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A two dimensional groundwater flow model for spring

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Abstract
The Bear's springflow model and the model with long transmission zone, assume the flow to be one-dimensional. However, in reality, the flow pertaining to a spring is three-dimensional. Using the Dupuit-Forchheimer assumptions, some three-dimensional flow process could be dealt as two-dimensional. A springflow domain can be visualized to have a recharge area, which may not be well defined, and a discharge area acts as the spring. Hantush (1967, vide Bouwer, 1978) has given solution for the rise of piezometric surface due to uniform recharge at a constant rate from a rectangular basin. The shape of the recharge area for a spring can be considered as rectangular. Similarly, a rectangular shape can be assumed for the spring's opening. Using the Hantush's basic solution for the rise of piezometric surface due to recharge from a rectangular area, a two-dimensional springflow model has been developed in this paper. The method of image is applied to convert the finite flow domain into an infinite one.

In this model, the spring aquifer system is an open system. Therefore, all the discharge does not appear as a springflow. The variation of logarithm of a springflow with during recession does not follow a straight line. Only towards a later part of the recession the variation is approximately linear. Using the random jump technique and the springflow model for an open flow domain, recharge area, spring opening, distance of the spring from the recharge area, transmissivity and storativity of the transmission zone and the recharge have been estimated from observed springflow data from the Kirkgoz spring in Turkey. Since the domain is an open one, the recharge computed by the model, which is based on Hantush's solution, is found higher than those computed using the model for a closed system.

STATEMENT OF THE PROBLEM

A schematic configuration of a spring flow domain is shown in Fig. 1(a). The corresponding idealized flow domain adopted for the development of the model is shown in Fig. 1(b). The recharge area of the spring is assumed to be a rectangle of size LxW_R and the spring opening conforms to a rectangle of size a x b. The aquifer which transmits water to the spring is homogeneous, isotropic, and has semi-infinite areal extent. It is aimed to find the temporal variation of the springflow due to time variant recharge through the entire recharge zone.

DEVELOPMENT OF THE MODEL

The basic saturated flow equation describing the flow in the spring aquifer system is the Boussinesq's equation:

$$\phi \frac{\delta s}{\delta t} - T \frac{\delta^2 s}{\delta x^2} - T \frac{\delta^2 s}{\delta^2 y} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (\xi, \nu, t)\, \delta_D (\xi - x, \nu - y)\, d\xi\, d\nu \qquad (1)$$

Where s is the rise in piezometric surface, t is time, x and y are the horizontal Cartesian coordinate, ϕ is the storage coefficient, T is the transmissivity, r (ξ,ν,t) is the recharge or discharge rate per unit area (positive for recharge and negative for discharge) and $\delta_D(\xi\text{-}x, \nu\text{-}y)$ is a Dirac delta function singular at the point of coordinates x, and y. The level of the initially rest piezometric surface coinciding with top of the aquifer is taken as the datum.

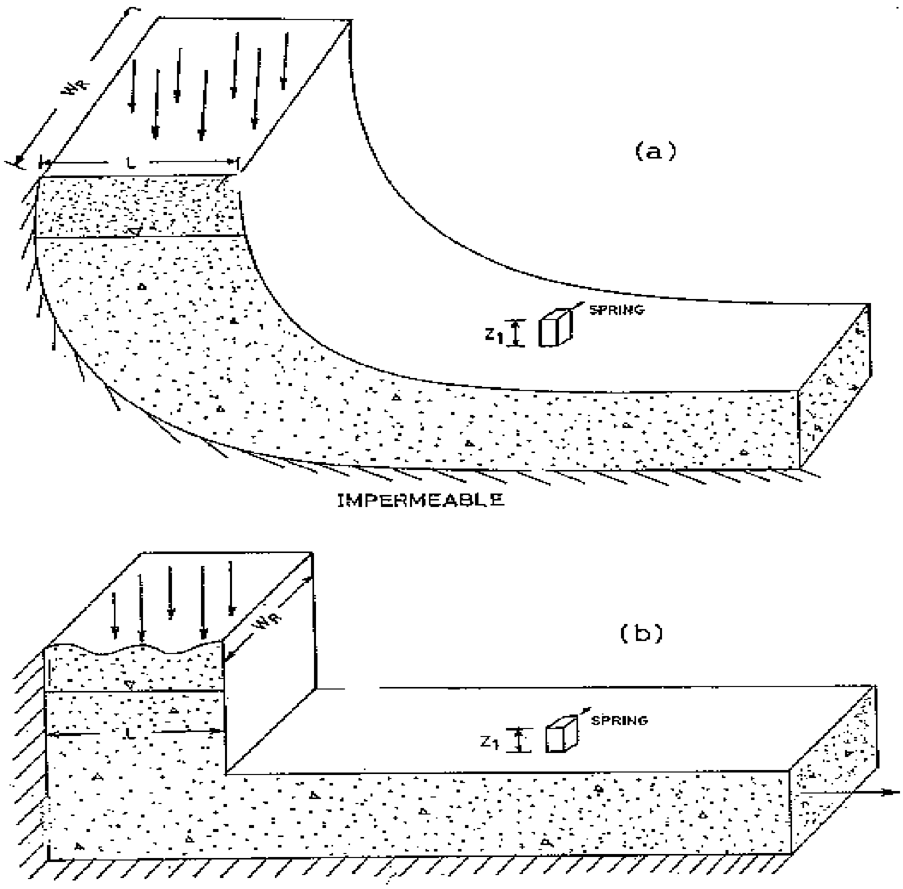

Figure 1(a) & (b). Schematic and idealised flow domain of a spring.

The required solution to the differential equation (1) for the spring needs to satisfy an initial condition S (x, y, 0) = 0. The boundary conditions to be satisfied are:

$$\left. \frac{\delta s}{\delta x} \right|_{x=0} = 0 \qquad (2)$$

$s(x, \pm \infty, t) = 0$ (3)

$s(\infty, y, t) = 0$ (4)

A spring gets activated when the piezometric surface tends to rise above its threshold. Once a spring gets activated, the rise in piezometric surfacce at the location of the spring remains invariant till the springflow becomes zero. Therefore, the other boundary condition to be satisfied is:

$S(x_1, y_1, t) = z_1, \quad t > t_1$
(5)

Where x_1, y_1 are the coordinate of the spring, t_1 is time of activation of the spring, z_1 is height of the threshold of the spring above the initially rest piezometric surface.

The method of image is applied to convert the finite flow domain into an infinite one. The boundary condition stated in equation (2) is thereby satisfied. The system of image and real springs is shown in Fig. 2. Hantush's basic solution being used in the present analysis, the boundary conditions stated in equation (3) and equation (4) and the initial conditions are automatically satisfied.

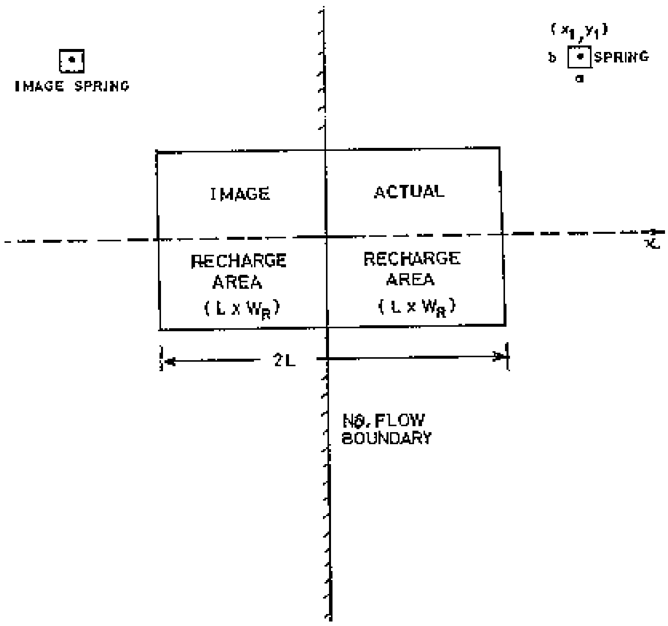

Figure 2. Flow domain of the proposed model based on theory of image.

Let the time span be discretised into uniform time steps of size Δt. Let during a time step, γ, the pulse recharge per unit area be $R_u(\gamma)$, and the pulse spring discharge per unit area of the spring opening be $q(\gamma)$. However, $q(\gamma)$ and $R_u(\gamma)$ may vary from one time step to next.

The rise in piezometric surface, $s(x_1,y_1,n\Delta t)$, at the spring at time $t=n\Delta t$, due to the time variant pulse recharge, $R_u(\gamma)$, $\gamma=1,2...,n$, through the recharge zone in the equivalent flow domain until the spring gets activated is given by

$$S(x_1, y_1, n\Delta t) = \sum_{\gamma=1}^{n} R_u(\gamma)\, \delta(2L, W_R, x_1, y_1,: \Delta t; n-\gamma+1) \qquad (6)$$

in which $\delta(A, B; X, Y; \Delta t; m)$ is a discrete kernel coefficient for rise in piezometric surface; A and B are length and width of the excitation zone; X and Y are the coordinate of the point of observation, the coordinate being measured from a local origin chosen at the center of the zone of perturbation; Δt is the time step size; 2L and W_R are length and width of the recharge area in the equivalent flow domain. The discrete kernel coefficient for rise in piezometric surface are the rise in piezometric surface at an observation point due to a unit pulse perturbation per unit area given to the system during the first unit time period. In the present problem, the zone of perturbation is either the area through which recharge takes place or the spring's opening. Let the spring get activated during Nth time step and the rising piezometric surface touches the spring's threshold at $t = (N-1)\,\Delta t$. Hence

$$\sum_{\gamma=1}^{N-1} R_u(\gamma)\; \delta(2L, W_R; x_1, y_1; \Delta t; N-\gamma) = Z_1 \qquad (7)$$

The time of activation of the spring can be predicted from equation 7 using an iteration procedure. As the spring gets activated at n=N, therefore, $q(\gamma) = 0$ for $\gamma = 1,2, \ldots N-1$.

The expression for rise in piezometric surface at $t = n\Delta t$ at the spring after its activation is given by

$$s(x_1,y_1,n\Delta t) = \sum_{\gamma-1}^{n} [R_U(\gamma)\, \delta(2L, W_R; x_1,y_1; \Delta t; n-\gamma+1)]$$

$$- \sum_{\gamma-1}^{n} [q(\gamma)\, \{d(a,b; 0,0; \Delta t; n-\gamma+1) + \delta(a,b; 2x_1, 0; \Delta t; n-\gamma+1)\}] \qquad (8)$$

The dimension a and b of the spring are in x and y direction, respectively and after activation of the spring, $s(x_1, y_1, n\Delta t) = z_1$

Splitting the second temporal summation into two parts, one part containing the summation up to (n-1)th term, and the other part the nth term and solving for q(n)

$$q(n) = [\ \sum_{\gamma=1}^{n} \{R_U(\gamma)\; \delta(2L, W_R; x_1, y_1; \Delta t; n-\gamma+1)\}$$

$$- \sum_{\gamma=1}^{n-1} \{q(\gamma)\; \{\delta(a,b; 0,0; \Delta t; n-\gamma+1) + \delta(a,b; 2x_1, 0; \Delta t; n-\gamma+1))\} - Z_1]/$$

$[\delta(a,b; 0,0; \Delta t;1) + \delta (a,b; 2x_1, 0; \Delta t; 1)]$ (9)

Since the spring gets activated during N^{th} time step, $q(\gamma)=0$ for $\gamma=1,2,\ldots.N-1$. $q(n)$, $n \geq N$ can be solved in succession starting from time step N.

The discrete kernel coefficients for rise of piezometric surface can be obtained from Hantush's solution for the rise of piezometric surface due to uniform recharge at a constant rate from a rectangular basin (Fig. 3). Making use of Hantush's solution, $\delta(A,B;x,y; \Delta t; m)$ is found to be

$$\delta(A,B; X, Y; \Delta t; m)= \frac{m}{4\phi} [F\{(A/2 + X) \eta_1,(B/2 + Y) \eta_1\} + F\{(A/2 + X)\eta_1 , (B/2 - Y) \eta_1\}$$

$$+ F\{(A/2 - X) \eta_1, (B/2 + Y) \eta_1\} + F\{(A/2 - X)\eta_1, (B/2 - Y) \eta_1\}]$$

$$- \frac{(m-1)}{4\phi} [F\{(A/2 + X) \eta_2, (B/2 + Y) \eta_2\} + F\{(A/2 + X) \eta_2, (B/2 - Y) \eta_2\}$$

$$+ F \{(A/2 - X) \eta_2 , (B/2 + Y) \eta_2\} + F\{ (A/2 - X) \eta_2, (B/2 - Y) \eta_2\}] \quad (10)$$

where

m = time step

η_1 = $(4Tm\Delta t /\phi)^{-0.5}$

η_2 = $\{4T(m-1) \Delta t/\phi\}^{-0.5}$

$F(\phi,\psi)$ = $\int_0^1 \mathrm{erf}(\phi \tau^{-0.5}) . \mathrm{erf} (\psi \tau^{-0.5}) d\tau$

Where $\phi = (A/2 \pm X) \eta_i$, and $\psi = (B/2 \pm Y) \eta_i$, i = 1,2

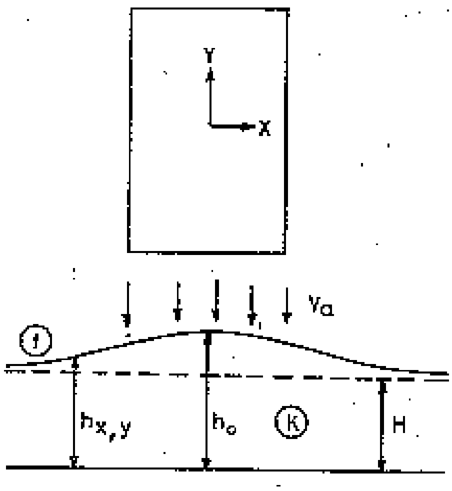

Figure 3. Recharge area and groundwater mound (After Hantush vide Bouwler, 1978).

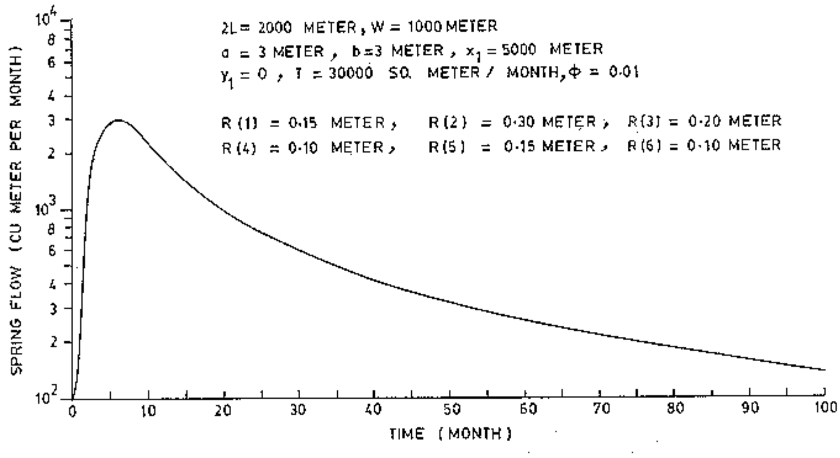

Figure 4. Variation of spring flow with time for a set of assumed model parameters and an assumed recharge per unit area.

RESULTS AND DISCUSSIONS

Assuming aquifer parameters and its geometry, springflow has been generated for a set of time variant recharge. The elevation z_1 has been assumed to be zero. The variation of $\log_{10} Q(t)$ versus time is represented in Fig. 4. As seen from the Figure, the graph during recession does not follow a straight line; the slope of the graph changes with time. For the assumed set of recharge, the recession starts from seventh month. The slope at time step 7 is – 0.0408. The slope decreases with time and reaches a minimum at time step 8 and then increases. The slope changes because the spring flow domain is not a closed system. In the example presented, a total of one-meter recharge per unit area takes place in a span of six month. The actual recharge area is 1 km x 1 km, which means that 10^6 cubic meter of water has been recharged. It is found that at the end of 120^{th} time step, only 6.36% of recharge appears as springflow. The remaining recharge has flown out as regional groundwater flow.

Using the random jump technique and springflow of the Kirkgoz spring, Turkey, aquifer parameters for the spring are estimated and are given below. The following initial guess of the upper and lower bounds of the model parameters has been made; $2L_u$ = 3000 meter, $2L_l$ = 1000 meter; W_{RU} = 3000 meter, W_{Rl} = 1000 meter; a_U = 15 meter, a_l = 2 meter; b_U = 15 meter, b_l = 2 meter; X_{1u} = 5000 meter, X_{1l} = 4000 meter; T_U = 40000 sq m/month, T_l = 20000 sq m/month; and ϕ_U = 0.001, ϕ_1 = 0.0001. The decay constant has been assumed to be equal to 1/6 $month^{-1}$ and Y = 0. The springflow for the month December, 1973, i.e. 27.53 X 10^6 cubic meter per month has been used to compute the springflow due to perturbation prior to the time origin. The estimated model parameter for the spring for which the objective function is the minimums are:

Length of the recharge zone (L) = 1155.50 meter
Width of the recharge zone (W_R) = 4046.00 meter
Length of the spring's opening (a) = 30.68 meter
Width of the spring's opening (b) = 30.68 meter
Distance of the spring from the no-flow boundary (X_1) = 4186 meter
Transmissivity of the aquifer in the flow domain (T) = 20590 m^2/month
Storage coefficient of the aquifer in the flow domain (ϕ) = 0.0013

There is a successive rapid decrease of the objective function in the search technique after end of each cycle.

Table 1. Observed and simulated springflow for the period of no recharge and no abstraction

Month	Observed springflow after deducting effect of prior perturbation (cu meter/month)	Simulated springflow (cu meter/month)
10	.1992E+08	.1559E+08
22	.2093E+08	.1900E+08
34	.2134E+08	.2085E+08
46	.1666E+08	.1820E+08
58	.2634E+08	.2334E+08
70	.2599E+08	.2886E+08
82	.2389E+08	.2579E+08
89	.4833E+08	.4119E+08

Using these parameters, the recharge has been computed by the model. As expected, the recharge, which is estimated in this model, would be more than the recharge estimated by a model, which considers a closed flow domain i.e., Bear's model. This is due to the reason that all recharge does not appear as springflow; part of the recharge flows out as regional groundwater flow. The observed and the simulated springflow for the periods of no recharge and no abstraction are similar in magnitude and are presented in Table 1.

CONCLUSIONS

The graph $\log_{10} Q(t)$ versus t during recession does not follow a straight line.

Parameters of the model can be estimated by the random jump technique.

The model based on Hantush's basic solution assumes the flow domain to be infinite. The Bear's model, with or without storage effect of the transmission zone, assumes that the flow domain of the spring is a closed one. Because of this difference in the characteristics of the flow domain, the recharge computed by the model which is based on Hantush's solution is more than those which is computed by either of the Bear's model. In the Bear's model, all recharge appears as springflow, whereas in the model based on Hantush's solution, only part of the recharge appears at the spring.

References

Bhar, A. K. (1996), "Mathematical modeling of spring flow" Ph. D. Thesis, University of Roorkee, Roorkee., U.P., India

Bouwer, H. (1978), “Groundwater Hydrology”, McGraw Hill, New York.