35 mm/hour, overland roughness of 0.03, the other parameters being the same as earlier. The simulated hydrograph is shown in Fig.6.5 as ungauged simulation, because the loss rate is adopted from the CWC report. This ungauged simulation gave a peak flow which is 13.7% less and a time to peak which is 20% earlier compared to the observed hydrograph. This may be treated as a fair agreement. The same simulation was repeated for Storm-4 also and the resulting hydrograph is as shown in Fig. 6.6 as ungauged simulation. It can be seen that the simulation of peaks and troughs is reasonably good with peak flow being 6.9% less and time to peak being 16.7% early. Despite the fact that 470% increase in loss rate and a 16.7% decrease in overland roughness, the peak flows decreased by 7.3% and 6.5% respectively for Storm-3 and storm-4, without any change in time to peak compared to the earlier simulation.

6.5 Sensitivity Analysis: It is important to check how the solution is sensitive to the changes in values of some parameters. Some times the solution is very sensitive to certain parameters and these are to be identified. Some numerical investigations are presented here.

Effect of Channel Width: It was found that the width of channel has very little influence on the peak flow and time to peak in the simulation of the hydrograph.

Effect of Overland Roughness: It was found that the rough surface will lead to damped flow and delayed peaks. An optimal roughness value of 0.039 for Pimpalgoan Joge catchment was arrived at by least squares solution. When the roughness value was chosen as 0.012, then the resulting peak flow is higher by 12.6% and the time to peak is advanced by 50% (2 hours) compared to the observed hydrograph. When the roughness value was chosen as 0.12, then the resulting peak flow is lesser by 20.8% and the time to peak is delayed by 25% (1 hour) compared to the observed hydrograph. This study indicates that roughness for overland flow are to be chosen carefully as they influence the peak flow and time to peak in the simulation of hydrograph.

Effect of Channel Roughness: Channel roughness also produces damped flow and delayed peaks. A value of 0.03 was taken as it is the minimum value suggested by Chow (1959) for Ghat streams. When the channel roughness value of 0.05 was chosen, then the peak flow reduced by 13.5% and time to peak is delayed by 25%. It was observed that the channel roughness has less influence on hydrograph compared to the overland roughness.

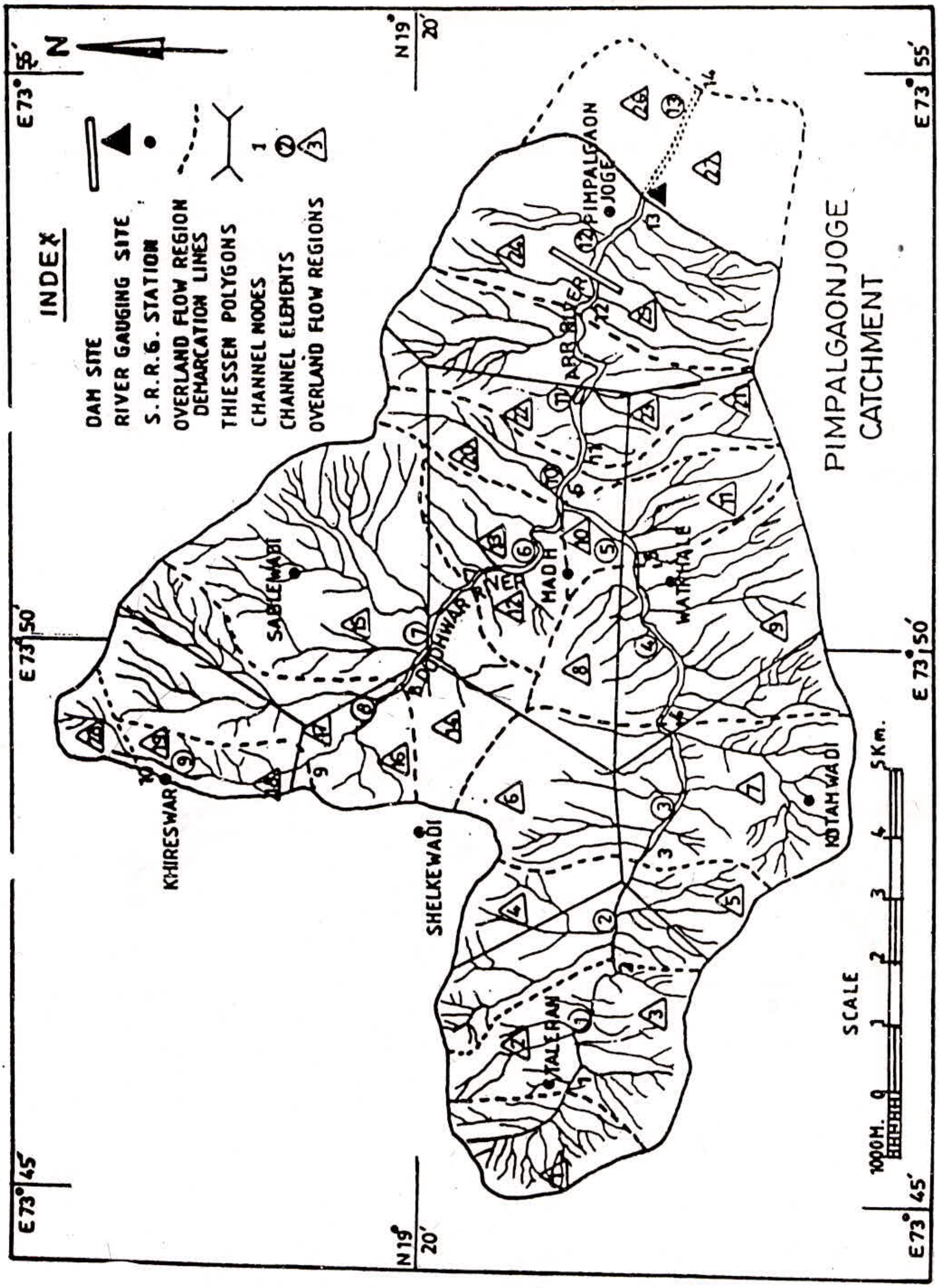
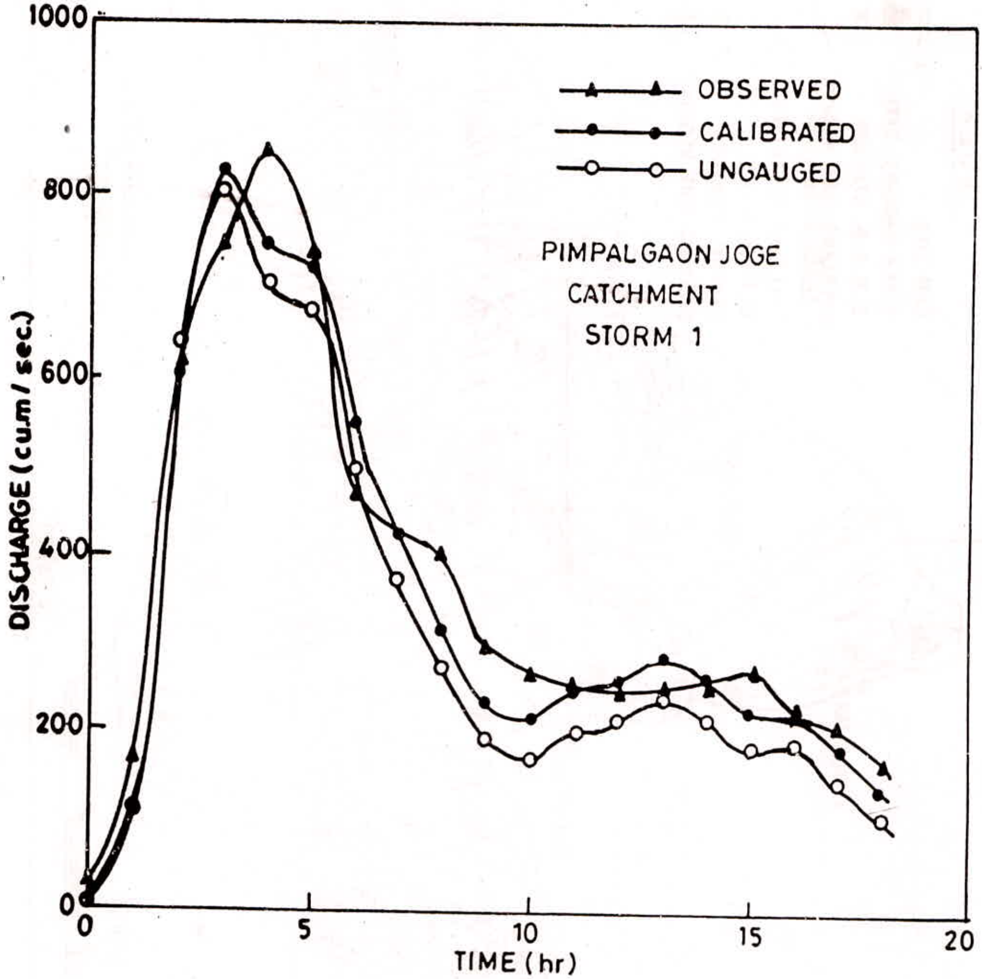

INDEX
DAM SITE
RIVER GAUGING SITE
S.R.R.G. STATION
OVERLAND FLOW REGION DEMARCATION LINES
THIESSEN POLYGONS
CHANNEL NODES
CHANNEL ELEMENTS
OVERLAND FLOW REGIONS
N
E 73° 45'
E 73° 50'
E 73° 55'
N 19° 20'
KHIRESWAR
SHELKEWADI
SABLEWADI
DOODHWAR RIVER
MADH
WATKHALE
ARB RIVER
PIMPALGAON JOGE
TALERAN
KOTAHWADI
PIMPALGAONJOGE CATCHMENT
SCALE
1000M.
5 Km.

FIG. 6.1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800
600
400
200
0
DISCHARGE (cu.m / sec.)
0
5
10
15
20
TIME (hr)
OBSERVED
CALIBRATED
UNGAUGED
PIMPALGAON JOGE
CATCHMENT
STORM 1

FIG. 6.2

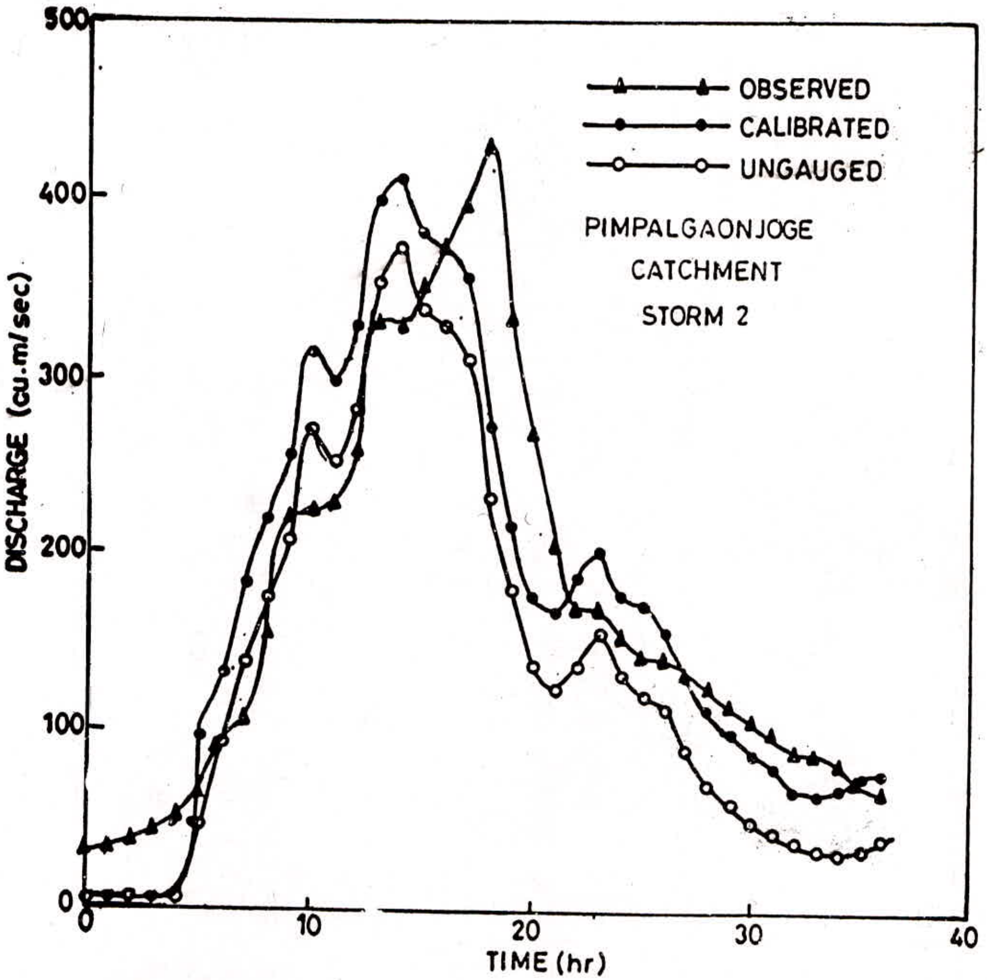
500
400
300
200
100
0
DISCHARGE (cu.m/sec)
0
10
20
30
40
TIME (hr)
OBSERVED
CALIBRATED
UNGAUGED
PIMPALGAONJOGE
CATCHMENT
STORM 2

FIG. 6.3

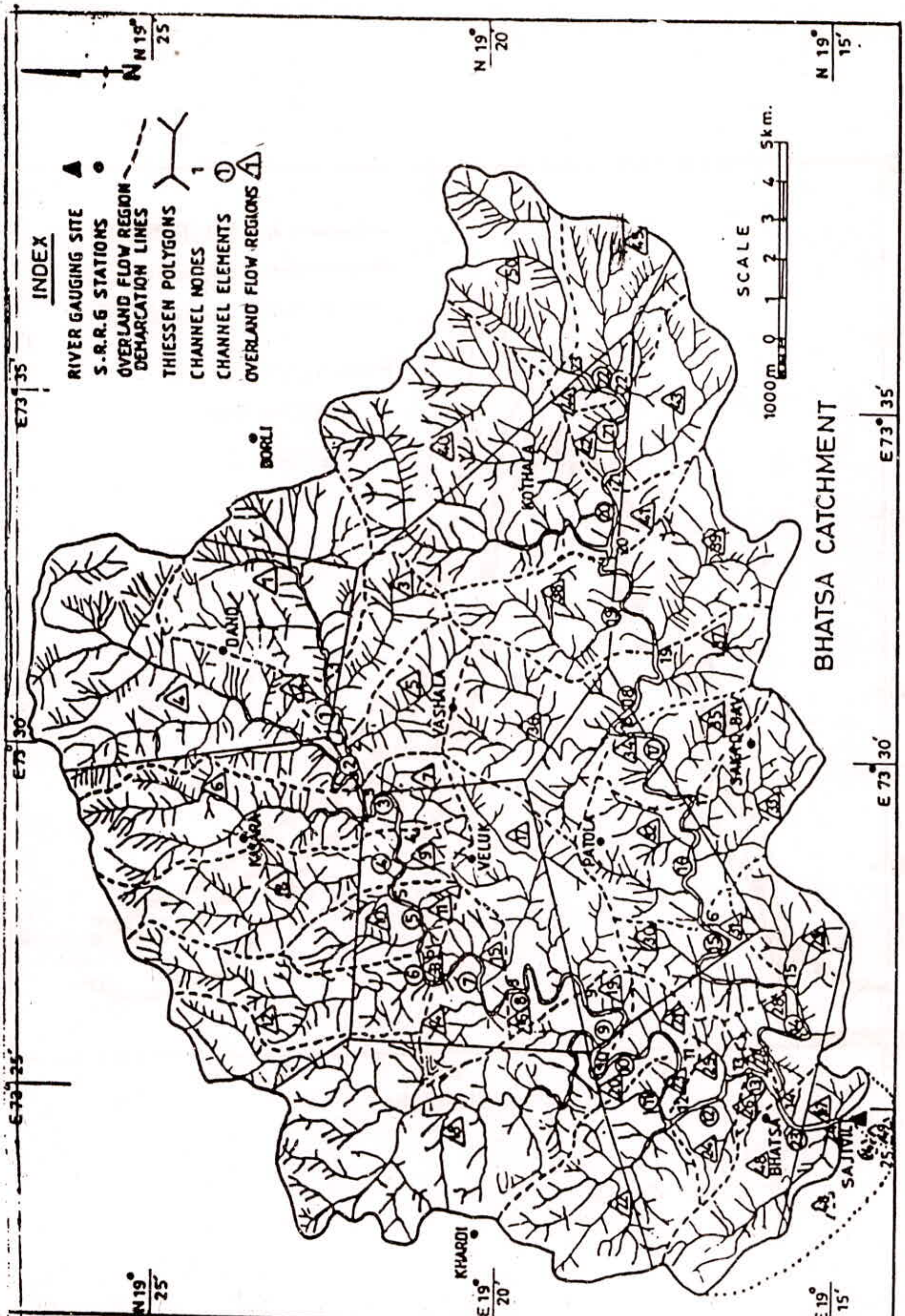
INDEX
RIVER GAUGING SITE
S.R.R.G STATIONS
OVERLAND FLOW REGION DEMARCATION LINES
THIESSEN POLYGONS
CHANNEL NODES
CHANNEL ELEMENTS
OVERLAND FLOW REGIONS
SCALE
1000m
5km.
BORLI
DAND
KASHALA
VELUK
PATOLA
KOTHALA
KHARDI
BHATSA
SAJIVLI
BHATSA CATCHMENT

FIG. 6.4

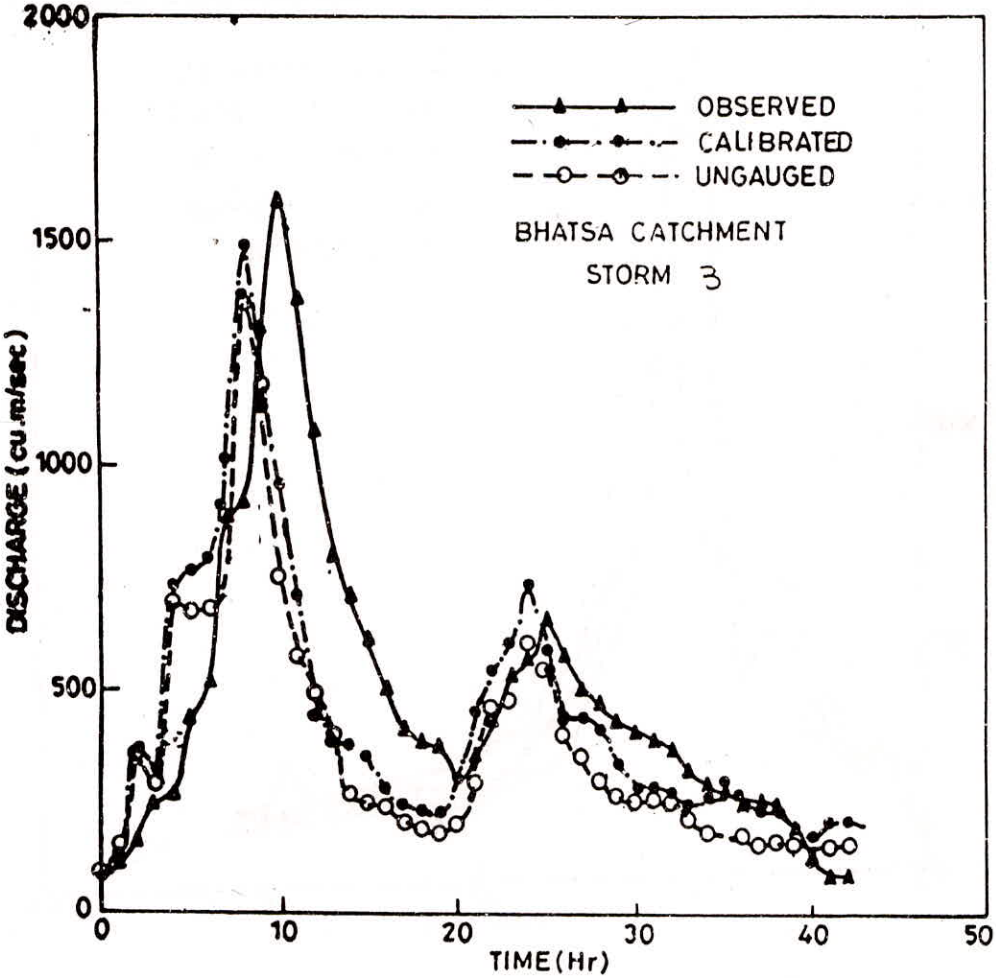
2000
1500
1000
500
0
DISCHARGE (cu.m/sec)
0
10
20
30
40
50
TIME (Hr)
OBSERVED
CALIBRATED
UNGAUGED
BHATSA CATCHMENT
STORM 3

FIG. 6.5

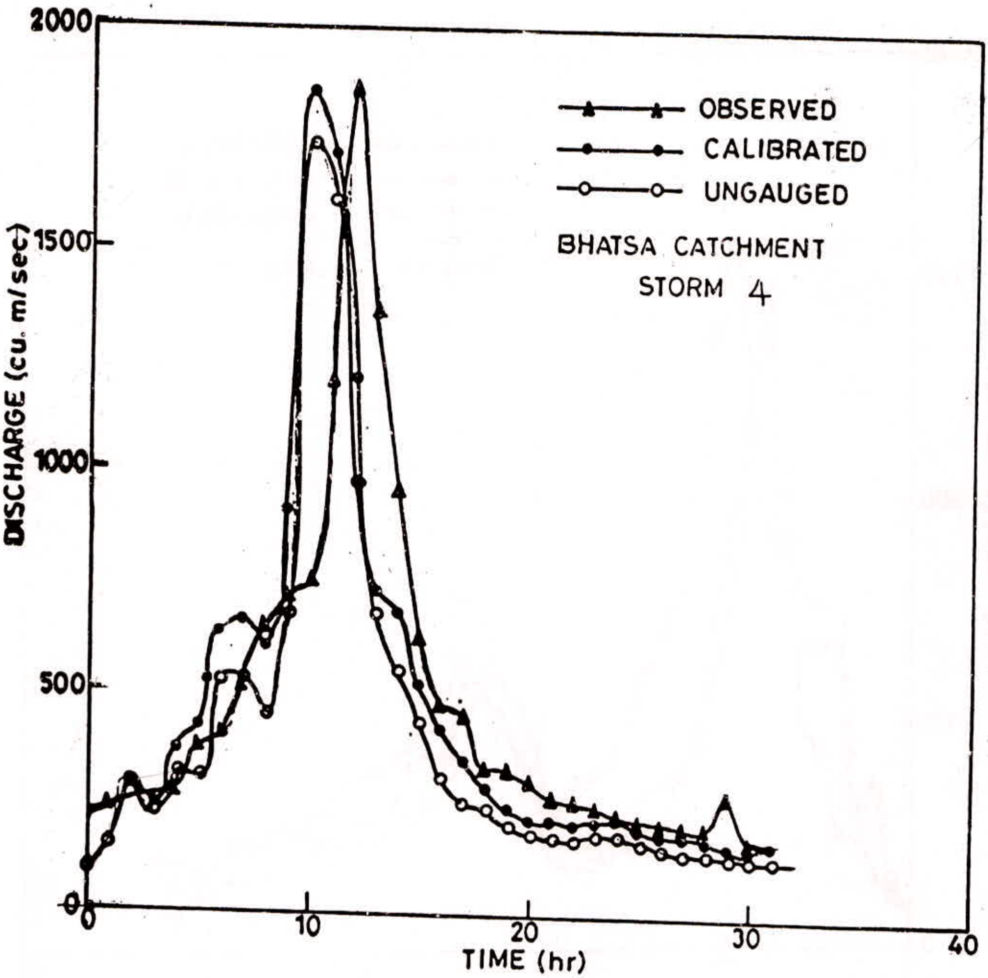
OBSERVED
CALIBRATED
UNGAUGED
BHATSA CATCHMENT
STORM 4
DISCHARGE (cu. m/sec)
2000
1500
1000
500
0
0
10
20
30
40
TIME (hr)

FIG. 6.6

Effect of tributary: When Dudhwar river catchment of Pimpalgoan Joge basin was considered as an overland plane, the peak flow is reduced by 26.2% and the time to peak is advanced by 25%. When the tributary was considered as a channel, rather than an overland plane, the results improved.

A distributed model has been presented in this section to simulate the runoff from the natural watersheds. Overland flow has been evaluated by using a simple formula based on mass balance equation. Channel flow was evaluated by using DWM based on FEM. Spatial and temporal variations of the parameters involved have been incorporated. The model has been applied to two natural catchments of size approximately 100 and 400 sq. km. in India. There is a fair degree of agreement between the observed and simulated hydrographs for both the catchments.

The study reported in this section indicates that lack of data regarding loss rate poses no serious difficulty in simulating runoff hydrographs for natural catchments. The study also showed that roughness of overland planes and channels affect the simulation results. Sensitivity of the solution with respect to changes in parameters like channel widths and tributary have been presented.

NOTATION:

A&A1 Global matrices
B Loop Matrix
C Consumption array
C_j Elements of consumption array, or consumption at node j, +ve if consumption, -ve if input.
D_i Diameter of pipe i (element i).
f Darcy-Weisbach friction factor.
F_j Nodal continuity equation at node j as a function of pressure heads in the network.
g Acceleration due to gravity.
$hf^{(e)}$ Pressure head loss in the element or pipe 'e' due to friction.
HF Array of pressure head losses in the elements of the network.
H_j Pressure head at node j.
δH_j Correction to the pressure head at node j.
K_e Conveyance factor of element e.
L_i Length of pipe i.
m Iteration counter.
N Number of nodes in the network.
NE Number of elements or pipes in the network.
NL Number of loops in the network, NL = NE - N + 1
Q_i Discharge in the element i.
$Q_j^{(e)}$ Discharge entering the node j of element e.
T_e Transmissivity of the element e, $T_e = K_e/|hf|^{1/2}$

(unsteady pipe flow)
A Area of cross section of pipe
C_D Coefficient of discharge

C_p Celerity of the pressure wave
D Diameter of pipe
n Number of elements
N =N(x), interpolation function
V Velocity of flow in pipe
φ Angle of inclination of pipe to the horizontal.

(dispersion)

[A] Global assembly of matrix.
C Concentration of dispersing mass in liquid phase.
C_* Concentration of source fluid.
C_o Input concentration.
C/C_c Relative concentration.
{C} Column vector of nodal values of C.
CN_x Courant number.
D, D_L, D_{xx} Longitudinal dispersion coefficient.
D_{ij} Hydrodynamic dispersion tensor.
{F} Right hand side column vector or force vector.
[J] Jacobian matrix.
[K] Stiffness matrix.
k_{ij} Element of the stiffness matrix.
k Constant of proportionality between concentrations in solid and liquid phases.
N =N(x), interpolation function
n Porosity of the medium.
[P] Capacitance matrix.
p_{ij} Element of the capacitance matrix.
PN_x Peclet number.
q Discharge rate of the source fluid.
R Retardation factor.
S Concentration of the dispersing mass in solid phase.
u, u_x Darcy velocity.
u_i Darcy velocity component in the i^{th} direction.
α_L Longitudinal dispersivity.
Δt Time step.
ε Tolerance criterion in the computations.
γ Coefficient in connection with the boundary conditions.
λ Radioactive decay coefficient.
φ Field variable.
$erfc(x) = 1 - erf(x)$.
$erf(x) = (2/\sqrt{\pi}) \int_0^x exp(-y^2)\, dy$.

(multi-aquifer basin)

$[A]$ & $[A_w]$ Global assembly of matrices for confined and unconfined aquifers
$[B]$ & $[B_w]$ Global assembly of matrices for confined and unconfined aquifers
$\{F\}$ & $\{F_w\}$ Right hand side column vector or force vector for confined and unconfined aquifers
N =N(x,y), interpolation function

q_L Leakage through the aquitard
Q&Q_w Strength of source or sink function for confined and unconfined aquifers
S Storage coefficient
S_s Specific storage of the aquitard
S_y Specific yield of the unconfined aquifer
T Transmissivity of the confined aquifer
e Tolerance criterion in the computations.

(two-dimensional stream flow modelling)

A Area of the Element.
a_i,b_i,c_i Constants of the triangular element.
C Chezy's coefficient applicable for open channels.
E Specific Energy
g Acceleration due to gravity.
h depth of flow, H - z.
h_m Mean depth of flow for an element.
H Water surface Elevation, H = H(x,y,t).
K_x,K_y Coefficients of conductance in open channel flow.
n Manning's roughness coefficient.
N Interpolation function, N = N(x,y).
Q Quantity of flow or discharge.
t Time elapsed.
Δt Time step.
u, v Velocities in the x- and y- directions.
U_m, V_m Mean velocities of flow for element in x- and y- directions.
x , y Coordinate directions.
z Bed level.

REFERENCES:

1. Akan A.O. and Yen B.C. (1981) 'Diffusion-Wave Flood Routing in Channel Networks', Journal of Hyd. Div., ASCE, 107(HY6): 719-732.

2. Bachamat, Y. and Bear J. (1964) 'The General Eqns. of Hydrodynamic Dispersion in Homogeneous, Isotropic Porous Medium' Journal of Geophysical Research, Vol. 69, No. 12, June 1964, pp. 2561-2567.

3. Bear J. (1961) 'On the Tensor Form of Dispersion in Porous Media' Journal of Geophysical Research, Vol 66, No. 4, April 1961, pp. 1185-1197.

4. Bear J (1972) 'Dynamics of Fluids in Porous Media', American Elsevier Environmental Sciences Series, New York.

5. Bennett J.P. (1974) 'Concepts of Mathematical Modelling of Sediment Yield', Water Resources Research, 10(3):485-492.

6. Brebbia C.A. (1988) 'Review of Some Fundamental Concepts and Milestone References on Boundary Elements', from 'Boundary Element Reference Book' edited by J. Mackerle and C.A.Brebbia, Computational Mechanics Publications, Springer Verlag, Berlin, 1988.

7. Bredehoeft, J.D. and Pinder G.F. (1970) 'Digital Analysis of areal Flow in Multiaquifer Ground Water Systems - A Quasi Three Dimensional Model' Water Resources Research' pp.883-888.

8. Chaudhry M.H. (1979) 'Applied hydraulic Transients', Van Nostrand Reinholdt Company, New York.

9. Chorley D.W. and Frind E.O. (1978) 'An Iterative Quasi Three Dimensional Finite Element Model for Heterogenous Multiaquifer System' Water Resources Research, p. 943.

10. Chow Ven Te (1959) 'Open Channel Hydraulics', McGraw-Hill Book Co., New York.

11. Cooley R.L. and Moin S.A. (1976) 'Finite Element Solution of St. Venant Equations', Journal of Hyd. Div., ASCE, 102(HY6) : 759-775.

12. Cunge J.A., Holly F.M. and Verwey A. (1980) 'Practical Aspects of Computational River Hydraulics', Pitmann, London.

13. CWC Report, (1973) 'Estimation of Design Flood Peak, A Method based on Unit Hydrograph Principle', Hydrology Directorate (Small Catchments). Central Water Commission, Government of India, New Delhi, 41pp.

14. CWC Report, (19820 'Flood Estimation Report for Krishna and Pennar Basins (Subzone-3h), A Method based on Unit Hydrograph Principle', Directorate of Hydrology (Small Catchments), Central Water Commission, Government of India, New Delhi, 36pp.

15. Dagan G. (1967) 'Hydrodynamic Dispersion in a Nonhomogeneous Porous Column', Journal of Geophysical Research, Vol. 72, No. 16, August 1967, pp. 4075-4080.

16. Daus A.D. and Find O.E. (1985) 'An Alternating Direction Galerkin Technique for Simulation of Contaminant Transport in Complex Ground Water Systems' Water Resources Research, Vol. 21, No. 5, May 1985, pp. 653-664.

17. Gray W.G. and Pinder G.F. (1974) 'Galerkin's Approximation of the Time derivative in the Finite Element Analysis of Ground Water Flow' Water Resources Research, pp. 821-828.

18. Huang Y.H. and Wu S.J. (1975) 'Simulation of Confined Aquifers by Three Dimensional Finite Elements' Advances in Computer Methods for Partial Differential Eqns., pp. 251-258.

19. Gupta S.K. and Tanji K.K. (1976) ' A Three Dimensional Galerkin Finite Element

Solution of Flow Through Multiaquifers in Sutter Basin, California' Water Resources Research, p. 155.

20. Fujinawa K. (1977) 'Finite Element Analysis of Ground Water Flow in Multiaquifer Systems - A Quasi-Three Dimensional Flow Model' Journal of Hydrology, pp. 349-362.

21. Frind E.O. and Verge M.J. (1978) ' Three Dimensional Modelling of Ground Water Flow Systems' water Resources Systems, pp. 844-855.

22. Freeze R.A. and Cherry J.A. (1979) 'Groundwater' Prentice-Hall Inc., Engelwood Cliffs.

23. Guymon G.L. (1970) 'A Finite Element Solution of the One-Dimensional Diffusion-Convection Equation', Water Resources Research, Vol. 6, No. 1, Feb. 1970, pp 204-210.

24. Freeze R.A. (1978) 'Mathematical Models of Hillslope Hydrology', in: Hillslope Hydrology (Edited by M.J.Kirkby), John Wiley and Sons, New York, pp 177-225.

25. Hantush, M.S. (1967) 'Flow of Groundwater in Relatively Thick Leaky Aquifers' Water Resources Research, pp. 583-590.

26. Huebner, K.H. (1975) 'The Finite Element Method for Engineers', John Wiley and Sons, New York.

27. Henderson F.M. (1966) 'Open Channel Flow', MacMillan, New York, 522 pp.

28. Hromadka II T.V., De Vries J.D. and Nestlinger A.J. (1986a) 'Comparison of Hydraulic and Hydrologic Routing Methods for Channel Flow', in: Hydrosoft 86 - Hydraulic Engg. Software (Edited by M. Radojkovic, C. Maksimovic and C.A.Brebbia), Proc. IInd International Conf., Southampton, U.K., pp. 73-83.

29. Hromadka II T.V., De Vries J.D. and Nestlinger A.J. (1986b) 'Comparison of Hydraulic Routing Methods for One Dimensional Channel Routing Problems', in: Hydrosoft 86 -Hydraulic Engg. Software (Edited by M. Radojkovic, C. Maksimovic and C.A.Brebbia), Proc. IInd International Conf., Southampton, U.K., pp. 85-98.

30. Jain S.K., Storm B., Bathurst J.C., Refsgaard J.C. and Singh R.D. (1992) 'Application of the SHE to Catchments in India. Part 2. Field Experiments and Simulation Studies with the SHE on the Kolar Subcatchment of the Narmada River', Journal of Hydrology, 140:25-47.

31. Jayawardena A.W. and White J.K. (1977) 'A Finite Element Distributed Catchment Model, I. Analytical Basis', Journal of Hydrology, 34(3/4):269-286.

32. Jayawardena A.W. and White J.K. (1979) 'A Finite Element Distributed Catchment Model, II. Application to Real Catchments', Journal of Hydrology, 42(3/4):231-249.

33. Jacoby S.L.S. (1968) 'Design of Optimal Hydraulic Networks',ASCE, Hyd. Div., Vol. 94, HY3, pp. 641-661.

34. Katopodes N.D. and Schamber D.R. (1983) 'Applicability of Dam Break Flood Wave Models', Journal of Hyd. Div., ASCE, 109(HY5):702-721.

35. Keuning D.H. (1976) 'Application of Finite Element Method to Open Channel Flow'; Journal of Hyd. Div., ASCE, 102(HY4) :459-468.

36. King I.P. (1976) 'Finite Element Models for Unsteady Flow Routing Through Irregular Channels', Proc. First Int. Conf. on FEM in Water Resources held at Princeton University, Princeton, New Jersey, pp 4.165-4.184.

37. Lai C. (1986) 'Numerical Modelling of Unsteady Open Channel flow', in: Advances in Hydrosciences (Edited by B.C.Yen), Vol. 14, Academic Press, New York, pp. 162-333.

38. Liang, T. (1971) 'Design of Conduit System by Dynamic Programming', ASCE, Hyd. Div., Vol. 97, HY3, pp. 383-393.

39. Marino M.A. (1974) 'Longitudinal Dispersion in Saturated Porous Media' Journal of the Hyd. Div., ASCE, Vol 100, No. HY1, January 1974, pp. 151-157.

40. Narasimhan T.N. and P.A. Witherspoon (1982) 'Overview of the Finite Element Method in Ground Water Hydrology', in Finite Elements in Water Resources, Edited by K.P.Holz, W.Zielke, C.A.Brebbia, G.Pinder and W.Gray, Proc. of the Fourth International Conference, Computational Mechanics Center, Southampton, U.K., 1982.

41. Neuman, S.P. and Witherspoon, P.A. (1969) 'Theory of Flow in a Confined Two-Aquifer System' Water Resources Research, pp. 803-816.

42. Nwaogazie F.I.L. and Tyagi A.K. (1984) 'Unified Stream Flow Routing by Finite Elements', Journal of Hyd. Div. ASCE, 110(HY11):1595-1611.

43. Oden, J.T., O.C.Zienkiewicz, R.H. Gallaghar and C.Taylor (Editors) (1974) 'Finite Element Methods in Flow Problems', Proceedings of the First International Symposium on finite element Methods in flow Problems', Swansea, U.K.

44. Panakala Rao E. (1982) 'Optimal Design of water distribution Networks', M.Tech. dissertation, department of Civil Engineering, Indian Institute of Technology, Bombay.

45. Panakala Rao E. (1990) 'Surface Runoff Modelling of Watersheds with Land Use from Remote Sensing', Ph.D. Thesis, Indian Institute of Technology, Bombay, India, 191pp.

46. Paygude D.G., B.Vasudeva Rao and S.G.Joshi (1985) 'Fluid Transients Following a Valve Closure by F.E.M.', Proc. of the International Conference on Finite Elements in Computational Mechanics, FEICOM-85, I.I.T., Bombay, December 1985.

47. Ponce V.M., Li R.M. and Simons D.B. (1978) 'Applicability of Kinematic and Diffusion Models', Journal of Hyd. Div., ASCE, 104(HY3):353-360.

48. Pinder, G.F. and Bredehoeft, J.D. (1968) 'Application of Digital Computer for Aquifer Evaluation' Water Resources Research, pp. 1069-1093.

49. Pinder G.F and Frind E.O (1972) 'Application of Galerkin's Procedure to Aquifer Analysis' Water Resources Research, pp. 108-120.

50. Refsgaard J.C., Seth S.M., Bathurst J.C., Erlich M., Storm B., Jorgensen G.H. and Chandra S. (1992) 'Application of SHE to Catchments in India. Part 1. General Results', Journal of Hydrology, 140:1-23.

51. Rich G.R. (1963) 'Hydraulic Transients', Dover Publications Inc, New York.

52. Ross B.B., Contractor D.N. and Shanholtz V.O. (1979) 'A Finite element Model of Overland and Channel Flow for Assessing the Hydrologic Impact of Landuse Change' Journal of Hydrology, 41(1/2):11-30.

53. Saraph V.R., B.Vasudeva Rao and J.T.Panikar (1979) 'Stability of Parallel Flows by Finite Element Method', International Journal of Numerical Methods in Engineering, Vol 14, No. 8, pp. 1257-1261.

54. Schaake, Jr. J.C. and Lai, D. (1969) 'Linear and Dynamic Programming Application to Water distribution Network design', Hydrodynamic Laboratory Report No. 116, Department of Civil Engineering, Massachusetts Institute of Technology, Cambridge, Mass., U.S.A., July 1969.

55. Scheidegger A.E. (1961) 'General Theory of Dispersion in Porous Media' Journal of Geophysical Research, Vol. 66, No. 10, October 1961, pp 3272-3278.

56. Shamir Uri (1974) 'Optimal design of Operation of Water Distribution Systems', Water Resources Research, american Geophysical Union, Vol. 10, No. 1, pp 27-36.

57. Streeter V.L. and Lai C. (1962) 'Water Hammer Analysis including Friction', Journal of Hyd. Div., ASCE, Vol. 88, No. HY3, May 1962, pp 79-112.

58. Streeter V.L. and Wylie E.B. (1981) 'Fluid Mechanics', McGraw-hill Book Co. Inc., New York.

59. Taylor C., Al-Mashidani G. and Davis J.M., (1974) 'A Finite Element Approach to Watershed Runoff', Journal of Hydrology, 21(3):231-246.

60. Vasudeva Rao B. (1981) 'Optimal Design of Urban water Distribution Systems', in Lecture notes on Optimal Design of Water Supply and Waste Water Removal Systems', Environmental Science and Engineering Group, Indian Institute of Technology, Bombay.

61. Vasudeva Rao B. and M.V.Patel (1982) 'Simulation of Multiaquifer Basin by Digital

Model', CSIR Research Project Report, Department of Civil Engineering, I.I.T., Bombay.

62. Vasudeva Rao, B., Joshi S.G. and Paygude D.G. (1986) 'Some Studies on the Dispersion of Chemical Nuclides in the Saturated Porous Media Flow by using FEM' Report submitted to the Board of Research in Nuclear Studies, Department of atomic Energy, Government of India, Bombay from Department of Civil Engineering, Indian Institute of Technology, Bombay.

63. Vasudeva Rao, B. (1987) 'Finite Element Analysis of Flow Networks', Journal of Engineering Analysis, Computational Mechanics Publications, Southampton, U.K., March 1987.

64. Vasudeva Rao B. and Medha J. Dixit (1987) 'A Simple Method in the Design of Water Distribution Networks', Proc. of the Afro-asian Conference on Integrated Water Management in Urban areas, Bombay 1987.

65. Vasudeva Rao B. and Panakala Rao E., (1988) 'Surface Runoff Modelling of Small Watersheds', Proc. International Seminar on Hydrology of Extremes held at Roorkee, India, pp. 145-155.

66. Vasudeva Rao, B. (1993) 'Two-Dimensional Stream Flow Modelling by F.E.M.', Mitteilungen des Leichtweiss Institut fur Wasserbau der Technichen Universitat, Braunschweig, Federal Republic of Germany, Heft 119, 1993.

67. Weinmann P.E. and Laurenson E.M., (1979) 'Approximate Flood Routing Methods - A Review', Journal of Hyd. Div., ASCE, 105(HY12):1521-1536.

68. Watt C.S., Hobbs J.M. and Boldy A.P. (1980) 'Hydraulic Transients Following a Valve Closure', Journal of Hyd. Div., ASCE, Vol 106, No. HY10, October 1980, pp. 1627-1640.

69. Wylie E.B. and Streeter V.L. (1978) 'Fluid transients', McGraw-hill Book Co. Inc., New York.

70. Yuang Y.H. and Sonnenfeld S.L. (1974) 'Analysis of Unsteady Flow Towards an Artesian Well by Three Dimensional Finite Elements' Water Resources Research, pp. 591-596.

71. Zia Hosseinipour and Michael Amein (1984) 'Finite Element Computation of Two-dimensional Unsteady Flow for River Problems', Finite Elements in Water Resources, Proceedings of the 5th International Conference, Burlington, Vermont, U.S.A., June 1984, pp. 457-466.

72. Zienkiewicz O.C. (1980) 'Finite element Method' (3rd Edition), Mcgraw-Hill Book Co. Inc., New York.

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