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COMPARATIVE STUDY OF DIFFERENT PARAMETER ESTIMATION TECHNIQUES FOR EV-1 DISTRIBUTION

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SYNOPSIS

Design of water resources structures needs estimation of hydrologic and meteorologic events for different return periods. This can be obtained through frequency analysis approach. The extreme value type 1 (EV-1) distribution is widely used for frequency analysis of extreme events in hydrology and meteorology. Its successful application depends upon the accuracy with which its parameters can be estimated. However, there is no universal agreed method of estimating its parameters.

This paper explains and statistically compares various parameter estimation techniques cited in literature using data of different sample sizes generated by Monte Carlo Simulation. The analysis of results reveals that method of mixed moments and incomplete mean procedures are least accurate methods for estimating the parameters and quantiles of EV-1 distribution. The methods based on probability weighted moments, principle of maximum entropy and maximum likelihood give nearly similar results and are recommended for general use.

1.0 INTRODUCTION

Design of water resources structures needs estimation of hydrologic and meteorologic events for different return periods. This can be achieved through frequency analysis approach, in which, in general a past record is fitted with a statistical distribution which is then used to make inferences about future flows. Many probability distributions and several parameter estimation techniques for each distributions have been proposed for at site flood analysis. The selection of appropriate parameter estimation technique for the distribution is as important as selecting the distribution itself, because several estimation techniques based on the same distribution can lead to widely differeing parameters and quantile estimates. This variability in estimation tends to be more pronounced as sample size decreases. The present study deals with assessment of the behaviour of several estimators available for the Gumbel 's extreme value type 1 (EV-1) distribution which is widely used for frequency analysis of extremes in hydrology and meteorology. Despite extensive use of EV-1 distribution, there is no generally accepted method for estimating its parameters. The primary objective of the study is to compare seven estimation techniques for EV-1 distribution, which have been advocated by various researchers.

2.0 METHODS OF PARAMETER ESTIMATION

The cumulative distribution function F(x) of EV-1 distribution is defined as $F(x) = \exp(-\exp(-(X-b)a))$, where b and a are location and scale parameters respectively. Keeping in view the objective of the study various methods are briefly described in subsequent sections.

2.1 Method of Moments (MOM)

This is one of the most popular methods of estimating the parameters a and b (Lowery and Nash, 1970; Landwehr et al, 1979). The estimates of a and b are given as:

$$a = 1.28255/S_x \qquad \dots . \quad (1)$$

$$b = \overline{X} - 0.450041\ S_x \qquad \dots . \quad (2)$$

where, $\overline{X}$ and S_x are the mean and standard deviation of the sample.

2.2 Method of Maximum Likelihood (MLE)

This method involves finding the values of a and b that maximise the likelihood, or equivalently the log likelihood function of the observed data. The parameters a and b can be obtained from the following two equations which can be solved readily by Newton-Raphson method:

$$\frac{1}{a} = \frac{\overline{X} - \Sigma X\ .\exp(-ax)}{\Sigma \exp(-ax)} \qquad \dots . \quad (3)$$

and, $$b = \frac{1}{a} \ln \left(\frac{n}{\Sigma \exp(-ax)}\right) \qquad \dots . \quad (4)$$

2.3 Method of Least Squares (LEAS)

The parameters a and b are estimated by the following equations:

$$\frac{1}{a} \quad \frac{n \quad \Sigma Z_i\ X_i - \Sigma X_i \quad \Sigma Z_i}{n\ \Sigma X_i^2 - (\Sigma X_i)^2} \qquad \dots . \quad (5)$$

and, $$b = \overline{X} - \quad Z_i/a.n \qquad \dots . \quad (6)$$

here, $Z_i = -\ln(-\ln(F(X_i)))$ is obtained for each data point from the appropriate plotting position formula. In the study Gringortan plotting position formula (Seth and Goel (1986)) has been used.

2.4 Method of Probability Weighted Moments (PWM)

Following Landwehr, et.al. (1979), the parameters a and b can be given as:

$$b = \bar{X} - 0.5772/a \qquad \dots . \quad (7)$$

$$\frac{1}{a} = \frac{\bar{X} - 2\,M_{101}}{1_n 2} \quad .... \quad (8)$$

where, M_{101} is the first probability weighted moment defined as:

$$M_{101} = \frac{1}{n\,(n-1)} \sum_{i=1}^{n} x_i(n-i) \quad .... \quad (9)$$

here i is the rank of the x_i when x_i are arranged in descending order.

2.5 Method Based on Principle of Maximum Entropy

This method is based on maximizing the information (entropy) content imbued in the EV-1 distribution subject to the constraints:

$$\int_0^\infty f\,(x)\,dx = 1 \quad .... \quad (10)$$

$$\int_0^\infty x\,f\,(x)\,dx = \bar{X} \quad .... \quad (11)$$

$$\int_0^\infty f\,(x)\,\exp\,(-a(x-b))\,dx = E\,(e^{-a(x-b)}) = 1 \quad .... \quad (12)$$

a and b can be obtained as:

$$E(y) = 0.5772 \quad .... \quad (13)$$

$$E(\exp\,(-y)) = 1 \quad .... \quad (14)$$

where, $y = a\,(x-b)$. Clearly a and b have to be estimated using an iterative procedure.

2.6 Incomplete Mean Procedure (ICM)

The ICM method uses means calculated over only parts of the data range. By arranging the sample in ascending order $x_1, x_2, \ldots, x_n$, first the sample mean ($\bar{x}$) is calculated. $\bar{x}$ is then used to devide the sample into two disjointed sets. The mean of the upper set having values greater than $\bar{x}$ is calculated and called first incomplete mean $\bar{x}_1$. Similarly the mean of all observations above $\bar{x}_1$ is calculated and is the second incomplete mean $\bar{x}_2$, and so on. For the EV-1 distribution, the first two incomplete means are:

$$\bar{X}_i = b - \frac{n}{a(n-n_i)}\left(\ln J(1-e^{-J}) - J + \frac{J^2}{4} - \frac{J^3}{18} + \frac{J^4}{96}\right), \quad i = 1, 2 \quad .... \quad (15)$$

where, $J = \ln\,(n/n_i)$, n is the sample size and n_i the number of observations corresponding to the lower limit of the range on which the incomplete mean is calculated. Jain and Singh (1987) have given the derivation of equation 15.

2.7 Method of Mixed Moments (MIX)

This method uses the first moment of the EV-1 distribution and the first moment of its logarithmic version. The parameters a and b are given by:

$$a = \frac{1.28255}{S_x} \quad \ldots\ldots \quad (16)$$

$$\text{and, } \exp(a\,b) = 1 + a\,\bar{x} + \frac{a^2}{2}\,(S_x^2 + \bar{x}^2) \quad \ldots\ldots \quad (17)$$

Equation (17) has been derived by Jain and Singh (1987).

2.8 Some Typical Studies

a. Raynal and Salas (1986) analyse and compare six methods, viz. i) Method of moments using biased estimator of variance, ii) Method of moments using unbiased estimator of variance, iii) Method of maximum likelihood, iv) Mode and interquantile range method, v) Least squares method, and vi) Best linear combination of order statistics, using data generating techniques. Considering criteria of bias, variance and mean square error of estimates of parameters and quantile points and ease of obtaining estimators they conclude that the best linear combination of order statistics compares favourably with the other methods for sample sizes smaller than 20. For larger samples the probability weighted moments method is preferred. Likewise considering all factors of comparison, the least squares and mode and interquantile range methods should not be used for fitting the Gumbel distribution.

b. Phien (1987) recommends suitable solution procedures for MOM, MLE, POME and PWM for EV-1 distribution and discusses related problems. On the basis of simulation study to evaluate the performance of these methods in terms of commonly used criteria, i.e. the bias, root mean square error and goodness of fit statistics he concludes that:

(i) MOM is not as good as the remaining three methods;

(ii) If one is interested in unbiased estimates then PWM is obviously the best choice;

(iii) If one is interested in having minimum values for the RMSE, then the MLE is the most appropriate method;

(iv) With respect to both the bias and the RMSE, the POME has almost the same performance as the MLE. As such one may choose the POME instead of MLE also.

c. Jain and Singh (1987) estimated parameters of EV-1 distribution for 55 annual flood data sets by seven methods, viz. methods of (1) moments, (2) probability weighted moments, (3) mixed moments, (4) Maximum likelihood estimation, (5) incomplete means, (6) principle of maximum entropy and, (7) least squares. They conclude:

(i) The MLE Method was the most accurate of all followed by the POME, PWM, MOM and LEAS methods;

(ii) The ICM and MIX Methods were the least accurate of all and could not be recommended;

(iii) The MLE, POME, MOM and PWM methods may be recommended for general use. However, the POME method would be preferable, as convergence in parameter estimation is guaranteed in each case.

d. Arora and Singh (1987) estimate and inter compare statistically the parameters and quantiles of EV-1 distribution using synthetically generated data for sample lengths 5 to 1000. They compare seven parameter estimation techniques for two sampling cases, i.e. (i) purely random process and (ii) serially correlated process. They also give a bias correction for the method of moments-quantile estimator. With regard to the intercomparison of parameters and quantile estimators some of their conclusions are as follows:

(i) The methods of mixed moments and incomplete means resulted in poor estimation of the parameters and the quantiles;

(ii) The method of least squares provided minimum bias and maximum efficiency estimate of the scale parameter a, for very small samples and also provided competative estimate of the location parameter b:

(iii) The maximum likelihood method generally provided most efficient quantile estimates followed closely by the entropy method. In fact POME performed practically in the same manner as MLE and was relatively easier to solve;

(iv) For small samples the method of probability weighted moments and the method of moments performed comparably in efficiency of estimating the quantiles. However, effeciency of PWM improved relative to MLE with increasing sample size. PWM also resulted in nearly unbiased quantile estimates;

(v) The incorporation of serial correlation in samples resulted in deterioration of the performance of all estimators. However, all the methods performed much more similarly in this case also.

3.0 PROPOSED METHODOLOGY

The seven methods described in section 2.1 to 2.7 to estimate the parameters and quantiles of EV-1 distribution have been compared on the basis of following eleven criteria: (i) Bias in location parameter, (ii) bias in scale parameter, (iii) standard deviation of location parameter, (iv) standard deviation of scale parameter, (v) efficiency of the method in estimating the location parameter as compared to maximum likelihood method, (vii) bias of quantile estimates, (viii) standard deviation of quantile estimates, (ix) efficiency of quantile estimates, (x) average of relative deviations between computed and expected value of reduced variates, and (xi) average of squares of relative deviations between computed and expected value of reduced variates.

The study has been carried out for different sample lengths of 10, 20, 30, 40, 50, 60, 70, 75, 80, 90 and 100 using synthetically generated 50,000 EV-1 distributed random numbers with location and scale parameters as 0.0 and 1.0 respectively.

3.1 Steps Involved

Various steps of the study are as follows:

1. Get samples of different lengths (10, 20, 30, 40, 50, 60, 70, 75, 80, 90 and 100) from the generated EV-1 distributed reduced variates e.g. if the sample length is 10 then one will be getting 50,000/10 i.e. 5000 samples;
2. Arrange the samples in descending order;
3. Estimate the parameters (a and b) of the EV-1 distribution by the seven methods using equation 1-17;
4. Calculate quantiles x(F) for F=0.001, 0.01, 0.02, 0.05, 0.1, 0.250, 0.5, 0.75, 0.9, 0.95, 0.98, 0.99 and 0.999 by the following equation:

$$x(F) = b - \frac{\ln(-\ln(F))}{a} \quad \text{.... (18)}$$

5. Calculate bias, standard deviation, and efficiency of location parameter b, scale parameter a and quantiles by the following procedure:

Let $\hat{\theta}$ denote an estimate of $\theta \varepsilon$ (a, b, x(F)). $\hat{\theta}$ is a random variable whose distribution function depends upon sample size, the method of parameter estimation, and of course, the distribution of the sample itself. The performance statistics of $\hat{\theta}$ are given as follows:

$$\text{Bias, BIAS}(\hat{\theta}) = \theta - E(\hat{\theta}) \quad \text{.... (19)}$$

$$\text{Variance, STD}^2(\hat{\theta}) = E(\hat{\theta} - E(\hat{\theta}))^2 \quad \text{.... (20)}$$

$$\text{Mean Square Error, MSE}(\hat{\theta}) = E(\theta - \hat{\theta})^2$$

$$= \text{BIAS}^2(\hat{\theta}) + \text{STD}^2(\hat{\theta}) \quad \text{.... (21)}$$

$$E(\hat{\theta}) = \mu(\hat{\theta}) = \sum_{i=1}^{N} \frac{\hat{\theta}_i}{N} \quad \text{.... (22)}$$

$$\text{STD}^2(\hat{\theta}) = \sum_{i=1}^{N} \left(\frac{(\hat{\theta}_i - \mu(\hat{\theta}))^2}{N-1}\right)^2 \quad \text{.... (23)}$$

$$\text{BIAS}(\hat{\theta}) = \theta - \mu(\hat{\theta}) \quad \text{.... (24)}$$

$$\text{MSE}(\hat{\theta}) = \text{BIAS}^2(\hat{\theta}) + \text{STD}^2(\hat{\theta}) \quad \text{.... (25)}$$

$$\text{EFF}(\hat{\theta}) = \frac{\text{MSE}(\hat{\theta} \mid \text{MLE})}{\text{MSE}(\hat{\theta} \mid \text{other method})} \quad \text{.... (26)}$$

A value of EFF ($\hat{\theta}$) < 1 implies that the method under consideration is less efficient (i.e. has higher mean square error) compared to MLE and vice versa.

6. Calculate average of relative deviation between computed and expected value of reduced variates for various methods;

$$DA = \sum_{i=1}^{N} \left| \frac{X_{ci} - X_{Ei}}{X_{Ei}} \right| X \ 100/N \qquad .... (27)$$

where,

N = Sample size;

X_{ci} = Computed value of i^{th} reduced variate using estimated value of a and b;

X_{Ei} = Expected value of i^{th} reduced variate

$$= \sum_{j=1}^{n} Y_{i,j/n} \qquad .... (28)$$

$Y_{i,j}$ = i^{th} reduced variate for the j^{th} sample;

n = number of samples

The number of reduced variates will be equal to the size of the sample.

7. Calculate average of squares of relative deviations between computed and expected value of reduced variates by following equation:

$$DR = \sum_{i=1}^{N} \left(\frac{X_{ci} - X_{Ei}}{X_{Ei}}\right)^2 (100/N) \qquad .... (29)$$

8. Compare various methods of parameter estimation on the basis of statistics calculated above.

4.0 ANALYSIS AND RESULTS

Seven parameter estimation techaniques for EV-1 distribution have been intercompared on the basis of above mentioned eleven criteria. The analysis of results is being given in two parts, i.e. (i) parameter estimates, and (ii) quantile estimates as follows:

4.1 Parameter Estimates

For two typical sample sizes of 20 and 30 generally encountered in frequency analysis, performance of statistics of parameters is given in table 1, though studies were conducted for sample sizes 10-100. From the table it is seen that: (i) the method of mixed moments and incomplete mean procedures can be prima facie rejected as unreliable estimators of Gumbel distribution parameters, (ii) methods based on probability weighted moments, maximum likelihood estimation and principle of maximum entropy are comparable and it is difficult to arrive at any definite conclusion regarding superiority of any one of them.

Table 1 : Performance of Statistics of Parameters

Method	Sample size	BIAS(A)	STD(A)	EFF(A)	BIAS(B)	STD(B)	EFF(B)
MLE	20	-0.071	0.200	1.000	-0.019	0.231	1.000
MOM		-0.074	0.239	0.719	-0.015	0.235	0.973
PWM		-0.040	0.216	0.937	0.00	0.231	1.008
POME		-0.067	0.203	0.987	-0.019	0.232	0.996
LEAS		-0.056	0.243	0.722	-0.024	0.366	0.401
MIX		-0.074	0.239	0.719	-0.930	0.267	0.057
ICM		-0.209	0.460	0.177	-0.068	0.420	0.297
MLE	30	-0.045	0.151	1.000	-0.013	0.188	1.000
MOM		-0.049	0.189	0.653	-0.010	0.192	0.964
PWM		-0.025	0.168	0.860	-0.001	0.188	1.004
POME		-0.042	0.154	0.973	-0.013	0.189	0.996
LEAS		-0.045	0.197	0.612	-0.022	0.299	0.397
MIX		-0.049	0.189	0.653	-0.935	0.218	0.039
ICM		-0.038	0.316	0.246	0.090	0.374	0.241

4.2 Quantile Estimates

The quantiles corresponding to probability of non-exceedence equal to 0.001, 0.01, 0.02, 0.05, 0.1, 0.25, 0.5, 0.75, 0.9, 0.95, 0.98, 0.99 and 0.999 have been estimated by various methods. The bias, standard deviation and efficiency of quantiles for various methods and various sample size were earlier given by authors in Seth and Goel (1987).

4.2.1 Bias of quantile estimates

MIX and ICM can be rejected as unreliable estimators of quanitles.

PWM provided unbiased quantile estimates for all N and F. MOM provided lesser bias than MLE and POME. POME resulted in slightly less bias than MLE. The bias of LEAS does not show any consistent trend for all the F values.

4.2.2 Standard deviation of quantile estimates

ICM can be rejected on the basis of standard deviation of quantiles. For the lower six quantiles i.e. for F = 0.001, 0.01, 0.02, 0.05, 0.1, 0.25, MIX gives lowest standard deviation of quantiles, while for upper six quantiles i.e. for F = 0.75, 0.9, 0.95, 0.98, 0.99 and 0.999 LEAS is giving lowest standard deviation nearly for all the sample sizes. MLE is better than MOM, PWM and POME. In general, the ranking in descending order can be:

1) MLE, 2) POME, 3)PWM, and 4)MOM

4.2.3 Efficiency of quantile estimates

ICM and MIX can be rejected as unreliable estimates. LEAS gives maximum efficiency for the top six quantiles i.e. quantiles corresponding to F = 0.75, 0.9, 0.95, 0.98, 0.99, and 0.999 while MLE is the second best for top six quantiles and best for lower six quantiles nearly for all the sample sizes. In general, POME is better than PWM and PWM is better than MOM.

It is difficult to decide about the method which gives minimum bias and standard deviation and maximum efficiency of all the quantiles for all the sample sizes. As such, it is proper to compare all the methods based on average of relative and average of squares of relative deviations between computed and expected value of reduced variates.

4.2.4 Average of relative deviations between computed and expected values of reduced variates (DA)

The values of DA and its rank are given in Table 2. The various methods can be in general ranked as given below in descending order for all the sample sizes:

Rank	1	2	3	4	5	6	7
Method	PWM	LEAS	MLE	POME	MOM	ICM	MIX

4.2.5 Average of squares of relative deviations between computed and expected value of reduced variates (DR)

The values of DR and its rank are given in Table 3. The various methods can be in general ranked as given below in descending order for all the sample sizes:

Rank	1	2	3	4	5	6	7
Method	PWM	LEAS	MLE	POME	MOM	ICM	MIX

5.0 CONCLUSIONS

The various methods have been statistically inter-compared using synthetically generated samples of EV-1 distribution. The sample sizes have been taken as 10, 20, 30, 40, 50, 60, 70, 75, 80, 90 and 100. Based on this study, following conclusions can be drawn:

1. Based on bias, standard deviation and efficiency of parameters and quantiles, it is difficult to arrive at any definite conclusion as no method is the best according to all the criteria. However, MIX and ICM are the least accurate methods. PWM, POME and MLE give nearly similar results and can be recommended for use in practice.

2. Based on average of relative deviations and square of deviations between expected and computed value of reduced variates, the ranking of different methods is as given below in descending order.

Rank	1	2	3	4	5	6	7
Method	PWM	LEAS	MLE	POME	MOM	ICM	MIX

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Table 2 : COMPARISON OF DIFFERENT PARAMETER ESTIMATION TECHNIQUES FOR EV-1 DISTRIBUTION ON THE BASIS OF AVERAGE OF RELATIVE DEVIATIONS BETWEEN COMPUTED AND EXPECTED VALUE OF REDUCED VARIATES (DA)

Method	DR and Rank for Sample Size																					
	Sample Size																					
	10		20		30		40		50		60		70		75		80		90		100	
MLE	60.9	4	10.8	3	7.8	3	32.1	3	5.4	3	4.7	3	6.3	3	5.6	3	4.1	3	3.6	3	4.9	3
MOM	52.2	3	10.8	3	8.3	5	37.4	5	6.4	5	5.6	5	7.9	5	7.1	6	5.3	5	4.7	5	6.6	5
PWM	0.9	1	0.2	1	0.3	1	1.7	1	0.3	1	0.4	1	0.7	1	0.8	1	0.5	1	0.5	1	0.7	1
POME	61.9	5	10.9	4	7.9	4	33.7	4	5.5	4	4.8	4	6.7	4	5.8	4	4.3	4	3.8	4	5.2	4
LEAS	33.4	2	6.8	2	5.5	2	25.9	2	4.3	2	4.0	2	5.5	2	4.8	2	3.5	2	3.3	2	4.6	2
MIX	930.5	7	307.1	6	318.1	7	1864.2	7	360.5	7	364.5	7	565.8	7	523.1	7	412.7	7	393.1	7	616.9	7
ICM	231.1	6	22.9	5	27.7	6	78.6	6	18.2	6	11.0	6	19.0	6	6.2	5	10.6	6	9.3	6	13.9	6

Table 3 : COMPARISON OF DIFFERENT PARAMTER ESTIMATION TECHNIQUES FOR EV-1 DISTRIBUTION ON THE BASIS OF AVERAGE OF SQUARES OF RELATIVE DEVIATION BETWEEN COMPUTED AND EXPECTED VALUE OF REDUCED VARIATES (DR)

Method	DR and Rank for Sample Size																					
	Sample Size																					
	10		20		30		40		50		60		70		75		80		90		100	
MLE	252.1	4	3.4	3	2.4	4	304.7	3	1.4	3	1.1	3	7.1	3	4.9	3	1.3	3	0.9	3	6.3	2
MOM	187.3	3	3.5	4	2.5	5	419.2	5	2.1	5	1.8	5	11.6	5	8.2	6	2.4	5	1.6	5	11.8	4
PWM	0.03	1	0.0008	1	0.02	1	0.9	1	0.005	1	0.009	1	0.1	1	0.1	1	0.02	1	0.02	1	0.1	1
POME	253.7	5	3.6	5	2.3	3	340.2	4	1.6	4	1.3	4	8.3	4	5.6	4	1.6	4	1.0	4	7.4	3
LEAS	79.8	2	1.6	2	1.2	2	208.1	2	1.0	2	1.0	2	6.0	2	4.2	2	1.1	2	0.8	2	6.3	2
MIX	62688.1	7	3384.5	7	4082.6	7	1068572.5	7	7495.3	7	8202.3	7	63065.0	7	48519.4	7	48519.4	7	12062.7	7	107894.2	6
ICM	3951.4	6	18.9	6	33.1	6	1909.7	6	20.08	6	7.8	6	73.8	6	6.7	5	10.9	6	7.1	6	56.2	5

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