CS-7

COMPARATIVE STUDY OF UNIT HYDROGRAPH METHODS

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ABSTRACT

This study has been conducted to investigate the suitability of using various methods for deriving unit hydrographs for the small catchments. Two approaches, parametric system synthesis (conceptual models) and non-parametric system analysis, have been used in the study In all five methods three in the former approach (Nash Model, Clark Model and Singh's Model) and two in the later approach (Collin's Method, Least Squares Method) have been tested. The flood events of six small catchments (Br. No.807/1, 51, 604/2, 969, 228 and 566) of Godavari basin subzone 3f have been analysed using the five methods. The average unit hydrographs for each catchment are estimated using the average parameters obtained by taking the geometric means of the parameters of the unit hydrograph for individual storm for the conceptual model approach. However, the standard averaging procedure is used to estimate the average unit hydrographs for the Collins and least squares methods. The performance of these methods in reproducing these storms of respective catchments are judged by comparing the values of error functions (for peak, time to peak and hydrograph) evaluated with the average unit hydrograph. The techniques which give best results depending upon the criterion of each of the error functions for a particular catchment are given below:

Catchment Br.No.	Catchment Area km^2	Best technique for error functions			
		*EFF	** SE	***PAEP	PAETP
807/1	823.62	N	N/C	M	CL
228	483.03	N N	N	M	N
604/2	340.52	N	N/C	CL	M/S
969	208.49	C	C	M/C	N
566	137.21	CL	CL	M	M/C
51	86.76	C/CL/S	CL	C	M

Where *EFF-Efficiency of the method, **SE-Standard error, ***PAEP-Percentage absolute error in peak, ****PAETP-Percentage absolute error in time to peak and

C- Collin's Method, M-Matrix Method, N-Nash Model, CL -Clark Model, and S-Singh's Model.

It can be sen from the above table that the Nash Model is relatively more efficient and gives less standard error for the catchments of large size (area ranges between 340-823 sq km). It has also been realized that the methods based on parametric system synthesis give non-oscillating unit hydrograph which are physically realizable. The methods based on the non-parametric system analysis approach sometimes involve the problem of fluctuations in the derived unit hydrograph making it difficult to decide the ordinates of the unit hydrograph.

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1.0 INTRODUCTION

The engineers, involved in the design of storm sewers, highway drainage, spillways, diversion works, bridges, culverts and other flood control works, often require the design peak rate of discharge at a certain location and time taken for the flood peak to reach that location. The majority of the hydraulic structures mentioned above are constructed on small watersheds. The size and cost of hydraulic structures are largely determined by the design flow. Over estimation of design flow can be costly for initial construction and under estimation of design flow can be costly for repairs and dangerous to people and down stream facilities. Therefore,the accurate estimation of the design flood hydrograph and peak discharge is an important task for the engineers and especially to the hydrologists. Unit hydrograph approach, which assumes the watershed as a linear time invariant system, is a simple tool to relate the excess rainfall as an input and the direct surface runoff as an output to the watershed as a system. The problem remains to identify the system which leads to the determination of the IUH(instantaneous unit hydrograph) or the TUH (T-hour unit hydrograph). The following two approaches are used in this study for the determination of the T-hour unit hydrograph:

(i) Parametric system synthesis

This requires the use of conceptual linear models. These models evaluate the system response in terms of certain number of parameters which can be estimated from the given input and output data.

(ii) Non-parametric system analysis

In this approach the input and output are related in the form of a mathematical expression through the system response function.

It is to be noted that the system response is a continuous mathematical function while using parametric system synthesis approach and since the system response is a continuous function, therefore it can be termed as an IUH or the impulse response. In the case of the non-parametric approach, the discritization interval Δt defines the response function and it becomes the unit pulse response rather than impulse response i.e.compute a Δt-duration unit hydrograph.

2.0 REVIEW

The unit hydrograph concept was given first in 1932 by Sherman since then a number of techniques for finding the IUH or TUH have been investigated by many hydrologists and engineers. These techniques have served to strengthen the approach as a practical tool rather than to alter the basic assumptions of the unit hydrograph. The methods available in literature are mainly based on two approaches-parametric systems synthesis and Non parametric system analysis approach. The methods based on non parametric system analysis approach may be classified in five groups:

(i) conventional methods,

(ii) matrix methods,

(iii) transform methods,

(iv) time series methods and

(v) linear programming methods.

The methods based on parametric system syntheis approach may be classified in two groups:

(i) Conceptual models and

(ii) Synthetic unit hydrograph

2.1 Non Parametric System Analysis

2.1.1 Conventional methods

(a) Sherman's unit hydrograph

The concept of the unit hydrograph, introduced by Sherman (1932), a member of the original Boston Committee on floods. He was the first to consider the existence of a unique direct runoff hydrograph for a storm of given duration and volume over a particular watershed. This unique response, termed the unit hydrograph or unit graph, was originally defined as the hydrograph representing a selected volume of surface runoff from a given basin for one day rainfall. The hypothesis on which Sherman(1932) based the unit hydrograph was"that in any drainage basin, surface runoff from rainfall that is distributed with satisfactory uniformity as to area and time that occurs in a given unit of time will produce hydrographs in which the bases are approximately equal and the ordinates vary directly with the intensity of net rainfall". The prefix 'unit' referred to the specified unit of time in which storm occurred and not to the runoff-volume which has often been chosen to be a unit depths over the watershed. Sherman(1932) further found that all ordinates of the hydrograph were proportional to the volume of surface runoff that direct runoff hydrographs were independent of previous runoff events and simple hydrographs could be linearly superimposed to constitute complex hydrographs. Bernard(1935) expressed the unit hydrograph in terms of the percentage of direct runoff which occured in successive time intervals. He termed his hydrograph a distribution graph. Smith(1941) modified Bernard's approach by employing a smooth curve through the discharge points and used instantaneous distribution coefficients.

(b) Unit hydrograph derived from S-curve

Mergan and Hullinghors(1939) introduced the concept of the S-curve to present the watershed response to a continuous uniform rainfall. The algebric difference between two S-curves lagged by a selected time interval was shown to yield a unit graph for a storm of duration equal to the lag interval. Cuenod(1956) referred to the S-curve as characteristics hydrograph. Like unit graphs, S-curves were indices of watershed response.

(c) Collin's method for unit hydrograph derivation

Collin's (1939), and later Fekette(1954), employed trial and error procedures for analysing complex hydrographs. An assumed unit graph was employed on all rainfall events except the largest. Then a unit graph was obtained from the largest event and compared with assumed one. The procedure were repeated until the assumed and computed unit graphs agreed favourably.

(d) Barne's method for unit hydrograph determination

Barne's (1959) used, what he termed, a method of progressive addition which involved successive estimation of the distribution co-efficients with continuous checking of the estimated and actual hydrographs.

2.1.2 Matrix methods

(a) Matrix inversion method

All matrix method utilize the equation [Y]=[Y][h] as a system of working equation, where [Y] is direct runoff vector, [h] is unit hydrograph vector and [X] is a matrix of effective rainfall which size depends upon no. of excess rainfall and direct surface runoff ordinates.

For this system a straight forward solution ca be obtained by inversion,

$$[h] = [X]^{-1} [Y] \qquad \ldots(1)$$

provided [X] is a square matrix and non singular. In general this method is not recommended when the number of equations is large (Burden, Faires and Reinalds(1978)). This method can be effective for computing the UH provided the system of equations is not ill conditioned and the data are error free. However, there is no way to insure satisfaction of these conditions apriori.

Further, a direct solution of equation: [Y]=[X][h] may give rise to the numerical problem of ill conditioning. To measure it a residual vector[r] is defined as:

$$[r] = [Y] - [x]\,[\tilde{h}] \qquad \ldots(2)$$

where $[\tilde{h}]$ is an approximation of the exact solution [h]. Intuitively when any norm of r, $||r||$,is a small number then $||h-\tilde{h}||$ will also be small. However, in extreme cases this may not be the case. Then the following inequality can be used as an upper limit of the norm.

$$||h-\tilde{h}|| = K(x) \frac{||r||}{||x||} \qquad \ldots(3)$$

where K(x) is the so called condition number equal to [$||x||\,||x^{-1}||$]. It indicates whether [x] is ill-behaved (big number) or well behaved (small number) matrix.

(b) Forward Substitution Method

Dooge(1973) employed the forward substitution method the computation of subsequent ordinates of the UH by going down the rows of the system equation i.e.

$$Y_1 = h_1 X_1$$

$$Y_2 = h_1 X_2 + h_2 X_1$$

$$Y_3 = h_1 X_3 + h_2 X_2 + h_3 X_1 \qquad \ldots(4)$$

etc.

The method is found highly sensitive to data errors.

(c) Successive Over-relaxation Method

Newton and Vinyard(1967) provided this method which is an iterative one having its working set of equations as:

$$h_i^{(K)} = (1-w)\, h_i^{(K-1)} + \frac{w}{X_i} \left[Y_i - \sum_{j=1}^{i-1} X_{i-j+1} h_j^{(K)} \right.$$

$$\left. \sum_{j=i+1}^{N} X_{i-j+1} h_j^{(K-1)} \right], \quad i = 1,2,.....N \qquad ...(5)$$

where w is a parameter whose optimal value must be known, K denotes the order of iteration and N the number of observed discharge. Note that X for zero or negative subscript will vanish. Dooge(1973) recommended to compute number of UH ordinates, N, using the equation:

$$M = N - i + 1$$

where N and i are numbers of ordinates of [Y] and [X] respectively. A significant improvement in the results was obtained by increasing M to N in this study.

(d) Least square method

Snyder (1955) first time used this method to obtain the distribution coefficients of distribution graph. Least square method can be used to determine the ordinates of an optimal UH when the number of equations exceeds the number of the UH ordinates. The equations to be solved for this may be written as:

$$[h] = [[X]^T[X]]^{-1}[X]^T\,[Y] \qquad ...(6)$$

It was seen by various investigations (Delluer and Rao, 1972, N.E.R.C. 1975 Papazafirious,1976) that the recession limbs of the unit hydrographs derived from isolated rainfall runoff events were having osciallations using least square method mainly due to ill conditioning of the set of equations. Bree(1978) studied the causes of ill conditoning of the set of equations and showed that the ill conditioning was produced by collenearity in the excess rainfall ordinates. Diskin and

Boneh(1975) proposed a method to determine the catchment unit hydrograph analysing the several rainfall-runoff events simultaneously with the objective of minimizing the sum of error squared differences between the predicted and observed quick response runoff for all the storm events. Bree(1978) found that by analysing more events simultaneously tendency to produce oscillations was reduced. Mawdslay J.A.and Tagg A.F.(1981) considered an alternative technique for solving the over determined set of simultaneous linear equations which describe the multi event procedure-this technique is known as the House Holder Method. Although they solved the unconstrained least squares minimization problem but suggested that the conditions i.e.non negative UH ordinates and equal volume of rainfall excess and predicted runoff could be imposed subsequently if they did not ocur naturally in the solution. Bruen and Dooge(1983) proposed an efficient and improved estimation algorithm to derive physically realizable unit hydrograph. They solved the equation: $[X + \alpha I][h]=[Y]$ instead of $[X] [h] = [Y]$ where α was termed as regularisation factor supplied by the users and I is an unit matrix. They suggested that the procedure may be used to derive unit hydrograph from single as well as multiple event data. The use of different values of regularization factor resulted the unit hydrograph without oscillations.

2.1.3 Transform methods

(a) Harmonic analysis

O'Donnel(1960,1966) proposed the IUH derivation by harmonic analysis. The effective rainfall hyetograph, the IUH and the surface runoff hydrograph were each represented by sum of a harmonic series, each series having the same fundamental time period equal to or greater than the time of storm runoff. He derived the harmonic coefficients of IUH from the harmonic coefficients of a curve of excess rainfall and the resultant runoff hydrograph after relating the coefficients of nth harmonic of the three series by simple equations. Let functions Y,X and h are having their fourier coefficients as $(A_i, B_i), (a_i, b_i)$ and (α_i, β_i), i=1,2...The coefficients (α_i, β_i) can be expressed in terms of (A_i, B_i) and (a_i, b_i) as:

$$\alpha_i = \frac{2}{T} \frac{a_i A_i + b_i B_i,}{a_i^2 + b_i^2} \quad i \geq 1 \ ; \alpha_0 = \frac{1}{T} \frac{A_0}{a_0} \qquad ...(7a)$$

$$\beta_i = \frac{2}{T} \frac{a_i B_i - b_i A_i}{a_i^2 + b_i^2}, \quad i = 1,2,..... \qquad ...(7b)$$

Analytical form of the UH resulting from harmonic analysis can then be expressed as

$$h(t) = \sum_{i=0}^{\infty} \alpha_i \cos \frac{2\pi t}{T} i + \sum_{i=1}^{\infty} \beta_i \sin \frac{2\pi t}{T} i \qquad ...(8)$$

where T is the assumed period of recurrence of rainfall runoff event and is normally equal to the duration of direct runoff.

(b) Laguerre polynomials

Laguerre polynomials have been investigated principally by Dooge (1965,1973). Laguerre orthogonal functions can be defined as

$$L_m(t) = \exp(-t/2) \sum_{k=0}^{m} (-1)^K \binom{m}{K} \frac{t^K}{K!} \quad m=0, 1,2, \qquad ...(9)$$

If the functions Y,X and h of equation(1) are expanded in terms of Laguerre functions as:

$$f(t) = \sum_{m=0}^{m} a_m \; L_m(t) \qquad \ldots(10)$$

for some number m, then the relation between Laguerre coefficients of h and those of X and Y can be derived as

$$\alpha_m \, a_o = \sum_{K=0}^{m} A_K - \sum_{K=1}^{m} a_K \, \alpha_{m-K}, \quad m = 0, 1,2\ldots \qquad \ldots(11)$$

where α_k are the Laguerre coefficients of h, a_k the coefficients of X and A_k the coefficients of Y.

(b) Meixner Polynomials

Dooge (1973) suggested that the Meixner Polynomials are the discrete analog of Laguerre Polynomials and have the form

$$M_n(S) = \sum_{K=0}^{n} (-1)^K \binom{n}{K} \binom{s}{K}, \quad n=0, \; 1,2,\ldots \qquad \ldots(12)$$

for s = o, 1,2,... If a function f(s) is expanded in terms of these polynomials,

$$f(s) = \sum_{m=0}^{n} \alpha_m \, (1/2)^{(s+m+1)/2} \, M_m(s), \quad s=0,1,2\ldots \qquad \ldots(13)$$

where α_m are the expansion coefficients. Thus for the UH these coefficients can be expressed as

$$\alpha_p \, a_0 = \sum_{k=0}^{P} (1/2)^{(p-k+1)/2} A_k - \sum_{k=0}^{p-1} a_{p-k}, \quad p = 0,1,2 \qquad \ldots(14)$$

where α_p are the Meixner coefficients of h, a_k the Meixner coefficients of x and A_k the Meixner coefficient of Y.

2.1.4 Time series methods

In this method, Mejia R and March (1966) used a black-blox analysis, where the effective rainfall and direct runoff were viewed as time series and were described in terms of their auto correlation and cross correlation functions. An analysis gave rise to a system of linear equations which can be written as

$$\sum_{k=1}^{n} h_k \sum_{i=1}^{2p+2-k} x_i X_{i+K-j} = \sum_{i=1}^{2p+1} x_{i-j+1} Y_i \ , \ j=1,2,.(2p-1) \qquad ...(15)$$

where n is the number of ordinates of the vector [h] and 2p+1 the number of ordinates of [Y]. Any ordinates of X with zero or negative argument is taken to vanish. Equation(15) can be solved by the least squares method.

Bayazit(1966) described the use of spectral analysis and its numerical application of IUH derivation.

2.1.5 Linear programming method

This method (K P Singh,1976; Mays and Coles,1980) computes the UH in discrete time form but, unlike preciding methods, it can use data of several events at a time. The resulting [h] is an optimal solution using simpler method for the specified constraints. This method can impose desirable constraints on the solution such as to insure non-negative values of h or preserve monotonic nature of h on both sides of the peak.

However, the simplex method can give rise to a computer storage problem. One reason is the introduction of slack variables. The system of equations can be written as

$$Y_j = h_1 X_j + h_2 X_{j-1} + \ldots .. h_j x_1 + \varepsilon_j, \quad j=1,2,\ldots \qquad ...(16)$$

$$\sum_{j=1}^{m} h_j = 1/\Delta t \qquad ...(17)$$

where ε is the difference between observed DSRO (direct surface runoff and computed DSRO and may be positive, zero or negative and Δt, the duration of the UH. However, for linear programming, it must be non-negative. This problem is circumvented by Deininger(1969) by using

$$\varepsilon_j = u_j + V_j \qquad \ldots(18)$$

where u_j and V_j are the two non negative slack variables that account for positive, zero and negative difference. The objective, then, is to minimize E

$$E = \sum_{j=1}^{n} (u_j + V_j) \quad -- \qquad \ldots(19)$$

Constraints can be imposed to insure non negative h as well as its monotonic behaviour before and after its peak.

2.2 Parametric System Synthesis Approach

2.2.1 Conceptual models

Any discussion of hydrologic models must start with a definition of the system that the models are supposed to represent. A system can be defined (Diskin, 1970) as an ordered assembly of interconnected elements that transform, in a given time reference, certain measurable inputs into output. Inputs and outputs are usually represented as functions of time. These functions may be continuous or discrete. The simple conceptual models for unit hydrograph derivation require the specification of a system for which excess rainfall is the input and direct surface runoff is the output. The system of this kind is known as the surface runoff system and the models used to synthesize the surface runoff system are known as surface runoff system models which may be classified in three groups: (i) Linear models,(ii) Quasi linear models, and (iii) NOn-linear storage models.

(i) Linear models

Linear models treat the system to be linear. The linear system obey principle of superimposition and proportionality. By considering the individual inputs to a linear system to be in the form of pulses of finite magnitude and short duration, it can be shown that any linear system can be represented by the convolution integral operator.

$$y(t) \int_{o}^{t} x(\tau)\, h(t-\tau)\, d\tau \qquad ...(20)$$

where h(t) is the unit impulse response.

If the same input, whenever it is applied to the system always to produce the same output, the system is time invariant system otherwise it is time variant system. The system may be termed as lumped system when input function does not involve spatial co-ordinates. The system whose input function involves spatial coordinates is termed as distributed system.Some of the conceptual linear models are briefly described as follows:

(a) Sato and Mikkawa model

Sato and Mikkawa (1956) published a runoff routing method for the transformation of successive hourly rates into a discharge hydrograph for a small river in Japan. This routing equation is based on the second order Gamma distribution as a fundamental runoff function:

$$u(t) = \frac{1}{K}\,\frac{t}{K}\, e^{-t/k} \qquad ...(21)$$

For an inflow of $f_o r$ unit per hour the flow rate ($t \geq 1$) can be written as:

$$q(t) = f_o r\, [\, e^{-\frac{(t-1)}{K}} \left(\frac{t-1}{K} + 1\right) - e^{-t/K} \left(\frac{t}{K} + 1\right)] \qquad ...(22)$$

here f_o is a runoff coefficient

Sato and Mikkawa found that a series of terms like equation (22) could well describe the discharge from the drainage basin caused by a one hour rain of depth r.

$$q = F_1(t) + F_2(t) \ldots\ldots$$

or

$$= f_1 r \{e^{-(t-1)/K_1} (\frac{t-1}{K_1} + 1) - e^{-t/K_1} (\frac{t}{K_1} + 1)\}$$

$$+ f_2 r \{e^{-(t-1)/K_2} (\frac{t-1}{K_2} + 1) - e^{t/K_2} (\frac{t}{K_2} + 1)\} \qquad \ldots(23)$$

This runoff system can be simulated by the hydraulic analog in figure 1. The term of this series of second order Gamma distributions

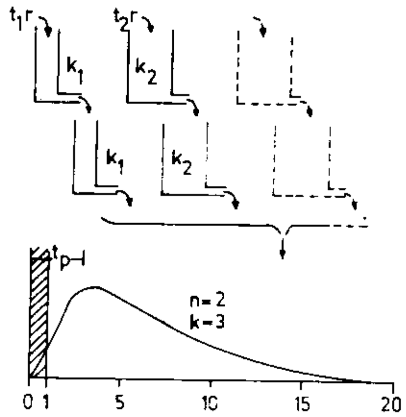


FIGURE 1 - SATO AND MIKKAWA MODEL

are of decreasing magnitude and Sato and Mikkawa found that two or three terms gave results of sufficient accuracy. In a final note the writers state that the n-th order Gamma distribution is a suitable element for the characterization of runoff in any river.

In their 1956 paper Sato and Mikkawa indicated that K_1 is to be found as the time to peak in the hydrograph . The writers apparently assumed that the time of inflow is so short that the hydrograph is practically identical to an instantaneous hydrograph and that the peak caused in the first branch of the model will be dominant. In 1959 Takenouchi describes Sato's method and he states that K_1 should be computed from the equation:

$$t_p = \frac{e^{1/K_1}}{e^{1/K_1} - 1} \qquad \ldots(24)$$

Which is the time to peak for a one hour hydrograph of the first branch of the model.

(b) Rational method

According to Dooge(1959) it was Mulvaney who in 1851 proposed a method that is known as the rational method based on translation approach. It is assumed in the method that the effect of rainfall on the most remote part of the basin takes a certain period, the time of concentration T_c, to arrive at the outlet. This time of concentration can either be derived from correlations with basin characteristics or it can be computed from the times of flow in successive "bankfull" reaches of the main channel. It is further assumed that a constant intensity of excess rainfall CI occurs, uniformly spread over the area A, where C is a runoff coefficient and I is rainfall intensity. If this rate of input, a step function, continues until the time of concentration T_c has expired, the excess rainfall that fell on the remotest point of the drainage basin will just begin to cause a reaction at the outflow so that the latter will have reached its ultimate and maximum rate Q = CI A.

If it is decided that the design flow rate Q may be exceeded on an average of once in N years, rainfali intensity/ duration formulas or graphs are used to find the average rainfall rate I for the period T_c to be exceeded with an average return period on N years (figure 2).

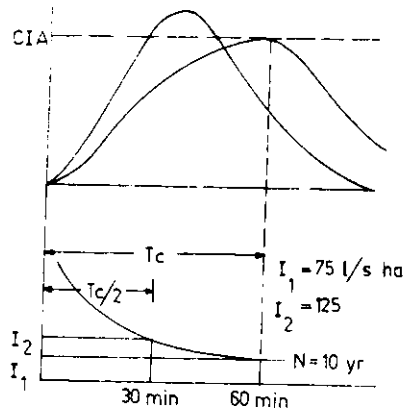


FIG 2-RATIONAL METHOD

One fundamental weakness of this method comes out when the growth of Q over T_c to its final value Q=CIA is considered. This growth can be represented by an s-curve the ordinates of which have been multiplied by CIA. The shape of this curve is determined by the basin's geometry and topography.

The T_c hydrograph and 1/2 T_c hydrograph, both caused by rainfall intensities of the same probability 1/N.are shown in figure 3. The outflow due to the average rainfall rate I_2 with the same recurrence interval of N years for a period 1/2 T_c will be higher than the rate I_1 for the total time of concentration T_c as the average rainfall rate I is coniderably higher than the rate I_1.

Later on, the rational method was modified and concept of time area diagram was introduced. The travel time to the outlet are determined for a number of points in the drainage basin using the hydraulic features of the bankfull channel system and time contour lines with equal time intervals are drawn. If it is assumed that an instantaneous excess rainfall of unit depth occurs simultaneously on all points of the basin, the excess rainfall on the elementary area between the time contour lines t and t+1 will arrive at the outlet between t and t+1 and will be represented by the appropriate part of the IUH situated over this interval. This hydrograph can be called the time area diagram or curve. Dividing all ordinates by the number of surface units A will yield the IUH according to the modified rational method.(Fig.3).

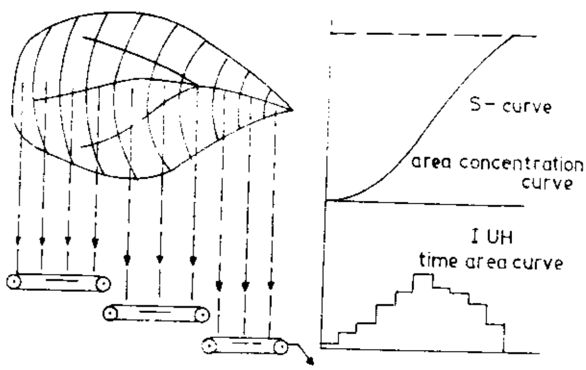


FIG. 3-MODIFIED RATIONAL METHOD

The modified rational method shows a marked improvement when compared with the rational method. The method can transform the design storm to an outflow hydrograph since it is not restricted to a constant input over the critical period.

The topography of the basin may indicate that a certain pattern of areal distribution instead of a uniform rain must be considered as critical. For that case the elementary areas between the time contour lines should be weighted accordingly and this will result in a time area diagram that is adjusted for the variation in rainfall intensity (Dooge,1959). It is to be noted that both the rational method and modified rational method consider the translation of excess rainfall through a system of linear channels for which the time of travels are independent of discharge rates. Nash (1958) applied the modified rational method to a number of natural drainage basins where actual time distributions of excess rain and outflow rates were available. Comparison of computed and observed hydrographs, however, showed a serious over estimation of flood peaks.

(e) Zoch model

Zoch (1934,1936, 1937) presented a runoff model which consists of a linear storage that was fed by a rectangular block input of uniform excess rain. He also presented solutions for triangular and elliptic inputs.

These inputs can be considered as the effect of translation in particular basins(which have the appropriate shape and topography) on an instantaneous excess rainfall. In that case the input diagrams represent the respective time area curves.

(d) Clark model

Clark(1945) used the idea given by Zoch and presented an IUH that was obtained by routing the time area curve through a linear storage. He first calculated translation times and drew the time contour lines by a bar diagram (figure 4) and the successive flow rates of this diagram can be routed through the linear storage by the use of the routing equation

$$q(t) = q(t-1)\ e^{-t/K} + p(t)\ (1-e^{-t/K}) \qquad \ldots(25)$$

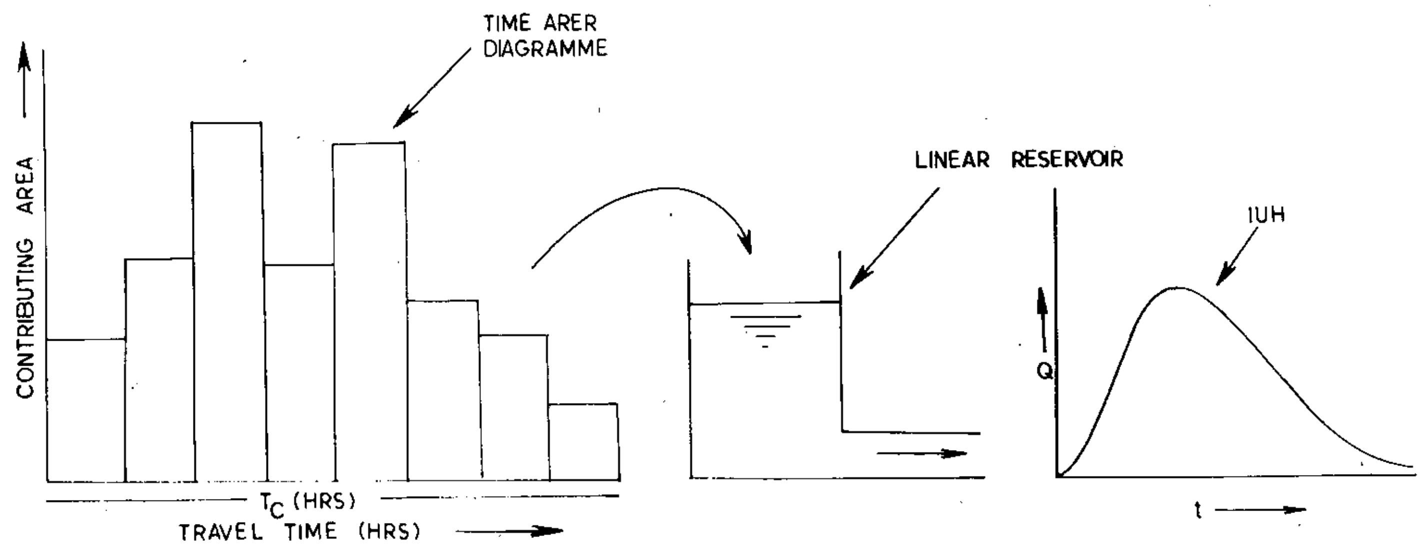


FIGURE 4 - CLARK MODEL

(e) Nash model

Nash (1957) derive the IUH routing the unit impulse input through N linear reservoirs of equal storage co-efficient and the impulse response i.e. IUH for Nash Model is given by the equation:

$$q(t) = \frac{1}{K \Gamma N} (t/K)^{N-1} e^{-t/K} \qquad \ldots(26)$$

Nash (1960) gave the method of moment to derive the parameters N and K. The diagrammatic representation of Nash Model is shown in figure 5.

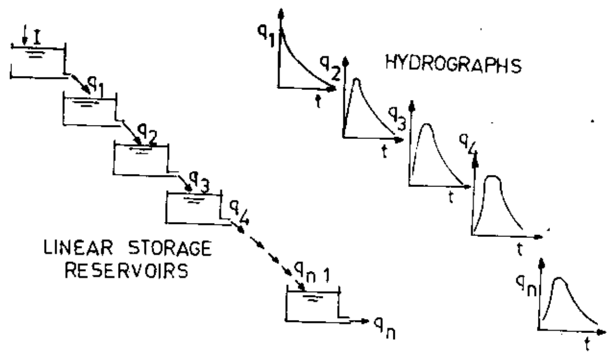
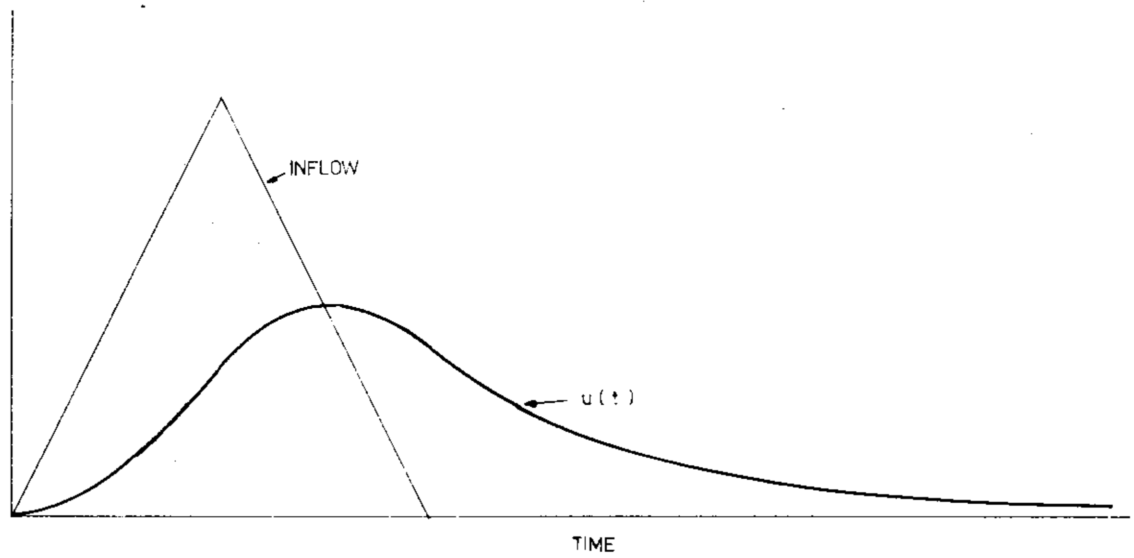


FIGURE 5 - NASH MODEL

(f) O'Kelly's model

O'Kelly (1955) concluded from his study of a number of Irish drainage basins that the smoothing effect of storage on the time area curve was so great that the latter could be replaced by an isosceles triangle without loss of accuracy. The base of this time-area diagram was the time of concentration T_c and its area represented the unit depth of input. O'Kelly routed this input through one linear storage in order to find the IUH. Figure 6 shows the diagrammatic representation of O'Kelly's model.



IUH BY ROUTING AN ISOSCELES TRIANGLE

FIGURE 6 - O'KELLY'S MODEL

(g) Dooge model

One of the criticism that can be made of the Nash Model is that it allows for reservoir storage effects but not for channel translation effects present in any catchment. Dooge(1959) developed a model technique which allows for translatory as well as storage effects in catchments

In brief Dooge's general analysis of linear catchment system postulates that the output from an element of a catchment incurs a translation delay time τ, and also passes through linear reservoirs in its passage to the catchment outlet, N being dependent on τ. Dooge shows that the following two assumptions simplify the general equation for the impulse response of a system without appreciable effects on the results.

(i) All elements having the same τ values have the same chain of linear storage to the outlet.

(ii) All the storages have the same storage characteristics, K.

The above assumptions yield a model system that can be represented diagrammatically as shown in figure 7.

Dooge's final equation for the impulse response of this model to a non-uniformly distributed input can be written:

$$q(t) = \frac{1}{A} \int_{o}^{t/K} \left[\frac{m^{N-1} e^{-m}}{(N-1)!} \right] \left(i \frac{dA}{d\tau} \right)_{\tau} dm \qquad \ldots(27)$$

where $m = \frac{t-\tau}{K}$,N and $\frac{dA}{d\tau}$ are functions of τ , $\left(\frac{dA}{d\tau} \right)_{\tau}$ is the ordinate of the time-area curve at time τ and i the average rainfall intensity along each isochrone (varying only with distance from the outlet).

Dooge(1959) presented a procedure for evaluating equation(20). The quantity in square brackets(the Poisson probability function) is available in tables and so the integral can be evaluated numerically for any given t value by taking increments of τ from 0 to t. The distribution of linear reservoirs (as a function of τ) must be known or assumed and the time-area curve has to be found (again as a function of τ) in order to carry out the integration. The N(τ) distribution may be found by moments technique outlined by Dooge(1959) or, possibly by a Laguerre function analysis.

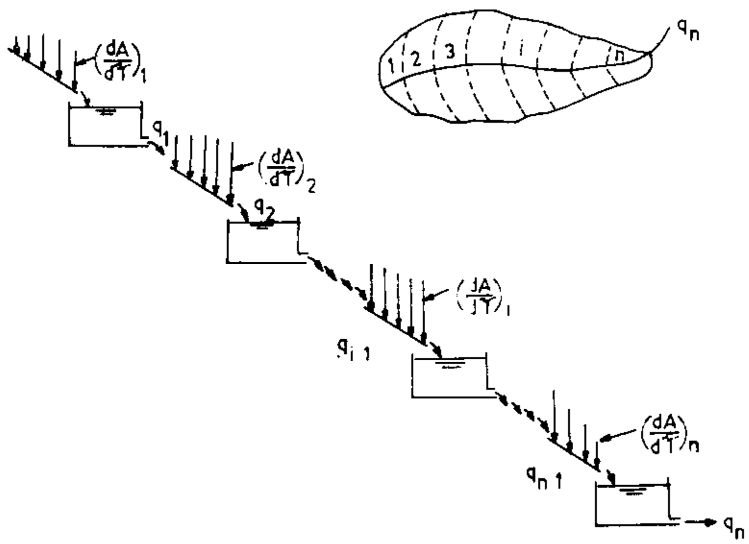


FIGURE 7 - DOOGE'S MODEL

2.2.2 Quasi linear models

Those non linear models of the surface runoff system are quasi linear models that retain many of the advantages of the linar models. The ordinary convolution integral may be used to estimate surface runoff from excess rainfall. However,the impulse response h(τ), which is constant for each storm event for linear models, is varied from one event to another. The form of the impulse response is selected on the basis of some property of the input function for that event, or as a consequence of initial stage of the system or one of its elements. The form of the impulse may be specified numerically or by reference to some conceptual model.

(a) Diskin's model

Diskin(1972) suggested one quasi-linear model based on results published by Minshall(1960). Dooge(1973) also suggested similar type of a quasilinear model. One single curve is fitted through the set of unit hydrographs which are plotted in a dimensionless form U.L.Versus (t/L) presented by Minshall as time function U(t), where L is the time to the centroid of the unit hydrograph. Furthermore, the time to the centroid of each unit hydrograph was found to be related to the average intensity of the rainfall excess for the event used for deriving that unit hydrograph. The dimensionless unit hydrograph and the curve relating the time to the centroid and the average rainfall excess constitute in this case the non-linear model.

(b) Singh's model

Other quasi linear models employ some conceptual model to derive the equation of the IUH, Singh(1964) proposed a quasilinear theory by assuming that each sub area might be represented by a two-element cascade. The basic equation for the IUH derived from Singh's approach is:

$$u(t) = \frac{1}{(K_2 - K_1)} \int_0^{t_1} (e^{-(t-\tau)/K_2} - e^{-(t-\tau)/K_1}) \omega(\tau)\, d\tau \qquad ...(28$$

where u(t) is the IUH ordinate at time t after occurrence of instantaneous unit rainfall excess

K_1 is the channel storage factor

K_2 is the overland flow storage discharge factor

$\omega(\tau)$ is the ordinate of the conc.time area diagram with base equal to time of concentration T_c

τ is the variable travel time

In applying the model, the author reduced the number of variables to to two, by assuming $K_1 \approx 0.25$. Further, the time-area diagram was considered to be isosceles trinagle or any other standard curve.

(c) Kulandaiswamy model

Another quasi-linear model for excess rainfall-surface runoff relationship was proposed by Kulandaiswamy that the process was non-linear but that the non-linear effects did not seem to be large The storage could be satisfactorily expressed by:

$$S = a_0 Q + a_1 \frac{dQ}{dt} + a_2 \frac{d^2Q}{dt^2} + b_1 \frac{di}{dt} + b_o\, i \qquad ...(29)$$

where the b coefficients vary randomly from storm to storm; the coefficients decrease with increase in Q_p, meaning that for major storms the peak is higher and the time to peak of the IUH lower than for minor ones. A satisfactory correlation was also established for rainfall excess and the peak of surface runoff.

2.2.3 Non linear storage models

Non linear models are the models that incorporate one or more non linear storage elements defined by an equation of the type:

$$S = C\,Q^m \qquad \ldots(30)$$

Where S is a storage element

Q is discharge and

C and m are the constants describing the element

The models fall under this category are described below:

(b) Laurenson model

Laurenson(1964) proposed a ~~model~~ which uses the non linear coefficient type routing equation similar to Muskingum method in order to rout the inflow through the series of concentrated storages. The model requires: (i) Temporal variations in rainfall excess,(ii) areal variations in rainfall excess, (iii) different elements pass through different amount of storage,(iv) catchment storage distributed, not concentrated, and (v) discharge v/s storage is non linear.

The overall routing procedure for the model is:

(i) Divide the catchment into sub-catchments.

(ii) Determine the hyetograph for farthest upstream sub-catchment with shape given by nearest recording gauge and scaled to make maximum ordinate equal to average rainfall for sub area.

(iii) Subtract the losses from the average rainfall hyetograph in order to get the excess rainfall hyetograph for the sub-catchment.

(iv) Convert the excess rainfall hyetograph using the equation Q =iA where i is rainfall hyetograph ordinates and A is catchment area.

(v) Rout the inflow hydrograph obtained from step (iv) through storage of the sub-catchment and hence the outflow from the sub-catchment is estimated.

(vi) Develop the excess rainfall hydrograph for another sub-catchment repeating the steps(ii),(iii) and(iv).

(vii) Add the rainfall hydrograph obtained from step(vi) (with time phase shift) to outflow hydrograph from upstream.

(viii) Rout the combined hydrograph through appropriate storage.

Mathematically, non linear routing method can be given from the continuity and storage equation as:

$$S = K(q)\, q \qquad \ldots(31)$$

$$\frac{(i_1 + i_2)}{2}\,\Delta t - \left(\frac{q_1 + q_2}{2}\right)\Delta t = S_2 - S_1 \qquad \ldots(32a)$$

or

$$q_2 = C_0\, i_2 + C_1\, i_1 + C_2\, q_1 \qquad \ldots(32b)$$

where

$$C_0 = C_1 = \Delta t/(2K_2 + \Delta t) \qquad \ldots(32c)$$

$$C_2 = (2K_1 - \Delta t)/(2K_2 + \Delta t) \qquad \ldots(32d)$$

Where K_1 and K_2 are the storage coefficients for the discharges q_1 and q_2 respectively.

An iterative procedure is used to solve the equation(32). Let $K_2 = K_1$ and find q_2, redetermine K_2 and iterate.

(b) Prasad model

Prasad (1967) proposed a non linear storage model representing the watershed system by a single non-linear storage reservoir, for which the relationship between the storage(S) and the outflow(Q) was given, considering the unsteady flow effects,by:

$$S = K_1\, Q^N + K_2 \frac{dQ}{dt} \qquad \ldots(33)$$

Differentiating this expression and substituting the result in continuity equation leads to the following expression.

$$K_2 \frac{d^2Q}{dt^2} + K_1 N Q^{N-1} \frac{dQ}{dt} + Q = R \qquad \ldots(34)$$

The equation represents the relationship between the input R(t) to the surface runoff system and the output Q(t) of the system. It can be solved numerically for any given input function R(t) by a numerically iterative procedure producing the output function Q(t) which is the surface runoff hydrograph of the watershed considered.

The equation parameters N, K_1 and K_2 were correlated with basin and rainfall characteristics by stepwise multiple correlation techniques.

2.3 Synthetic Unit Hydrograph

Bernard (1935) assumed that the peak of the unit hydrograph should be inversely proportional to the time of concentration which is to be proportional more or less to the length of the longest channel divided by the square root of the slope. With this formula he calculated a factor U for each catchment which was assumed to be proportional to the time of concentration and plotted U against the ordinates of the one day unit hydrograph at one day, two days etc. after the rainfall. These plottings were made on logarithmic paper and the points approximated by parallel straight lines. One straight line was drawn to approximate all the points representing ordinates of the one day unit hydrograph on the first day after the rainfal, another line was drawn for points representing the second day ordinates etc. To obtain the one day unit hydrograph for a catchment, the value of U for that catchment is calculated and entered on the chart. Then the ordinates on the first day, the second day etc. after the rainfall can be read off using the appropriate lines on the chart. Since only one measure of the catchment was used, therefore, the basic shape and scale of the unit hydrograph must be determined based on this measure. One day unit hydrograph having the same peak are identifical in Bernard's work and hence the method is one parameter. Bernard used the data of six catchments of areas ranging from 500 to 6000 square miles of United States.

McCarthy(1938) used the basic data consisted of twenty two 6 hour unit hydrographs for catchments of areas ranging from 74 to 716 squares miles. The catchment characteristics used were area, overland and stream pattern. Area was accounted for by converting the unit hydrographs and the catchment characteristics to the corresponding quantitites for 'model' catchment of 10 square miles by using the Froude model law. In order to calculate the overland slope, the area above each contour was plotted against the level of that contour, and the mean slope of this area-elevation curve was taken as the overland slope. This quantity being area divided by length, has the dimension of length and has, therefore, to be multiplied by the length scale in converting to the model catchment. The stream pattern number has been assumed to be unity if no stream has a tributary draining more than 25% of the total catchment area, to be two if these are two tributaries of approximately equal size draining at least 50% of the total catchment area,to be three if these tributaries drain 75% of the total catchment area.

The peaks of the unit hydrographs for each catchment has been plotted on logarthmic paper against the slope of the respective catchment, and each point is marked with the appropriate stream pattern number. A best fit curve has been drawn for each of the three stream numbers through the points of that stream number. McCarthy did not attempt to correlate any characteristics of the unit hydrograph with the catchment characteristics; instead, the correlated the lag from the beginning of rainfall to the peak, and the time base of the unit hydrograph separately, with the unit hydrograph peak. In expressing the lag and time base of the unit hydrograph as functions of the peak he implicitly assumed that all unit hydrographs having same peak are identical. The resulting relationship may be classified as one parameter. The use of model law implies that the response of small catchments are similar to the large catchments. This assumption is hardly tenable when one considers that the time of flow through soil or overland before entering defined channel is probably much the same in small as in large catchments, the only difference in delay time deriving from larger periods of channel flow on the large catchments.

Snyder FF(1938) sought relations between the lag from the centre of area of the effective rainfall diagram to the peak of the storm runoff and the product L'_{cA} where L denotes the length of the longest water course to the catchment boundary and L_{cA} denotes the length, by the channels, to the centre of area of the catchment. Like McCarthy, Snyder found relations between the lag and other UH parameters. As only a single UH parameter was used, the relationship is 'one parameter'.

Taylor and Schwarz(1952) used the catchment characteristics those were used by Snyder, with the addition of the average slope of the main channel. They had used the data of twenty catchments ranging from 20 to 1,600 square miles in area and for each catchment, several unit hydrographs of different durations. The peaks of the unit hydrographs of different durations for each catchment were correlated with the duration of the unit hydrographs and a set of curves obtained of form:

$$U(T)_p = U(o)_p\, e^{mT} \qquad \ldots(35)$$

Where $U(T)_p$ is the peak of the unit hydrograph of duration T; $U(o)_p$ is the peak of the IUH, and m is an empirical parameter, constant for each catchment. Since the relation between the peak of the unit hydrograph of duration T and the peak of the IUH is a function of the shape of the IUH, the value of m is clearly an index of the shape of the IUH.

In the next step the peak of the IUH and the shape factor m were plotted separately against the catchment characteristics. The equations were $U(o)_p \propto 1/S^{1/2}$ and $m \propto (LL_{CA})^{0.3}$ i.e.the peak of the instantaneous unit hydrograph was found to be a function of the main channel slope, and the shape of the instantaneous unit hydrograph was found to be a function of the catchment length. This, therefore, is a 'two parameter' relationship.

O'Kelly(1955) assumed that the instantaneous unit hydrograph could be obtained by routing an isosceles triangular inflow of the unit volume and of base length T hours through storage described by S=KQ. T and K are two parameters to derive the instantaneous unit hydrograph based on O'Kelly's approach. The catchment characteristics used were area and overland slope. O'Kelly, like McCarthy, assumed that area could be allowed for by the Froude model law. All values of T and K were modified to correspond with a catchment of 100 sq. miles area. These modified values of T and K were plotted against the overland slope which was defined as the median value of the maximum slope occur-ingring at the intersections of a grid of square mesh imposed on a map of the catchment. O'Kelly's conclusion was that the modified T and K could both be expressed as nominal powers of the slope i.e. $T=As^B$ and $K=Cs^D$ where s denotes the slope, and A,B,C and D are empirically derived constants. If B and D were equal, then T/K would be a constant A/C, and the shape of the instantaneous unit hydrograph would have been fixed as suggested by Commons. Infact O'Kelly used slightly different values of B and D and so obtained a basic shape which varied slightly with the catchment slope, and consequently with the unit hydrograph parameter K. However, the evidence for varying the basic shape with catchment slope was inadequate and practically equally good results could be obtained by using Common's approach.

Clark(1945) suggested a procedure to derive instantaneous unit hydrograph by routing the time-area diagram of the catchment having base length equal to time of concentration of the catchment through a single linear reservoir. Therefore, the method requires knowledge of two quantities, T and K in addition to the time-area diagram of the catchment. Clark pointed out that the parameters T and K might be related but relating T and K separately with the catchment characteristics would provide the set of parameter values, T and K, which in addition to time area diagram could be used to derive the instantaneous unit hydrograph for the catchment.

Minshal(1960) used the two parameters, peak rate and time to peak, of the unit hydrograph and pointed out that these two parameters were dependent on rainfall intensity and storm pattern. He presented a method for constructing a synthetic unit hydrograph for small drainage basins involving emperical relationships for the percentage of the peak rate at times before and after the peak rate in terms of the rainfall intensity and drainage area.

Nash (1960) related the first and second moments of the IUH with the topographical characteristics of the catchment for some English basins. He tried various forms of the relationships using different catchment characteristics but the final relationships obtained were of the form:

$$m_1 = 20.7\ A^{0.3}\ S^{-0.3} \text{ and } m_2 = 1.0\ m_1^{-0.2}\ S^{-0.1} \qquad ...(36)$$

where, m_1 is the first moment about the origin,

m_2 is the ratio of the second moment about the centroid to m^2_1,

A is the drainage area (square mile) and

S is a measure of overland slope

The relationships between model parameters and the moments of IUH could be solved in order to get the parameter values, N and K, for the catchment. Then, the general equation for the unit hydrograph would be used to obtain the unit hydrograph of the catchment. The form of the equation is:

$$u(T,T)= \frac{1}{T}\{I(N,t/K) - I\ (N,\frac{t-T}{K})\}$$

where (N,t/K) is the value of the incomplete gamma function of order N at t/K, and T is duration of effective rainfall.

Gray (1961) used two parameter gamma distribution equivalent to the expressions developed by Edson(1951) and Nash(1958) in order to fit dimensionless unit hydrographs. The form of the relationship suggested by Gray is:

$$\frac{Q_t}{P_R} = \frac{25.0(r')^q}{\Gamma q}\,(e^{-r't/P_R})\,(\frac{t}{P_R})^{q-1} \qquad \ldots(38)$$

where $\frac{Q_t}{P_R}$ is the % of flow/0.25 P_R at any given t/P_R value,

P_R is the period of rise from the beginning of surface runoff to the peak discharge,

r' is a dimensionless parameter equal to the product r P_R,

q is a shape parameter,

r is a scale parameter,

Γ denotes the gamma function and

e is the base of the natural lograithm

The time of rise, P_R was found to be a significant parameter.

The storage factor K or P_R/r', was significantly correlated with the watershed characteristics $L/\sqrt{S_c}$, where L is the length of the stream & S_c is the channel slope. Then the parameter r' was purely emperically related to the time of rise P_R. As a result, it was found that for uniformly distributed, short duration, high intensity storm over small watershed areas, the unit-hydrographs could be derived from the water shed characteristics, $L/\sqrt{S_c}$.

Eagleson(1962) related the lag time, peak discharge and width of the unit hydrograph at some percentage of the maximum discharge to basin and sewer characteristics. The excess rainfall was determined from R.E.=(1-A)P, where A is the slope of a precipitation loss curve i.e.losses v/s P.Lag time was theoretically computed by means of a weighted Mannings relationship and the mean travel distance. Peak discharge was correlated with mean basin slope; and the unit hydrograph base width, W_o and the widths at 50% and 75% of Q_{max} were expressed analytically as functions of Q_{max}.

3.0 STATEMENT OF THE PROBLEM

The unit hydrograph is a simple tool being widely used in deterministic hydrology to study the peak as well as time distribution of the runoff resulting from the rainfall records. Since the large number of techniques for finding the unit hydrograph are available in literature, therefore, some detailed investigations are required, for some of the methods which are frequently being used in practice for deriving the U.H., in order to see their applicability and limitations. The main purpose of the present study is to predict the discharge hydrograph for different storms of a particular catchment using its representative unit hydrograph derived from the different methods and to compare their performances on the basis of some criteria. The following methods are used in the study:

(a) Parametric System Synthesis

- (i) Nash's model
- (ii) Clark's model
- (iii) Singh's model

(b) Non-Parametric system analysis

- (i) Collin's method
- (ii) Least square method

The rainfall-runoff data of six catchment of Godavari basin subzone 3f are analysed using the above methods.

4.0 DESCRIPTION OF THE STUDY AREA

4.1 Topography

The lower Godavari subzone 3f is essentially a sub humid region having mean annual rainfall varying between 1000mm to 1600 mm. The sub zone 3f covers parts of areas in the states of Maharashtra, Madhya Pradesh, Andhra Pradesh and Orissa. The subzone 3f extends from longitude 96^{o} to 83^{o} east and latitude 17^{o} to 23^{o} North, and is approximately L-shaped. The lower Godavari subzone 3f has a complex relief. Plains of medium heights upto 150m exist near the main Godavari river in its lower reaches. Higher plains between heights of 150 m to 300 m cover most of the upper reaches. The western part of the subzone and north of Nagpur is the zone of the low plateau in the range of 300 m to 600m. The south-east and north-west portions of the subzone cover high plateaus in the ranges of 600 to 900 metres, and there are hills and higher plateaus ranges from 900 to 1350 m in the south eastern part of the sub zone.

4.2 Meteorology and Climatology

4.2.1 Rainfall

The subzone having a continental type of climate cold in winter and very hot in summer received most of the rainfall from the south west monsoon(June to September). A small portion of the sub zone on the south-east wind gets rain from north east monsoon (November-December) besides short duration thunder storms.

4.2.2 Temperature

The greater part of the sub zone has an average annual temperature varying from 25°C to 27.5°C. The minimum temperature in the sub zone varies from 2.5°C to 12.5°C. The maximum temperature recorded varies from 48°C to 47.5°C. The minimum temperature is recorded in the month of December and the maximum temperature is recorded in April.

4.2.3 Soil

The broad soil groups in the subzone are red soils and black soils. The red soils are either classified in to red sandy, red loamy and red yellow soils. Black soils are classified as deep black, medium black and shallow black soils. The black soils are clayey in texture and are derived from trap rocks. The texture of the red soils vary considerably from place to place and are derived from all types of rocks. Sandy textures predominates the red soil groups. The soil type may vary considerably from catchment to catchment.

4.2.4 Land use

More than 50% of the area is covered by forest and only 25% of the area is arable land.

5.0 DATA AVAILABLE FOR THE STUDY

The data available from the source may be described in the following form:-

(a) Catchment plans and catchment characteristics

(b) Hourly rainfall data from raingauge in the catchments specially installed for the purpose

(c) Hourly discharge values

(d) Thiessen weight of each raingauge stations

5.1 Catchment Characteristics

The location of Lower Godavari basin subzone 3f in the map of India is shown in figure 8. The maps showing catchment plans of catchment Br.No.807/1,51,604/2, 228, 566 and 969/1 are given in figure 9, 10,11,12, 13 and 14 respectively. The catchment characteristics namely length of the main streams, distance of centre of gravity to the outlet along the main stream, catchment area and average slopes for each six catchments considered for the study are given in Table 1.

5.2 Raingauge Stations

The complete details of raingauge stations for each storms of different catchments may be found in table 2 and corresponding Thiessen weights are also mentioned in the same table.

5.3 Rainfall-Runoff Data

The data of the rainfall and runoff at hourly intervals are collected from C.W.C.,New Delhi for the six catchments. The number of storms which are analysed for each catchment may be found in table 3.

Converted hourly discharge values from stage-discharge curve are supplied along with the measured stage. The rainfall is measured in inch/mm and discharge is expressed in cusec/cumec. Single peak hydrographs having fairly uniform rainfall are considered for the present analysis.

Table 1 - Catchment Characteristics

Catch. No.	Catchment Area(A)	Length of main stream (L)	Distance of C.G. to out let (L_c)	Slope S	$LL_c/\sqrt{S}$
	Km^2	km	km	m/m	$\times 10^4$
807/1	823.62	67.20	25.75	0.00228	3.624
51	86.76	23.74	10.06	0.001299	0.663
604/2	340.52	45.95	20.44	0.00193	2.138
969/1	208.48	24.94	6.76	0.00207	0.371
228	483.03	42.00	18.00	0.0038	1.23
566	137.21	19.55	8.37	0.004917	0.233

Table 2 - Details of Rain Gauge Stations

Catchment	No.of Rain gauge stations	Name of Rain Gauge stations	Theissen Weight
807/1	5	1. Bazargaon	0.156
		2. Mohgaon	o.181
		3. Kanholi	0.272
		4. Khairee	0.171
		5. Gumgaon	0.220
	4	1. Bazargaon	0.156
		2. Mohgaon	0.22
		3. Kanholi	0.337
		4. Khairee	0.287
	7	1. Bridge site	0.0614
		2. Takal Ghat	0.986
		3. Amgaon	0.15594
		4. Kanholi	0.16599
		5. Mohgaon	0.13078
		6. Khairee	0.23135
		7. Bazargaon	0.15594
	8	1. Bridge site	0.0404
		2. Takal ghat	0.0905
		3. Gumgaon	0.1308
		4. Amgaon	0.1056
		5. Kanholi	0.170
		6. Mohgaon	0.1358
		7. Khairee	0.1710
		8. Bazar Gaon	0.1559
51	1	Nishat Khera	1.00
	2	1. Nishatkhera	0.932
		2. Bridge site	0.068
604/2	4	1. Dongargaon	0.134
		2. Walni	0.380
		3. Bhilvargondi	0.316
		4. Dhanoli	0.170
	6	1. Katol	0.257
		2. Metpangara	0.119
		3. Bhilwargondi	0.177
		4. Saenga	0.189

		5. Kendhali	0.155
		6. Dhanoli	0.103
969/1	2	1. Isapur	0.422
		2. Rajni	0.578
	3	1. Isapur	0.479
		2. Gobargondi	0.256
		3. Bridge site	0.265
	3	1. Isapur	0.406
		2. Rajni	0.484
		3. Br.site	0.110
228	4	1. Chindwara	0.28
		2. Gangiwara	0.25
		3. Kanhr gaon	0.30
		4. Chabri	0.17
	3	1. Chabri	0.265
		2. Chindwara	0.372
		3. Kanhar gaon	0.363
	5	1. Bridge site	0.104
		2. Chindwara	0.200
		3. Ganjiwara	0.245
		4. Kanhargaon	0.285
		5. Chabri	0.167
566	5	1. Karpa	0.199
		2. Kharwari	0.2021
		3. Barkhar	0.196
		4. Wagholi	0.22
		5. Ghat Biroli	0.181
	3	1. Karpa	0.28
		2. Lipariya	0.47
		3. Ghat Biroli	0.25

Table 3 - Rainfall-Runoff Data

Catchment No.	No.of storms analysis	Date of storms
807/1	9	i.22.7.66 at 9 hrs. to 23.1.66 at 4 hrs
		ii.24.7.67 at 19 hrs to 25.7.68 at 17 hrs
		iii.4.7.68 at 17 hrs to 5 .7.68 at 2 hrs.
		iv.6.9.69 at 1 hr to 7.9.69 at 7 hrs.
		v.10.8.70 at 1 hr to 10.8.70 at 20 hrs
		vi.22.6.71 at 23 hrs to 23.8.71 at 22 hrs
		vii.22.8.73 at 12 hrs to 23.8.73 at 11 hrs
		viii.16.9.73 at 14 hrs to 17.9.73 at 13 hrs
		ix.9.8.73 at 12 hrs to 10.8.73 at 13 hrs
51	6	i.17.9.69 at 13 hrs to 18.9.69 at 6 hrs
		ii.5.9.66 at 17 hrs to 6.9.66 at 19 hrs
		iii.5.10.71 at 21 hrs to 6.10.71 at 19 hrs
		iv.4.8.73 at 2 hrs to 4.8.73 at 20 hrs.
		v.9.8.73 at 12 hrs to 10.8.73 at 11 hrs.
		vi.13.7.73 at 17 hrs to 14.7.73 at 18 hrs
604/2	9	i.26.8.59 at 13 hrs to 27.8.59 at 1 hr
		ii.5.9.59 at 12 hrs to 6.9.59 at 15 hrs
		iii.12.9.59 at 16 hrs to 13.9.59 at 8 hrs
		iv.3.8.60 at 22 hrs to 4.8.60 at 16 hrs
		v.11.8.62 at 4 hrs to 12.8.62 at 3 hrs
		vi.1.7.63 at 16 hrs to 3.7.63 at1 7 hrs
		vii.2.7.63 at 17 hrs to 3.7.63 at 7 hrs
		viii.28.7.63 at 15 hrs to 29.7.63 at 13 hrs
		ix.29.7.63 at 13 hrs to 30.7.63 at 12 hrs
969/1	4	i.3.7.69 at 16 hrs to 4.7.69 at 12 hrs
		ii.14.7.73 at 11 hrs to 15.7.73 at 9 hrs
		iii.7.9.69 at 22 hrs to 8.9.21 hrs
		iv.11.6.70 at 1 hr to 11.6.70 at 20 hrs
228	12	i.17.7.66 at 13 hrs to 18.7.66 at 12 hrs
		ii.22.7.66 at 8 hrs to 23.7.66 at 8 hrs
		iii.2.8.67 at 24 hrs to 3.8.67 at 20 hrs
		iv.13.9.68 at 2 hrs to 14.9.68 at 19 hrs
		v.15.8.69 at 15 hrs to 16.8.69 at 16 hrs
		vi.21.9.69 at 14 hrs to 22.9.69 at 14 hrs
		vii.9.8.70 at 17 hrs to 10.8.70 at 6 hrs
		viii.23.8.71 at 14 hrs to 24.8.71 at 13 hrs
		ix.27.6.72 at 16 hrs to 28.6.72 at 15 hrs
	x.	x.4.7.73 at 10 hrs to 5.7.73 at 14 hrs
		xi.7.7.73 at 18 hrs to 8.7.73 at 24 hrs
		xii.17.7.73 at 22 hrs to 18.7.73 at 24 hrs

566	6	i.25.7.61 at 1 hr to 25.7.61 at 12 hrs ii.8.8.62 at 6 hrs to 8.8.62 at 24 hrs iii.10.8.62 at 6 hrs to 10.8.62 at 24 hrs iv.10.8.62 at 24 hrs to 11.8.62 at 17 hrs v.24.7.64 at 12 hrs to 25.7.64 at 15 hrs vi.15.8.65 at 17 hrs to 16.8.65 at 11 hrs

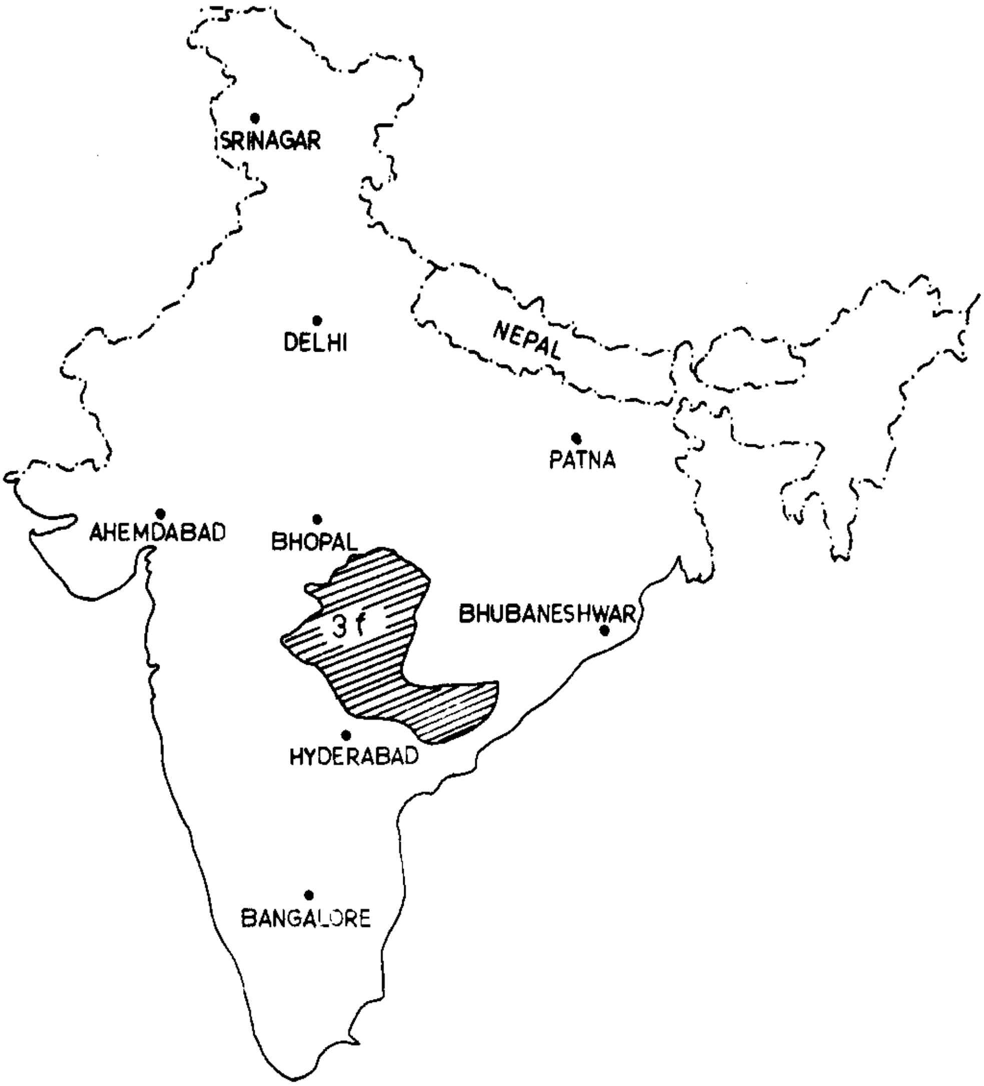


FIG. 8 -MAP OF INDIA
SHOWING SUBZONE 3 f

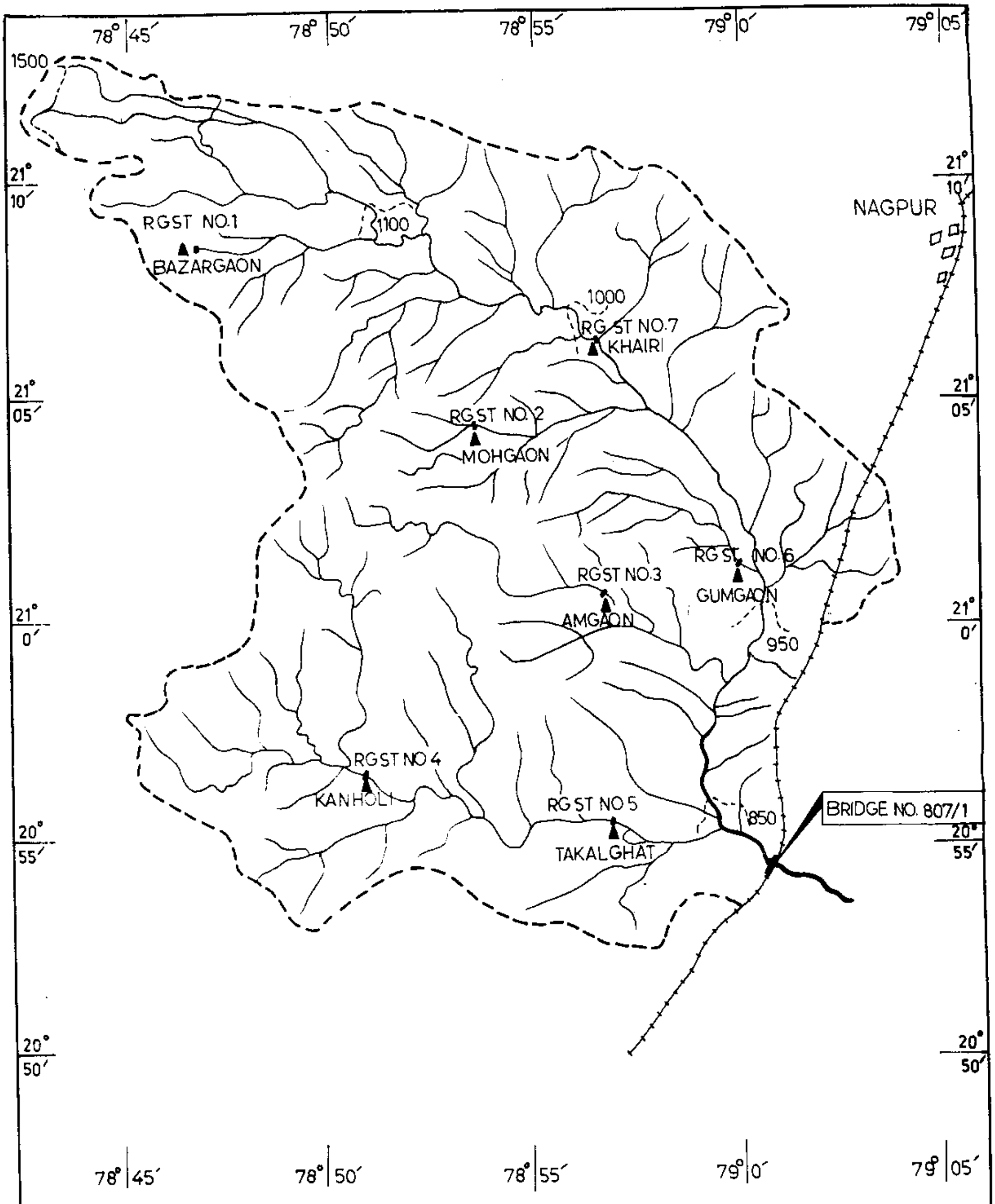


FIGURE 9 - CATCHMENT AREA PLAN OF BRIDGE NO.807/1 SUBZONE 3f

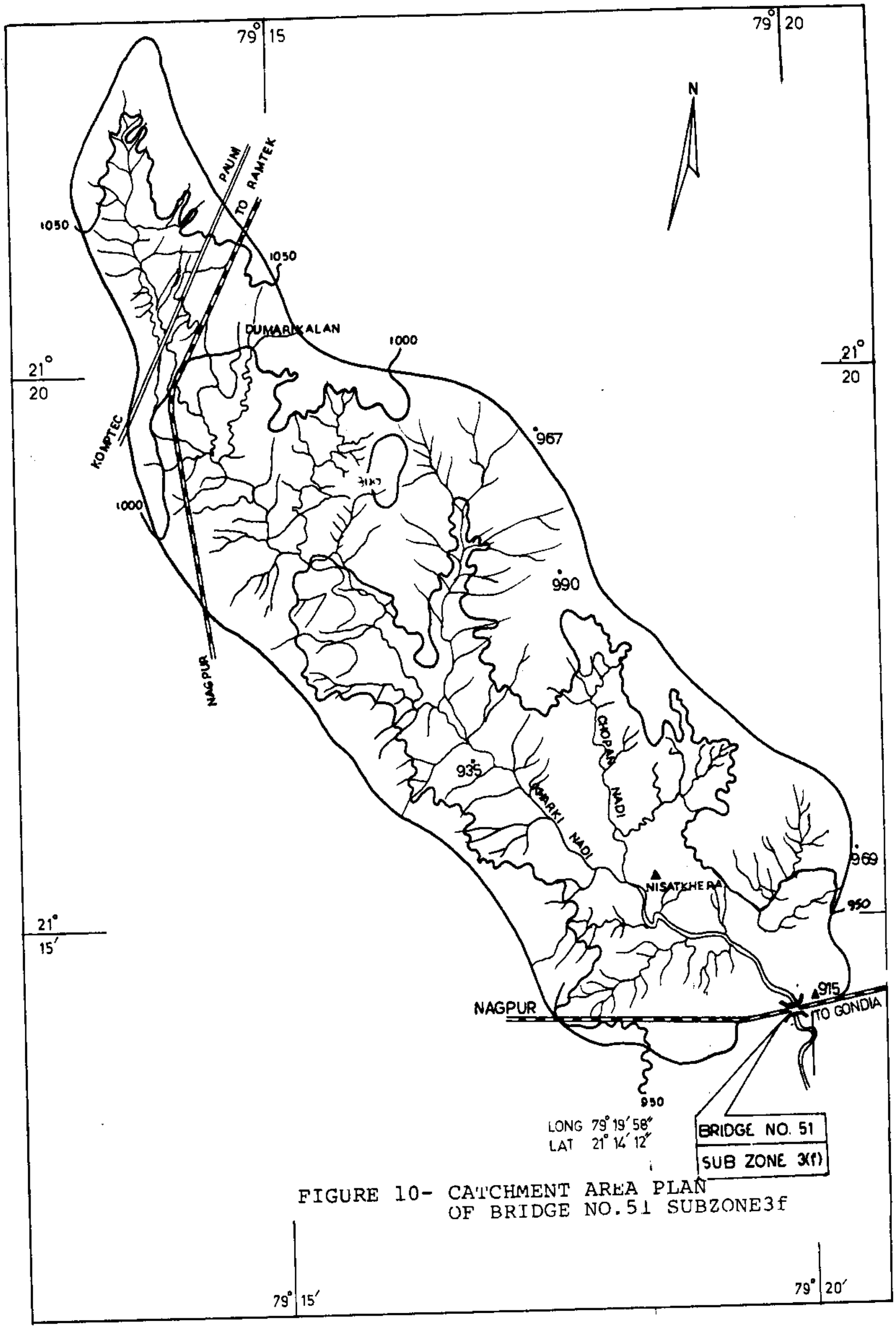


FIGURE 10- CATCHMENT AREA PLAN OF BRIDGE NO.51 SUBZONE3f

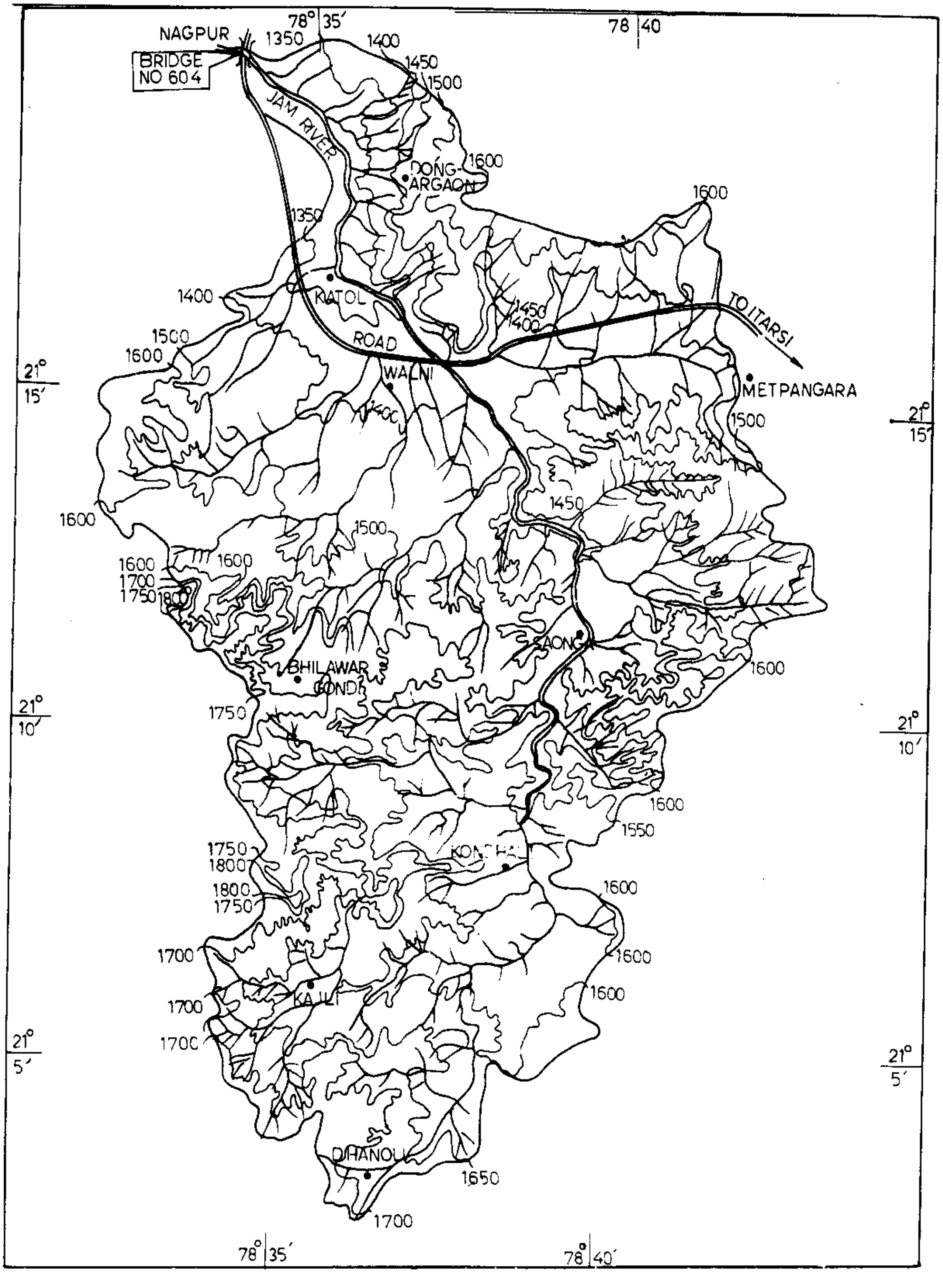


FIGURE NO- 11 CATCHMENT AREA PLAN OF BRIDGE NO.604/2 SUBZONE 3f

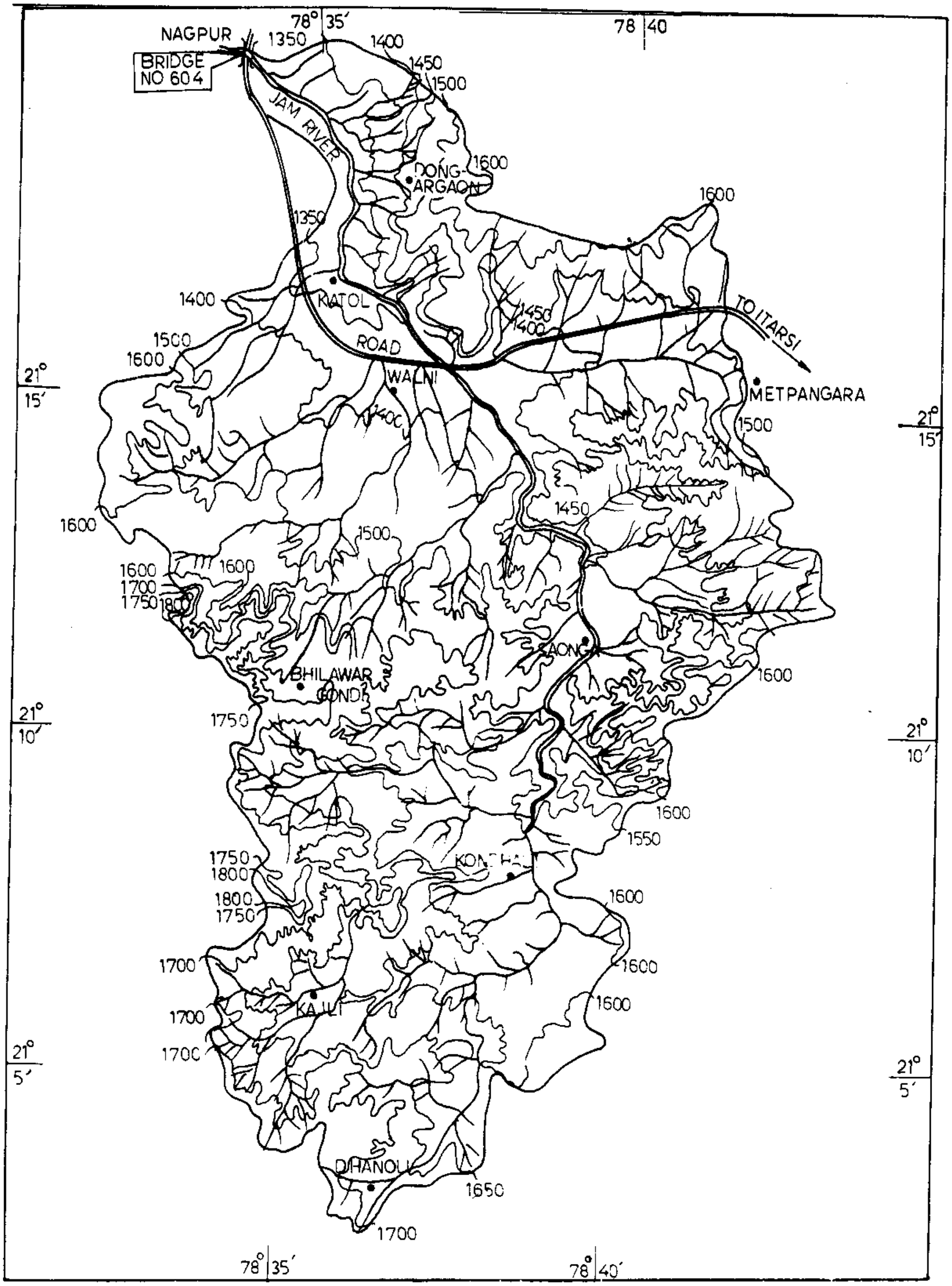


FIGURE NO- 11 CATCHMENT AREA PLAN OF BRIDGE NO.604/2 SUBZONE 3f

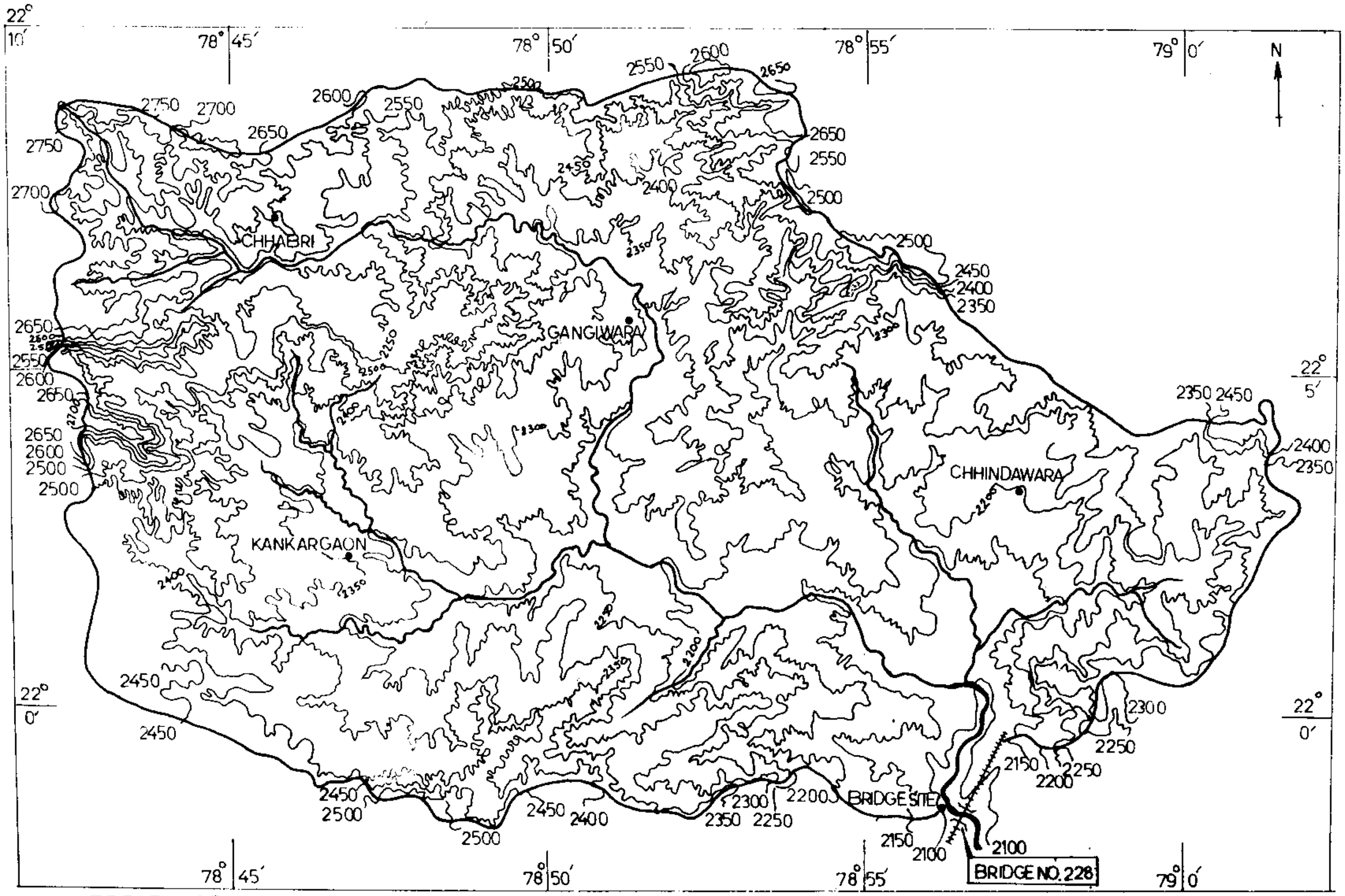


FIGURE 12- CATCHMENT ARE PLAN OF BRIDGE NO.228 SUBZONE 3f

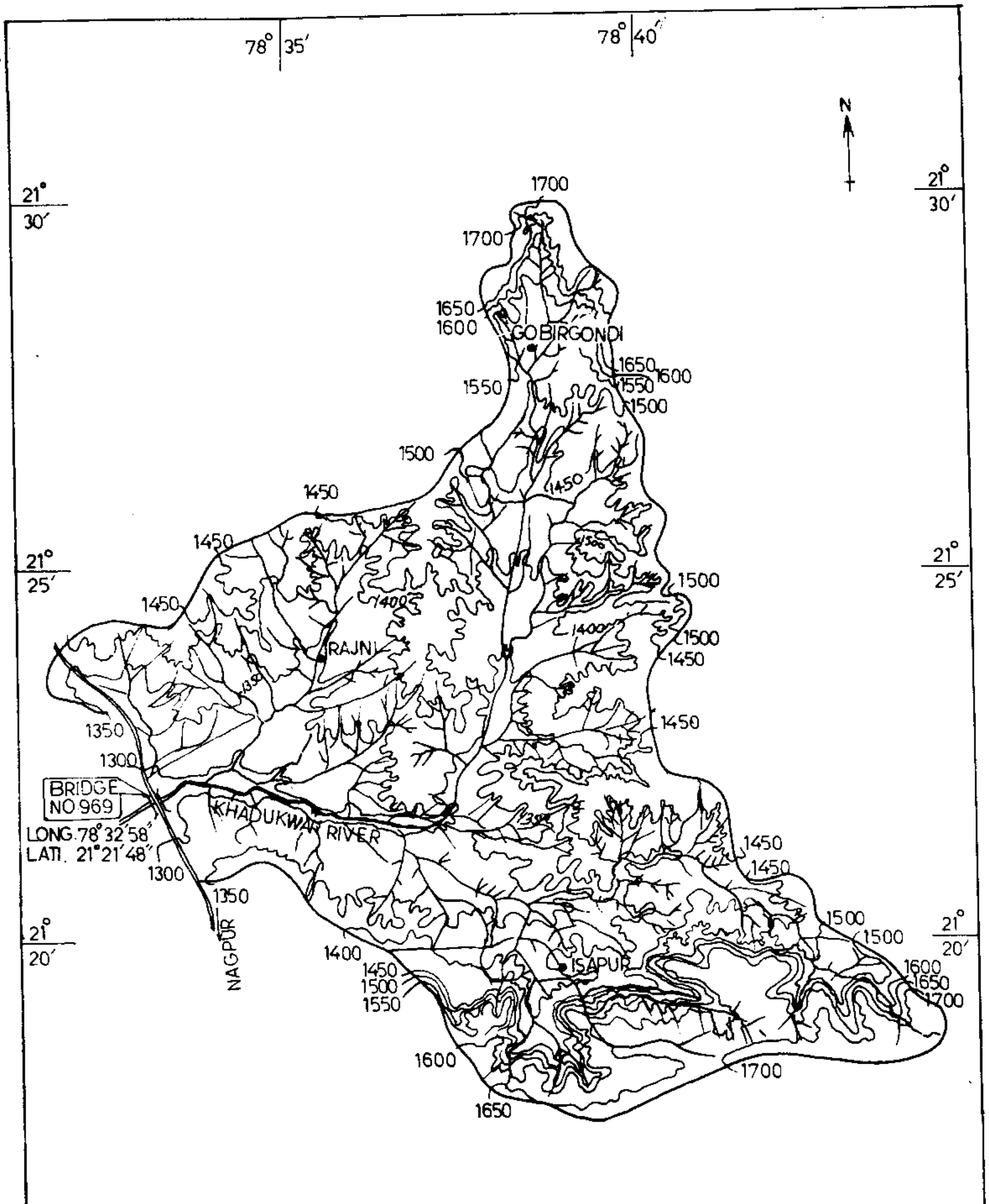


FIGURE 13 - CATCHMENT AREA PLAN OF BRIDGE NO.969
SUBZONE 3f

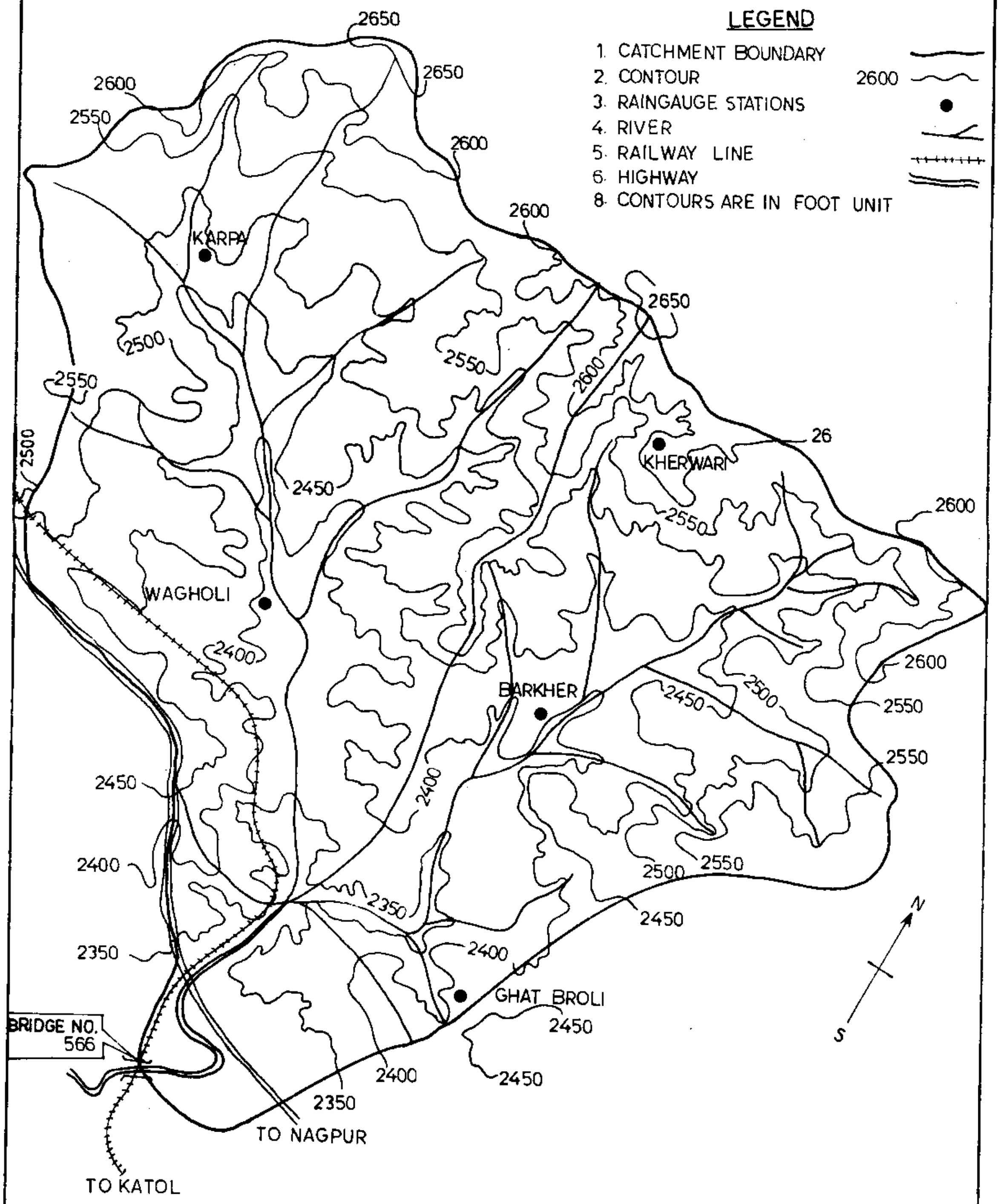


FIGURE 14 - CATCHMENT AREA PLAN OF BRIDGE NO.566
SUBZONE 3f

6.0 METHODOLOGY

6.1 Derivation of Unit Hydrograph Using Methods Other Than Clark Model

6.1.1 Estimation of effective rainfall and direct surface runoff

Since continuous rainfall is measured at different points within or near the catchments by means of standard autographic recorders, the data of rainfall are available at hourly interval for different sites within or near the catchments. Thiessen polygons have been drawn around the locations of raingauge stations and Thiessen weights have been calculated(The weights for different stations were also provided by CWC, New Delhi along with the data). Mean hourly values of rainfall during the storm are obtained by taking the weighted mean of the observed values at different stations. The next step is to separate the storm runoff from the total flood hydrograph. A number of methods for baseflow separation have been proposed in the literature. They are almost all lacking in any physical justification and are generally arbitrary. The method used in the analysis was to draw a line from the rising point of the hydrograph to the recession point on the falling limb of the hydrograph. A trial and error procedure was adopted to locate the starting point of rising hydrograph to be the same as the start of the effective rainfall. The ϕ index method was used to separate effective rainfall and infiltration rate was determined by a trial and error procedure such that the volume of effective rainfall equalled the volume of direct runoff.

6.1.2 Estimation of Unit Hydrograph

(a) Collin's method

It is a method based on trial and error techniques. The method was suggested by Collin's in 1939. The technique is very useful since it may be used to derive the unit hydrograph from complex storms i.e. the storms having consecutive periods of varying rainfall intensity. This technique involves the finding of a set of co-efficients for a rain occuring in more than one time interval by a series of trial and error approximations. The basic steps of the method are:

(i) Assume a unit graph by considering all rains combined into one average rain and apply it to all effective rains except the largest.

(ii) Subtract the resulting hydrograph from the actual hydrograph of of the surface runoff and reduce the residuals to unit graph terms.

(iii) Use the weighted co-efficients as a revised approximation for the next trial. The weights are given as the ratio of all effective rains except the largest one to the total rainfall for the first trial coefficients and the ratio of largest rainfall to the total rainfall for the unit graph obtained from step(ii).

(iv) Repeat step (i),(ii) and (iii) until the residual unit graph is in agreement with the assumed unit hydrograph.

(b) Nash model

Nash considered that the IUH could be obtained by routing the unit impulse input through a cascade of linear reservoirs with equal storage coefficient. The outflow from the first reservoir is considered as inflow to the second reservoir and so on. The equation for the IUH requires the estimation of the two parameters, N and K, to obtain the

IUH ordinates. The unit hydrograph for a specified duration may be obtained using the relation between IUH and UH. The parameters N and K are estimated from observed rainfall and discharge using the following procedure:

(i) Estimate effective rainfall and direct surface run-off using the procedure as described earlier.

(ii) Estimate the moments of effective rainfall and direct surface runoff. The general equation for the n^{th} moment of effective rainfall about the origin is:

$$MERH_n = \sum_{i=1}^{m} I_i \, t_i^n / \sum_{i=1}^{m} I_i \qquad \ldots(39)$$

and for the n^{th} moment of direct surface runoff about the origin is:

$$MDRH_n = \sum_{i=1}^{m} \frac{Q_i + Q_{i+1}}{2} \, t_i^n / \sum_{i=1}^{M} \frac{Q_i + Q_{i+1}}{2} \qquad \ldots(40)$$

where

Q_i is the direct surface runoff for the i^{th} interval,

M is the number of direct surface runoff ordinates,

m is the number of excess rainfall blocks,

n is the n^{th} moment about the origin and

t_i is the time to the mid point of the i^{th} interval from the origin.

(iii) Compute the first and second moment of effective rainfall and direct runoff using the equations mentioned in step(ii). Solve the following two equations to estimate N and K:

$$MDRH_1 - MERH_1 = NK \qquad \ldots(41)$$

and

$$MDRH_2 - MERH_2 = N(N+1)\,K^2 + 2NK\,MERH_1 \qquad \ldots(42)$$

After getting the parameters, N and K, the IUH ordinates may be obtained using the equation:

$$IUH(t) = \frac{1}{K\Gamma N} (t/K)^{N-1} e^{-t/K} \quad ...(43)$$

and the unit hydrograph of duration T hours is given as:

$$U(T,t) = \frac{1}{T}[I(N,t/K) - I(N,(t-T)/K)] \quad ...(44)$$

where I(N,t/K) is the incomplete gamma function of order N at (t/K). A subroutine for estimating the incomplete gamma function is available on VAX-11/780 system.

(c) Singh's model

Singh, K.P. proposed the routing of time area diagram having base equal to time of concentration through two unequal linear reservoirs to get IUH for the watershed. Thus the model is three parameter model i.e. storage coefficients K_1 and K_2 and time of concentration T_c provided time area diagram for the catchment is known. Singh(1964) suggested that the parameter K_1 may be considered equal to 0.25 and hence the model's parameters reduce to two from three. The discretely coincident form of Singh's model for pulsed inputs are given by the equation (O'Conner 1982):

$$\phi(B)\, y_m = \theta(B)\, X_m \quad ...(45a)$$

where $\phi(B) = (1-q_1B)(1-q_2B)$ ...(45b)

$$\theta(B) = \theta_0 + \theta_1 B,\ q_1 = e^{-T/K_1}\ ;\ q_2 = e^{-t/K_2} \quad ...(45c)$$

$$\theta_0 = \left[\frac{K_1(q_1-1) - K_2(q_2-1)}{K_2 - K_1}\right] \quad ...(45d)$$

$$\theta_1 = \left[\frac{K_2(q_2-1)\, q_1 - K_1(q_1-1)q_2}{K_2 - K_1}\right] \quad ...(45e)$$

where

K_1 = 0.25 (first reservoir storage coefficient),

K_2 is second reservoir storage coefficient,

T is the sampling interval,

X_m is the intensity of the (m+1) st input of pulse of duration T, m=0,1,2 and

Y_m is the sampled output function

In the equation (45a), X_m represents the time area diagram sampled at T-hour sampling interval in the form of pulsed input and Y_m represents the sampled pulse response of the Singh's model. The base length of the pulse form of time-area diagram is the time of concentration of the sub-basin. The time-area diagram for the sub-basin is estimated using the relations of HEC-1 programme i.e.

$$AI = 1.414(T_R)^{1.5} \text{for } 0 \leq T_R < 0.5 \qquad ...(46)$$

$$1-AI = 1.414\ (1-T_R)^{1.5} \text{ for } 0.5 \leq T_R \leq 1 \qquad ...(47)$$

Where AI is the cumulative area as a fraction of total sub-basin area and T_R is the fraction of time of concentration.

The two parameter K_2 and T_c are estimated optimizing the sum of squares of the difference between observed and computed direct surface runoff using Quasi-Newton Method optimization where search of the parameters are made in the direction of gradient of the error function in order to get optimum value of the parameters.

(d) Least square approach(matrix method)

The equation for the least square estimates of TUH with isolated event data is (Bruen 1983):

$$(X^T X + \alpha I)h = X^T y \qquad ...(48)$$

where

X is the convolution matrix,

h is the ordinate of unit hydrograph for the sub-basin derived using the rainfall -runoff data,

α is the regularisation factor and

y is the vector of the direct runoff

Subroutine KER6TM is used to estimate the pulse response for a discrete, linear, time-invariant single input, single output system. The data may consist of one or many records of input and output time series for the system. The methodology used for estimating unit hydrographs using the above approach are breifly described here as below:

(i) Estimate excess rainfall and direct surface runoff for each storm events of the sub-basin.

(ii) Produce the auto and cross product series required for definition of the matrix X^TX and vector X^Ty.

(iii) Use Farden's algorithm to estimate the pulse response.

(iv) If the derived pulse response is not realistic in shape, the value of K may be increased and previous steps may be repeated. However, the realistic pulse response is estimated after smoothening the pulse response obtained from step (iii) manually in the analysis.

6.2 Derivation of Unit Hydrograph using Clark Model

6.2.1 Method of estimating excess rainfall

There are different options available in HEC-1 to estimate excess rainfall excluding the component of rainfall which has been lost. Out of these, the simple option i.e.Uniform loss rate option has been used for the present study. In this option, there are two parameters STRTL and CNSTL. Here STRTL represents the initial loss rate and CNSTL represents the uniform loss rate. These parameters may be fixed or optimized alongwith other parameters depending upon the

User's desire. The objective function is weighted sum of squares of the difference between observed and computed runoff. Mathematically, the objective function may be expressed as:

$$STDER = \sum_{i=1}^{n} (QOBS_i - QCOMP_i)^2 \times WT_i/n \qquad ...(49)$$

where, STDER is an objective function which is to be minimised.

$QOBS_i$ is the observed runoff hydrograph ordinate i,

$QCOMP_i$ is the runoff hydrograph ordinate for time period i computed by HEC-1

n is the total number of hydrograph ordinates and

WT_i is the weight for the hydrograph ordinate i computed from the following equation

$WT_i = (QOBS_i + QAVE)/(2* QAVE)$

where QAVE is the average computed discharge

For the present study, the initial loss is assumed to be zero and constant loss parameters CNSTL has been optimised.

6.2.2 Method of estimation of direct surface runoff

The method used for separating the base flow from discharge hydrograph in order to get direct surface runoff is somewhat different from the method used in case of the other techniques. The procedure used in HEC-1 for separating the baseflow require three input parameters, STRTQ,QRCSN and RTIOR. The relation between the stream flow hydrograph and these variables are shown in figure 15.

Here, the variable STRTQ represents the initial flow in the river and it is affected by the long term contribution of ground water releases in the absence of precipitation and is a function of antecedent conditions. The variable QRCSN indicates the flow at which an expo-

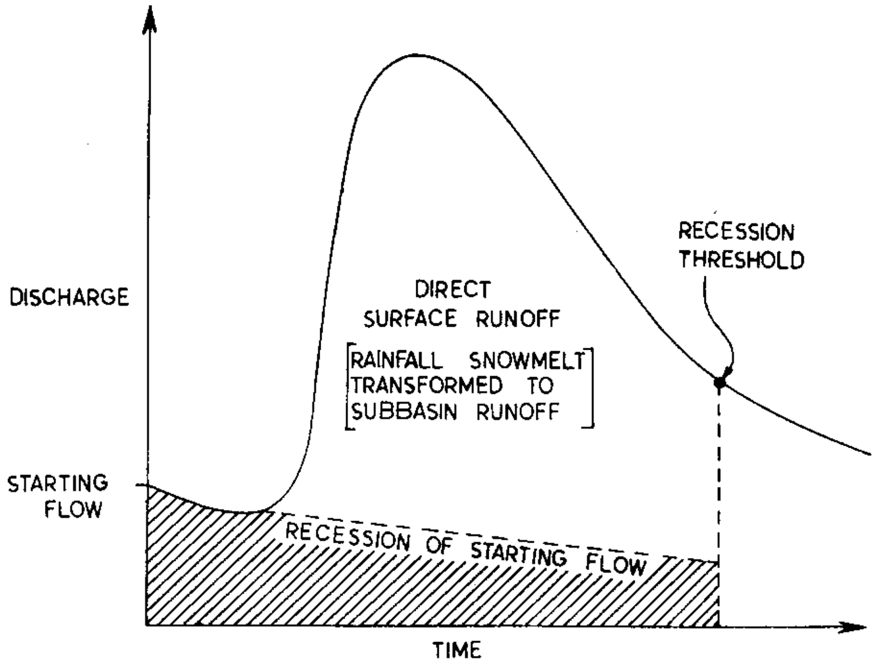


FIGURE 15 - BASE FLOW SEPARATION (HEC-1 APPROACH)

nential recessions begins on the recession limb on the computed hydrograph. Recession of the starting flow and falling limb follow a specified exponential decay rate, RTIOR, which is assumed to be a characteristics of the basin. RTIOR is equal to the ratio of recession limb flow to the recession limb flow occurring one hour later.

The program adjust RTIOR to the time step of the particular simulation and computes the recession flow Q as:

$$Q = Qo\ (RTIOR)^{n}$$

Where Qo is STRTQ or QRCSN, and n is the number of time intervals since recession was initiated. QRCSN and RTIOR can be obtained by plotting the recession limb observed flow on log scale and time on simple scale. The points at which the recession limb begins to depart from straight line is used to define RTIOR. Other alternative available in the programme is that QRCSN can be specified aś a ratio of peak flow. For example, the user can specify that exponential recession is to begin when the falling limb discharge drops to 0.1 of the calculated peak discharge.

The rising limb of the stream flow hydrograph is adjusted for baseflow by adding the recessed starting flow to the computed direct runoff flows. The falling limb is determined in the same manner until the computed flow is determined to be less than QRCSN. At this point the time at which the value of QRCSN is reached is estimated from the computed hydrograph. From this point, the stream flow hydrograph is computed using the recession equation unless the computed flow rises above the base flow recession. This is the double peaked hydrograph case where a rising limb of the second peak is computed by combining the starting flow recessed from the beginning of the simulation and direct runoff.

6.2.3 Estimation of time area diagram

In HEC-1 there are two options available for time area diagram. One can supply the time area diagram and, if not, the programme itself estimates the time area diagram using the equations(46) and (47).

6.2.4 Estimation of unit hydrograph

The resulting hyetograph form of time-area diagram is routed through a linear reservoir to simulate the storage effects of the basin and the outflow at the out let of the basin represents the IUH. The linear reservoir routing is accomplished using the general equation:

$$u(t + 1) = CA^{*} I(t) + CB^{*} u(t) \qquad \ldots(51)$$

where CA and CB are the routing coefficients.

u(t+1) is instantaneous flow at end of period

u(t) is the instantaneous flow at beginning of period

I(t) is the time area diagram

The routing coefficient CA and CB are calculated from:

$$CA = \frac{\Delta t}{R + 0.5\,\Delta t} \qquad \ldots(52$$

$$CB = 1-CA \qquad \ldots(53)$$

where Δt is the computational interval (hrs) and R is the basin storage factor in hrs.

The ordinates for one hour unit hydrograph are estimated averaging the ordinates of IUH ordinates at hourly interval.

6.3 Estimation of Direct Surface Runoff Using Unit Hydrograph

If the excess rainfall as an input consists of a series of pulses of duration T and U(j) is the system response(UH) to a unit pulse input of duration T, then the computed direct surface runoff is obtained using convolution summation:

$$Q_i = \sum_{i=1}^{m} \sum_{j=1}^{i} U(j) \times X(i-j+1) \qquad \ldots(54)$$

where Q_i is the computed direct surface runoff,

$U(j)$ is the j^{th} ordinate of UH of duration T hours,

$X(i)$ is the excess rainfall for computation interval i which is of T hour duration and m is the number of rainfall blocks.

Since the rainfall data available is at hourly interval, therefore, it is assumed as a pulse form input of duration one hour and convolution summation is used.

6.4 Comparison of Different Methods

The methodologies of different methods which are used for the present analysis are compared as follows:

(i) The Collin's method and least square method derive unit hydrograph relating excess rainfall-direct surface runoff of different storms while Nash, Clark and Singh models derive the instantaneous unit hydrograph using certain number of parameter values estimates from the observed records. Then the relationship between IUH and UH is utilised to estimate the unit hydrograph.

(ii) The methodology adopted for base flow separation in Clark Model is different than that of adopted for the other methods.

(iii) UH estimation procedures are different for each of the methods.

(iv) Uniform loss rate is optimised in Clark Model while it is an initial estimate in other methods.

6.5 Performance Criteria for Comparison

The same set of data of each catchments are used for the five methods in this study. Naturally the best criterion is how well these methods produce the events. This has been achieved by convoluting the effective rainfall increments with the derived unit hydrograph and comparing the reproduced hydrograph with the observed hydrograph. As time to peak and peak discharge are two important parameters of the observed hydrograph, these have also been included as criteria for comparison. Inspite of some differences in methodologies of the different methods as discussed in section 6.4, it is worthwhile to compare the results on the basis of some criteria for comparison and draw some conclusions from that. The following criteria have been employed for studying the relative merits of the different methods:

(i) The efficiency of the method which is defined, mathematically, as:

$$EF = \frac{F_0 - F_1}{F_0} \qquad \text{...(55a)}$$

$$F_0 = \sum_{i=1}^{n} (Q_{0i} - \bar{Q})^2 \qquad \text{...(55b)}$$

$$F_1 = \sum_{i=1}^{n} (Q_{0i} - Q_{Ci})^2 \qquad \text{...(55c)}$$

where

EF = Efficiency of the method,

Q_{0i} = i^{th} observed ordinate,

$\bar{Q}$ = the mean of the observed hydrograph,

Q_{Ci} = i^{th} computed hydrograph ordinate using the particular method, and

n = number of discharge hydrograph ordinates

(ii) The percentage errors in reproducing the peak which is the ratio of the absolute difference between observed and computed peak and observed peak i.e. mathematically:

$$\text{PAEP} = |Q_{PO} - Q_{PC}| / Q_{PO} \times 100 \qquad \ldots(56)$$

where, PAEP is the percentage absolute error in reproducing the peak

Q_{PO} is the observed peak, and

Q_{PC} is the computed peak

(iii) The percentage errors in reproducing the time to peak which is defined as the ratio of the absolute difference between observed and computed time to peak and observed time to peak, i.e. mathematically:

$$\text{PAETP} = |Q_{TPO} - Q_{TPO}| \; Q_{TPO} \times 100 \qquad \ldots(57)$$

where, PAETP is the percentage absolute error in time to peak,

Q_{TPO} is the observed time to peak (hrs), and

Q_{TPC} is the computed time to peak(hrs)

(iv) The average standard error which is defined as the root mean squared of sum of the differences between observed and computed hydrographs. Mathematically,

$$SE = \sqrt{\sum_{i=1}^{n} (Q_{oi} - Q_{ci})^2 / n} \qquad \ldots(58)$$

where

SE is the average standard error,

Q_{oi} is the i^{th} ordinate of observed hydrograph,

Q_{ci} is the i^{th} ordinate of computed hydrograph, and n is the number of ordinates.

7.0 ANALYSIS

7.1 Analysis Using Collin's Method

Storms, excess rainfall,direct surface runoff data of catchments are used to estimate the representative unit hydrograph which is then used to predict the discharge hydrograph of calibration storms and the independent storms not used in calibration. The steps used in the analysis are:

(i) Estimate unit hydrograph using Collin's method as described in section 6.1.2(a) for each storms used in calibration.

(ii) Calculate average peak and time to peak of the unit hydrograph and an average unit hydrograph is derived having the average peak & time to peak.

(iii) Use the average unit hydrograph obtained from step (ii) to to reproduce the storms used in calibration.

(iv) Use the average unit hydrograph to test the reproduction of the independent storms.

(v) Calculate the error functions in reproducing the storm hydrographs.

(vi) Repeat step (i) to (v) for each of the six catchments.

7.2 Analysis Using Nash Model

The available set of data is used for the model calibration and testing in the following steps:

(i) Estimate excess rainfall and direct surface runoff for each storm of the catchment using the procedure described in section 6.1.2(b).

(ii) Calculate N and K for each of the storms selected for calibration applying the method of moments for storm excess rainfall and runoff data.

(iii) Estimate average values of N and K taking geometric mean of the values N and K for the storms considered in the calibration. The parameter values which are not consistent, in comparison to the parameters of the most of the storm events, are discarded while taking the geometric mean.

(iv) Test the performance of the model reproducing the independent storms hydrographs, which are not included for calibration, with average values of the parameters N and K.

(v) Reproduce the hydrographs of those storm events, which are used in calibration, with average parameter values.

(vi) Compute the error functions in reproducing the storms.

(vii) Repeat from step (i) to (vi) for each catchment.

7.3 Analysis using Singh's Model

The calibration and testing of Singh's model has been done as follows:

(i) Estimate excess rainfall and direct surface runoff for each storm of the catchment using the procedure as described earlier.

(ii) Take the parameter K_1-0.25 for each storm.

(iii) Estimate the optimised parameters K_2 and T_c for the catchment minimizing the sum of square differences between observed and computed direct surface runoff for all of the storms used in calibration using Quasi Newton method for optimization

(iv) The optimized parameters K_2 and T_c alongwith K_1=0.25 are used to derive unit hydrograph. Obtain the representative unit hydrograph using the average parameters.

(v) Use the representative unit hydrograph obtained from step (iv) to reproduce the storms used in calibration.

(vi) Use these parameters to test the reproduction of independent storms.

(vii) Calculate the error function in reproducing the storms.

(viii) Repeat the steps (i) to (vii) for each catchment.

7.4 Analysis using Least Squares Approach

The analysis procedure using least square approach are as follows:

(i) Estimate excess rainfall and direct surface runoff for each storm of the catchments using the procedure described earlier.

(ii) Use the methodology discussed in section 6.1.2(d) to estimate average unit hydrograph using the data for calibration storms.

(iii) Predict the storms used in calibration with the average unit hydrograph derived from step (ii).

(iv) Use the average unit hydrograph to reproduce the test storms.

(v) Calculate the error functions in predicting the storms.

(vi) Repeat step (i) to (v) for each catchment.

7.5 Analysis using Clark Model (HEC-1 Approach)

HEC-1 loss rate optimization programme is used to calibrate and test the model as in following steps:

(i) Estimate excess rainfall and direct surface runoff for each storm of the catchment as described in sections 6.2.1 and 6.2.2

(ii) Calculate $R/(T_c+R)$ for the storms used in calibration optimising both T_c and R.

(iii) Calculate average value of $R/(T_c+R)$ for the catchment.

(iv) Optimize both T_c and R, having average value of $R/(T_c+R)$ for each storms used in calibration. Hence estimate (T_c+R).

(v) Calculate average value of (T_c+R) by taking the geometric mean of the consistent values.

(vi) Having average value $R/(T_c+R)$ and using average (T_c+R) value of a catchment, calculate average T_c and R.

(vii) Use the average T_c and R obtained from step (vi) to predict the storms used in calibration.

(viii) Use the average T_c and R to test the reproduction of independent storms.

(ix) List the error functions in reproducing the storms.

(x) Repeat the step (i) to (ix) for each catchment.

8.0 DISCUSSION OF RESULTS

8.1 Comparison of Unit Hydrograph

The unit hydrographs are derived by the five methods for the six catchments of Godavari basin subzone 3f using the rainfall-runoff data of different events for each catchment. The unit hydrographs, thus obtained, are used to obtain a representative unit hydrograph using the averaging procedure for each method. The peak characteristics of the representative unit hydrographs for different methods corresponding to each catchments are compared in table 4.

The computed unit hydrographs using Collin's and Matrix (least square) methods oscillate for most of the storms. However, the representative unit hydrographs for these methods are obtained averaging only those unit hydrographs having comparatively less oscillations. The unit hydrographs derived from five different methods are quite comparable.

8.2 Comparison of Predicted Discharge Hydrographs

The representative unit hydrographs of each method are utilized to predict the direct surface runoff hydrographs for the calibration storms and the test storms of each catchments. Table 5 shows the comparison of observed and predicted discharge hydrograph peak and time to peak for the storms of different catchments, where the last two storms of each catchments are test storms and remaining are the calibration storms. The suitability of different methods for the storms of different catchments on the basis of the error functions are given in table 6. The suitable techniques for the unit hydrograph derivation for different catchments, based on the study conducted for the available rainfall-runoff data of different events are given in table 7 depending upon the different criterion of error functions. The agreement between observed and predicted hydrographs overall is the best for Nash Model followed respectively Clark Model, Colin's Method, Matrix Method and Singh's Model. However, as indicated in table 7, different methods are showing better performance for different catchments in predicting the hydrographs. The performance of Nash Model is promising even with non-optimized parameters N and K, whereas the parameters of Clark Model and Singh's Model are being optimized.

9.0 CONCLUSIONS

Five methods of the unit hydrograph derivation have been compared and Nash Model is found to be the best technique based on the efficieny and standard error, specially for comparatively larger catchments. The Collin's Method and Matrix Method are having the problem of fluctuations in the derived unti hydrograph making difficult to decide the ordinates of unit hydrograph. When data errors are unusually high, Collin's and Matrix Method give unrealistic shape of the unit hydrograph.

Table 4 - Comparison of Representative Unit Hydrograph Peak Characteristics for Different Methods

Catchment	Representative U H Peak (m^3/S)					Representative U H time to peak(hrs)				
Br.No.	C	M	N	CL	S	C	M	N	CL	S
807/1	45	53	36	35	38	4	3	5	3	3
228	20	21	18	14	22	3	3	5	4	3
604/2	20	21	21	19	21	3	3	3	2	3
969/1	18	18	15	15	19	2	2	2	2	2
566	12	16	12	10	11	2	2	2	2	2
51	7	6	5	6	6	2	3	2	2	3

Table 5 - Comparison of Observed and Predicted Discharge Hydrograph Peak, 'Q_p' and Time to Peak, 'T_p'.

Catchment	Storm	Comparison of discharge hydrograph peak						Comparison of discharge hydrograph time to peak					
Br.No.	No.	Observed	Predicted (m^3/s)					Observed	Predicted (hrs)				
		(m^3/s)	C	M	N	CL	S	(hrs)	C	M	N	CL	S
1	2	3	4	5	6	7	8	9	10	11	12	13	14
807/1	1	440	470	554	428	469	436	9	7	6	7	6	6
	2	600	423	474	399	367	386	8	9	8	10	8	8
	3	255	154	159	158	130	143	10	11	7	10	9	9
4	4	490	416	485	418	373	389	10	11	10	12	10	11
	5	370	266	328	258	256	252	7	6	5	7	5	5
	6	1432	1259	1406	1316	1196	1206	6	9	7	9	8	8
	7	455	399	499	392	405	385	7	7	6	8	6	6
	8	315	330	355	305	319	295	9	11	10	11	9	10
	9	950	605	759	605	571	571	7	7	6	8	7	7
228	1	179	116	127	117	102	128	3	6	6	7	6	6
	2	372	302	340	303	262	341	5	7	7	9	8	7
	3	120	122	129	113	110	131	4	4	4	6	5	4
	4	339	211	234	237	229	229	13	10	9	11	10	10
	5	540	310	362	330	271	358	6	6	6	8	7	6
	6	324	204	210	208	227	208	10	9	9	10	9	9
	7	220	134	155	146	127	154	7	6	6	8	7	6
	8	162	162	177	158	156	179	11	6	6	8	7	6
	9	70	39	42	38	37	42	10	7	7	8	7	7
	10	185	174	196	189	163	188	11	11	11	12	11	11
	11	430	311	336	297	287	342	8	6	6	8	7	6
	12	384	296	357	321	261	350	11	6	6	8	7	6

Table 5 .....Contd

1	2	3	4	5	6	7	8	9	10	11	12	13	14
604/2	1	115	103	104	104	120	106	6	5	5	5	4	5
	2	293	225	229	228	238	233	10	9	9	9	8	9
	3	139	138	138	140	183	138	6	6	6	6	5	6
	4	176	128	126	127	149	125	5	5	5	5	4	5
	5	205	245	250	250	247	290	4	5	5	5	4	5
	6	346	276	281	280	358	290	4	5	4	5	4	5
	7	321	204	202	203	250	209	4	5	4	5	4	4
	8	249	193	194	195	196	197	9	9	9	9	8	9
	9	173	127	130	130	167	134	8	7	7	7	6	7
969/1	1	190	184	184	163	204	194	12	12	12	11	11	12
	2	238	237	226	198	211	228	4	4	3	4	3	3
	3	41	56	56	49	57	58	6	4	4	4	4	4
	4	291	303	279	244	254	294	7	8	8	8	8	8
566	1	92	79	95	80	76	75	4	4	4	4	4	4
	2	112	140	176	137	134	125	4	5	5	5	5	5
	3	352	233	263	235	234	227	7	7	7	7	6	7
	4	347	203	254	194	216	183	3	3	3	3	3	3
	5	400	321	409	307	354	286	4	4	4	4	4	4
	6	197	133	153	144	130	135	3	4	4	5	4	5
51	1	180	155	129	118	130	140	6	3	4	4	3	4
	2	100	101	87	80	83	86	7	4	5	4	4	4
	3	205	181	158	151	229	165	5	5	5	5	5	6
	4	110	108	98	89	91	10	6	7	7	7	6	7
	5	68	105	84	79	85	91	4	4	4	4	4	5
	6	41	41	34	33	47	37	12	12	12	12	12	13

Table 6 - Suitability of Different Methods for the Storms of Different Catchments on the basis of the Error Functions

Catch-ment	Total No. of storms	Methods	No. of the storms out of total storms for which the methods perform better on the basis of the error functions			
			EF	PAEP	PAETP	SE
807/1	9	C	2	0	3	3
		M	1	6	3	1
		N	5	1	3	3
		CL	3	1	4	2
		S	1	1	2	0
228	12	C	2	2	5	2
		M	3	5	5	4
		N	5	2	6	5
		CL	1	1	3	1
		S	2	4	4	3
604/2	9	C	2	2	6	3
		M	2	0	7	2
		N	4	0	5	3
		CL	1	7	3	2
		S	1	1	7	0
969/1	4	C	1	2	3	1
		M	3	2	0	3
		N	0	0	4	0
		CL	0	0	0	0
		S	0	1	2	0
566	6	C	0	0	6	1
		M	1	5	6	1
		N	1	0	5	1
		CL	4	0	5	4
		S	0	1	5	1
51	6	C	2	4	3	1
		M	0	1	5	2
		N	1	1	4	1
		CL	2	1	4	3
		S	2	0	1	2

Table 7 - Suitable Techniques for the Unit Hydrograph Derivation for Different Catchments Depending upon the Criterion of Error Functions.

Catchment Br No.	Catchment Area (km^2)	Best technique based on the error functions			
		EF	SE	PAEP	PAETP
807/1	823.62	N	N/C	M	CL
228	483.03	N	N	M	M
604/2	340.52	N	N/C	CL	M/S
969/1	208.49	C	C	M/C	N
566	137.21	CL	CL	M/C	M/C
51	86.76	C/CL/S	CL	C	M

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