International Seminar on Hydrology of Extremes

1-3 December, 1988
Roorkee (India)

STOCHASTIC MODELLING OF LOW FLOWS IN MAHANADI BASIN

P. Govinda Rao
Senior Scientific Officer—Gr. I.
Department of Science & Technology, New Delhi-110 016

SYNOPSIS

The study of low flows or droughts in a river basin is of interest to hydro-power and irrigation engineers, landuse managers and water resource planners and scientists. Past studies on low flows indicated that stochastic models using double exponential distribution function gives better fit to these type of data. Hence in the present study an attempt has been made to fit the low flows observed at Sambalpur and Naraj in Mahanadi Basin using the above type of stochastic model. The stochastic models obtained for seasonal low flows for Sambalpur and Naraj are:

$$X_i = 56.3 - 52.9 \ln\{\ln(T_i)\} \quad \text{for Sambalpur}$$

$$X_i = 97.6 - 73.3 \ln\{\ln(T_i)\} \quad \text{for Naraj.}$$

Where X_i in cumecs and Ti in years.

1.0 INTRODUCTION

1.1 Study of low flow frequencies and their magnitudes are very important in water resources planning and management. They are often required for quantitative assessment of capability of a river or a stream to meet the water supply demands and water quality requirements during drought periods. Prolonged spells of subnormal/deficit precipitation, low humidity content and high evaporation rates results in low stream flows in a river/stream over a period of time in which surface streamflow is sustained mainly by groundwater recharge and upstream releases.

1.2 Earlier, many investigators have attempted to estimate low streamflows by statistical and stochastic models. These studies indicated that stochastic models gives better fit to these type of data and thereby better estimates. The present paper attempts the analysis of low flows in Mahanadi basin along the main stream at Sambalpur and Naraj by a stochastic model using a double exponential distribution function.

2.0 PAST STUDIES

2.1 Statistical methods have been widely used by several investigators for estimating low flows (3,4,5,8). Gumbel (2) suggested asymptotic distribution for drought flows and found it very suitable. He analysed droughts by the asymptotic theory of smallest values of limited statistical variate (3).

2.2 The importance of low flow forecasting to the adoption of water quality criteria has been explained by Ray and Walker (6). Dawdy et al (1) explained results of physically based stochastic modelling of droughts. In a case study Sharma et al (7) compared the results of Gumbel's and Stochastic models and concluded that stochastic models gives better fit to low flow series.

3.0 DATA USED

3.1 Daily discharge measurements at Sambalpur (1926-47) and Naraj (1926-45) have been used for the present study. From these data, the occurrence of low flows in each year on a day, 7-day average and one month's mean minimum flows have been compiled and are given in Table 1 and Table 2 for Sambalpur and Naraj respectively. These data formed as data base to the model.

Table 1

One-day, 7-day average low flow and one month's average minimum flows for Mahanadi river at Sambalpur (cumecs).

Year	One-day	7-day	One month
1926	51.0(6)	70.8(6)	81.8(6)
1927	51.4(4)	51.0(4)	51.0(4)
1928	51.0(4)	51.0(4)	64.4(5)
1929	25.5(5)	25.5(5)	25.5(5)
1930	25.5(6)	25.5(6)	46.5(6)
1931	25.5(5)	25.5(5)	37.8(5)
1932	51.0(5)	51.0(5)	51.0(5)
1933	90.6(5)	101.9(5)	131.1(4)
1934	90.6(5)	101.9(5)	131.1(4)
1935	101.9(6)	101.9(6)	112.7(6)
1936	90.6(4)	99.6(4)	101.9(5)
1937	90.6(6)	113.2(6)	163.8(6)
1938	25.5(4)	25.5(5)	40.2(5)
1939	25.5(5)	25.5(6)	44.9(5)
1940	25.5(5)	25.5(5)	32.1(5)
1941	51.0(5)	51.0(5)	51.1(5)
1942	25.5(5)	25.5(5)	28.0(5)
1943	40.9(5)	45.9(5)	51.0(5)
1944	70.8(6)	82.4(6)	108.7(6)
1945	51.0(6)	51.0(6)	58.0(6)
1946	42.5(6)	44.1(6)	51.1(5)
1947	6.0(6)	7.1(6)	14.0(5)

Figures shown in the brackets are the months in which low flow occured.

Table 2

One-day, 7-day average low flow and one month averages of minimum daily flows in cumecs for Mahanadi river at Naraj.

Year	One-day	7-day	One-month
1926	85.0(6)	87.8(6)	129.3(6)
1927	85.0(5)	85.0(5)	85.0(5)
1928	85.0(4)	85.0(5)	104.3(5)
1929	56.6(5)	56.6(5)	56.6(5)
1930	56.6(5)	56.6(5)	80.4(5)
1931	56.6(5)	56.6(6)	70.3(5)
1932	85.0(6)	85.0(6)	87.9(6)
1933	141.6(5)	155.7(5)	211.0(4)
1934	90.6(6)	107.6(6)	137.0(5)
1935	155.7(6)	155.7(6)	172.0(6)
1936	141.6(4)	141.6(5)	152.6(5)
1937	141.6(6)	169.9(6)	309.6(4)
1938	56.6(4)	56.6(5)	74.0(5)
1939	56.6(5)	56.6(5)	78.6(5)
1940	56.6(4)	56.6(5)	63.9(5)
1941	85.0(5)	85.0(5)	85.0(5)
1942	56.6(5)	56.6(5)	59.4(5)
1943	79.3(5)	80.9(6)	85.0(5)
1944	113.3(6)	130.8(6)	194.6(5)
1945	85.0(5)	85.0(5)	99.6(5)

Figures shown in the brackets are the months in which low flow occured.

4.0 METHOD ADOPTED

4.1 The annual low flow is commonly defined as the lowest consecutive seven day mean flow in the year. Usually it occurs in the same season although not necessarily in the same month, each year and thus this value may be considered as seasonal low flow. But it is felt that this criterion may not represent the real drought conditions in the basin but a period of one month may be more reliable and more realistic representative of drought conditions in the basin.

4.2 The fitted stochastic model using double exponential distribution function of non-dimensional variable (Yi)

$$Y_i = \frac{x_i - \mu}{\lambda - \mu} \quad ----(1)$$

Estimation of magnitude of Xi of a low flow for a given recurrence interval Ti is given by

$$x_i = c(\lambda - \mu) + \mu + m(\lambda - \mu)\ln\{\ln(T_i)\} \quad ----(2)$$

Where

$$\mu = \bar{X}_i - 3\sigma_{x_i}$$

$$\lambda = \bar{X}_i + 3\sigma_{x_i}$$

and m and c are distribution parameters, $\overline{X}_i$ *and* σ_{x_i} *are the mean and standard deviation of the Xi series.* $T_i = n+1/m$ *, where n is the total number of years of data used and m is the rank number. When series are arranged in ascending order, the first value which is lowest will have rank 1 and the largest value will have largest rank number.*

5.0 RESULTS

5.1 *The values of constants m,c,* μ *and* γ *have been computed and substituted in equation (2) to obtain the stochastic models for all the three series of the low flows both at Sambalpur and Naraj.*

5.2 *The models for Sambalpur are as follows:*

$$x_i = 71.3 - 39.8 \ln\{\ln(T_i)\} \quad \text{for 1-day}$$

$$x_i = 93.8 - 86.4 \ln\{\ln(T_i)\} \quad \text{for 7-days}$$

$$x_i = 56.3 - 52.9 \ln\{\ln(T_i)\} \quad \text{for monthly}$$

In case of Naraj they are as below:

$$x_i = 165.7 - 112.4 \ln\{\ln(T_i)\} \quad \text{for 1-day}$$

$$x_i = 147.1 - 86.3 \ln\{\ln(T_i)\} \quad \text{for 7-days}$$

$$x_i = 97.6 - 73.3 \ln\{\ln(T_i)\} \quad \text{for monthly}$$

Acknowledgements:

The required data has been made available by the Statistics Directorate, Central Water Commission, New Delhi.

References:

1. *Dawdy, D.R., V.K. Gupta and V.P. Singh, (1979), 'Stochastic Modelling of drought' Proc. Int. Symp. on Hydrological aspects of droughts, New Delhi, India.*

2. *Gumbel, E.J., (1954) 'Statistical theory of extreme values and some practical applications', National Bureau of Standards, Applied Mathematics Series, 33, U.S. Deptt. of Commerce.*

3. *Gumbel, E.J. (1958) 'Statistics of Extremes' Columbia University Press, New York.*

4. *Nicholas C. Matalas (1963) 'Probability distribution of low flows', United States Geological Survey Professional Paper, 434-A, U.S. Govt. Printing Office, Washington*

5. *O'Conner, D.J., (1959) 'Statistical Analysis of Drought flows in rivers in New York State', New York State Water Pollution Control Board, Research Report 1.*

6. *Ray, W.C. and W.R. Walker (1970). 'Low flow criteria for stream standards', Journal of the Sanitary Engineering Division, A.S.C.E., pp.1331-1344.*

7. *Sharma, H.D., S.V. Singh and S.R. Jindal (1975), 'Stochastic Modelling for low flows' Proc. of the Seminar on Hydrology held at Roorkee.*

8. *Velz, C.J., J.J. Gannon (1960) 'Drought flow characteristics of Michigan Streams', Michigan Water Resources Commission, Lausing, Michigan.*