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DETERMINATION OF REACH TRANSMISSIVITY UNDER VARIOUS HYDROLOGIC BOUNDARY CONDITIONS

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CONTENTS

LIST OF SYMBOLS

b	=	half width of canal at water surface
b'	=	half bottom width of canal
e	=	a parameter
D_i	=	depth to permeable layer below the canal bed
D_w	=	depth to water table at large distance from the canal measured from the level of water surface in the canal
$F(\frac{\pi}{2}, k)$	=	complete elliptic integral of 1st kind with modulus k
$F(\sin^{-1} t, k)$	=	incomplete elliptic integral of 1st kind with argument $\sin^{-1} t$ and modulus k.
H	=	depth of water in the canal
i	=	$\sqrt{-1}$
K	=	coefficient of permeability
K_1	=	complete elliptic integral of the first kind with modulus k
K_1'	=	complete elliptic integral of the first kind with modulus $\sqrt{(1-k^2)}$
M, M'	=	constants
N	=	constant
p	=	pressure
q	=	half of seepage loss from the canal
q'	=	half of seepage occurring through bed of the canal
w	=	complex potential plane ($w = \phi + i\Psi$)
x	=	abscissa
y	=	ordinate
z	=	$x + iy$
α	=	a parameter

β	=	a parameter
ν_w	=	unit weight of water
ϕ	=	velocity potential function
ψ	=	stream function
θ	=	zhukovskii function $z + \frac{iw}{k}$

LIST OF FIGURES

LIST OF TABLE

ABSTRACT

Reach transmissivity for a canal or stream is a site specific constant which depends on canal geometry and aquifer boundary. The reach transmissivity, when multiplied with depth to water table position from the water surface in the canal measured at soame observation ploint in the vicinity the canal, gives the seepage rate. In the present report, the reach transmissivity for various sub-soil conditions have been reviewed. Analytical solution for reach transmissivity pertaining to a canal embeded in a porous medium underlain by a highly permeable layer, has been obtained using Zhukovsky's function and conformal mapping. Results have been presented for various position of water table above the highly permeable layer. The active seepage zone is under the canal and the phreatic lines merge with the water table within a distance of thrice the width of water surface from the centre of the canal.

1.0 INTRODUCTION

Prediction of seepage from canals and stream is based on the hydraulic properties of soils of which the flow domain is comprised of and the prevailaing boundary and the initial conditions. Seepage from a body of surface water under a steady state condition can be estimated by solving Laplace equation satisfying the pertinent boundary conditions. The study on seepage from canals and streams under steady state conditions are exhaustive and well documented. However, the unsteady seepage problems being complicated have been solved for idealized boundary conditions such as: exchange of flow between a fully penetrating river and an infinite homogeneous confined aquifer. For a stream or canal hydraulically connected with an aquifer,it has been assumed that the exchange flow rate is linearly dependent on the potential difference between the aquifer and the stream. The constant of propertionalaity has been designated by Morel Seytoux et al (1973) as reach transmissivity. The reach transmissivity for particular flow boundary could be derived assuming a steady state condition but it can be used to predict seepage under unsteady state situation as unsteady states cana be assumed to be succession of steady states. In the present report, the reach transmissivity, for various hydrogeological condition, available in literature has been enlighted. Expression for reach transmissivity for a partially penetrating canal in a homogeneous aquifer underlain by a highly prervious layer at a finite depth has been derived.

2.0 REVIEW

There are three basic aconditions to which the natural profiles of soil hydraulic conductivity can be reduced for theoritical treatment of seepage flow system (Bouwer, 1965). These are:

i) The soil in which the channel is embeded is uniform and underlain by a less permeable (considered as impermeable material) soil,

ii) The soil in which the channel is embeded is uniform and underlain by a more permeable (considered as infinitely permeable) soil,

ii) The soil in which the chananel is embeded is of much lower hydraulic conductivity than the original soil for a relatively short distance normal to the channel perimeter (clogged soil, semipermeable linings).

The geometry of a channel cosnstructed in an aquifer conforming to situation 1 is shown in Figure 1. The channel is hydraulically connected with the aquifer. For a specific case in which the channel is rectangular and the bottom of the chananel extends to the impermeable layer, the seepage loss is given by (Bouwer, 1965b).

$$Q = 2\,K(H_w - 0.5D_w)/(L - 0.5w_b)D_w \qquad \ldots (2.1)$$

Half of the above seepage quantity enters to the aquifer to the right of the canal and the other half enters to the aquifer in the left. The reach transmissivity for a fully penetrating canal of reach length L_r is

$$\Gamma_r = 2KL_r(H_w - 0.5D_w)/(L - 0.5w_b) \qquad \ldots (2.2)$$

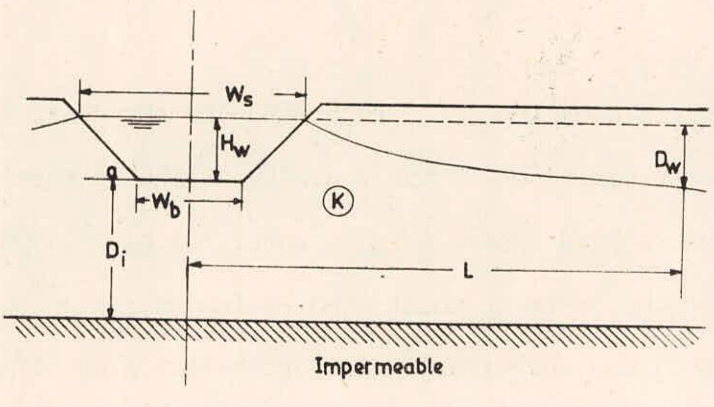


FIGURE 1. GEOMETRY FOR CHANNELS IN SOIL UNDERLAIN BY IMPERMEABLE MATERIAL

L can be regarded as the distance of the observation well where the drawdown 'D_w' is observed.

An approximate expression for seepage from a chananel shown in Figure 1 is given by (Bouwer, 1969)

$$Q = K2(H_w + D_i - 0.5D_w)/(L - 0.25w_b - 0.25w_s)D_w \quad \dots (2.3)$$

Hence, the approximate expression for rach transmissivity for a canal conforming to the configuration depicted in Figure 1 is

$$\Gamma_r = 2KL_r(H_w + D_i - 0.5D_w)/(L - 0.25w_b - 0.25w_s) \quad \dots (2.4)$$

The above expression is not exact and the error in Γ_r will increase with increasing D_i.

The error in equation (2.4) is due to the curvature and divergence of the stream lines in the vicinity of the chananel. To account for the extra head losses in this zone, Dachler (1936) divided the flow system on the basis of model studies into a region with curvilinear flow (region I) and one with Dupuit Forchheimer flow (Fig. 2), the dividing line being at a distance

$$L_1 = \frac{w_s + H_w + D_i}{2} \quad \dots (2.5)$$

from the centre of the canal. The flow in region I was analysed with an approxiaimte equation for the potential and the stream line distribution under a plane source of finite width. A factor 'F' has been determined to estimate flow in region I as

$$Q_I = 2FK\Delta H \quad \dots (2.6)$$

where ΔH is the vertical distance between the water surface in the canal and the groundwater table at the dividing line between the two flow regions. Values of F given by Dachler are presented in Figure 3.

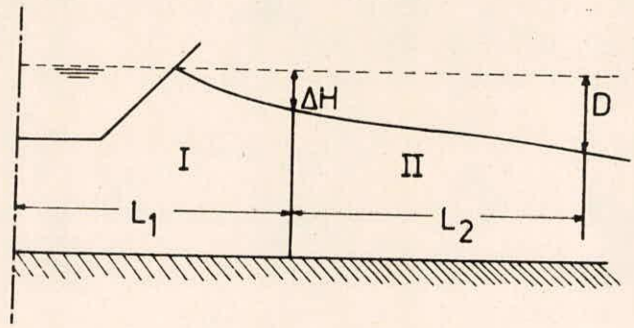


FIGURE 2. DIVISION OF FLOW SYSTEM IN REGIONS I AND II FOR DACHLER'S ANALYSIS

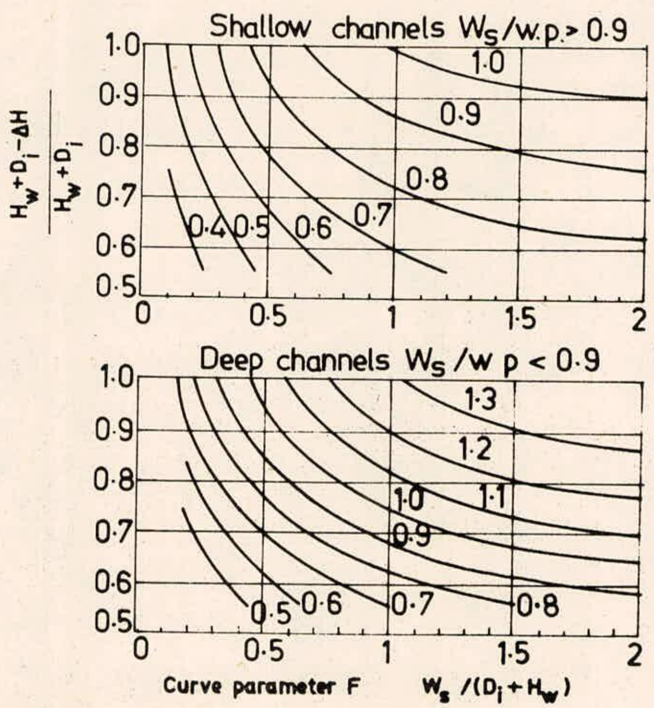


FIGURE 3. DACHLER'S VALUES OF F FOR SHALLOW AND FOR DEEP CHANNELS

The flow in region II has been expressed with Dupuit Forchheimer theory as

$$Q_{II} = \frac{2K(D_w - H)}{L_2}[D_i + H_w - 0.5\Delta H - 0.5D_w] \qquad \dots (2.7)$$

Since it is requaired to calculate the seepage for a given value of D_w at a distance $L_1 + L_2$ from the chananel centre, ΔH will not be known initially. ΔH is found by trial error which satisfies the condition $Q_I = Q_{II}$. The reach transmissivity for a canal of length L_r will be given by

$$\Gamma_r = \frac{2KL_r}{L_2}[1 - \frac{\Delta H}{D_w}][D_i + H_w - 0.5\Delta H - 0.5D_w] \qquad \dots (2.8)$$

Bouwer has applied Ernst's approach to analyse seepage from a canal constructed in a porous medium of finite depth underlain by an impervious layer. Following Ernst's approximate solution for potential distribution pertainaing to flow to a line sink, the head loss, h_r, due to radial flow in the vicinity of the canal has been expressed by Bouwer as

$$h_r = \frac{Q}{\pi K} \log_e(\frac{D_i + H_w}{w_p}) \qquad \dots (2.9)$$

The head loss, h_h, due to horizontal flow in the region away from the canal has been expressed by Bouwer as

$$h_h = \frac{Q}{2K} \frac{L}{D_i + H_w - 0.5D_w} \qquad \dots (2.10)$$

Since $D_w = h_r + h_h$, Bouwer dhas combined equations (2.9) and (2.10) to obtain the relation

$$Q = \frac{KD_w}{\frac{1}{\pi}\log_e[(D_i + H_w)/w_p] + [\frac{0.5L}{D_i + H_w - 0.5D_w}]} \qquad \dots (2.11)$$

The reach transmissivity for a canal reach of length L_r from equation (2.11) can be obtained as

$$\Gamma_r = \frac{KL_r}{\frac{1}{\pi}\log_e[(D_i+H_w)/w_p] + [\frac{0.5L}{D_i+H_w - 0.5D_w}]} \qquad \text{... (2.12)}$$

Using a simple potential theory Morel Seytoux et al (1979) have derived the following expression of reach transmissivity for a canal embeded in a porous medium underlain by an impervious layer:

$$\Gamma_r = -\frac{TL_r}{e} \; \frac{0.5w_p + e}{5w_p + 0.5e} \qquad \text{... (2.13)}$$

in which

L_r = length of a canal reach,

T = transmissivity,

w_p = wetted perimeter of the canal, and

e = saturated thickness below the canal bed .

Herbert (1970) has related the flow from a partially penetrating river having semicircular cross section (Fig. 4) to the potential difference between the river and in the aquifer below the river bed. The expression is given by

$$Q_r = \pi L_r K (h_r - h_o)/\log_e(0.5m/r_r) \qquad \text{... (2.14)}$$

in which

L_r = length of river reach,

h_r = potential at the river boundary,

h_o = potential in the aquifer below the river bed,

m = saturated thickness of the aquifer, and

r_r = radius of the semicircular river cross section.

The reach transmissivity which could be obtained from equation (2.14) is

$$\Gamma_r = \pi L_r K/\log_e(0.5m/r_r) \qquad \ldots (2.15)$$

For a rectangular chananel shown in Fig.5,Aravin (1965) has derived the following expression for flow to the chananel:

$$Q = \frac{K(H+h)(H-h)}{L - \frac{B}{2}} + \frac{K(H - h)}{\frac{L}{2T} - \frac{1}{\pi}\log_e \sin h(\frac{\pi B}{4T})} \qquad \ldots (2.16)$$

Thus reach transmissivity for a canal reach of length L_r could be written as

$$\Gamma_r = \frac{KL_r(H+h)}{L - 0.5B} + \frac{KL_r}{\frac{0.5L}{T} - \frac{1}{\pi}\log_e \sin h(\frac{\pi B}{4T})} \qquad \ldots (2.17)$$

Seepage flow from a canal embeded in a porous medium of finite depth underlain by a highly pervious layer (Figure 6), has been analysed for simplified canal geometry by Hammad (1959). The analysis is valid for the situation in which the piezometric head in the underlying highly pervious layer is very near the canal water level. According to Hammd

$$Q = KD_w \frac{2\,K_1}{K_1' - C} \qquad \ldots (2.18)$$

in which

K_1 and K_1' are the complete elliptic integral of the first kind corresponding to modulus k_1 and comaplementary modulus k_1' respectively. The moduli are defined as

$$k_1 = 0.5\,[\frac{w_s'}{2} + (\frac{w_s'}{4} - 2H_w'^2)^{\frac{1}{2}}]$$

$$k_1' = (1 - k_1^2)^{\frac{1}{2}}$$

The other constants are:

$$C = H_w'/k_1$$

$$H_w' = \tan\,[\frac{H_w}{2(H_w+D_p)}] \text{ for } H_w < D_p$$

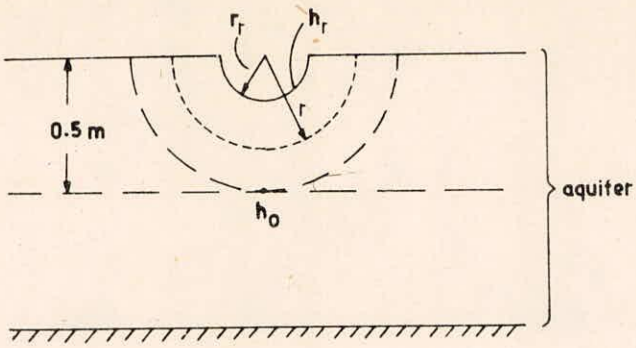


FIGURE 4. REPRESENTATION OF PARTIALLY PENETRATING RIVER

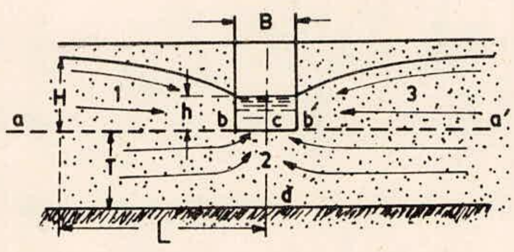


FIGURE 5. FLOW TO A RECTANGULAR DITCH

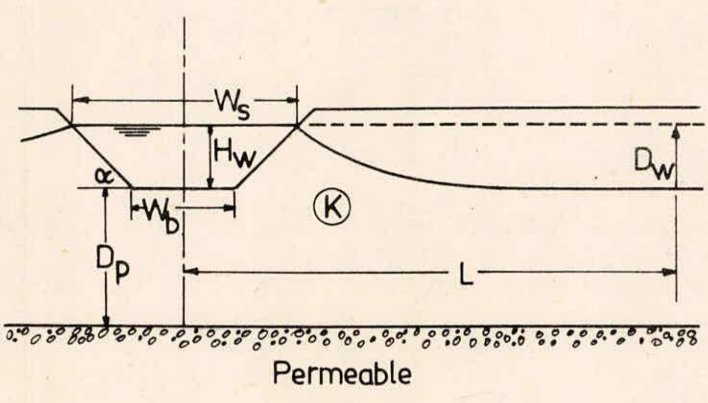


FIGURE 6. GEOMETRY AND SYMBOLS FOR CHANNELS IN SOIL UNDERLAIN BY PERMEABLE MATERIAL

and

$$w'_s = 2 \tan h \left[\frac{\pi w_s}{4(H_w + D_p)}\right] \text{ for } H_w < D_p$$

The reach transmissivity for a canal reach of length l_r can be written as

$$\Gamma_r = KL_r \; \frac{2K_1}{K' - C} \qquad \ldots (2.19)$$

Aravin has analysed the seepage from a canal which has very shallow water depth in it. The water table lies above the highly permeable layer as shown in Fig. (7). The analysis has been carried out using Zhukovsky's function and conformal mapping. The seepage quantity is given by

$$Q = K(T - H)K'_1/K_1 \qquad \ldots (2.20)$$

in which K_1 is the complete elliptic integral of first kind with mudulus $k = \exp\left(\frac{-(b + \frac{Q}{K})}{2H}\right)$, K'_1 is comaplete elliptic integral of first kind with modulus k', where k' is given by

$$k' = \sqrt{(1 - k^2)}$$

when k is very near to zero

$$Q = K(T - H)(b + .882H)/T$$

Thus

$$\Gamma_r = KL_r(b + .882H)/T \qquad \ldots (2.21)$$

The case of seepage from a canal in a two layered soil (Fig.8) underlain by an impermeable layer has been analysed by Ernst (vide Bower, 1969). Following Ernst's solution, the reach transmissivity pertaining to a two layered soil system can be written as:

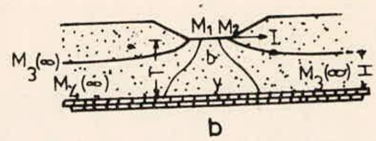


FIGURE 7. SEEPAGE FROM A CANAL WITH SHALLOW WATER DEPTH EMBEDED IN A POROUS MEDIUM UNDERLAIN BY A HIGHLY PERMEABLE LAYER

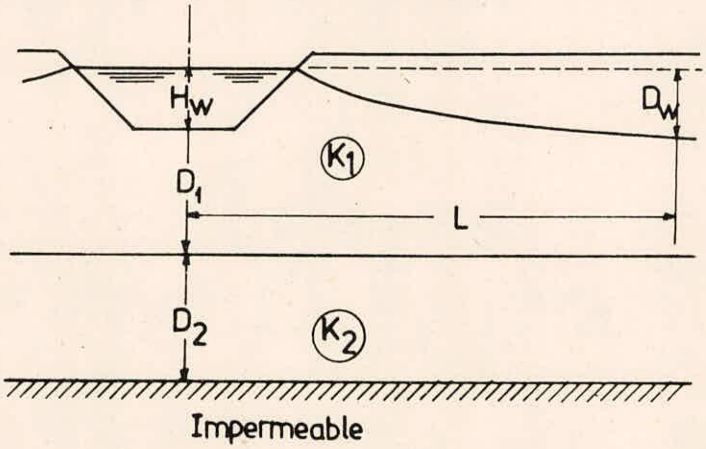


FIGURE 8. CANAL IN A TWO LAYERED SOIL SYSTEM

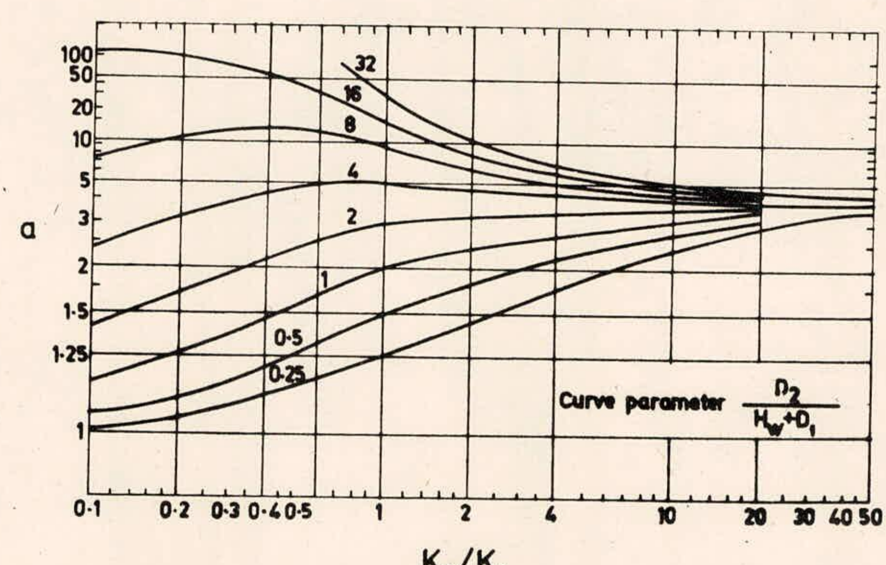


FIGURE 9. PARAMETER α FOR CALCULATING SEEPAGE LOSS FROM A CANAL IN A TWO LAYERED SOIL SYSTEM

$$\Gamma_r = \frac{K_1 L_r}{\dfrac{0.5 K_1 L}{K_1(D_1 + H_w - 0.5 D_w) + K_2 D_2} + \dfrac{1}{\pi} \ln \dfrac{\alpha (H_w + D_1)}{w_p}} \qquad \ldots (2.22)$$

in which K_1 and K_2 are permeabilities of the top and bottom layer respectively. The parameter α given by Van Beer (vide Bouwer, 1969) is shown in Fig. 9.

3.0 PROBLEM DEFINITION AND METHODOLOGY

3.1 Statement of the Problem

Figure 10(a) shows a schematic cross section of a canal in Z plane. A highly pervious stratum is underlying at a depth D_i below the canal bed. The depth of water in the canal is H. The bed width of the canal is 2b'. The water table in the aquifer is at a depth D_w below the level of water in the canal. It is required to find the quantity of seepage from the canal to the aquifer.

3.2 Methodology

The pertinent complex potential plane w, where $w = \phi + i\Psi$, is shown in Figure 10(b), in which Ψ is the stream function and ϕ is the velocity potnetial function defined as

$$\phi = -K\left(\frac{p}{\nu_w} + y\right) + c \qquad \dots (1)$$

In equation 1,

K = coefficient of permeability,

p = pressure,

ν_w = unit weight of water,

y = elevation head, and

c = constant which has been takend as zero.

At large distance from the canal the water table is at a depth D_w below the water level in the canal. Therefore, the potential at large distance from the canala is $-K(H - D_w)$. Since the underlying layer is highly pervious, the interface of the pervious and highly pervious layer cana be regarded as an equipotential line. The potential along the interface is assumed to be $-K(H - D_w)$.

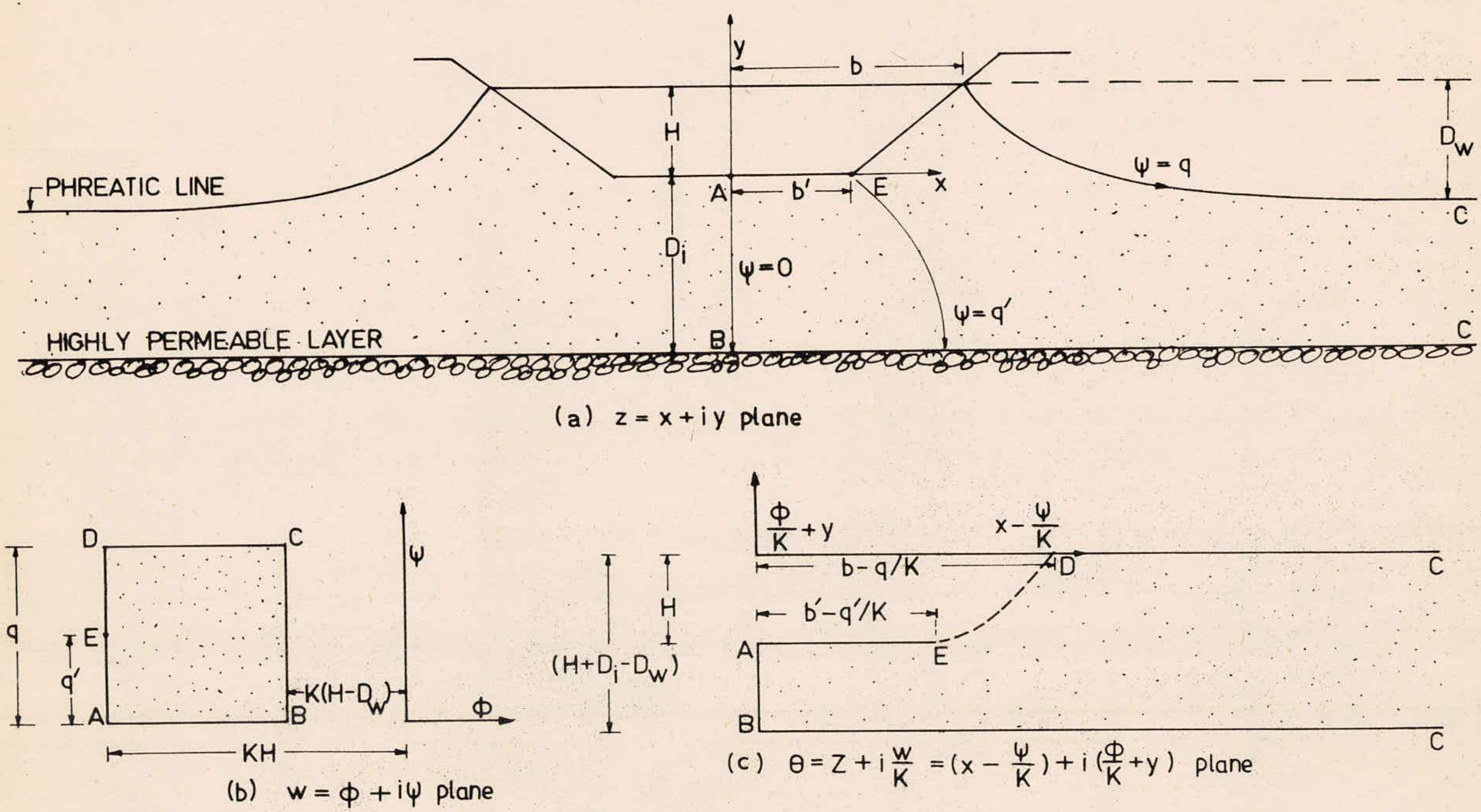


FIGURE 10. STEPS OF MAPPING

The flow domain consists of phreatic line which is curvilinear and unknown a priori. Conformal mapping can be applied to analyze the unconfined flow after transforming the flow domain to Zhukovskey's θ plane. The pertinent θ plane, in which $\theta = Z + \frac{iW}{K} = (x - \frac{\Psi}{K}) + i(\frac{\Phi}{K} + y)$, is shown in Figure 10(c). The locus of DE is not known though the locations of points D, E are known. In Figure 10(d), the idealized θ plane plane has been shown. According to Schwarz-Christoffel transformation the conformal mapping of the polygon in θ plane on to the upper half of the auxiliary t plane (Fig. 10(e)) is given by (vide Harr, 1962)

$$\theta = M \int_{o}^{t'} \frac{(e - t)^{\alpha} dt}{t^{\alpha}(1-t)^{\frac{1}{2}}(\beta - t)^{\frac{1}{2}}} + N \qquad \ldots (2)$$

the vertices C,D,E,A,B being mapped onto points $-\infty$, o, e, 1, β respectively on the real axis of the t plane. M is a comaplex constant to be evaluated. The lower limit of integration determines the other constrant N. For point D, $t = 0$ and $\theta = b - q/K$. In equation 2, as the lower limit of integration has been chosen to be zero, accordingly the constant $N = b - q/K$. For point E, where $\theta = b' - q'/K - iH$ and $t = e$ equation (2) becomes

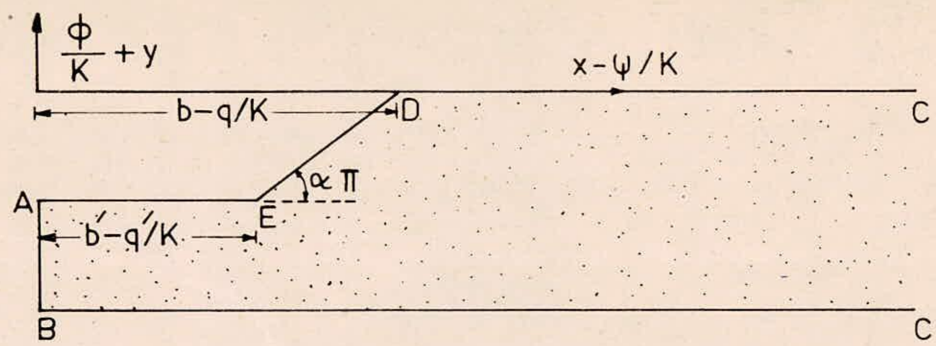
$$b' - \frac{q'}{K} - b + \frac{q}{K} - iH = M \int_{o}^{e} \frac{(e-t)^{\alpha} dt}{t^{\alpha}(1-t)^{\frac{1}{2}}(\beta - t)^{\frac{1}{2}}} \qquad \ldots (3)$$

Equating the arguments of terms on either side of equation (3)

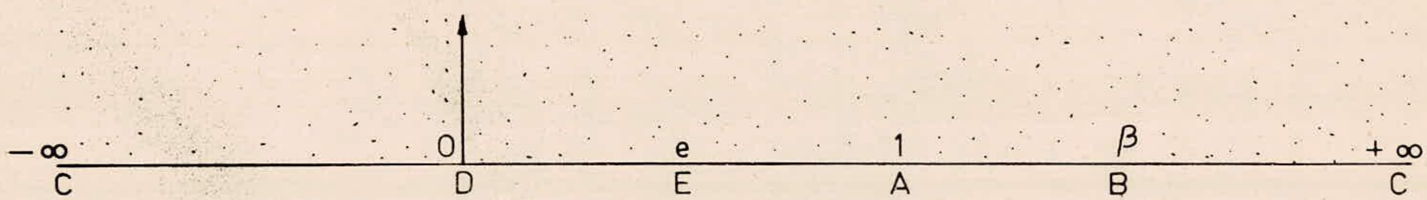
$$[(b - b' - \frac{q}{K} + \frac{q'}{K})^2 + H^2]^{\frac{1}{2}} = |M| \int_{o}^{e} \frac{(e-t)^{\alpha} dt}{t^{\alpha}(1-t)^{\frac{1}{2}} (\beta - t)^{\frac{1}{2}}} \qquad \ldots (4)$$

For $e \leq t \leq 1$, the relation between θ and t planes is given by

$$\theta = M \int_{e}^{t'} \frac{(e-t)^{\alpha} dt}{t^{\alpha}(1-t)^{\frac{1}{2}}(\beta - t)^{\frac{1}{2}}} + b' - \frac{q'}{K} - iH \qquad \ldots (5)$$



(d) IDEALISED θ-plane



(e) t plane

FIGURE 10. STEPS OF MAPPING

For point A, where $t=1$ and $\theta = -iH$, equation (5) becomes

$$-b' + \frac{q'}{k} = M \int_e^1 \frac{(e-t)^{\alpha} dt}{t^{\alpha}(1-t)^{\frac{1}{2}} (\beta - t)^{\frac{1}{2}}} \qquad \ldots (6)$$

Equating the arguments of terms on either side

$$b' - \frac{q'}{K} = |M| \int_e^1 \frac{(t-e)^{\alpha} dt}{t^{\alpha}(1-t)^{\frac{1}{2}}(\beta - t)^{\frac{1}{2}}} \qquad \ldots (7)$$

For $1 \le t \le \beta$ the relation between θ and t planes is

$$\theta = M \int_1^{t'} \frac{(e-t)^{\alpha} dt}{t^{\alpha}(1-t)^{\frac{1}{2}}(\beta - t)^{\frac{1}{2}}} - iH \qquad \ldots (8)$$

For point B, $t=\beta$ and $\theta = - i(H+D_i-D_w)$. Making use of this condition in equation (8) and equating the argument of terms on either sie the following relationship is derived

$$D_i-D_w = |M| \int_1^{\beta} \frac{(t-e)^{\alpha} dt}{t^{\alpha}(t-1)^{\frac{1}{2}}(\beta - t)^{\frac{1}{2}}} \qquad \ldots (9)$$

Form equation 2

$$d\theta = \frac{M(e-t)^{\alpha} dt}{t^{\alpha}(1-t)^{\frac{1}{2}} (\beta - t)^{\frac{1}{2}}}$$

Let $\quad t = Re^{ir}$

$\quad dt = Re^{ir}\, idr$

when one moves around point C in t plane at infinite there is a jumap equal to $i(H+D_i-D_w)$ in θ.

Therefore,

$$i(H+D_i-D_w) = \lim_{R\to\infty} \int_0^{\pi} \frac{M(e-Re^{ir})^{\alpha} Re^{ir} idr}{(Re^{ir})^{\alpha} (1-Re^{ir})^{\frac{1}{2}}(\beta-Re^{ir})^{\frac{1}{2}}}$$

$$= (\pm)\ (-1)^{\alpha}\, i\, M\pi$$

or

$$|M| = \frac{H+D_i-D_w}{\pi} \qquad \ldots (10)$$

The conformal mapping of the polygon in the complex potnetial w plane to upper half of the t plane is given by

$$w = M' \int_{o}^{t'} \frac{dt}{t^{\frac{1}{2}}(1-t)^{\frac{1}{2}}\ (\beta - t)^{\frac{1}{2}}} - KH + iq \qquad \ldots (11)$$

$$= M'\frac{2}{\sqrt{\pi}}\, F(\sin^{-1}\sqrt{t'}, \sqrt{\frac{1}{\beta}}) - KH + iq$$

in which M' is a constant.

For point A, w - KH and t=1. Using this relation in equation (11), the following relation is obtained:

$$-iq = \frac{2M'}{\sqrt{\beta}}\ F\ (\frac{\pi}{2}, \sqrt{\frac{1}{\beta}}) \qquad \ldots (12)$$

in which $F(\frac{\pi}{2}, \sqrt{\frac{1}{\beta}})$ is complete elliptic integral of first kind with modulus equation to $\sqrt{\frac{1}{\beta}}$.

For point E, t = e, and w = -KH + iq'. Using this relation in equation (11)

$$iq' - iq = \frac{2M'}{\sqrt{\beta}}\ F(\sin^{-1}\sqrt{e}, \frac{1}{\sqrt{\beta}}) \qquad \ldots (13)$$

For $1 \le t \le \beta$, the relation between w and t planes is given by

$$w = M' \int_{1}^{t'} \frac{dt}{t^{\frac{1}{2}}(1-t)^{\frac{1}{2}}\ (\beta - t)^{\frac{1}{2}}} - KH \qquad \ldots (14)$$

For point B, t = β, and w = $-K(H-D_w)$. Making use of this relationship in equation (14) the constant, M' is found to be

$$M' = \frac{-KD_w i}{\frac{2}{\sqrt{\beta}}\, F\,(\frac{\pi}{2}, \sqrt{(\frac{\beta - 1}{\beta})})} \qquad \ldots (15)$$

Substituting for M' in equation (12) the expression for seepage loss from the canal is found to be

$$\frac{q}{KD_w} = F(\frac{\pi}{2}, \frac{1}{\sqrt{\beta}})/F(\frac{\pi}{2}, \sqrt{(\frac{\beta - 1}{\beta})}) \qquad \ldots (16)$$

The seepage loss through the bed of the canal is found from equation (13) to be

$$q' = \frac{KD_w F(\frac{\pi}{2}, \frac{1}{\sqrt{\beta}})}{F(\frac{\pi}{2}, \sqrt{(\frac{\beta-1}{\beta})})} - \frac{KD_w F(\sin^{-1}\sqrt{e}, \frac{1}{\sqrt{\beta}})}{F(\frac{\pi}{2}, \frac{1}{\sqrt{\beta}})} \qquad \ldots (17)$$

From geometry of Figure 1(d)

$$\alpha\pi = \tan^{-1}\left(\frac{H}{b + \frac{q'}{K} - \frac{q}{K} - b'}\right) \qquad \ldots (18)$$

The locus of the phreatic line is determined as follows:

For $t = t' < 0$, $\theta = x - \frac{q}{K}$ and

$$\theta = M \int_0^{t'} \frac{(e-t)^{\alpha}\, dt}{t^{\alpha}\,(1-t)^{\frac{1}{2}}(\beta - t)^{\frac{1}{2}}} + b - \frac{q}{K} \qquad \ldots (19)$$

Hence,

$$x = b + |M| \left[\int_0^{t'} \frac{(e-t)^{\alpha}\, dt}{(-t)^{\alpha}\,(1-t)^{\frac{1}{2}}(\beta - t)^{\frac{1}{2}}}\right] \qquad \ldots (20)$$

Values of t' are negative for the phreatic line, subsituting $t = -T$ in equation (20)

$$x = b + |M| \int_0^{t'} \frac{(e+T)^{\alpha}\, dT}{T^{\alpha}(1+T)^{\frac{1}{2}}(\beta + T)^{\frac{1}{2}}} \qquad \ldots (21)$$

For $t = t' < 0$, $w = -Ky + q$ and

$$w = -M' \int_{t'}^{0} \frac{dt}{t^{\frac{1}{2}}(1-t)^{\frac{1}{2}}(\beta - t)^{\frac{1}{2}}} - KH + iq \qquad \ldots (22)$$

Hence,

$$y = H - |M| \int_{t'}^{o} \frac{dt}{(\beta - t)^{\frac{1}{2}} (1 - t)^{\frac{1}{2}} (o - t)^{\frac{1}{2}}} \quad \ldots (23)$$

or

$$y = H - D_w \frac{F\left(\sin^{-1} \sqrt{\left(\frac{t'}{1+t'}\right)}, \sqrt{\left(\frac{\beta - 1}{\beta}\right)}\right)}{F\left(\frac{\pi}{2}, \sqrt{\left(\frac{\beta - 1}{\beta}\right)}\right)} \quad \ldots (24)$$

Thus for a given balue of t', x and y co-ordinates of phreatic line can be known from equations (21) and (24)

4.0 RESULTS AND DISCUSSIONS

Let the integrals

$$\int_{0}^{e} \frac{(e-t)^{\alpha}\,dt}{t^{\alpha}(1-t)^{\frac{1}{2}}(\beta - t)^{\frac{1}{2}}} = A_1 ,$$

$$\int_{e}^{1} \frac{(t-e)^{\alpha}\,dt}{t^{\alpha}(1-t)^{\frac{1}{2}}(\beta - t)^{\frac{1}{2}}} = A_2 ,$$

and

$$\int_{1}^{\beta} \frac{(t-e)^{\alpha}\,dt}{t^{\alpha}(t-1)^{\frac{1}{2}}(\beta - t)^{\frac{1}{2}}} = A_3 .$$

Substituting for M in equation (9) and dividing terms on either side by H

$$\frac{D_i}{H} - \frac{D_w}{H} = \frac{A_3}{\pi}\left[1 + \frac{D_i}{H} - \frac{D_w}{H}\right]$$

or

$$\frac{D_i}{H} = \frac{A_3}{\pi - A_3} + \frac{D_w}{H} \qquad \ldots (25)$$

Considering equation (18)

$$b - \frac{q}{K} - b' + \frac{q'}{K} = H \cot \alpha\pi$$

Using this relation in equation (4) the following relation is obtained:

$$\frac{D_i}{H} = \frac{\pi}{A_1} \operatorname{cosec} \alpha\pi - 1 + \frac{D_w}{H} \qquad \ldots (26)$$

Equating equations (25) and (26)

$$\frac{A_3}{\pi - A_3} = \frac{\pi}{A_1} \operatorname{cosec} \alpha\pi \quad - 1 \qquad \ldots \quad (27)$$

Equation (27) indicates that if e and β are assigned values, then α has to be determined by an iteration procedure. Thus the three parameters e, β , and α cannot be assigned independent values . For numerical calculations, e and β are assumed and α is obtained by iteration procedure using equation (27). It may be noticed that A_1 and A_3 are also function of α . With assumed valaues of e,β and D_w, q/K and q'/K are evaluated using equation (16) and (17) respectively. Using equation (7) and (4)

$$\frac{b' - \frac{q'}{K}}{[(b' - b + \frac{q}{K} - \frac{q'}{K})^2 + H^2]^{\frac{1}{2}}} = \frac{A_2}{A_1} \qquad \ldots \quad (28)$$

Using the relation given at equation (18) and equation (28) the following relation is obtained:

$$\frac{b' - \frac{q'}{K}}{[(H \cot \alpha\pi \ )^2 + H^2]^{\frac{1}{2}}} = \frac{A_2}{A_1} \qquad \ldots \quad (29)$$

For the assumed values of e, β , and iterated valaue of α , b' is obtained from equation (29) since all other terms are known except b'. The width of the canal b is calculated from equation (18) and the depth to highly permeable layer, D_i, is calculated marking use of equation (26).

The seepage rate for various canal configurationa and for various depth to the highly permeable layer, and the reach transmissivity per unit length of canal are presented in Table 1. The implicit nature

TABLE 1

SEEPAGE QUANTITY AND REACH TRANSMISSIVITY FOR VARIOUS CANAL SECTIONS

α	β	e	b	b'	D_i	H	D_w	q'/K	q/K	$\Gamma_r = 2q/D_w$
0.25	1.605	0.8	28.04	10.19	25.03	0.01	25.00	10.18	28.02	2.24K
	1.0086	0.6	38.36	29.82	17.83	2.0	10.00	17.42	23.96	4.79K
	1.0031	0.5	50.21	42.6	20.59	2.0	10.00	21.58	27.19	5.44K
0.4	1.59	0.5	15.7	10.30	18.88	2.0	10.00	11.258	6.492	2.251
0.45	1.614	0.7	12.81	6.27	14.52	2.0	10.00	4.95	11.17	2.234K
0.48	1.51	0.6	13.79	8.09	15.22	2.0	10.00	6.07	11.65	2.33K
0.42	1.51	0.6	2.05	1.51	3.54	1.0	0.5	0.3037	0.5824	2.33
0.42	1.51	0.6	0.263	.182	.404	0.1	0.1	.0607	.1165	2.33
0.42	1.51	0.6	1.819	1.395	3.347	1.0	0.3	.1822	0.3494	2.329

of the equations prohibits to present the variation of seepage quantity with respect to one of the variables keeping other variable constant.

If the depth of water in the canal is small, the seepage quantity estimated by the present analytical method compares well with the seepage quantity calculated by Aravin's solution. For example if $b = 28.04$ m, $D_w = 25$m, $H = 0.01$m, $D_i = 25.03$m, the seepage quantity, q, obtained from the present analaysis is 28.02K. According to Aravin's solution the seepage occurring from half of the canal section is 28.017K. Hammad has analysed seepage from canal for small valaue of D_w. The seepage quantity, q, estimated by the present method compares well with that of Hammad's solution for small value of D_w. For example if $b = .2635$m, $b' = .1820$m, $D_w = 0.1$m, $H = 0.1$m, $D_i = .405$, the seepage quantity, q, obtained from the present analaysis is 0.1165K. The corresponding seepage quantity estimated using Hammad's solution is 0.1187K.

The loci of the phreatic line for various canal geometry and for different depths to the highly permeable layer are presented in Figure 11. Because of the existence of the draining layer, the phreatic lines merge with the water at a finite distances from the canal.

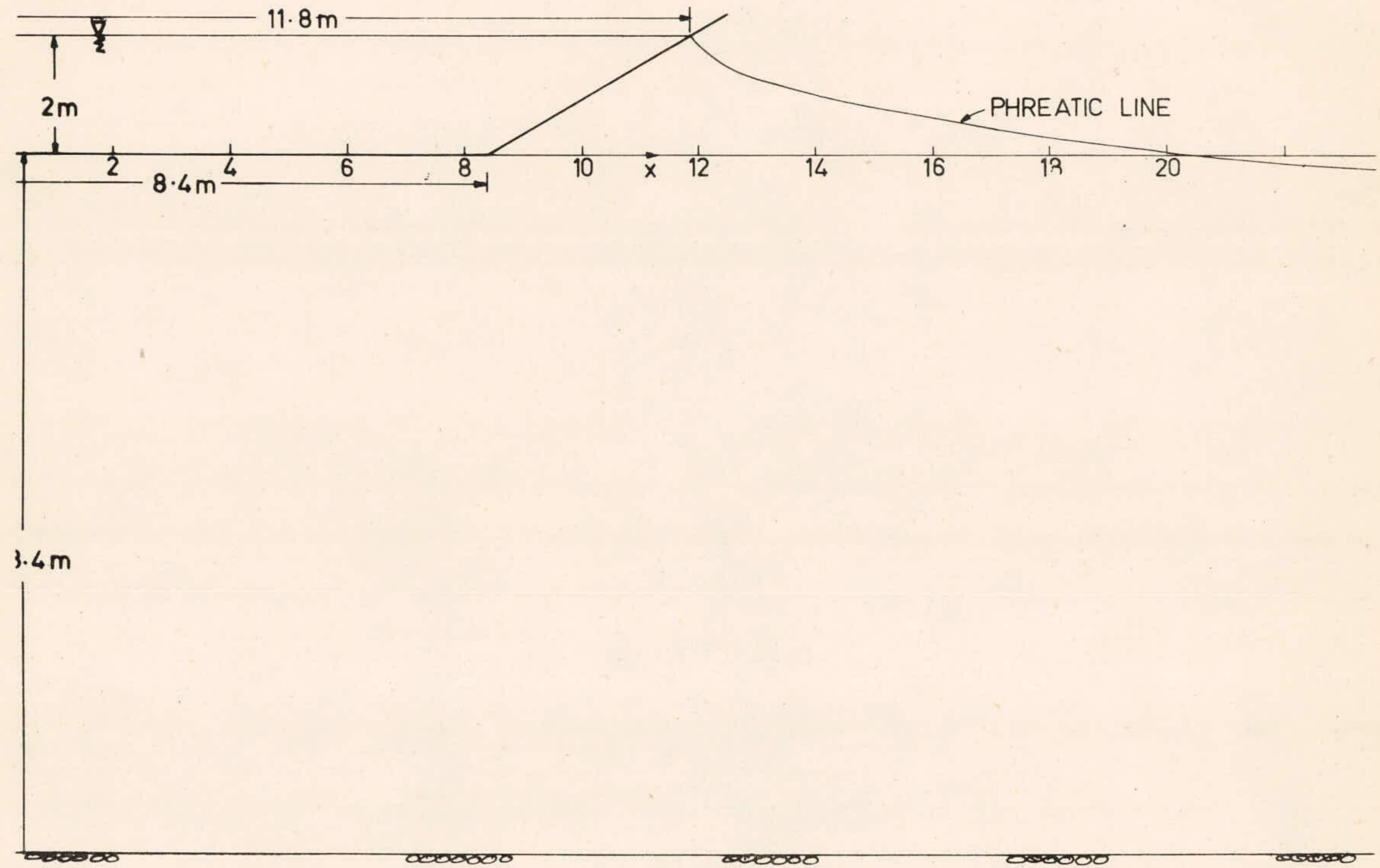


FIGURE 11(a). LOCUS OF PHREATIC LINE FOR D_i=18.4m, D_w=3m, b'=8.4m, b=11.8m and H=2m

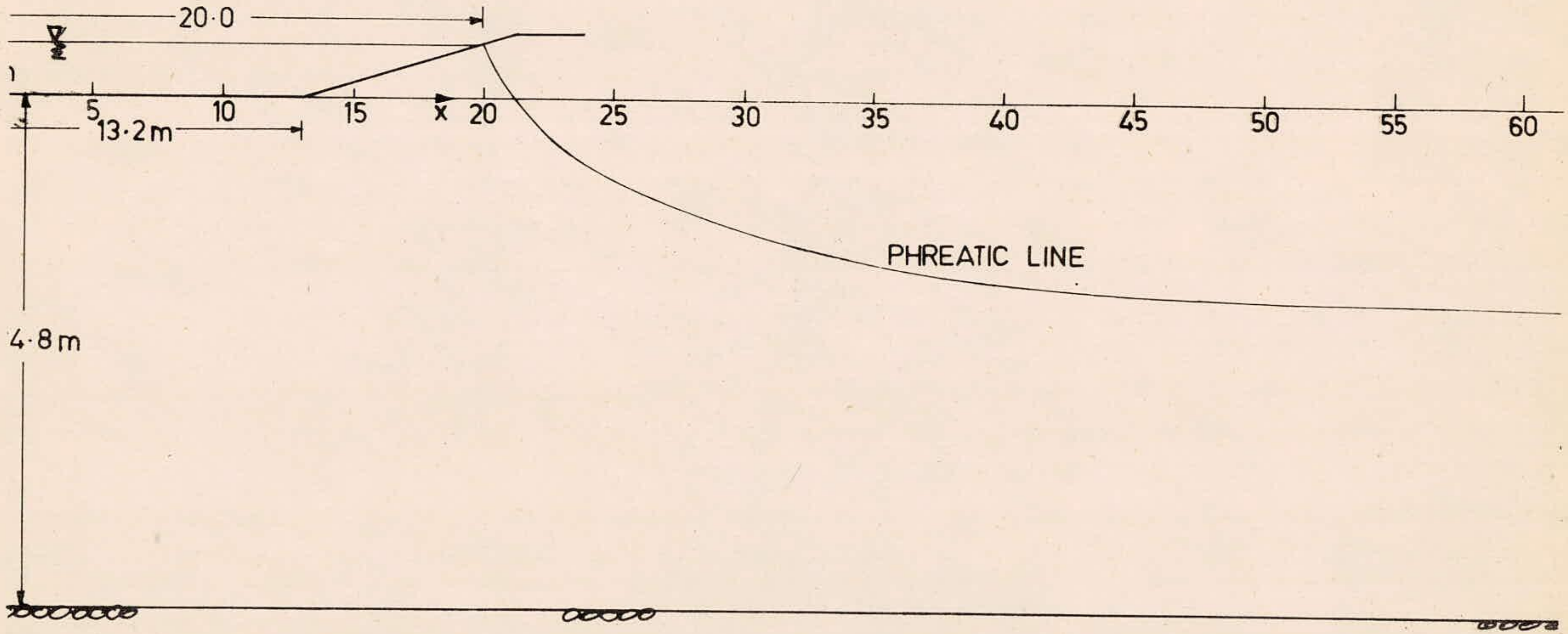


FIGURE 11(b). LOCUS OF PHREATIC LINE FOR D_i=24.8m, D_w=10m
b'=13.2m, b=20m and H=2m

5.0 CONCLUSION

An analytical solution for estimating seepage from a canal embeded in a porous medium of finite depth underlain by a highly permeable layer has been obtained using Zhukovsky's function and conformal mapping for any position of water table above the highly permeable layer. The depth of canal, bottom width of canal, and width of canal at the water surface have been preserved in the analysis. For small depth of water in the canal, the results obtained from the present analysis compares with the result given by Aravin. If the difference in potentials at the canal surface and in the aquifer at large distance from the canal is small, the seepage quantity estimated by the present analysis compares with Hammad's solution. The reach transmissivity has been quantified for a canal which is underlain by a highly permeable layer at a finite depth. The locus of phreatic line has been determined. The phreatic lines merge with the water table at finite distances from the canal because of the presence of the highly permeable layer.

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