THEME 4

MODELLING OF FLOODS AND FLOOD DAMAGE ASSESSMENT

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SURFACE RUNOFF MODELLING OF SMALL WATERSHEDS

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SYNOPSIS

The work reported herein deals with the formulation of a mathematical model to estimate the surface runoff hydrograph-for a given net-rainfall hyetograph over small watersheds. With the precipitation excess as input into the model, overland flow was computed using the governing physical laws relating to the overland flow. This overland flow has been used as input while solving the stream flow equations (gradually varied unsteady flow equations) to obtain the hydrograph. The stream flow equations were solved by using Finite Element Method (FEM) in space domain and Finite Difference Method (FDM) in the time domain. The model has been applied to a small catchment in the western peninsular India of size 100 square kilometers and the results were compared with the actual observed hydrograph.

INTRODUCTION

For many of the catchments in India, the observed runoff data for a given rainfall is either scanty or nil, thereby affecting the reliability of the runoff predictions. Some statistical models are available to predict the extreme values of runoff or generation of synthetic stream flow values based on time series analysis. However, these predictions are based on the chances of occurence and not based on any of the physical laws governing the phenoomenon. Any computations based on the physical laws would give the reliable peaks and the time-wise distribution of runoff for a given net-rainfall input. The nonlinearity of the governing differential equations and the complexity of the solution procedure have discouraged many of the investigators to attempt any solution. But the advent of the computers in recent times and the development of efficient numerical schemes have paved the way to attempt any solution for real life problems.

Though the literature available on flood routing in channels is quite extensive [3,5], the available literature on flood routing in natural streams is rather rare, more so is the case with the simulation of hydrographs for small catchments in general and especially the ungauged basins in particular. Synthetic hydrographs have been simulated using the geometrical features of hydrographs of the hydrologically homogeneous watersheds. Little or no attempt has been made on the simulation of hydrographs based on the physical laws governing the overland flow (OLF) and the stream flow. Few investigators have used the simplified overland flow equations with the excess rainfall as input. The sizes of the catchments used in their study were as small as

225 hectares of agricultural watershed [4] and 1.9 sq. km. of hypothetical watershed [6]. The model formulation presented herein was applied to a catchment of nearly 100 square kilometers and the computed hydrograph was compared with the observed hydrograph for a given rainfall.

FORMULATION OF THE MATHEMATICAL MODEL

The model presented here has two distinct parts, namely (i) the computation of overland flow for a given net-rainfall pattern, (ii) the computation of stages and discharges at different points along the stream with the computed overland flow as the input. The overland flow can be computed as follows [7] :

The overland flow is mainly due to the excess precipitation occuring over the catchment. It can be evaluated by considering the mass balance equation which is of the form

$$\text{Inflow} - \text{Outflow} = \text{Increment in storage}$$

$$\text{or} \qquad P - q.L = \Delta \forall / \Delta t \tag{1}$$

where P is the inflow into the catchment which is mainly the excess precipitation, q is the overland flow per unit length of the stream flow from the region in the form of cross flow into the stream, $\Delta\forall$ is the increment in detention storage, Δt is the time chosen and L is the length of stream segment intercepting the overland flow. If d is the depth of overland flow, A_c is the area of the catchment segment, then

$$0.5\,(P_{t+\Delta t} + P_t)A_c - 0.5\,(q_{t+\Delta t} + q_t)L = A_c\,(d_{t+\Delta t} - d_t)/\Delta t \tag{2}$$

The variables with subscripts t and $t+\Delta t$ are at the beginning and end of the time period respectively. Assuming that the slope of the overland plane S = friction slope S_f in the case of overland flow, it can be written as

$$q = d.d^{2/3}.S^{1/2}/n \tag{3}$$

Where n is the Manning's roughness coefficient applicable to the overland flow. If P is expressed in mm/hour, d in mm, L in meters, then

$$\frac{\Delta t(P_{t+\Delta t} + P_t)}{2(3600)\,(1000)} - \frac{\Delta t.\, L.S.^{1/2}\,(d^{5/3}_{t+\Delta t} + d^{5/3}_t)}{2\ A_c\ n\,(100000)} = \frac{(d_{t+\Delta t} - d_t)}{1000} \tag{4}$$

Simplifying the equation 4, the resulting expression is

$$K1 \cdot d^{5/3}_{t+\Delta t} + 100 \; d_{t+\Delta t} = K2 \qquad (5)$$

where $K1 = L \; S^{1/2} \; \Delta t / \; (2 \; n \; A_c)$ (6a)

$$K2 = 100 \; d_t + \Delta \; t(P_t + P_{t+\Delta t})/72 \; - \; K1.d_t^{5/3} \qquad (6b)$$

The equation 5 can be solved iteratively as follows :

1. Set t = 0
2. Let t = t + Δt
3. Set the iteration counter, m=0
4. Let m=m + 1
5. Evaluate K1 and K2 using equations 6a and 6b respectively
6. Let $d_1 = d_t$
7. Compute $d2 = (K2 - K1.d1^{5/3})/100$
8. If (ABS (d2-d1) .LESS THAN OR EQUAL TO. 1.OE-5), then goto 10
9. Set d1=d2 and goto 7
10. Put $d_{t+\Delta t}$ = d2, compute $q1 = (S^{1/2}/n)d^{5/3}_t$; $q2=(S^{1/2}/n)d^{5/3}_{t+\Delta t}$
    compute q=0.5 (q1 + q2), print or store the value of q for further use.
11. If t > tmax, then stop
12. Goto step 2

The parameters required in the stream flow computation are the stage and the discharge at any point along the stream at any time step. These can be evaluated by making use of the continuity and momentum equations available for onedimensional stream flow. These equations are

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} - q = 0 \qquad (7)$$

$$\frac{\partial Q}{\partial t} + \frac{\partial (VQ)}{\partial x} + gA\left(\frac{\partial H}{\partial x} + S_f\right) = 0 \qquad (8)$$

where A is the area of cross section of flow in the stream, Q is the discharge in the stream, q is the lateral inflow into the channel per unit length, V is the mean velocity of flow, S_f is the friction slope, H is the water surface elevation, x is the distance measured along the stream. The momentum equation (8) can be rewritten as

$$g \; A \; [a + (\partial H/\partial x) + S_f] = 0 \qquad (9)$$

where $a = [(\partial Q/\partial t) + \{ \partial (VQ)/\partial x \}]/gA$ (10)

which gives $S_f = - [(\partial H/\partial x) + a]$ (11)

By using the Manning's formula, $Q = A.R^{2/3}.S^{1/2}/n$ (12)

The expression for discharge is given as, $Q = K\,[(\partial H/\partial x) + a]$ (13)

where, $K = -(A.R^{2/3}/n)/\,|\,[(\partial H/\partial x) + a]\,|^{1/2}$ (14)

In this model, if gAa = $\partial (VQ)/\partial x$, then it is known as convective acceleration model,

= $\partial Q/\partial t$, then it is known as local acceleraton model,

= $[(\partial Q/\partial t) + \partial (VQ)/\partial x]$, then it is known as coupled model,

= 0 , then it is known as diffusion hydrodynamic model.

Combining equations 7 and 13, the resulting equation is

$$\frac{\partial A}{\partial t} = \frac{\partial}{\partial x} \quad K\left(\frac{\partial H}{\partial x} + a\right) + q \tag{15}$$

Assume that a = 0, then equation 15 becomes

$$\frac{\partial}{\partial x}\left(K\frac{\partial H}{\partial x}\right) + q = \frac{\partial A}{\partial t} \tag{16}$$

For a segment of constant width W, the equation 16 becomes [2]

$$W\frac{\partial H}{\partial t} - \frac{\partial}{\partial x}\left(K\frac{\partial H}{\partial x}\right) - q = 0 \tag{17}$$

The equation 17 can be linearised over the domain of interest by using the Galerkin's criterion, that is

$$\int [N]^T \left\{W\frac{\partial H}{\partial t} - \frac{\partial}{\partial x}\left(K\frac{\partial H}{\partial x}\right) - q\right\} dx = 0 \tag{18}$$

where [N] is the interpolation function defined as

$$H \approx [N]\ \{H\} \tag{19}$$

Using the linear interpolation functions for line element, the [N] will be [1-(x/L), (x/L)]. Assuming that H can be written as $H=(H_{t+\Delta t} + H_t)/2$, the equation 18 for one element reduces to :

$$\begin{bmatrix} \frac{2LW}{6\Delta t} + \frac{K}{2L} & \frac{LW}{6\Delta t} - \frac{K}{2L} \\ \frac{LW}{6\Delta t} - \frac{K}{2L} & \frac{2LW}{6\Delta t} + \frac{K}{2L} \end{bmatrix} \begin{Bmatrix} H_1 \\ H_2 \end{Bmatrix}^{t+\Delta t} = \begin{Bmatrix} \frac{LW}{6\Delta t}(2H_1^t + H_2^t) \\ \frac{LW}{6\Delta t}(H_1^t + 2H_2^t) \end{Bmatrix}$$

$$+ \begin{Bmatrix} \frac{qL}{2} + \frac{K}{2L}(H_2^t - H_1^t) + Q_{in} \\ \frac{qL}{2} + \frac{K}{2L}(H_1^t - H_2^t) - Q_{out} \end{Bmatrix} \quad (20)$$

The element equations can be formed and asembled into global form. The global matrix can be solved only after substituing proper boundary conditions. This can be in the form of specifying either the upstream water surface elevation or the down stream water surface elevation if they are known., If no information is available, it is safe to assume that critical or normal flow conditions exist at the downstream end [1] and accordingly, the water surface elevation can be fixed and put into the global matrix. Though the procedure is finite element method in space domain, it is essentially the finite difference method in the time domain and hence the selecton of time step plays vital role on the convergence of the results. A time step, Δt=300 sec. was used in this procedure. Computations were carried out using time steps upto Δt=900 sec. and the results are close to those obtained by using a time step Δt=300 sec.

SOLUTION PROCEDURE

The computational procedure is as follows :

1. Discretize the overland domain into smaller elements according to their land use and surface conditions.
2. Discretize the stream into proper segments.
3. Read the stream and overland parameters, initial flow conditions, net-rainfall hyetograph. Let t=0, choose proper value of time step.
4. Put t=t+ Δt.
5. Compute the overland flow, q.
6. Formulate the element matrices and assemble them into global form.
7. Compute the critical or normal depth downstream and substitute it as boundary condition. Solve for $\{H\}$ values at time step t + Δt.
8. If t>tmax, then stop.
9. Put $\{H\}_t = \{H\}_{t+\Delta t}$, print results and goto step 4.

CASE STUDY

The model developed here was applied to a catchment in Western Peninsular India of area about 102.2 sq. km. with rain gauge stations as shown in Fig. 1. for which the observed data is available. The catchment is discretized into 15 overland elements and 7 stream segments with 8 nodes as shown in Fig. 2. Considering one particular storm event and the corresponding observed hydrograph, the Manning's roughness coefficient to be used for overland flow and stream flow were calibrated. The observed hydrograph, the weighted average excess rainfall hyetograph and the computed.

hydrograph are shown in Fig. 3. The computed hydrograph for spatially varied excess rainfall is also shown in Fig. 3.

For the second storm event, the spatially varied rainfall excess and the same roughness coefficients as inputs, the hydrograph was generated which is as shown in Fig. 4 along with the observed hydrograph. It can be seen that the peak of the computed hydrograph agree reasonably well with the observed one. In larger catchments, the infiltration appears as delayed runoff which sometimes may not be ignored. Hence a coupled model using surface runoff and subsurface runoff may yield better results but the data base should be on a grand scale to achieve this objective.

In conclusion, it can be said that the model depicted here can be used to simulate the hydrograph using the net rainfall and catchment parameters as input for small catchments.

ACKNOWLEDGEMENTS

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NOTATION USED

a - Parameter to describe the nature of the model
A - Area of cross section of the channel
A_c - Area of the catchment segment for overland flow
d - Depth of overland flow in mm,
g - Acceleration due to gravity
H - Water surface elevation
K - Conveyance parameter
K1 & K2 - Coefficients in the overland equations
L - Length of the stream segment
n - Manning's roughness coefficient
N - Interpolation function
P - Excess precipitation after accounting for infiltration and other losses in mm.
q - Overland flow into the channel pet unit length of stream segment
Q - Discharge in the stream
R - Hydraulic radius in channel flow
S - Slope of the overland plane
Sf - Friction slope
t - Time elapsed
Δt - Time step used
V - Velocity of stream flow
$\forall$ - Volume of water detained over the overland plane
W - Width of the stream segment
x - Coordinate measured along the stream.

REFERENCES

1. Freeze R.A. (1978) 'Mathematical Models of Hill-Slope Hydrology', in "Hill-Slope Hydrology" Edited by Kirkby M.J., John-Wiley and Sons, New York.

2. Hromadka O.T.V., Nestilinger A.J. and Devries J.D. (1986) 'Comparisons of Hydraulic Routing Methods for Channel Routing Problems', HYDROSOFT-86, Hydraulic Engineering Software, Proceedings of the Second International Conference held at Southampton, England.

3. King I.P. (1976) 'Finite Element Models of Flow Routing Through Irregular Channels', Proceedings of the First International Conference on Finite Elements in Water Resources held at Princeton University, New Jersey, USA.

4. Li, E.A., Shanholtz, V.O., Cntractor, D.N. and Carr, J.C. (1977) 'Generating Rainfall Excess Based on Readily Determinable Soil and Landuse Characteristics',Trans. American Society of Agricultural Engineers, Vol. 20, No. 6, pp 1070-1077.

5. Nwaogazie I.L.F. and Tyagi A.K. (1984) 'Unified Stream Flow Routing by Finite Elements', journal of Hydraulics Div., ASCE, Vol. 110, No. HY11, PP 1595-1611.

6. Ross, B.B., Contractor, D.N. and Shanholtz, V.O., (1979) 'A Finite Element Model of Overland and Channel flow for Assessing the Hydrological Impact of Landuse Change', journal of Hydrology, Vol. 40 ; pp 11-30.

7. Vasudeva Rao B. (1986) 'Design of Drainage Channel in Kalamboli Region', Report Submitted to the City and Industrial Development Corporation (CIDCO), New Bombay.

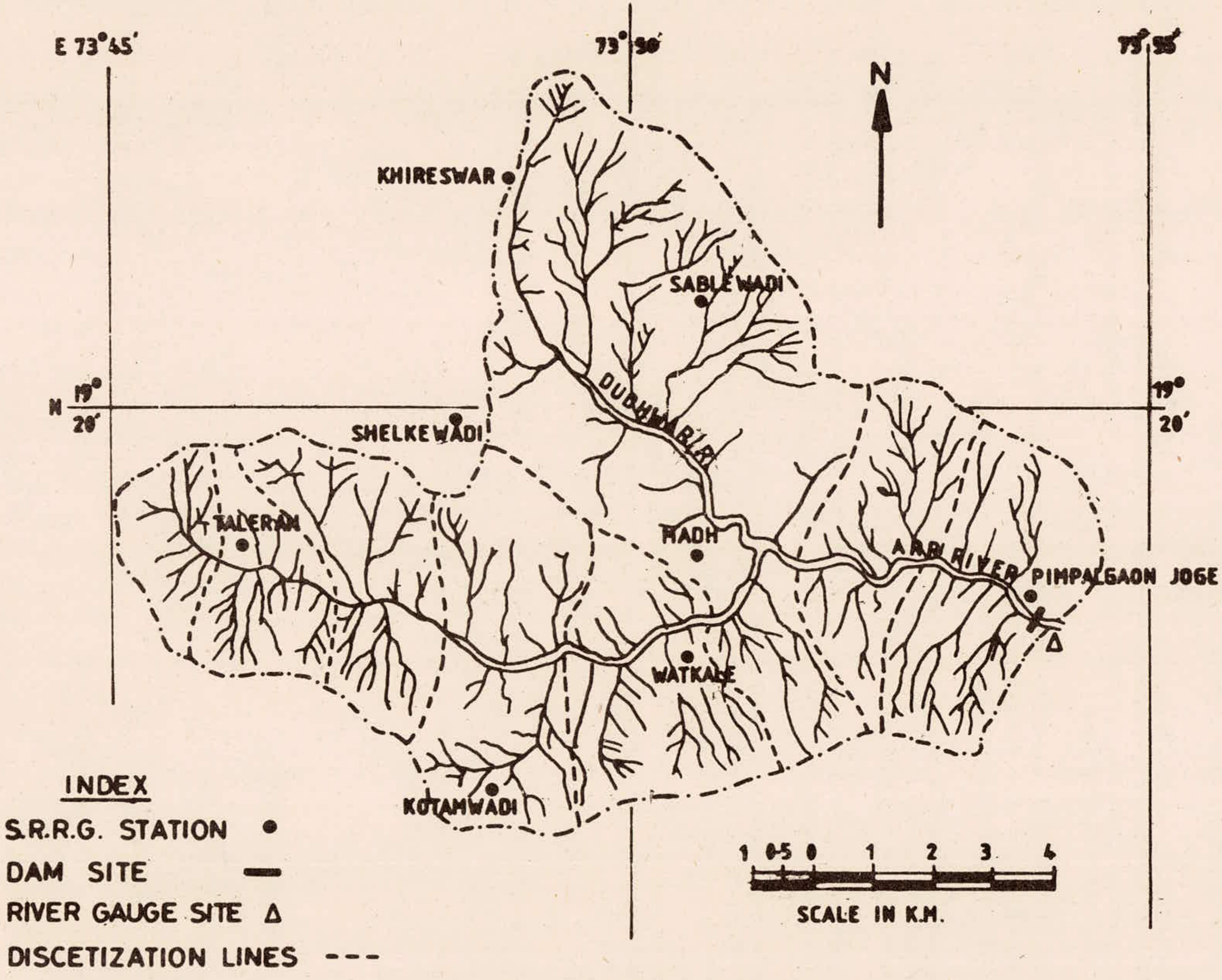

FIG. 1. PIMPALGAON JOGE CATCHMENT SHOWING RAINGAUGE STATIONS

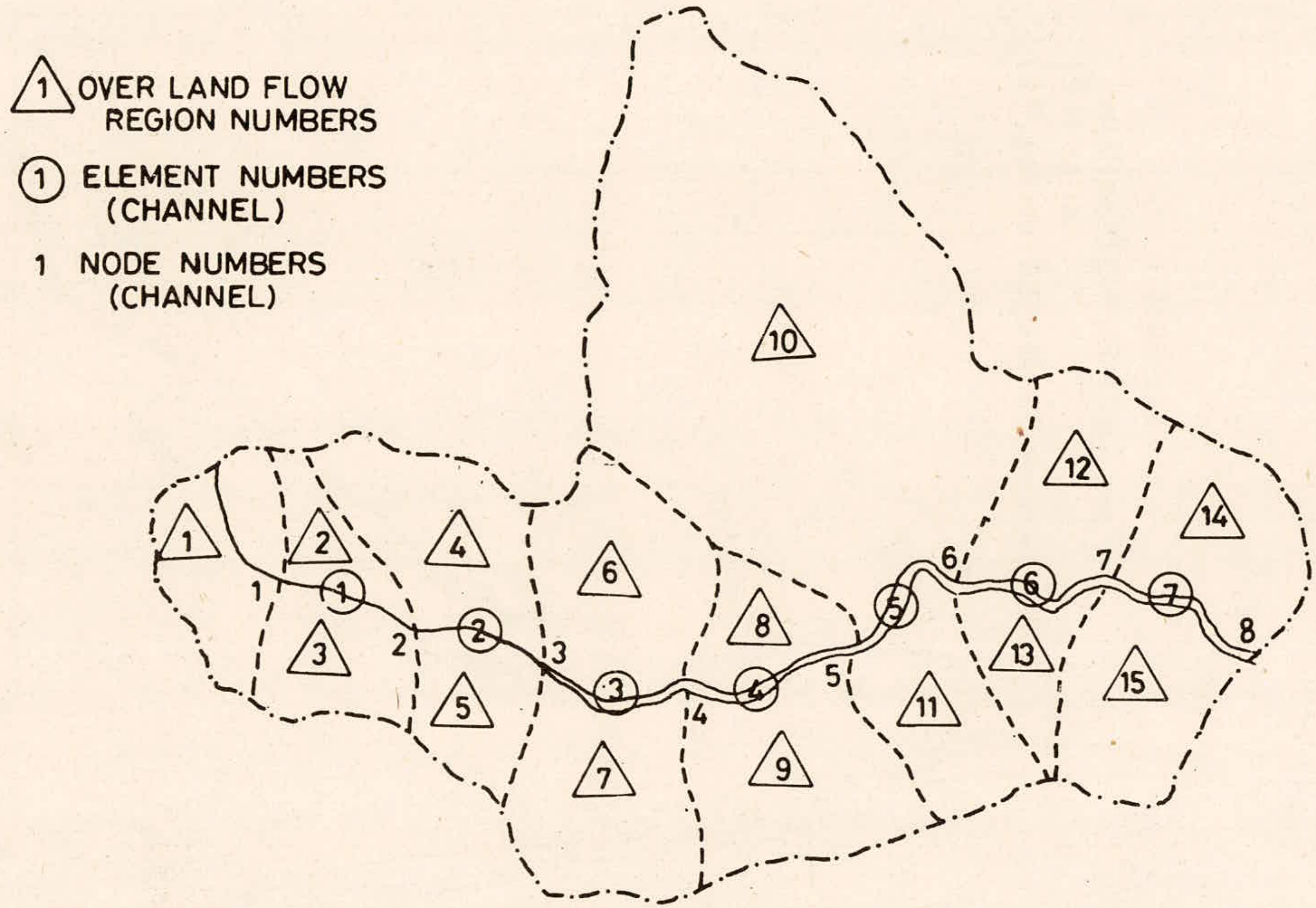

FIG. 2. CATCHMENT DISCRETIZATION

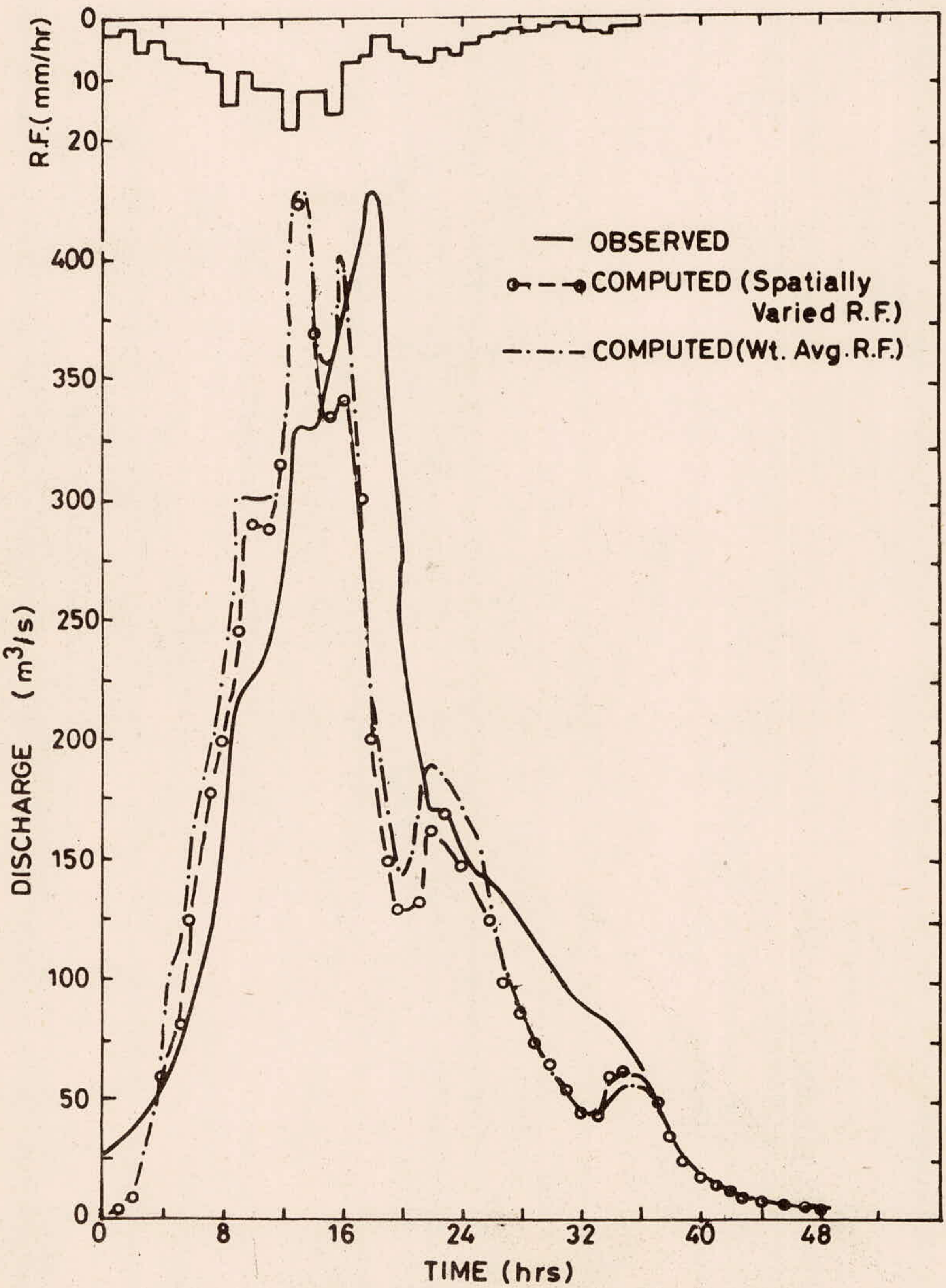

FIG. 3. COMPARISION OF OBSERVED AND COMPUTED HYDROGRAPHS FOR STORM 1.

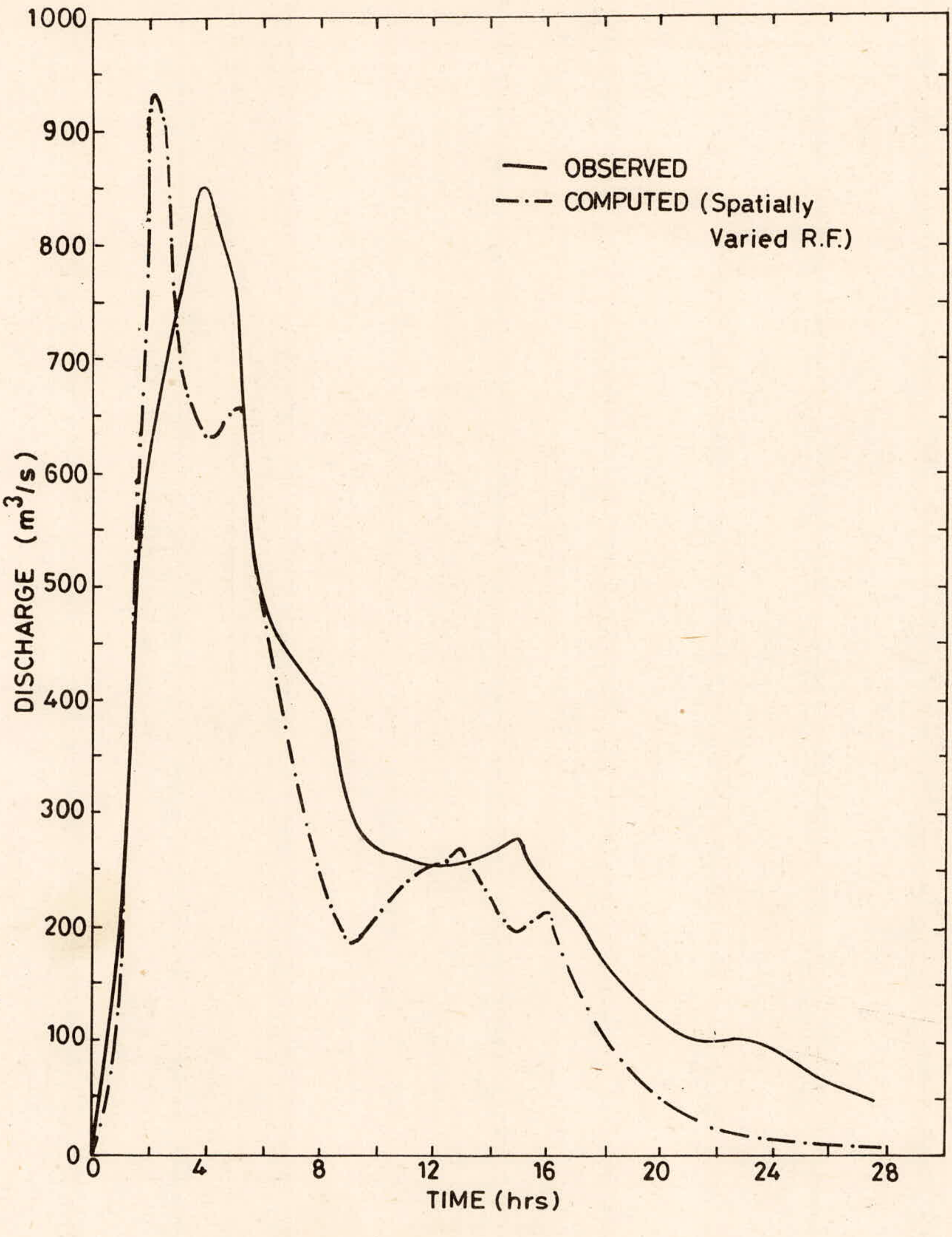

FIG. 4. COMPARISION OF OBSERVED AND COMPUTED HYDROGRAPHS FOR STORM 2.