ICIWRM – 2000, Proceedings of International Conference on Integrated Water Resources
Management for Sustainable Development, 19 – 21 December, 2000, New Delhi, India

Optimal operation of multiobjective multiple-reservoir system

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Abstract

In the present paper Preemptive Goal Programming (PGP) model has been developed (MRPPGP) and applied to Mahanadi Reservoir Project (MRP) system, situated in the state of Madhya Pradesh, India. The objective of the present work is to find out a better operation model for MRP system at its ultimate development stage. For the justifiable comparison of performance of MRPPGP model with the earlier reported models (HEC-3, MRPSIM, and CDDP), the same data set has been utilized. The MRPPGP model shows better performance as compared with the other models and is identified as the most suitable model for MRP system. In the second section of the study the PGP model is applied to MRP with the modified demand and inflow data. The irrigation demands are computed as per the guidelines by FAO-24 considering the effective rainfall in the command area. Based on the concept of net-inflow the inflow series were reconstructed. Furthermore the study was extended to explore the possibility of increase in the designed net culturable command area (CCA). It was observed from the results that by the application of MRPPGP model utilizing the modified data, an additional 10% net CCA can be irrigated. It is concluded that the MRPPGP model can deal effectivelly with the priorized demands and in case of MRP system its application shows significant improvement in the operation policy as compared with that given by other reported models.

INTRODUCTION

The concept of Goal Programming (GP) was first introduced by Charnes and Cooper (1961). An extensive treatment to GP was given by Ignizio (1982). Can and Houck (1984) used GP for real-time reservoir operation. Different GP schemes were presented and applied by Loganathan and Bhattacharya (1990).

In the present paper, Preemptive Goal Programming (MRPPGP) model has been developed and applied for the monthly operation of a multipurpose multi-reservoir system, Mahanadi Reservoir Project (MRP) system in Madhya Pradesh, India. The deviations from the demands in accordance with their priorities and from the total sum of maximum capacities of the reservoirs in the system, has been minimised as the objective function sequentially, subject to the system constraints.

For commensurable comparison of this approach, the data set and period of operation (1949-1978) was kept the same as used in the previous works viz. HEC-3 model (Khaliquzzaman, 1987), MRPSIM model (Sharma, 1988) and CDDP model (Shrivastava, 1989) and hereafter this data set is refereed as Bench-Mark (BM) data. HEC-3 model developed by Hydrologic Engineering Centre (1971) is considered to be one of the best-documented general reservoir simulation models. Mahanadi Reservoir Project Simulation

(MRPSIM) model utilises preemptive goal programming algorithm developed by Tennessee Valley Authority (TVA) for HYDROSIM model. Constrained Differential Dynamic Program (CDDP) is an extension of Dynamic Programming, which alleviate the dimensionality problem. This technique finds the optimal solution, which minimize the loss functions subject to the linear constraints.

In the remainder of the paper, the MRP system is described next, followed by the development of preemptive goal programming for MRP system and a comparison of the operation policy resulting from preemptive goal programming with the results of HEC-3, MRPSIM and CDDP models. Furthermore the developed MRPPGP model is applied to MRP with modified demand data and net-inflow (Verma et al, 1999, 2000) concept-based reconstructed inflow data. Finally, conclusions concerning usefulness of the preemptive goal programming approach to reservoir management are presented.

MRP-COMPLEX

The Mahanadi Reservoir Project (MRP) Complex consists of Pairi basin and Ravishankar Sagar Project (RSP) basin and situated on the upper reaches of Mahanadi River in the state of Madhya Pradesh, India. Fig. 1, shows the index map of MRP system at its ultimate development stage (i.e. post Pairi stage).

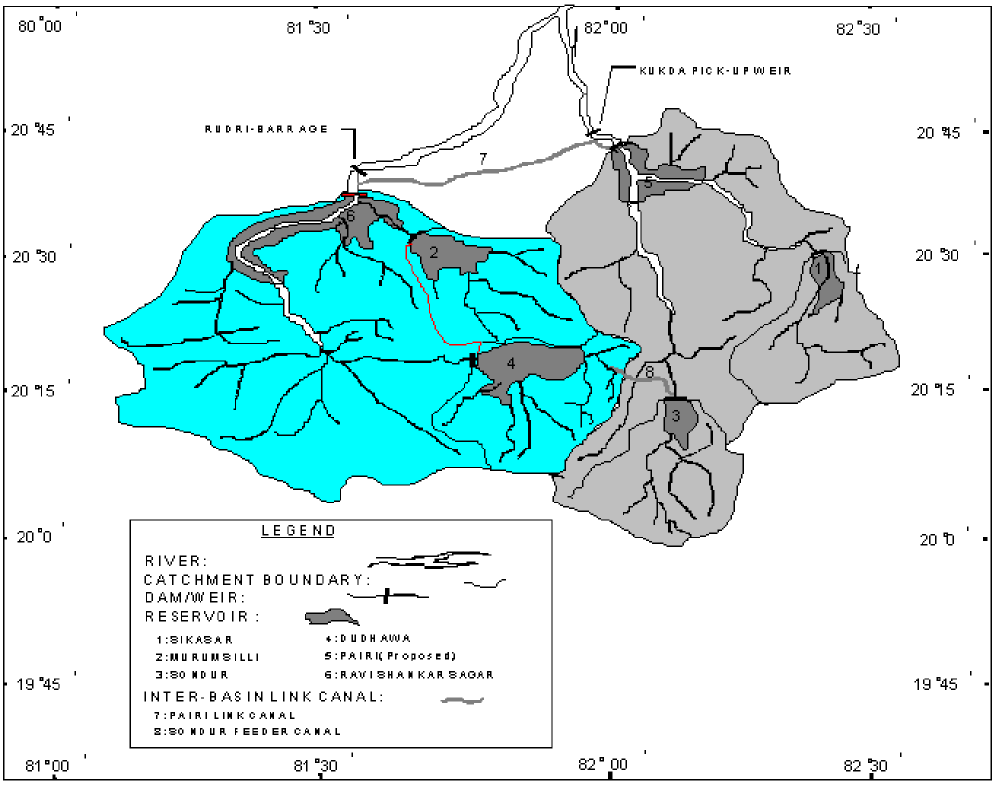

Figure 1. Index Map of Mahanadi Reservoir Project (MRP) Complex.

PREEMPTIVE GOAL PROGRAMMING (PGP)

In PGP, the underlying philosophy is based on ‘satisfying’ rather than ‘optimizing’. Instead of attempting to minimize or maximize various objective functions, PGP is concerned with the condition of achieving prespecified targets or goals. The solution procedure for the PGP model consists of first minimizing the deviational variables with the highest priority level, P_1, to the fullest possible extent. The algorithm proceeds hierarchically to lower priority levels, i.e. P_2, P_3, P_4,......, P_k. In the working with a particular priority level, the solution is obtained in such a way that it should not leads to degradation of the achievements in one or more higher priority goals.

General Formulation of PGP Model

A preemptive goal programming model with the prioritized objective function may be stated as,

$$\text{minimize} \sum_{k=1}^{K} P_k \sum_{i=1}^{M} (d_i^+ + d_i^-) \qquad (1)$$

subject to:

(a) goal constraints, $\sum_{j=1}^{N} a_{ij} \, x_j + d_i^- - d_i^+ = Gi$ for i=1,2,..,P (2)

(b) absolute constraints: continuity equations, storage/releases bounds (3)

(c) non-negativity condition, $x_j , d_i^+ , d_i^- \geq 0$ (4)

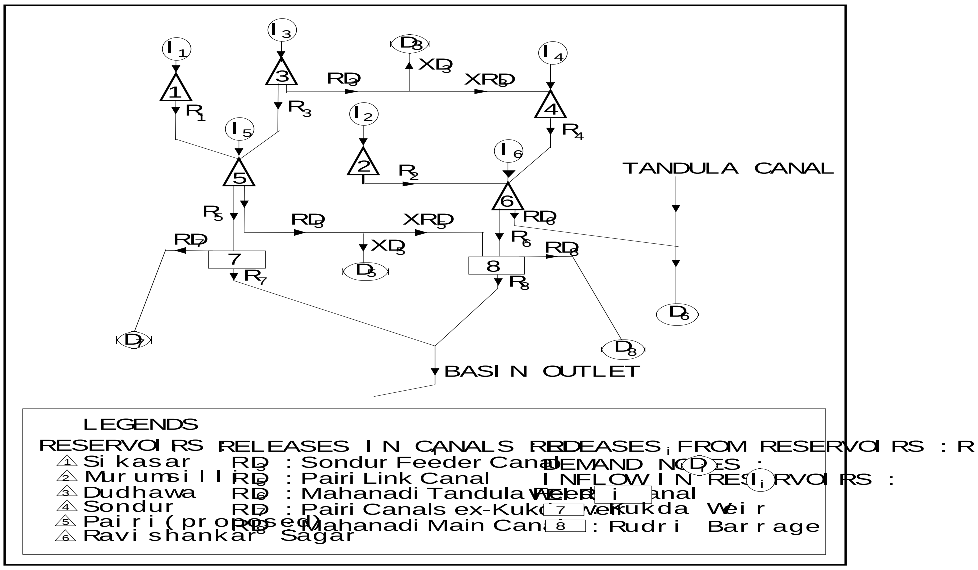

Figure 2. Schematic representation MRP-Complex.

PGP MODEL FORMULATION FOR MRP SYSTEM

In this section the model formulation for MRP system is presented. All the notations used in the formulation are depicted from Figure 2, which represents the schematic diagram of MRP system. The absolute constraints are discussed first. Then the goal constraints along with the objective function have been framed.

Absolute Constraints

The continuity equations, limits on storage in reservoirs and the flow constraints in channels are considered as absolute constraints.

Continuity Equations

Continuity equations for reservoirs and weirs have been derived in accordance with the system configuration, and can be given as,

At reservoir-sites:

$$S_{1,t} + R_{1,t} + SL_{1,t} * EV_{1,t} * (S_{1,t} + S_{1,t-1})/2 = S_{1,t-1} + I_{1,t} \qquad (5)$$

$$S_{2,t} + R_{2,t} + SL_{2,t} * EV_{2,t} * (S_{2,t} + S_{2,t-1})/2 = S_{2,t-1} + I_{2,t} \qquad (6)$$

$$S_{3,t} + R_{3,t} + SL_{3,t} * EV_{3,t} * (S_{3,t} + S_{3,t-1})/2 + RD_{3,t} = S_{3,t-1} + I_{3,t} \qquad (7)$$

$$S_{4,t} + R_{4,t} + SL_{4,t} * EV_{4,t} * (S_{4,t} + S_{4,t-1})/2 - XRD_{3,t} = S_{4,t-1} + I_{4,t} \qquad (8)$$

$$S_{5,t} + R_{5,t} + SL_{5,t} * EV_{5,t} * (S_{5,t} + S_{5,t-1})/2 + RD_{5,t} - R_{1,t} - R_{3,t} = S_{5,t-1} + I_{5,t} \qquad (9)$$

$$S_{6,t} + R_{6,t} + SL_{6,t} * EV_{6,t} * (S_{6,t} + S_{6,t-1})/2 + RD_{6,t} - R_{2,t} - R_{4,t} = S_{6,t-1} + I_{6,t} \qquad (10)$$

where, $SL_{i,t}$ and $EV_{i,t}$ slope of area-storage curve and average depth of evaporation during the time period *'t'*, for reservoir *'i'* . $S_{i,t}$ and $S_{i,t-1}$ storage in reservoirs *'i'* at time *'t'* and *'t-1'* respectively.

At weir-sites :

$$R_{5,t} - RD_{7,t} - R_{7,t} = 0 \qquad (11)$$

$$R_{6,t} + XRD_{5,t} - RD_{8,t} - R_{8,t} = 0 \qquad (12)$$

At demand-nodes in the link canals:

$$XRD_{3,t} = RD_{3,t} - XD_{3,t} \qquad (13)$$

$$XRD_{5,t} = RD_{5,t} - XD_{5,t} \qquad (14)$$

Storage Constraints

The storage ($S_{i,t}$) in any reservoir is bounded by upper and lower limits corresponding to their maximum storage capacity and dead storage capacity respectively, i.e. $S_{i,dead} \leq S_{i,t} \leq S_{i,max}$. The storage constraint equations for various reservoirs with respect to their dead storage and maximum storage capacities (in Mm^3) for the system are given as :

$$S_{1,t} \geq 18.00 \qquad (15)$$

$S_{1,t} \leq 217.00$ (16)

$S_{2,t} \geq 3.00$ (17)

$S_{2,t} \leq 165.00$ (18)

$S_{3,t} \geq 18.00$ (19)

$S_{3,t} \leq 180.00$ (20)

$S_{4,t} \geq 4.00$ (21)

$S_{4,t} \leq 288.00$ (22)

$S_{5,t} \geq 120.00$ (23)

$S_{5,t} \leq 540.00$ (24)

$S_{6,t} \geq 144.00$ (25)

$S_{6,t} \leq 909.00$ (26)

Channel Capacity Constraints

The total flow in the canal for a month should be less than the total volume of water corresponding to its maximum permissible capacity. The capacity constraints for canals of the system (in Mm^3), can be expressed as :

$RD_{3,t} \leq 73.35$ (27)

$RD_{5,t} \leq 243.0$ (28)

$RD_{6,t} \leq 51.30$ (29)

$RD_{7,t} \leq 400.0$ (30)

$RD_{8,t} \leq 513.7$ (31)

Preemptive Goals

Priority levels to various goals have been assigned according to the priorities set for the MRP system.. The actual goal framed is that the releases ($RD_{i,t}$) in the canals should be equal to or greater than the demand ($D_{i,t}$) on respective canal. The M&I demand (D_6) supplied by MFC, is assigned higher most priority (P_1), over the other demands as,

$$RD_{6,t} + d^{-}_{6,t} - d^{+}_{6,t} = D_{6,t} \quad (32)$$

The hierarchy of irrigation demands to be fulfilled is as follows:

Priority (P_2), is the demand D_8 to be met from MMC,

$$RD_{8,t} + d^{-}_{8,t} - d^{+}_{8,t} = D_{8,t} \quad (33)$$

Priority (P_3), is the demand D_7 to be supplied by Pairi canals ex-Kukda weir,

$$RD_{7,t} + d^{-}_{7,t} - d^{+}_{7,t} = D_{7,t} \quad (34)$$

Priority (P_4), is the demand D_5 to be met from PMLC,

$$XD_{5,t} + d^{-}_{5,t} - d^{+}_{5,t} = D_{5,t} \quad (35)$$

Priority (P_5), is the demand D_3 to be met from SFC,

$$XD_{3,t} + d^{-}_{3,t} - d^{+}_{3,t} = D_{3,t} \quad (36)$$

In the above equations $d^-_{3,t}$, $d^-_{5,t}$, $d^-_{6,t}$, $d^-_{7,t}$, and $d^-_{8,t}$ are the negative deviational variables corresponding to their target demands, which shows the under-achievement of a goal (i. e. demands, $D_{i,t}$) by the decision variables (Rd_i's and XRD_i's). $d^+_{3,t}$, $d^+_{5,t}$, $d^+_{6,t}$, $d^+_{7,t}$, and $d^+_{8,t}$ are the positive deviational variables. These variables show the over-achievement of a goal by the decision variables.

Priority (P_6), is to maximize the total storage in the system,

$$S_{1,t} + S_{2,t} + S_{3,t} + S_{4,t} + S_{5,t} + S_{6,t} + d^-_{s,t} - d^+_{s,t} = 2301.0 \qquad 37)$$

where, $d^+_{s,t}$ and $d^-_{s,t}$ represents the positive and negative deviational variables from the total storage capacity of the reservoirs in the system (i.e. 2301.00 Mm^3).

Objective Function

The objective function corresponding to the above priorities is framed as,

$$\text{minimize } \{P_1(d^-_{6,t}) + P_2(d^-_{8,t}) + P_3(d^-_{7,t}) + P_4(d^-_{5,t}) + P_5(d^-_{3,t}) + P_6(d^-_{s,t})\} \qquad (38)$$

The complete PGP model consisting of the equations from Eq. (5) to Eq. (38) is termed as MRPPGP. In this model after satisfying all the goal priority wise, at last the total storage of the system is maximized. This is a necessary virtual priority, which is included to assure maximum storage in the system after all the specified goals have been considered. In absence of this goal there is no control for maximization of total storage in the system.

SOLUTION ALGORITHM

The dead storage in reservoirs is taken as the initial storage for starting period of operation. The solution algorithm of PGP is explained in the following paragraphs.

Step 1: The objective corresponding to first priority goal (minimize $d^-_{6,t}$) is considered as the objective function with the other constraint equations (Eq. 5 to 37). This constitute a problem of optimization similar to linear programming with the flexibility to allow deviations in goals. This problem is optimized with the minimization of the deviation using the modified Simplex method.

Step 2. The value of the deviation variable ($d^-_{6,t}$) obtained from the previous step is substituted in the respective goal constraint equation. The substitution of the value of the deviational variable in the corresponding goal constraint equation insures that the solution obtained at this stage will not be degraded while optimizing for the lower priority goals. Then the deviation variable corresponding to next priority goal (minimize $d^-_{8,t}$) is considered in the objective function. This new objective of minimization of deviation variable corresponding to the second priority goal constitutes the objective function for second round of optimization along with the other constraint equations (Eq. 5 to 37). This new problem is then optimized in the similar manner as in step 1.

Step 3. Taking the next priority goals one after another and following the step 1 and step 2 for each of the goal, till the last priority goal is taken over.

Step 4. Finally, the result corresponding to the last priority goal yields in the values of all the decision (releases from reservoirs) and deviational variables.

Step 5. For next period, the same solution procedure is adopted with storage obtained from the solution of previous period as the initial storage. The procedure is repeated till the end period of operating horizon.

Table 1. Comparison of performance of various models.

Demands	MRPSIM	HEC-3	CDDP	PGP
M & I Demand :				
Average. Annual Demand	299.00	299.00	299.0	299.00
Average Annual Supply	297.90	299.00	299.0	299.00
Supply as % of Demand	99.60	100.00	100.00	100.00
Nos. of Successful Years*	30	30	30	30
Kharif Demand :				
Average. Annual Demand	1315.60	1358.00	1357.61	1357.61
Average Annual Supply	968.40	976.00	1144.30	1217.75
Supply as % of Demand	73.60	72.00	84.30	89.69
Nos. of Successful Years*	10	13	23	26
Rabi Demand :				
Average. Annual Demand	1023.76	998.47	988.38	988.38
Average Annual Supply	857.69	852.10	870.98	902.42
Supply as % of Demand	83.78	85.34	88.12	91.30
Nos. of Successful Years*	24	23	26	26

- Failure of less than 10% are not considered as failures.

RESULTS AND ANALYSIS OF MRPPGP MODEL USING THE BM DATA SET

The monthly operation policy for the individual year is determined using PGP model. The reservoirs were considered at the dead-storage level at the beginning of June of the year 1949. For each subsequent year the storage in each of the reservoirs at the end of May was considered as the beginning storage. The release sequences thus obtained are totaled for each year to determine the total water supplied in individual year. The results obtained with the PGP model have been compared (Table 1) with the results of HEC-3 model (Khaliquzzaman, 1987), MRPSIM model (Sharma, 1988), and CDDP model (Shrivastava, 1989). The operation policy from 1949 to 1978 by different models have been represented in terms of average annual demand & supply; supply as percentage of total demand and number of years in which the M & I, Kharif and Rabi demands met successfully. These values of total amount of water supplied in individual year have been plotted against total annual demand in MRP complex for 100% Kharif and 30% Rabi cropping pattern. These plots are shown in Figs. 3 and 4.

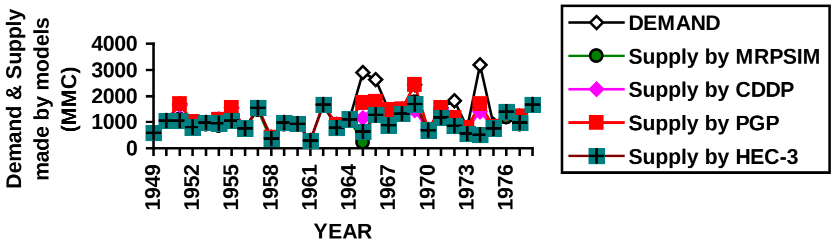

Figure 3. Comparison of Performance of various Models for Kharif Supply on BM-Data.

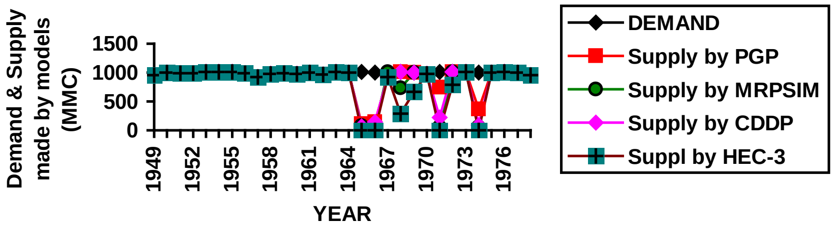

Figure 4. Comparison of Performance of various Models for Rabi Supply on BM-Data.

Analysis of the results in Table 1, shows that the M&I demand are met successfully by all the models. The effectiveness of various models is clearly observed in meeting the Kharif demands. The number of years in which the Kharif demands were successfully met by MRPSIM, HEC-3, CDDP and MRPPGP models are 10, 13, 23, and 26 respectively. Thus the MRPPGP model results in significant improvement in operation policy in terms of number of successful years as compared with other models. It can also be noted from the Table 1, that the maximum supply as 89.69 % of the total Kharif demand were insured by the MRPPGP model. For the Rabi demands the MRPPGP model meets successfully for 26 years which is same as that of suggested by CDDP model. But an improvement is observed in the average annual supply of 902.42 Mm^3 (91.30 %) in case of MRPPGP model over 870.98 Mm^3 (88.12 %) as suggested by CDDP model. The supplies as the percentage of demand for various models are shown in Fig. 5.

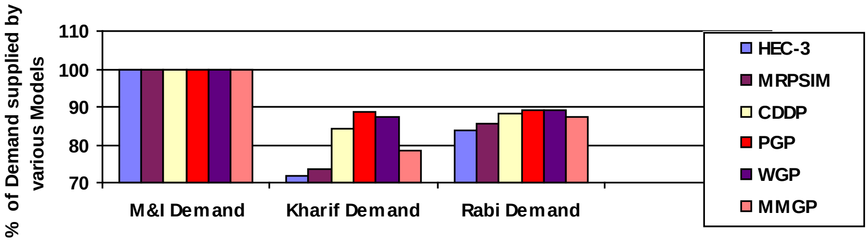

Figure 5. Chart showing the percentage of demand supplied by various Models on BM-Data.

The study has been restricted to the perfect mode of operation with the aim to test the effectiveness of the various GP models with the earlier reported models. The application of the PGP model to the MRP Complex using the BM-data for same period of operation, shows significant improvement in the operation policies as compared with other optimal operating models reported earlier. Thus from the study, It can be concluded that the PGP model is a better model for MRP complex.

Table 2. Results of 50 years' operation with modified data (increase in command area).

Sr. no.	% increase in command area	M & I requirement		Kharif requirement		Rabi requirement	
		successful years (%)	% of demand met	successful years (%)	% of demand met	successful years (%)	% of demand met
1	0 %	100%	100 %	98%	99.52 %	90%	97.98 %
2	3 %	100%	100 %	96%	99.34 %	88%	97.03 %
3	5 %	100%	100 %	96%	99.08 %	86%	96.38 %
4	7 %	100%	100 %	94%	98.75 %	84%	95.81 %
5	10 %	100%	100 %	94%	98.39 %	84%	94.83 %
6	13 %	100%	97.8 %	92%	98.34 %	82%	93.73 %
7	15 %	100%	96.6 %	92%	98.19 %	80%	92.86 %

ANALYSIS OF RESULTS OF MRPPGP MODEL WITH MODIFIED DATA

The monthly operation policies for MRP system with modified demand data corresponding to the post Pairi designed command area and reconstructed inflow series is derived for the period 1946 to 1995. The results have been shown in Table 2. It is observed from Table 2, that the model fulfils the M&I requirements for all the 50 years of operation period successfully. The M&I demands are the first priority in the model. The next priority is the irrigation requirements for Kharif season crops. It is observed from the Table 2, that the Kharif requirements are successfully met for 98% of the operation period. On the other hand the irrigation requirements for Rabi crops is fulfilled for 90% of the operation period.

It is revealed from Table 1, that the percentage of successful years for Kharif and Rabi crop requirements are 83.33% and 86.67% respectively with the BM data. Another important fact is that these results are pertaining to operation period of 30 years only, while the results with modified data are for 50 years of operation period. The improvement in the percentage success from 83.33% to 98% and from 86.67% to 90% for Kharif and Rabi crops is due to the modified demand computations and net inflow based reconstructed inflow computations. The importance of net inflow concept lies in the fact that these are the actual inflows measured in the reservoirs in post construction stage. Thus the computed inflow series for reservoirs, based on the net inflow observations provides a more authentic inflow information as compared to the rainfall runoff model based inflow information. In case of MRP system the previously available inflow information are based on the single rainfall runoff model developed corresponding to the observation made at Baronda gauge-discharge station in the Pairi basin and applied to the reservoirs of Pairi and adjacent Ravishankar basin.

Another study has been conducted to explore the possibility of increasing the designed command area corresponding to the post Pairi stage by application of developed PGP model with modified demand data and reconstructed inflow data for operation period from 1946 to 1995 (i.e. for 50 years). The results has been presented in Table 2, as percentage of operation period in which demands are met successfully and percentage of demand met by the PGP model corresponding to various percentage increase in the command area. It can be observed from the Table that up to 10% increase in the command area the M&I demand are met cent-percent successfully. The Kharif and Rabi demands are met for 90% and 86% of time respectively corresponding to 5% increase in the command area. The percentage of demand met or supplied for Kharif is 99.08% and for Rabi is 96.38% corresponding to 5% increase in command area. Similarly it can be observed that corresponding to 10% increase in the command area the percentage of time successfully met the Kharif and Rabi requirements are 94% and 84% respectively, and percentage of Kharif and Rabi demands met are 96.59% and 94.83% respectively. For more than 10% increase in the command area the M&I demand is required to sacrifice by more than 2%. The details of the command area in the system for various operating policies are presented in Table 8.8. Since the M&I is placed at the higher most priority level, the increase beyond 10% in command area may not be acceptable. Thus, it can be concluded that by application of developed PGP model for operation of MRP system at post Pairi stage with the modified demand and reconstructed inflow data the additional 10% command area can be irrigated.

CONCLUSION

It may be concluded that the preemptive goal programming model performs better than the HEC-3, MRPSIM, and CDDP models when applied to MRP system. It is also found that by the application of MRPPGP model 10% more command area can be irrigated in MRP system.

References

Can Emre K. and Mark H. Houck (1984), "Real-Time Reservoir Operation by Goal Programming", Journal of Water Resources Planning and Management, Vol. 110, No. 3, pp 2970-309.

Charnes, A., and Cooper, W. f. (1961), "Management Models and Industrial Applications of Linear Programming", Water Resources Bulletin, Vol. 14, No. 5, pp 1048-1063.

Cohon, J. L. (1978), "Multiobjective Programming and Planning", Academic Press, New York, N. Y.

Ignizio, J. P. (1982), "Linear Programming in Single and Multiple Objective Systems", Prentice-Hall, Englewood Cliffs, N. J.

Khaliquzzaman (1987), "Derivation of Operation Guidelines for a System of Reservoirs in MRP complex", Proceedings of Seminar on Application of Systems Analysis for Water Resources Development, System Engineering Organisation, Central Water Commission, New Delhi, pp 341-358.

Loganathan, G. V. and D. Bhattacharya (1990), "Goal Programming Techniques for Optimal Reservoir Operations", Journal of Water Resources Planning and Management, Vol. 116, No. 6, pp 820-838.

Sharma, P. K. (1988), "Development of Mathematical Model using Goal Programming and its Application to Mahanadi Reservoir Project-Complex", M. Tech. Thesis, Dept. of Civil Engg., IIT, New Delhi.

Shrivastava R. K. (1989), "An On-Line Reservoir Operation Model for Multi-Reservoir Systems", Ph. D. Thesis, Dept. of Civil Engg., IIT, New Delhi.

Verma, M. K. and Shrivastava R. K. (1999), "Conceptualisation of Net-Inflow in Reservoir Operation", Proceedings, South –East Asian Countries Conference on Challenges to Architects and Civil Engineers during Twenty First century’, Kathmandu, Nepal, Vol. II. pp. 798-804.

Verma, M. K. and Shrivastava R. K. (2000), "Rainfall Net-Inflow modelling for Mahanadi Reservoir Project Complex", Journal of IWRS, Vol 20, January 2000, pp. 19-26.