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Multilinear discrete cascade model for stage hydrograph routing

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Abstract
A method for routing stage hydrographs in rigid bed channels using multilinear modelling approach is presented. A discrete cascade model, which uses stage as the operating variable has been proposed in this study and it is used as the sub-model for this multilinear stage hydrograph routing method. The parameters of the sub-model are related to channel and flow characteristics by adopting the same relationships established for the corresponding parameters of the discrete cascade model operating on discharge variable. The suitability of this flood routing method is studied by routing a number of hypothetical stage hydrographs for a reach length of 40 km in uniform rectangular channels of different configurations. The solutions obtained using this method are compared with the corresponding solutions of the St.Venant equations. The study reveals that the proposed multilinear stage hydrograph routing model reproduces the St.Venant solutions closely when the rating curve corresponding to the input stage hydrograph is characterised by a narrow loop.

INTRODUCTION

Information based on discharge is the basis of analysis and decision making in many hydrological applications. However, for flood forecasting, design of flood control levees, automatic operation of canal networks, and consideration of environmental flow issues, particularly for aquatic-habitat needs, stream stage or flow depth is a more relevant variable than discharge. For these reasons and because stage can be measured easily and economically, considerable advantage is gained in routing a stage hydrograph rather than a discharge hydrogrpah, except for cases like rainfall-runoff modelling. Few simplified routing methods using stage as the operating variable are available. Hayami's diffusion analogy method(Hayami, 1951) is the only simplified stage-routing method well known in literature. Recently a variable-parameter stage hydrograph routing method directly derived from the St.Venant equations has been introduced by Perumal and Ranga Raju(1998a and 1998b). This method enables to account for the nonlinear characteristics of the flood wave propagation process by varying the parameters at each routing time interval. This method closely reproduces the St.Venant solutions subject to compliance of the assumptions of the method. The added advantage of this method is it allows the simultaneous computation of the discharge hydrographs corresponding to a given input stage hydrograph and/or routed stage hydrograph – which is a cumbersome feature while using the Hayami's method.

In this study an alternative variable parameter stage hydrograph routing method is investigated using multilinear modelling approach. The proposed multlinear method uses a discrete cascade sub-model operating on stage variable. This sub-model is analogous to the discrete cascade model introduced by O'Connor(1976) for operating on discharge variable. Multilinear discharge hydrograph channel routing method using discrete cascade model as the sub-model has been already proposed by Perumal(1994). In the proposed method, the parameters of the sub-model are related to channel and flow characteristics by adopting the same relationships established for the corresponding parameters of the discrete cascade model operating on discharge variable. The suitability of this method is studied by routing hypothetical stage hydrographs for a reach length of 40 km in uniform rectangular channels of different configurations, and comparing its solutions with the corresponding solutions of the St.Venant equations. The study reveals that the proposed multilinear stage hydrograph routing model reproduces the St.Venant solutions closely when the rating curve corresponding to the input stage hydrograph is characterised by a narrow loop.

MULTILINEAR MODELLING APPROACH

In the present work, multilinear modelling of flood wave movement in channels using stage as the operating variable is achieved by dividing the input stage hydrograph into blocks of constant duration equal to the routing time interval, each of which is subsequently routed through a discrete cascade sub-model. The parameters of this sub-model depend upon the magnitude of the input stage in a manner consistent with the physical interpretation of the model.

The discrete cascade model (O'Connor, 1976) is derived from the difference equation governing the Nash model(Nash, 1960) in discrete time domain, and its impulse response is analogous to the sampled pulse response of the Nash model. The nature of the input to the model is assumed pulsed (uniform blocks of duration Δt) and the output is sampled at intervals of Δt . The discrete cascade model directly estimates the pulse response unlike in the case of the Nash model wherein it is estimated by convolution of the Instantaneous Unit Hydrograph (IUH) with the unit pulse input, thus making it computationally more efficient than the Nash model.

The impulse response of the discrete cascade model operating on discharge variable, for unit time Δt , is expressed as (O'Connor,1976):

$$h_{1,q} = \left(\frac{\Delta t}{\Delta t + K_{d,q}} \right)^{n_{d,q}} \tag{1a}$$

$$h_{m,q} = \frac{(m + n_{d,q} - 2)}{(m-1)} \left(\frac{K_{d,q}}{\Delta t + K_{d,q}} \right) h_{(m-1),q} \quad \text{when} \quad m>1 \tag{1b}$$

where, $h_{1,q}$ is the discrete impulse response ordinate at t= Δt at which the discrete discharge impulse is applied, $h_{m,q}$ is the discrete impulse response ordinate at $t = m\Delta t$, and m=2,3,……, and $n_{d,q}$ and $K_{d,q}$ are the model parameters. The parameter $n_{d,q}$ may assume integer or non-integer values. The notation q subscripted to these parameters and variable imply that discharge is the operating variable of this model.

While equations (1a) and (1b) together represent the discrete cascade model for a unit discharge input, the corresponding form of the model for unit stage input needs to be established before proceeding to its use in the proposed multilinear stage hydrograph routing model.

DISCRETE CASCADE MODEL FOR STAGE INPUT

The differential equation of the cascade model governing the stage hydrograph movement in channel is derived from the Approximate Convection-Diffusion (ACD) equation in stage formulation(Perumal and Ranga Raju,1999), and it is expressed as

$$\frac{\partial y}{\partial t} + c\frac{\partial y}{\partial x} = 0 \qquad (2)$$

where, y is the stage or flow depth, c is the wave celerity, and x and t, respectively, denote distance along the channel and time.

The differential equation governing the flood movement process in a reach is deduced from equation (2) as

$$\left.\frac{dy}{dt}\right|_d = -\frac{(y_d - y_u)}{\left(\frac{l}{c}\right)} \qquad (3)$$

in which l denotes the reach length. The subscripts u and d, respectively, denote the upstream and downstream location of this reach. Equation (3) can be re-written in a form similar to that of the conventional input-output relationship of a single linear reservoir as

$$K\frac{dy_d}{dt} = y_u - y_d \qquad (4)$$

where, K is expressed as

$$K = \frac{l}{c} \qquad (5)$$

which denotes the propagation time of the stage hydrograph through the reach conceptualised by a linear reservoir. Therefore, following the same approach in the development of Nash model(Nash,1960), the equation governing the stage hydrograph movement in the channel reach conceptualised by a cascade of n number of such reservoirs can be expressed as

$$(1 + KD)^n y_d = y_u \qquad (6)$$

where, D is the notation denoting the differential operator.

The form of equation (6) is the same as that of the differential equation governing the Nash model(1960), with the inflow and outflow replaced by the respective stages. Due to this reason, it may be deduced that the discrete impulse response of the stage hydrograph model is of the same form as equations (1a) and (1b), and it is expressed as

$$h_1 = \left(\frac{\Delta t}{\Delta t + K_d} \right)^{n_d} \tag{7a}$$

$$h_m = \frac{(m + n_d - 2)}{(m-1)} \left(\frac{K_d}{\Delta t + K_d} \right) h_{m-1} \tag{7b}$$

where, K_d is the propagation time of the stage hydrograph in that channel reach conceptualised by a linear reservoir and n_d is the number of such reservoirs in the cascade conceptualising the given routing reach.

Since the same flood wave propagation phenomenon can be described either in stage or in discharge formulation using the respective ACD equations (Perumal and Ranga Raju, 1999), and because the form of discrete impulse response in stage formulation is the same as that of the discharge formulation, the parameter relationships for n_d and K_d with channel and flow characteristics remain the same as that for $n_{d,q}$ and $K_{d,q}$ respectively. Accordingly, the parameters n_d and K_d of the discrete impulse response in stage formulation are linked to the corresponding parameters in continuous time domain as (Perumal,1994)

$$K_d = K - \Delta t \tag{8}$$

$$n_d = \frac{nK - \Delta t}{(K - \Delta t)} \tag{9}$$

The parameters n and K are related to channel and flow characteristics as given by Dooge (1973) and adopted by Perumal (1994) as

$$n = \frac{S_0 B c_0 \Delta x}{Q_0} \tag{10}$$

$$K = \frac{Q_0}{S_0 B c_0^2} \tag{11}$$

where, Δx is the reach length, S_0 is the bed slope, B is the channel width, Q_0 is the reference discharge estimated using the reference stage y_0, and c_0 is the wave celerity corresponding to y_0.

Note that when $K < \Delta t$, a situation which arises while routing floods in steep bed slope reaches leading to kinematic flood wave movement, one has to resort to the use of equations (10) and (11) rather than equations (8) and (9).

While Q_0 is the primary variable used in the estimation of parameters of discrete cascade model based on discharge formulation, y_0 is the primary variable used in the estimation of corresponding parameters of the model based on stage formulation. The reference discharge Q_0 is estimated using y_0 as

$$Q_0 = A_0 v_0 \tag{12}$$

and

$$A_0 = f(y_0) \tag{13}$$

where, A_0 denotes the cross-sectional area of flow corresponding to y_0 ,and v_0 is the corresponding velocity determined either using Manning's or Chezy's friction law. The reference wave celerity c_0 can be estimated for any prismatic channel cross-section, using Manning's friction law as(Perumal and Ranga Raju, 1998a)

$$c_0 = \left(1 + \frac{2}{3}\left(\frac{P\partial R/\partial y}{\partial A/\partial y}\right)\right)v_0 \tag{14}$$

where, P is the wetted perimeter, R is the hydraulic radius and A is the cross-sectional area, and for uniform rectangular cross-sectional channel reach, equation(14) reduces to the form given by Price(1973) as

$$c_0 = \left(\frac{5}{3} - \frac{4}{3}\frac{y_0}{(B + 2y_0)}\right)v_0 \tag{15}$$

APPLICATION

Each ordinate of the input stage y_u at time t is routed through a discrete cascade submodel for which the parameters n_d and K_d are determined using equations (8) and (9) respectively. The reference stage y_0 needed to estimate Q_0 in these expressions is estimated as

$$y_0 = y_b + a(y_u - y_b) \tag{16}$$

where, y_b is the initial steady stage in the reach before the arrival of the flood, and a is a coefficient with limits 0<a<1.

For each ordinate of input stage y_u, the component stages are computed at the outlet of the reach using the discrete unit hydrograph given by equation (7) for a sufficient duration equal to the memory of the system. The overall stage at the outlet point at time $m\Delta t$ is estimated by adding all the component output stages estimated at time $m\Delta t$.

The method was tested by routing hypothetical stage hydrographs in uniform rectangular channels with no lateral flow. In each test run, the input stage hydrograph was routed in a channel using the proposed method for a specified distance of 40 km, and the output solution was compared with the corresponding solution of the St.Venant equations. The following form of input stage hydrograph was used:

$$y_t = y_b + (y_p - y_b)\left(\frac{t}{t_p}\right)^{\frac{1}{(\gamma-1)}} \exp\left[\frac{\left(1-\frac{t}{t_p}\right)}{(\gamma-1)}\right] \tag{17}$$

where, y_b is the initial flow depth corresponding to the initial steady flow of 100 m^3/s, y_p is the peak stage, γ is the skewness factor which defines the shape of the hydrograph, and t_p is the time to peak. Three types of rectangular channels, each having a width of 50 m as given in Table 1 were used in the study. Using these three channel configurations and the input stage hydrograph defined by γ =1.15, t_p =10 hr and y_p =10m, a total of 9 test runs were made to evaluate the proposed method. The details of these runs are presented in Table 2.

Table 1. Channel configurations used in this study.

Channel type	Width (m)	Slope	Manning's roughness
1	50	0.0002	0.04
2	50	0.0002	0.02
3	50	0.002	0.02

Table 2. Summary of test runs results

Channel Type	No.of equal sub-reaches	Peak flow and time to peak St.Venant soln.		Proposed method		y_{per} in %	t_{per}	VAREXP (in %)
		y_{pst} (m)	t_{pst}(hr)	y_{pc}(m)	t_{pc}(hr)			
1	1	8.11	14.75	7.51	15.00	8.00	-0.25	96.85
1	2	8.11	14.75	7.59	14.75	7.00	0.00	97.25
1	3	8.11	14.75	7.65	14.50	6.00	0.25	97.16
2	1	9.01	12.50	8.96	12.50	1.00	0.00	99.27
2	2	9.01	12.50	9.00	12.00	0.00	0.50	99.07
2	3	9.01	12.50	9.07	11.75	0.00	0.75	98.25
3	1	9.92	11.75	10.09	11.50	-2.00	0.25	99.62
3	2	9.92	11.75	9.93	11.50	0.00	0.25	99.76
3	3	9.92	11.75	9.89	11.50	0.00	0.25	99.78

Throughout the routing studies, the value of coefficient a used in equation(16) for computing the reference stage was taken as 0.5. This best value was arrived at by trial and error approach in reproducing the St.Venant solutions closely by the proposed method based on Nash-Sutcliffe criterion(Nash and Sutcliffe, 1970). A routing interval of 15 minutes was used in all the test runs.

THE PERFORMANCE CRITERIA

The performance of the method is evaluated by comparing its solution with the corresponding solution of the St.Venant equations based on the following criteria:

Accuracy of the reproduction of the hydrograph shape and size, and (2) accuracy of the reproduction of the peak stage with reference to its magnitude and time.

The criteria are quantified using the following measures:

The accuracy of the reproduction of the hydrograph's shape and size is estimated using the Nash-Sutcliffe criterion(Nash and Sutcliffe,1970) and it is denoted by the factor VAREXP in percentage, and

The accuracy of the reproduction of the peak stage is estimated by the measures of attenuation error in percentage, y_{per} which is expressed as

$$y_{per} = \frac{(y_{pst} - y_{pc})}{(y_p - y_0)} x100 \tag{18}$$

and by the measure of the time to peak error, t_{per} expressed as

$$t_{per} = \left| t_{pst} - t_{pc} \right| \tag{19}$$

where, the subscripts pst and pc denote, respectively, the peaks of the solutions of the St.Venant equations and of the proposed method. The method is considered as accurate in reproducing the solutions of the St.Venant equations closely, when all the above criteria together satisfy the following limits:

VAREXP> 99% ; $\left| y_{per} \right| \leq 1\,\%$ and $\left| t_{per} \right| \leq \Delta t$

DISCUSSION OF RESULTS

Table 2 shows the summary of results of the reproduction of pertinent characteristics of St.Venant solutions for all test runs made in this study. These pertinent characteristics are the peak and time to peak of the St.Venant solution, and that of the proposed method, variance explained by the proposed method in reproducing the St.Venant solution, and the attenuation error. These results have been obtained by considering the entire 40 km channel reach used for routing in each of the test runs as single, two and three sub-reaches. The reproductions of the St.Venant solutions using the proposed method are shown in Figs.1-3. It is inferred from the results given in Table 2 and Figs.1-3, that various characteristics of the St.Venant solutions are closely reproduced by the proposed method, as per the accuracy criteria set in this study, while routing in channel types 2 and 3. The reason for this may be linked to the magnitude of the non-dimensional longitudinal gradient of the water surface, denoted by $(1/S_0)\partial y/\partial x$, computed at the inlet of the reach using the given input hydrograph. The same reason was inferred in the case of multilinear discrete cascade model operating on discharge variable(Perumal,1994) and also in the case of variable parameter stage hydrograph routing method(Perumal and Ranga Raju,1998a).The absolute magnitudes of $(1/S_0)\partial y/\partial x$ computed at the inlet of channel type-1, channel type-2 and channel type-3 for the present study were 0.94, 0.60 and 0.05 respectively. It has been indicated in the earlier studies also that when $\left|(1/S_0)\partial y/\partial x\right|$ >>0, such simplified methods do not serve the intended purpose. It is seen that when the actual attenuation is more that 12%, the proposed method does not reproduce the St.Venant solutions accurately as per the set criteria. This inference is based on the results of a number of test runs made for this study by varying γ and t_p. The details of these additional test runs are not presented herein due to space limitation. However, it may be pointed out that the stringent accuracy criteria set forth in this study is arbitrary, and it is possible that a lesser stringent criteria may be sufficient for the application of the proposed method from practical considerations.

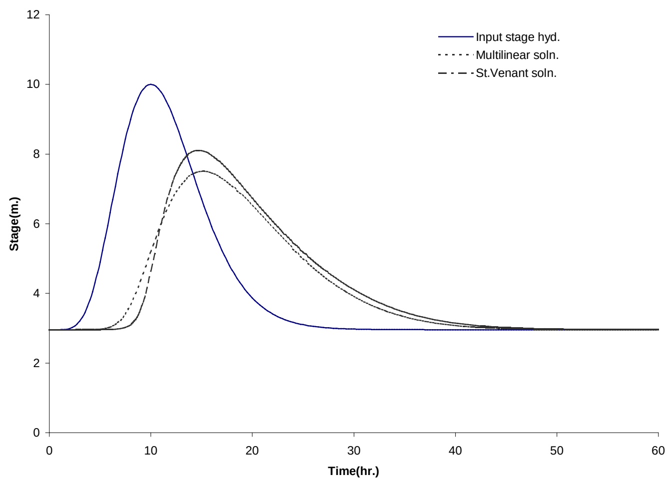

Figure 1. Routing result for run no. 1

The results pertinent to routing in channel type-1 reveal that routing solutions obtained by dividing the routing reach into two sub-reaches, yield marginally improved results than the single reach solution. However, the improved solution still does not meet the accuracy criteria set forth. But for some test run cases in channel type-3, routing using two sub-reaches yielded improved results, bringing them within the accuracy limits. For the other cases of channel type-3 routing, single reach routing was sufficient to yield accurate reproduction results. For test runs in channel type-2, it was found that routing based on single reach consideration yielded accurate results.

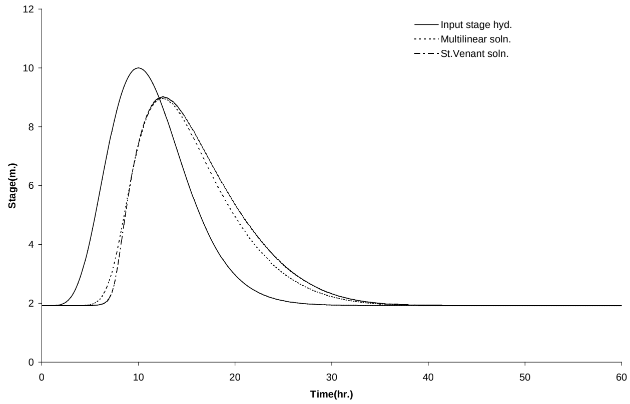

Figure 2. Routing result for run no. 4

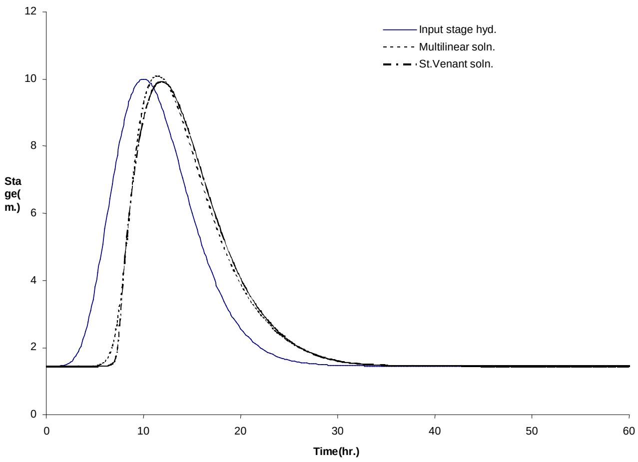

Figure 3. Routing results for run no. 7

CONCLUSION

The following conclusions are drawn from this study:

The multilinear discrete cascade model proposed in this study for routing stage hydrographs in channels can account for nonlinearity in flood wave movement in channels, and

The method is able to reproduce the solutions of the St.Venant equations closely when the rating curve corresponding to the inflow hydrograph is characterised by a narrow loop.

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