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MULTIPLE LINEAR REGRESSION

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ABSTRACT

The derivation of relationships among hydrologic variables is of great importance for the transfer of information from few gauged sites to many ungauged sites. This relationship may be between one dependent variable and one or more independent variables. The general form of the multiple linear regression is:

$$X_1 = B_1 + B_2X_2 + B_3X_3 \ldots\ldots + B_m X_m + \varepsilon$$

where:

X_1 is dependent variable

X_2 ,.....,X_m are independent variables and

B_1,. . . . .,B_m are regression coefficients

The user's manual gives the details of the computer programme for multiple linear regression. The selection of the different sets of independent variables and designation of a dependent variable can be made as many times as desired.

The output of the programme includes means and standard deviations of dependent and independent variables, correlation coefficients between independent variables and dependent variable, regression coefficients, standard error of regression coefficients, computed t-values, intercept, multiple correlation coefficient, standard error of estimate, analysis of variance for multiple regression and table of residuals.

The manual describes various statistics given in output with test example and input data specifications. The programme is written in FORTRAN . The manual also gives the limitations of the programme.

1.0 INTRODUCTION

The derivation of relationships among hydrological variables is of great importance for the transfer of information from few gauged sites to many ungauged sites. This relationship may be between one dependent variable and one independent variable. This may also be among one dependent variable and several independent variables. The regression analysis is useful in the estimation of missing data in hydrologic series. The use of correlation and regression analysis has been enhanced greatly by the advent of computers.

The association of three or more variables can be investigated by multiple linear regression and correlation analysis. The general form of the multiple linear regression is given below:

$$X_1 = B_1 + B_2X_2 + \ldots\ldots + B_mX_m + \varepsilon \qquad \ldots(1)$$

In equation (1) X_1 is a dependent variable and $X_2 \ldots\ldots X_m$ are independent variables.

If equation (1) is linear i.e. if all the variables (dependent and independent) are in linear form, the regression is referred as the multiple linear regression. The association between the variables of nonlinear relations in hydrology is often transformed to linear form for the multiple regression analysis as it is easier to treat linear equations.

If a regression equation with m parameters is fitted to a set of N data points of variables $X_1, X_2 \ldots\ldots X_m$,the number of degrees of freedom will be N-m. If the number of parameters is equal to the sample size the regression equation will pass through all the points as there is no degree of freedom. It can not be used for prediction as the errors of parameters are inversely proportional to the number of degrees of freedom.

In designing the multiple linear and nonlinear regression relations the selection of dependent and independent variables is very important. The depe-

ndent variable is defined by the problem itself. The independent variables are selected by the following two criteria:

(i) They have been observed in the past concurrently with the dependent variable so that the regression equation may be established, and they will continue to be observed in the future also so that X_1 may be predicted from them when necessary.

(ii) The variables are selected for investigation by a subjective method on the grounds that the analysis of physical phenomenon indicates an effect on dependent variable by the independoent variables.

These criteria are necessarily subjective when the services or surveys have been established for gathering the hydrologic data. Various variables to be observed are decided by some criterion of subjective nature.

Many variables X_{m+1}, X_{m+2} are neglected as they are not known to affect X_1. The random term ε i.e. the difference between predicted X_1, and the observed X_1 encompasses all neglected variables and all errors.

1.1 Purpose and Capabilities of the Programme

Multiple linear regression analysis is performed for a set of independent variables and a dependent variable. In the programme the selection of different sets of independent variables and designation of a dependent variable can be made as many times as desired. The programme carries out the following operations:

1. Reads the title of the problem.
2. Reads subset selections i.e. different sets of selections of independent and dependent variables for multiple regression.
3. Calls various subroutines to calculate means, standard deviations, simple and multiple correlation coefficients, regression coefficients, T values and analysis of variance for multiple regression.

4. Prints the results.

The programme for multiple linear regression can deal with upto 40 variables including both independent and dependent variables.

1.2 Definitions of the Terminology Related to Topic

(a) Standard deviation of residuals: This is defined as the standard deviation of the differences between the observed and estimated/predicted values of dependent variable. This parameter is a measure of the closeness with which the estimated values approach the observed values.

(b) Multiple correlation coefficient: The correlation coefficient is a measure of the linear association of two variables. In the case of multiple regression, the two variables are the estimated and observed values of the dependent variable. Thus the multiple correlation coefficient is the ratio of the standard deviation of estimated values to that of observed values, i.e.

$$R = \frac{S_e}{S_o} \qquad \ldots(2)$$

(c) Coefficient of determination : The square of the multiple correlation coefficient(R^2)is called coefficient of determination. This indicates part of the variance which has been accounted for by multiple regression.

(d) Partial correlation coefficients : The partial correlation coefficient measure the association of the dependent variable X_1 with any given independent variable X_i. The partial correlation coefficient may be determined by the following equation:

$$r_i^2 = \frac{(1 - R_2^2) - (1 - R_1^2)}{(1 - R_2^2)}$$

$$= 1 - \frac{1 - R_1^2}{1 - R_2^2} \qquad \ldots (3)$$

where :

r_i : Partial correlation coefficient for i^{th} independent variable

R_i : Multiple correlation coefficient between X_1 and all the independent variables

R_2: Multiple coefficient between X_1 and all the independent variables excedpt X_i

(5) Beta coefficients

The effect of individual independent variable on the dependent variable may be measured by a dimensionless form of the regression coefficient. Each variable can be expressed in the following form:

$$X_1 = B_1 + B_2 X_2 + \ldots\ldots + B_m X_m$$

or

$$\frac{X_1}{S_1} = \frac{B_1}{S_1} + \frac{B_2}{S_1} X_2 + \ldots\ldots \frac{B_m}{S_1} X_m$$

or

$$\frac{X_1}{S_1} = \frac{B_1}{S_1} + \frac{B_2 S_2}{S_1} \frac{X_2}{S_2} + \ldots\ldots \frac{B_m S_m}{S_1} \frac{X_m}{S_m}$$

or

$$\frac{X_1}{S_1} = B'_1 + B'_2 \frac{X_2}{S_2} + \ldots\ldots B'_m \frac{X_m}{S_m} \qquad \ldots (4)$$

where, $B'_1, \ldots\ldots, B'_m$ are the beta coefficients.

1.3 Scope

The programme for multiple linear regression can be used where the association of three or more variables has to be investigated.

1.4 Hardware and Software Requirements

FORTRAN compiler and simple FORTRAN instructions are required to run

the programme . The programme has been taken from IBM's scientific subroutine package and implemented and tested on DEC-2050 and VAX-11/780 system. However, the same programme can be used with little or no modification on any other computer system also. The memory requirement depends upon the length of the data which will modify the dimension statements of the programme.

2.0 SPECIFIC METHOD

(i) Parameter Estimation

The parameters of the multiple regression are estimated by method of least squares for the sum of squares of residuals. The sum of squares of residuals is given by the following equation:

$$\varepsilon^2 = \sum_{i=1}^{N} (X_1 - X_1')^2 \qquad \ldots (5)$$

where :

X_1: Observed value of dependent variable

X_1': Estimated value of dependent variable

Estimated value of the dependent variable is given by:

$$X_1 = B_1 + B_2X_2 + B_3X_3 + \ldots\ldots + B_mX_m$$

For $\varepsilon^2 = 0$

$$[Y] = [X][B]$$

$$[X]^T[Y] = [X]^T[X][B]$$

$$[X^TX]^{-1}[X]^T[Y] = [X^TX]^{-1}[X]^T[X][B]$$

$$[B] = [X^TX]^{-1}[X]^T[Y]$$

where:

[B]: Matrix containing M regression coefficients

[X]: (NxM) matrix containing independent variables

[Y]: (Nx1) matrix containing dependent variable

(ii) Statistics Calculated in the Programme for Multiple Regression

Formulae used for the calculation of various statistics are given in subsequent sections:

(a) Mean of the variable (dependent or independent)

where:

$$\bar{X} = \frac{\sum_{i=1}^{N} X_i}{N} \quad \ldots (7)$$

$\bar{X}$: Mean

X_i: i^{th} value of variable

N : Total number of observations

(b) Standard deviation

$$\sigma = \left(\frac{\sum_{i=1}^{N} (X_i - \bar{X})^2}{N - 1} \right)^{1/2} \quad \ldots (8)$$

where :

σ : Standard derivation

(c) Correlation X v/s Y

$$r_{x,y} = \frac{S_{xy}}{S_x S_y}$$

$$= \frac{\sum_{i=1}^{N} (X_i - \bar{X})(Y_i - \bar{Y}) / N-1}{\sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (X_i - \bar{X})^2} \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (Y_i - \bar{Y})^2}}$$

$$= \frac{\sum_{i=1}^{N} (X_i - \bar{X})(Y_i - \bar{Y})}{\left(\sum_{i=1}^{N} (X_i - \bar{X})^2 \sum_{i=1}^{N} (Y_i - \bar{Y})^2 \right)^{1/2}} \quad \ldots (9)$$

where :

$r_{x,y}$: Correlation X v/s Y

(d) Regression coefficient

Regression coefficients are calculated by the method of least squares

(e) Standard error of regression coefficient

where

$$S_{b_i} = \frac{S_1}{S_i} \sqrt{\frac{1}{(N - M)(1 - R_i^2)}} \quad \ldots (10)$$

S_{b_i} Standard error of i^{th} regression coefficient

S_1 : Standard deviation of residuals

S_i : Standard deviation of X_i

R_i : Multiple correlation coefficient of X_1 with respect to all variables except the variable X_i

(f) T value

$$\text{T value} = \frac{\text{regression coefficient}}{\text{standard error of regression coefficient}} \quad \ldots(11)$$

(g) Sum of squares due to regression

$$SSDR = \sum_{i=1}^{N} (X_1' - \bar{X}_1)^2 \quad \ldots(12)$$

where :

SSDR : Sum of squares due to regression

X_1' : Estimated value of dependent variable

$\bar{X}_1$: Mean of dependent variable

(h) Sum of squares from regression

$$SSFR = \sum_{i=1}^{N} (X_1 - X_1')^2 \quad \ldots(13)$$

where:

SSFR : Sum of squares from regression

X_1 : Observed value of dependent variable

(i) Mean squares due to regression

$$\text{Mean squares due to regression} = \frac{\text{Sum of squares due regression}}{\text{Number of degrees of freedom}} \quad \ldots(14)$$

Number of degrees of freedom will be equal to number of independent variables.

(j) Mean squares from regression

$$\text{Mean squares from regression} = \frac{\text{Sum of squares from regression}}{\text{Number of degrees of freedom}} \quad \ldots(15)$$

Number of degrees of freedom will be equal to N-M

(k) Multiple correlation coefficient

This is the square root of coefficient of determination.

Multiple correlation coefficient = $\sqrt{R^2}$

$$= \sqrt{\frac{\text{Sum of squares due to regression}}{\text{Sum of squares about mean}}}$$

$$= \sqrt{\frac{\sum_{i=1}^{N} (X_1' - \bar{X})^2}{\sum_{i=1}^{N} (X_1 - \bar{X}_1)^2}} \qquad \ldots(16)$$

Sum of squares about the mean is the sum of squares due to regression and sum of squares from the regression.

(l) Standard error of estimate

Standard error of estimate = $\sqrt{\text{Mean square from regression}}$

$$= \sqrt{\sum_{i=1}^{N} (X_1 - X_1')^2 / (N - M)} \qquad \ldots\ (17)$$

(m) F value

$$\text{F value} = \frac{\text{Mean squares due to regression}}{\text{Mean squares from regression}} \qquad \ldots\ (18)$$

(iii) Tests of Hypothesis

F-Test: A test of the hypothesis that the regression equation is not explaining a significant amount of variation in X_1 can be made by calculating the F value, which is the ratio of mean squares due to regression to mean squares from regression.

The hypothesis is rejected if F value for the regression equation exceeds

$F_{1-\alpha, M-1, N-M}$ where $(1-\alpha)$ is the confidence level. In other words regression equation explains significant amount of variation in X_1 if F value is greater than $F_{1-\alpha, M-1, N-M}$.

The values of cumulative F distribution for ν_1 (numerator (M-1) and ν_2 (denominator (N-M)) are given in table 2.

T-Test: A test of hypothesis that the i^{th} independent variable is not contributing significantly to explain the variation in dependent variable is made by calculating the T-value. The hypothesis is rejected at $1-\alpha$ probability level if t value exceeds $t_{1-\alpha/2, N-M}$. In other words i^{th} variable is significant at $1-\alpha$ level if t value is greater than $t_{1-\alpha/2, N-M}$.

Percentile values $(t_{\alpha, \nu})$ for the t distribution with ν degrees of freedom are given in table 1.

2.1 General Description/Programme Description

2.1.1 Programme description

The multiple linear regression programme consists of the main routine named MREG, a special input subroutine DATA and four other subroutines. The subroutines have been described below:

(a) SUBROUTINE DATA (M,D)

The purpose of this subroutine is to read an observation from input device. This subroutine is called by the subroutine CORRE and must be provided by the user. If size and location of data fields are different from problem to problem, this subroutine must be recompiled with proper format statements. Various calling arguments are:

M : The number of variables in an observation

D : Output vector of length M containing the observation data

(b) SUBROUTINE CORRE(N,M,IO,X,XBAR,STD,RX,R,B,D,T)

This subroutine computes means, standard deviations, sums of cross products of deviations, and correlation coefficients. Various calling arguments are :

N : Number of observations N must be greater than or equal to 1

M : Number of variables M must be greater than or equal to 1

IO : Option code for input data

0 if the data are to be read in from subroutine DATA

1 if data are already in CORRE

X If IO=0, X is 0.0, if IO=1, X is input matrix (NxM) containing data

XBAR Output vector of length M containing means

STD Output vector of length M containing standard deviations

RX Output matrix (MxM) containing sums of cross products of deviations from means

R Output matrix containing correlation coefficients

B Output vector of length M containing the diagonal of the matrix of sums of cross products of deviations from mean

D Working vector of length M

T Working vector of length M

c) SUBROUTINE ORDER (M,R,NDEP,K,ISAVE,RX,RY)

The purpose of this subroutine is to construct from larger matrix of correlation coefficients a subset matrix of intercorrelations among independent variables and a vector of intercorrelations of independent variables with dependent variable.

The calling arguments are :

M : Number of variables and order of matrix R

R : Input matrix containing correlation coefficients

NDEP : The subscript number of the dependent variable

K : Number of independent variables to be included in the forthcoming regression, K must be greater than or equal to 1.

ISAVE : Input vector of length K + 1 containing, in ascending order, the subscript numbers of K independent variables to be included in the forthcoming regression, upon returning to the calling routine this contains in addition, the subscript number of the dependent variable in K+1 position

RX : Output matrix (KxK) containing intercorrelations among independent variables to be used in forthcoming regressior

RY : Output vector of length K containing intercorrelations of independent variables with dependent variable

(d) SUBROUTINE MINV (A,N,D,L,M)

The subroutine is used for matrix inversion. Various calling arguments are:

A : Input matrix destroyed in computation and replaced by resultant inverse

N : Order of matrix A

D : Resultant determinant

L : Work vector of length N

M : Work vector of length N

(e) SUBROUTINE MULTR(N,K,XBAR,STD,D,RX,RY,ISAVE,B,SB,TANS)

This subroutine performs a multiple linear regression analysis for a dependent variable and a set of independent variables. The calling arguments are:

N : Number of observations

K : Number of independent variables in the regression

XBAR : Input vector of length M containing means of all the variables

STD : Input vector of length M containing standard deviations

of all the variables

D : Input vector of length M containing the diagonal of the matrix of sums of cross products of deviations from means for all variables

RX : Input matrix (KxK) containing the inverse of intercorrelations among independent variables

RY : Input vector of length K containing intercorrelations of independent variables with dependent variable

ISAVE: Input vector of length K + 1 containing subscripts of independent variables in ascending order. The subscript of the dependent variable is stored in the last K + 1 position

B : Output vector of length K containing regression coefficients

SB : Output vector of length K containing standard deviation of regression coefficients

T : Output vector of length K containing t-values

ANS : Output vector of length 10 containing the following information

ANS(1) : Intercept

ANS(2) : Multiple correlation coefficient

ANS(3) : Standard error of estimate

ANS(4) : Sum of squares attributable to regression (SSAR)

ANS(5) : Degree of freedom associated with SSAR

ANS(6) : Mean squares of SSAR

ANS(7) : Sum of squares of deviations from regression SSDR

ANS(8) : Degrees of freedom associated with SSDR

ANS(9) : Mean squares of SSDR

ANS(10) : F Value

2.1.2 Programme Modification

Programme capacity can be increased or decreased by making changes in dimension statements. Input data in a different format can also be handled

by providing a specific format statement in subroutine DATA. In order to familiarize the user with the programme modification, the following are the general rules.

(1) Changes in the dimension statements of the main programme MREG :

(a) The dimension of arrays XBAR,STD,D,RY,ISAVE,B,SB,T and W must be greater than or equal to the number of variables M.

(b) The dimension of array RX must be greater than or equal to the product of MxM.

(c) The dimension of array R must be greater than or equal to (M+1)M/2.

(2) Changes in the input format statement of the special input subroutine,DATA :

The subroutine DATA is normally written by the user to handle different formats for different problems. The user may modify this subroutine to perform testing of input data, transforming of data and so on.

2.2 Data requirement

Data of various variables among which multiple linear regression has to be performed are required. The pairs of dependent and independent variables should be complete.

2.3 Analysis

The following model has been used to correlate discharge with area and average annual maximum 24 hour rainfall depth:

$$Q = B_1 + B_2A + B_3I \qquad \ldots (19)$$

where :

Q : Mean annual flood in thousands of cusecs

A : Watershed area in thousands of square miles

I : Average annual maximum 24 hour rainfall depth in inches

B_1, B_2, B_3 are the regression coeffficients.

The calculations for various statistical parameters are described in subsequent sections:

(a) Means

The mean of Q,A and I have been calculated using equation(7) and are 21.772,1.524,2.450 respectively.

(b) Standard deviations

Using equation (8) the standard deviations of Q,A and I are 32.050, 2.422 and 0.477 respectively.

(c) Correlation coefficients between A and Q and I and Q using equation(9) are

$$r_{A, Q} = 0.994$$
$$r_{I,Q} = 0.590$$

(d) Regression coefficients

The regression coefficients are calculated by the method of least squares. The regression coefficients and the intercept are 13.151 , 0.011 and 1.65700 respectively.

(e) Standard error of regression coefficients

Using equation(10) standard errors of regression coefficients are 0.562 and 2.853 .

(f) T values

$$\text{T value} = \frac{\text{Regression Coefficient}}{\text{Standard error of regression coefficient}}$$

$$\text{T value for A} = \frac{13.151}{0.562}$$
$$= 23.414$$

$t_{1-\alpha/2,\ N-M}$ or $t_{.975,11}$ is 2.201 from table [1], value for A is greater than $t_{.975,11}$. So the variable A is significant at 95%

probability level.

$$\text{T value for I} = \frac{0.011}{2.853}$$

$$= 0.004$$

$|T|$ value for I is less than $t_{.975,11}$. So the variable is not significant at 95% probability level.

(g) Sum of squares due to regression

$$SSDR = \sum_{i=1}^{14} (Q_i' - \bar{Q})^2$$

$$= 13182.63$$

(h) Sum of squares from regression

$$SSFR = \sum_{i=1}^{14} (Q_i' - Q_i)^2$$

$$= 171.099$$

(i) Mean squares due to regression

$$\text{Mean squares due to regression} = \frac{13182.62}{2}$$

$$= 6591.312$$

(j) Mean squares from regression

$$\text{Mean squares from regression} = \frac{171.098}{11}$$

$$= 15.554$$

(k) F value

Using equation (18), F value is 423.758. F value is greater than $F_{1-\alpha}$, M-1, N-M or $F_{.95,2,11}$ or 3.98. So the model Q = 13.15105 A + .0011191 is explaining significant amount of variation in Q.

(l) Multiple correlation coefficient $= \sqrt{\frac{\text{Sum of squares due to regression}}{\text{Sum of squares about mean}}}$

$$= \sqrt{\frac{\sum_{i=1}^{14} (Q_i - \bar{Q})^2}{\sum_{i=1}^{14} (Q_i - \bar{Q})^2}}$$

$$= \left(\frac{13182.62305}{13353.72168} \right)^{1/2}$$

$$= .99357$$

(m) Standard error of regression = $\sqrt{\text{Mean squares from regression}}$

$$= \sqrt{\frac{\sum_{i=1}^{N} (Q_i - Q_i')^2}{N-M}}$$

$$= \sqrt{15.554}$$

$$= 3.94391$$

2.4 Advantages and Limitations

2.4.1 Advantages

(i) The parameters of a linear model or a model which can be transformed to linear can be estimated.

(ii) The programme for multiple regression analysis can deal with upto 40 variables (included both independent and dependent variables).

(iii) Any variable in the set of original variables can be designated as a dependent variable and any number of remaining variables can be specified as independent variables.

(iv) Selection of a dependent variable and a set of independent variables can be made as many times as desired.

2.4.2 Limitations

(i) All the observations of independent and dependent variables should be complete.

(ii) Subroutine DATA should be written by the user depending upon the problem.

(iii) The regression equation should not be extrapolated beyond the range of independent variables because of two reasons. Firstly, the confidence intervals in the regression line become very wide as the distance from mean value is increased. Secondly,the relation may be different in the extrapolated range.

2.5 Programme Details

The source programme listing and input specifications have been given in Appendix I and II. Various statistics printed in the output are described in Appendix III. Test data is given in Appendix IV. Input data(data file) and output file are given in Appendix V and VI respectively.

3.0 RECOMMENDATIONS

The programme for multiple regression analysis can deal with upto 40 variables (dependent and independent both). The format for input data has been set free. For the input data, the subroutine DATA should be written by the user. If there are more than 40 variables, the dimension statements in the main programme must be modified to handle the particular problem. The selection of different sets of independent variables and designation of a dependent variable can be made as many times as desired.

The regression analysis should be carried out carefully and the dependent and independent variables should be chosen on the basis of their physical significance. In regression analysis, it is possible to improve the model by including cross product terms by multiplying together two independent variables to form a new variable. Before carrying out the regression analysis the form of the model should be decided on the basis of rational analysis.

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3. Scientific Subroutine Package, International Business Machines, White Plains, N.Y.

TABLE - 1

Percentile values($t_{\alpha,\nu}$) for the t distribution with ν degrees of freedom

	α							
ν	0.75	0.80	0.85	0.90	0.95	0.975	0.995	0.9995
1	1.0005	1.376	1.963	3.078	6.314	12.706	63.657	636.619
2	0.816	1.061	1.386	1.886	2.920	4.303	9.925	31.598
3	0.765	0.978	1.250	1.638	2.353	3.182	5.841	12.941
4	0.741	0.941	1.190	1.533	2.132	2.776	4.604	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	4.032	6.859
6	0.718	0.906	1.134	1.440	1.943	2.447	3.707	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	3.499	5.405
8	0.706	0.889	1.108	1.397	1.860	2.306	3.355	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	3.250	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	3.169	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	3.106	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	3.055	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	3.012	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.977	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.947	4.073
16	0.690	0.866	1.071	1.337	1.746	2.120	2.921	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.898	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.878	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.861	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.845	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.831	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.819	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.807	3.767
24	0.685	0.857	1.059	1.318	1.711	2.064	2.797	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.787	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.779	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.771	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.763	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.756	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.750	3.646
35	0.682	0.852	1.052	1.306	1.690	2.030	2.724	3.591
40	0.681	0.851	1.050	1.303	1.684	2.021	2.704	3.551
45	0.680	0.850	1.048	1.301	1.680	2.014	2.690	3.520
50	0.680	0.849	1.047	1.299	1.676	2.008	2.678	3.496
55	0.679	0.849	1.047	1.297	1.673	2.004	2.669	3.476
60	0.679	0.848	1.046	1.296	1.671	2.000	2.660	3.460
70	0.678	0.847	1.045	1.294	1.667	1.994	2.648	3.435
80	0.678	0.847	1.044	1.293	1.665	1.990	2.638	3.416
90	0.678	0.846	1.043	1.291	1.662	1.987	2.632	3.402
100	0.677	0.846	1.042	1.290	1.661	1.984	2.626	3.390
200	0.676	0.844	1.039	1.286	1.653	1.972	2.601	3.340
300	0.676	0.843	1.038	1.285	1.650	1.968	2.592	3.323
400	0.676	0.843	1.038	1.284	1.649	1.966	2.588	3.315
500	0.676	0.843	1.037	1.284	1.648	1.965	2.586	3.310
1000	0.675	0.842	1.037	1.283	1.647	1.962	2.581	3.301
∞	0.67449	0.84162	1.03643	1.28155	1.64485	1.95996	2.57582	3.29053

TABLE 2

Cumulative Distribution for ν_1 (Numerator) and ν_2 (Denominator) Degrees of Freedom

ν_2	ν_1 / p	1	2	3	4	5	6	7	8	9	10	11	12
1	.0005	.0^{3}62	.0^{3}50	.0^{3}38	.0^{3}94	.016	.022	.027	.032	.036	.039	.042	.045
	.001	.0^{3}25	.0^{3}10	.0^{3}60	.013	.021	.028	.034	.039	.044	.048	.051	.054
	.005	.0^{3}62	.0^{3}51	.018	.032	.044	.054	.062	.068	.073	.078	.082	.085
	.010	.0^{3}25	.010	.029	.047	.062	.073	.082	.089	.095	.100	.104	.107
	.025	.0^{3}15	.026	.057	.082	.100	.113	.124	.132	.139	.144	.149	.153
	.05	.0^{3}62	.054	.099	.130	.151	.167	.179	.188	.195	.201	.207	.211
	.10	.025	.117	.181	.220	.246	.265	.279	.289	.298	.304	.310	.315
	.25	.172	.389	.494	.553	.591	.617	.637	.650	.661	.670	.680	.684
	.50	1.00	1.50	1.71	1.82	1.89	1.94	1.98	2.00	2.03	2.04	2.05	2.07
	.75	5.83	7.50	8.20	8.58	8.82	8.98	9.10	9.19	9.26	9.32	9.36	9.41
	.90	39.9	49.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9	60.2	60.5	60.7
	.95	161	200	216	225	230	234	237	239	241	242	243	244
	.975	648	800	864	900	922	937	948	957	963	969	973	977
	.99	405^{1}	500^{1}	540^{1}	562^{1}	576^{1}	586^{1}	593^{1}	598^{1}	602^{1}	606^{1}	608^{1}	611^{1}
	.995	162^{2}	200^{2}	216^{2}	225^{2}	231^{2}	234^{2}	237^{2}	239^{2}	241^{2}	242^{2}	243^{2}	244^{2}
	.999	406^{2}	500^{2}	540^{2}	562^{2}	576^{2}	586^{2}	593^{2}	598^{2}	602^{2}	606^{2}	609^{2}	611
	.9995	162^{4}	200^{4}	216^{4}	225^{4}	231^{4}	234^{4}	237^{4}	239^{4}	241^{4}	242^{4}	243^{4}	244^{4}
2	.0005	.0^{3}50	.0^{3}50	.0^{3}42	.011	.020	.029	.037	.044	.050	.056	.061	.065
	.001	.0^{3}20	.0^{3}10	.0^{3}68	.016	.027	.037	.046	.054	.061	.067	.072	.077
	.005	.0^{3}50	.0^{3}50	.020	.038	.055	.069	.081	.091	.099	.106	.112	.118
	.01	.0^{3}20	.010	.032	.056	.075	.092	.105	.116	.125	.132	.139	.144
	.025	.0^{3}13	.026	.062	.094	.119	.138	.153	.165	.175	.183	.190	.196
	.05	.0^{3}50	.053	.105	.144	.173	.194	.211	.224	.235	.244	.251	.257
	.10	.020	.111	.183	.231	.265	.289	.307	.321	.333	.342	.350	.356
	.25	.133	.333	.439	.500	.540	.568	.588	.604	.616	.626	.633	.641
	.50	.667	1.00	1.13	1.21	1.25	1.28	1.30	1.32	1.33	1.34	1.35	1.36
	.75	2.57	3.00	3.15	3.23	3.28	3.31	3.34	3.35	3.37	3.38	3.39	3.39
	.90	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.40	9.41
	.95	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4
	.975	38.5	39.0	39.2	39.2	39.3	39.3	39.4	39.4	39.4	39.4	39.4	39.4
	.99	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4
	.995	198	199	199	199	199	199	199	199	199	199	199	199
	.999	998	999	999	999	999	999	999	999	999	999	999	999
	.9995	200^{1}	200^{1}	200^{1}	200^{1}	200^{1}	200^{1}	200^{1}	200^{1}	200^{1}	200^{1}	200^{1}	200^{1}
3	.0005	.0^{3}46	.0^{3}50	.0^{3}44	.012	.023	.033	.043	.052	.060	.067	.074	.079
	.001	.0^{3}19	.0^{3}10	.0^{3}71	.018	.030	.042	.053	.063	.072	.079	.086	.093
	.005	.0^{3}46	.0^{3}50	.021	.041	.060	.077	.092	.104	.115	.124	.132	.138
	.01	.0^{3}19	.010	.034	.060	.083	.102	.118	.132	.143	.153	.161	.168
	.025	.0^{3}12	.026	.065	.100	.129	.152	.170	.185	.197	.207	.216	.224
	.05	.0^{3}46	.052	.108	.152	.185	.210	.230	.246	.259	.270	.279	.287
	.10	.019	.109	.185	.239	.276	.304	.325	.342	.356	.367	.376	.384
	.25	.122	.317	.424	.489	.531	.561	.582	.600	.613	.624	.633	.641
	.50	.585	.881	1.00	1.06	1.10	1.13	1.15	1.16	1.17	1.18	1.19	1.20
	.75	2.02	2.28	2.36	2.39	2.41	2.42	2.43	2.44	2.44	2.44	2.45	2.45
	.90	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.22
	.95	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74
	.975	17.4	16.0	15.4	15.1	14.9	14.7	14.6	14.5	14.5	14.4	14.4	14.3
	.99	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	27.1
	.995	55.6	49.8	47.5	46.2	45.4	44.8	44.4	44.1	43.9	43.7	43.5	43.4
	.999	167	149	141	137	135	133	132	131	130	129	129	128
	.9995	266	237	225	218	214	211	209	208	207	206	204	204

Read .0^{3}56 as .00056, 200^{1} as 2000, 162^{4} as 1620000, etc.

ν_2	p \ ν_1	15	20	24	30	40	50	60	100	120	200	500	∞
1	.0005	.051	.058	.062	.066	.069	.072	.074	.077	.078	.080	.081	.083
	.001	.060	.067	.071	.075	.079	.082	.084	.087	.088	.089	.091	.092
	.005	.093	.101	.105	.109	.113	.116	.118	.121	.122	.124	.126	.127
	.01	.115	.124	.128	.132	.137	.139	.141	.145	.146	.148	.150	.151
	.025	.161	.170	.175	.180	.184	.187	.189	.193	.194	.196	.198	.199
	.05	.220	.230	.235	.240	.245	.248	.250	.254	.255	.257	.259	.261
	.10	.325	.336	.342	.347	.353	.356	.358	.362	.364	.366	.368	.370
	.25	.698	.712	.719	.727	.734	.738	.741	.747	.749	.752	.754	.756
	.50	2.09	2.12	2.13	2.15	2.16	2.17	2.17	2.18	2.18	2.19	2.19	2.20
	.75	9.49	9.58	9.63	9.67	9.71	9.74	9.76	9.78	9.80	9.82	9.84	9.85
	.90	61.2	61.7	62.0	62.3	62.5	62.7	62.8	63.0	63.1	63.2	63.3	63.3
	.95	246	248	249	250	251	252	252	253	253	254	254	254
	.975	985	993	997	100^{1}	101^{1}	101^{1}	101^{1}	101^{1}	101^{1}	102^{1}	102^{1}	102^{1}
	.99	616^{1}	621^{1}	623^{1}	626^{1}	629^{1}	630^{1}	631^{1}	633^{1}	634^{1}	635^{1}	636^{1}	637^{1}
	.995	246^{2}	248^{2}	249^{2}	250^{2}	251^{2}	252^{2}	253^{2}	253^{2}	254^{2}	254^{2}	254^{2}	255^{2}
	.999	616^{2}	621^{2}	623^{2}	626^{2}	629^{2}	630^{2}	631^{2}	633^{2}	634^{2}	635^{2}	636^{2}	637^{2}
	.9995	246^{4}	248^{4}	249^{4}	250^{4}	251^{4}	252^{4}	252^{4}	253^{4}	253^{4}	253^{4}	254^{4}	254^{4}
2	.0005	.076	.088	.094	.101	.108	.113	.116	.122	.124	.127	.130	.132
	.001	.088	.100	.107	.114	.121	.126	.129	.135	.137	.140	.143	.145
	.005	.130	.143	.150	.157	.165	.169	.173	.179	.181	.184	.187	.189
	.01	.157	.171	.178	.186	.193	.198	.201	.207	.209	.212	.215	.217
	.025	.210	.224	.232	.239	.247	.251	.255	.261	.263	.266	.269	.271
	.05	.272	.286	.294	.302	.309	.314	.317	.324	.326	.329	.332	.334
	.10	.371	.386	.394	.402	.410	.415	.418	.424	.426	.429	.433	.434
	.25	.657	.672	.680	.689	.697	.702	.705	.711	.713	.716	.719	.721
	.50	1.38	1.39	1.40	1.41	1.42	1.42	1.43	1.43	1.43	1.44	1.44	1.44
	.75	3.41	3.43	3.43	3.44	3.45	3.45	3.46	3.47	3.47	3.48	3.48	3.48
	.90	9.42	9.44	9.45	9.46	9.47	9.47	9.47	9.48	9.48	9.49	9.49	9.49
	.95	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5	19.5	19.5	19.5	19.5
	.975	39.4	39.4	39.5	39.5	39.5	39.5	39.5	39.5	39.5	39.5	39.5	39.5
	.99	99.4	99.4	99.5	99.5	99.5	99.5	99.5	99.5	99.5	99.5	99.5	99.5
	.995	199	199	199	199	199	199	199	199	199	199	199	200
	.999	999	999	999	999	999	999	999	999	999	999	999	999
	.9995	200^{1}	200^{1}	200^{1}	200^{1}	200^{1}	200^{1}	200^{1}	200^{1}	200^{1}	200^{1}	200^{1}	200^{1}
3	.0005	.093	.109	.117	.127	.136	.143	.147	.156	.158	.162	.166	.169
	.001	.107	.123	.132	.142	.152	.158	.162	.171	.173	.177	.181	.184
	.005	.154	.172	.181	.191	.201	.207	.211	.220	.222	.227	.231	.234
	.01	.185	.203	.212	.222	.232	.238	.242	.251	.253	.258	.262	.264
	.025	.241	.259	.269	.279	.289	.295	.299	.308	.310	.314	.318	.321
	.05	.304	.323	.332	.342	.352	.358	.363	.370	.373	.377	.382	.384
	.10	.402	.420	.430	.439	.449	.455	.459	.467	.469	.474	.476	.480
	.25	.658	.675	.684	.693	.702	.708	.711	.719	.721	.724	.728	.730
	.50	1.21	1.23	1.23	1.24	1.25	1.25	1.25	1.26	1.26	1.26	1.27	1.27
	.75	2.46	2.46	2.46	2.47	2.47	2.47	2.47	2.47	2.47	2.47	2.47	2.47
	.90	5.20	5.18	5.18	5.17	5.16	5.15	5.15	5.14	5.14	5.14	5.14	5.13
	.95	8.70	8.66	8.63	8.62	8.59	8.58	8.57	8.55	8.55	8.54	8.53	8.53
	.975	14.3	14.2	14.1	14.1	14.0	14.0	14.0	14.0	13.9	13.9	13.9	13.9
	.99	26.9	26.7	26.6	26.5	26.4	26.4	26.3	26.2	26.2	26.2	26.1	26.1
	.995	43.1	42.8	42.6	42.5	42.3	42.2	42.1	42.0	42.0	41.9	41.9	41.8
	.999	127	126	126	125	125	125	124	124	124	124	124	123
	.9995	203	201	200	199	199	198	198	197	197	197	196	196

n_2	p \ n_1	1	2	3	4	5	6	7	8	9	10	11	12
4	.0005	.0^{4}44	.0^{3}50	.0^{3}46	.013	.024	.036	.047	.057	.066	.075	.082	.089
	.001	.0^{3}18	.0^{3}10	.0^{3}73	.019	.032	.046	.058	.069	.079	.089	.097	.104
	.005	.0444	.0^{3}50	.022	.043	.064	.083	.100	.114	.126	.137	.145	.153
	.01	.0^{3}18	.010	.035	.063	.088	.109	.127	.143	.156	.167	.176	.185
	.025	.0^{3}11	.026	.066	.104	.135	.161	.181	.198	.212	.224	.234	.243
	05	.0^{3}44	.052	.110	.157	.193	.221	.243	.261	.275	.288	.298	.307
	.10	.018	.108	.187	.243	.284	.314	.338	.356	.371	.384	.394	.403
	.25	.117	.309	.418	.484	.528	.560	.583	.601	.615	.627	.637	.645
	.50	.549	.828	.941	1.00	1.04	1.06	1.08	1.09	1.10	1.11	1.12	1.13
	.75	1.81	2.00	2.05	2.06	2.07	2.08	2.08	2.08	2.08	2.08	2.08	2.08
	.90	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.91	3.90
	.95	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91
	.975	12.2	10.6	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.79	8.75
	.99	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.4
	.995	31.3	26.3	24.3	23.2	22.5	22.0	21.6	21.4	21.1	21.0	20.8	20.7
	.999	74.1	61.2	56.2	53.4	51.7	50.5	49.7	49.0	48.5	48.0	47.7	47.4
	.9995	106	87.4	80.1	76.1	73.6	71.9	70.6	69.7	68.9	68.3	67.8	67.4
5	.0005	.0^{4}43	.0^{3}50	.0^{3}47	.014	.025	.038	.050	.061	.070	.081	.089	.096
	.001	.0^{3}17	.0^{3}10	.0^{3}75	.019	.034	.048	.062	.074	.085	.095	.104	.112
	.005	.0443	.0^{3}50	.022	.045	.067	.087	.105	.120	.134	.146	.156	.165
	.01	.0^{3}17	.010	.035	.064	.091	.114	.134	.151	.165	.177	.188	.197
	.025	.0^{3}11	.025	.067	.107	.140	.167	.189	.208	.223	.236	.248	.257
	.05	.0^{3}43	.052	.111	.160	.198	.228	.252	.271	.287	.301	.313	.322
	.10	.017	.108	.188	.247	.290	.322	.347	.367	.383	.397	.408	.418
	.25	.113	.305	.415	.483	.528	.560	.584	.604	.618	.631	.641	.650
	.50	.528	.799	.907	.965	1.00	1.02	1.04	1.05	1.06	1.07	1.08	1.09
	.75	1.69	1.85	1.88	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89
	.90	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.28	3.27
	.95	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.71	4.68
	.975	10.0	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.57	6.52
	.99	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.96	9.89
	.995	22.8	18.3	16.5	15.6	14.9	14.5	14.2	14.0	13.8	13.6	13.5	13.4
	.999	47.2	37.1	33.2	31.1	29.7	28.8	28.2	27.6	27.2	26.9	26.6	26.4
	.9995	63.6	49.8	44.4	41.5	39.7	38.5	37.6	36.9	36.4	35.9	35.6	35.2
6	.0005	.0^{4}43	.0^{3}50	.0^{3}47	.014	.026	.039	.052	.064	.075	.085	.094	.103
	.001	.0^{3}17	.0^{3}10	.0^{3}75	.020	.035	.050	.064	.078	.090	.101	.111	.119
	.005	.0^{4}43	.0^{3}50	.022	.045	.069	.090	.109	.126	.140	.153	.164	.174
	.01	.0^{3}17	.010	.036	.066	.094	.118	.139	.157	.172	.186	.197	.207
	.025	.0^{3}11	.025	.068	.109	.143	.172	.195	.215	.231	.246	.258	.268
	.05	.0^{4}43	.052	.112	.162	.202	.233	.259	.279	.296	.311	.324	.334
	.10	.017	.107	.189	.249	.294	.327	.354	.375	.392	.406	.418	.429
	.25	.111	.302	.413	.481	.524	.561	.586	.606	.622	.635	.645	.654
	.50	.515	.780	.886	.942	.977	1.00	1.02	1.03	1.04	1.05	1.05	1.06
	.75	1.62	1.76	1.78	1.79	1.79	1.78	1.78	1.78	1.77	1.77	1.77	1.77
	.90	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.92	2.90
	.95	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00
	.975	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.41	5.37
	.99	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.79	7.72
	.995	18.6	14.5	12.9	12.0	11.5	11.1	10.8	10.6	10.4	10.2	10.1	10.0
	.999	35.5	27.0	23.7	21.9	20.8	20.0	19.5	19.0	18.7	18.4	18.2	18.0
	.9995	46.1	34.8	30.4	28.1	26.6	25.6	24.9	24.3	23.9	23.5	23.2	23.0

n_2	p \ n_1	15	20	24	30	40	50	60	100	120	200	500	∞
4	.0005	.105	.125	.135	.147	.159	.166	.172	.183	.186	.191	.196	.200
	.001	.121	.141	.152	.163	.176	.183	.188	.200	.202	.208	.213	.217
	.005	.172	.193	.204	.216	.229	.237	.242	.253	.255	.260	.266	.269
	.01	.204	.226	.237	.249	.261	.269	.274	.285	.287	.293	.298	.301
	.025	.263	.284	.296	.308	.320	.327	.332	.342	.346	.351	.356	.359
	.05	.327	.349	.360	.372	.384	.391	.396	.407	.409	.413	.418	.422
	.10	.424	.445	.456	.467	.478	.485	.490	.500	.502	.508	.510	.514
	.25	.664	.683	.692	.702	.712	.718	.722	.731	.733	.737	.740	.743
	.50	1.14	1.15	1.16	1.16	1.17	1.18	1.18	1.18	1.18	1.19	1.19	1.19
	.75	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08
	.90	3.87	3.84	3.83	3.82	3.80	3.80	3.79	3.78	3.78	3.77	3.76	3.76
	.95	5.86	5.80	5.77	5.75	5.72	5.70	5.69	5.66	5.66	5.65	5.64	5.63
	.975	8.66	8.56	8.51	8.46	8.41	8.38	8.36	8.32	8.31	8.29	8.27	8.26
	.99	14.2	14.0	13.9	13.8	13.7	13.7	13.7	13.6	13.6	13.5	13.5	13.5
	.995	20.4	20.2	20.0	19.9	19.8	19.7	19.6	19.5	19.5	19.4	19.4	19.3
	.999	46.8	46.1	45.8	45.4	45.1	44.9	44.7	44.5	44.4	44.3	44.1	44.0
	.9995	66.5	65.5	65.1	64.6	64.1	63.8	63.6	63.2	63.1	62.9	62.7	62.6
5	.0005	.115	.137	.150	.163	.177	.186	.192	.205	.209	.216	.222	.226
	.001	.132	.155	.167	.181	.195	.204	.210	.223	.227	.233	.239	.244
	.005	.186	.210	.223	.237	.251	.260	.266	.279	.282	.288	.294	.299
	.01	.219	.244	.257	.270	.285	.293	.299	.312	.315	.322	.328	.331
	.025	.280	.304	.317	.330	.344	.353	.359	.370	.374	.380	.386	.390
	.05	.345	.369	.382	.395	.408	.417	.422	.432	.437	.442	.448	.452
	.10	.440	.463	.476	.488	.501	.508	.514	.524	.527	.532	.538	.541
	.25	.669	.690	.700	.711	.722	.728	.732	.741	.743	.748	.752	.755
	.50	1.10	1.11	1.12	1.12	1.13	1.13	1.14	1.14	1.14	1.15	1.15	1.15
	.75	1.89	1.88	1.88	1.88	1.88	1.88	1.87	1.87	1.87	1.87	1.87	1.87
	.90	3.24	3.21	3.19	3.17	3.16	3.15	3.14	3.13	3.12	3.12	3.11	3.10
	.95	4.62	4.56	4.53	4.50	4.46	4.44	4.43	4.41	4.40	4.39	4.37	4.36
	.975	6.43	6.33	6.28	6.23	6.18	6.14	6.12	6.08	6.07	6.05	6.03	6.02
	.99	9.72	9.55	9.47	9.38	9.29	9.24	9.20	9.13	9.11	9.08	9.04	9.02
	.995	13.1	12.9	12.8	12.7	12.5	12.5	12.4	12.3	12.3	12.2	12.2	12.1
	.999	25.9	25.4	25.1	24.9	24.6	24.4	24.3	24.1	24.1	23.9	23.8	23.8
	.9995	34.6	33.9	33.5	33.1	32.7	32.5	32.3	32.1	32.0	31.8	31.7	31.6
6	.0005	.123	.148	.162	.177	.193	.203	.210	.225	.229	.236	.244	.249
	.001	.141	.166	.180	.195	.211	.222	.229	.243	.247	.255	.262	.267
	.005	.197	.224	.238	.253	.269	.279	.286	.301	.304	.312	.318	.324
	.01	.232	.258	.273	.288	.304	.313	.321	.334	.338	.346	.352	.357
	.025	.293	.320	.334	.349	.364	.375	.381	.394	.398	.405	.412	.415
	.05	.358	.385	.399	.413	.428	.437	.444	.457	.460	.467	.472	.476
	.10	.453	.478	.491	.505	.519	.526	.533	.546	.548	.556	.559	.564
	.25	.675	.696	.707	.718	.729	.736	.741	.751	.753	.758	.762	.765
	.50	1.07	1.08	1.09	1.10	1.10	1.11	1.11	1.11	1.12	1.12	1.12	1.12
	.75	1.76	1.76	1.75	1.75	1.75	1.75	1.74	1.74	1.74	1.74	1.74	1.74
	.90	2.87	2.84	2.82	2.80	2.78	2.77	2.76	2.75	2.74	2.73	2.73	2.72
	.95	3.94	3.87	3.84	3.81	3.77	3.75	3.74	3.71	3.70	3.69	3.68	3.67
	.975	5.27	5.17	5.12	5.07	5.01	4.98	4.96	4.92	4.90	4.88	4.86	4.85
	.99	7.56	7.40	7.31	7.23	7.14	7.09	7.06	6.99	6.97	6.93	6.90	6.88
	.995	9.81	9.59	9.47	9.36	9.24	9.17	9.12	9.03	9.00	8.95	8.91	8.88
	.999	17.6	17.1	16.9	16.7	16.4	16.3	16.2	16.0	16.0	15.9	15.8	15.7
	.9995	22.4	21.9	21.7	21.4	21.1	20.9	20.7	20.5	20.4	20.3	20.2	20.1

n_2	n_1 / p	1	2	3	4	5	6	7	8	9	10	11	12
7	.0005	.0^{42}	.0^{350}	.0^{248}	.014	.027	.040	.053	.066	.078	.088	.099	.108
	.001	.0^{317}	.0^{310}	.0^{276}	.020	.035	.051	.067	.081	.093	.105	.115	.125
	.005	.0^{242}	.0^{250}	.023	.046	.070	.093	.113	.130	.145	.159	.171	.181
	.01	.0^{317}	.010	.036	.067	.096	.121	.143	.162	.178	.192	.205	.216
	.025	.0^{310}	.025	.068	.110	.146	.176	.200	.221	.238	.253	.266	.277
	.05	.0^{242}	.052	.113	.164	.205	.238	.264	.286	.304	.319	.332	.343
	.10	.017	.107	.190	.251	.297	.332	.359	.381	.399	.414	.427	.438
	.25	.110	.300	.412	.481	.528	.562	.588	.608	.624	.637	.649	.658
	.50	.506	.767	.871	.926	.960	.983	1.00	1.01	1.02	1.03	1.04	1.04
	.75	1.57	1.70	1.72	1.72	1.71	1.71	1.70	1.70	1.69	1.69	1.69	1.68
	.90	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.68	2.67
	.95	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57
	.975	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.71	4.67
	.99	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.54	6.47
	.995	16.2	12.4	10.9	10.0	9.52	9.16	8.89	8.68	8.51	8.38	8.27	8.18
	.999	29.2	21.7	18.8	17.2	16.2	15.5	15.0	14.6	14.3	14.1	13.9	13.7
	.9995	37.0	27.2	23.5	21.4	20.2	19.3	18.7	18.2	17.8	17.5	17.2	17.0
8	.0005	.0^{42}	.0^{350}	.0^{248}	.014	.027	.041	.055	.068	.081	.092	.102	.112
	.001	.0^{317}	.0^{310}	.0^{276}	.020	.036	.053	.068	.083	.096	.109	.120	.130
	.005	.0^{242}	.0^{250}	.027	.047	.072	.095	.115	.133	.149	.164	.176	.187
	.01	.0^{317}	.010	.036	.068	.097	.123	.146	.166	.183	.198	.211	.222
	.025	.0^{310}	.025	.069	.111	.148	.179	.204	.226	.244	.259	.273	.285
	.05	.0^{242}	.052	.113	.166	.208	.241	.268	.291	.310	.326	.339	.351
	.10	.017	.107	.190	.253	.299	.335	.363	.386	.405	.421	.435	.445
	.25	.109	.298	.411	.481	.529	.563	.589	.610	.627	.640	.654	.661
	.50	.499	.757	.860	.915	.948	.971	.988	1.00	1.01	1.02	1.02	1.03
	.75	1.54	1.66	1.67	1.66	1.66	1.65	1.64	1.64	1.64	1.63	1.63	1.62
	.90	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.52	2.50
	.95	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28
	.975	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.24	4.20
	.99	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.73	5.67
	.995	14.7	11.0	9.60	8.81	8.30	7.95	7.69	7.50	7.34	7.21	7.10	7.01
	.999	25.4	18.5	15.8	14.4	13.5	12.9	12.4	12.0	11.8	11.5	11.4	11.2
	.9995	31.6	22.8	19.4	17.6	16.4	15.7	15.1	14.6	14.3	14.0	13.8	13.6
9	.0005	.0^{41}	.0^{350}	.0^{248}	.015	.027	.042	.056	.070	.083	.094	.105	.115
	.001	.0^{317}	.0^{310}	.0^{277}	.021	.037	.054	.070	.085	.099	.112	.123	.134
	.005	.0^{242}	.0^{350}	.023	.047	.073	.096	.117	.136	.153	.168	.181	.192
	.01	.0^{317}	.010	.037	.068	.098	.125	.149	.169	.187	.202	.216	.228
	.025	.0^{310}	.025	.069	.112	.150	.181	.207	.230	.248	.265	.279	.291
	.05	.0^{240}	.052	.113	.167	.210	.244	.272	.296	.315	.331	.345	.358
	.10	.017	.107	.191	.254	.302	.338	.367	.390	.410	.426	.441	.452
	.25	.108	.297	.410	.480	.529	.564	.591	.612	.629	.643	.654	.664
	.50	.494	.749	.852	.906	.939	.962	.978	.990	1.00	1.01	1.01	1.02
	.75	1.51	1.62	1.63	1.63	1.62	1.61	1.60	1.60	1.59	1.59	1.58	1.58
	.90	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.40	2.38
	.95	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07
	.975	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.91	3.87
	.99	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.18	5.11
	.995	13.6	10.1	8.72	7.96	7.47	7.13	6.88	6.69	6.54	6.42	6.31	6.23
	.999	22.9	16.4	13.9	12.6	11.7	11.1	10.7	10.4	10.1	9.89	9.71	9.57
	.9995	28.0	19.9	16.8	15.1	14.1	13.3	12.8	12.4	12.1	11.8	11.6	11.4

n_2	n_1 / p	15	20	24	30	40	50	60	100	120	200	500	∞
7	.0005	.130	.157	.172	.188	.206	.217	.225	.242	.246	.255	.263	.268
	.001	.148	.176	.191	.208	.225	.237	.245	.261	.266	.274	.282	.288
	.005	.206	.235	.251	.267	.285	.296	.304	.319	.324	.332	.340	.345
	.01	.241	.270	.286	.303	.320	.331	.339	.355	.358	.366	.373	.379
	.025	.304	.333	.348	.364	.381	.392	.399	.413	.418	.426	.433	.437
	.05	.369	.398	.413	.428	.445	.455	.461	.476	.479	.485	.493	.498
	.10	.463	.491	.504	.519	.534	.543	.550	.562	.566	.571	.578	.582
	.25	.679	.702	.713	.725	.737	.745	.749	.760	.762	.767	.772	.775
	.50	1.05	1.07	1.07	1.08	1.08	1.09	1.09	1.10	1.10	1.10	1.10	1.10
	.75	1.68	1.67	1.67	1.66	1.66	1.66	1.65	1.65	1.65	1.65	1.65	1 65
	.90	2.63	2.59	2.58	2.56	2.54	2.52	2.51	2.50	2.49	2.48	2.48	2.47
	.95	3.51	3.44	3.41	3.38	3.34	3.32	3.30	3.27	3.27	3.25	3.24	3.23
	.975	4.57	4.47	4.42	4.36	4.31	4.28	4.25	4.21	4.20	4.18	4.16	4.14
	.99	6.31	6.16	6.07	5.99	5.91	5.86	5.82	5.75	5.74	5.70	5.67	5.65
	.995	7.97	7.75	7.65	7.53	7.42	7.35	7.31	7.22	7.19	7.15	7.10	7.08
	.999	13.3	12.9	12.7	12.5	12.3	12.2	12.1	11.9	11.9	11.8	11.7	11.7
	.9995	16.5	16.0	15.7	15.5	15.2	15.1	15.0	14.7	14.7	14.6	14.5	14.4
8	.0005	.136	.164	.181	.198	.218	.230	.239	.257	.262	.271	.281	.287
	.001	.155	.184	.200	.218	.238	.250	.259	.277	.282	.292	.300	.306
	.005	.214	.244	.261	.279	.299	.311	.319	.337	.341	.351	.358	.364
	.01	.250	.281	.297	.315	.334	.346	.354	.372	.376	.385	.392	.398
	.025	.313	.343	.360	.377	.395	.407	.415	.431	.435	.442	.450	.456
	.05	.379	.409	.425	.441	.459	.469	.477	.493	.496	.505	.510	.516
	.10	.472	.500	.515	.531	.547	.556	.563	.578	.581	.588	.595	.599
	.25	.684	.707	.718	.730	.743	.751	.756	.767	.769	.775	.780	.783
	.50	1.04	1.05	1.06	1.07	1.07	1.07	1.08	1.08	1.08	1.09	1.09	1.09
	.75	1.62	1.61	1.60	1.60	1.59	1.59	1.59	1.58	1.58	1.58	1.58	1.58
	.90	2.46	2.42	2.40	2.38	2.36	2.35	2.34	2.32	2.32	2.31	2.30	2.29
	.95	3.22	3.15	3.12	3.08	3.04	3.02	3.01	2.97	2.97	2.95	2.94	2.93
	.975	4.10	4.00	3.95	3.89	3.84	3.81	3.78	3.74	3.73	3.70	3.68	3.67
	.99	5.52	5.36	5.28	5.20	5.12	5.07	5.03	4.96	4.95	4.91	4.88	4.86
	.995	6.81	6.61	6.50	6.40	6.29	6.22	6.18	6.09	6.06	6.02	5.98	5.95
	.999	10.8	10.5	10.3	10.1	9.92	9.80	9.73	9.57	9.54	9.46	9.39	9.34
	.9995	13.1	12.7	12.5	12.2	12.0	11.8	11.8	11.6	11.5	11.4	11.4	11.3
9	.0005	.141	.171	.188	.207	.228	.242	.251	.270	.276	.287	.297	.303
	.001	.160	.191	.208	.228	.249	.262	.271	.291	.296	.307	.316	.323
	.005	.220	.253	.271	.290	.310	.324	.332	.351	.356	.366	.376	.382
	.01	.257	.289	.307	.326	.346	.358	.368	.386	.391	.400	.410	.415
	.025	.320	.352	.370	.388	.408	.420	.428	.446	.450	.459	.467	.473
	.05	.386	.418	.435	.452	.471	.483	.490	.508	.510	.518	.526	.532
	.10	.479	.509	.525	.541	.558	.568	.575	.588	.594	.602	.610	.613
	.25	.687	.711	.723	.736	.749	.757	.762	.773	.776	.782	.787	.791
	.50	1.03	1.04	1.05	1.05	1.06	1.06	1.07	1.07	1.07	1.08	1.08	1.08
	.75	1.57	1.56	1.56	1.55	1.55	1.54	1.54	1.53	1.53	1.53	1.53	1.53
	.90	2.34	2.30	2.28	2.25	2.23	2.22	2.21	2.19	2.18	2.17	2.17	2.16
	.95	3.01	2.94	2.90	2.86	2.83	2.80	2.79	2.76	2.75	2.73	2.72	2.71
	.975	3.77	3.67	3.61	3.56	3.51	3.47	3.45	3.40	3.39	3.37	3.35	3.33
	.99	4.96	4.81	4.73	4.65	4.57	4.52	4.48	4.42	4.40	4.36	4.33	4.31
	.995	6.03	5.83	5.73	5.62	5.52	5.45	5.41	5.32	5.30	5.26	5.21	5.19
	.999	9.24	8.90	8.72	8.55	8.37	8.26	8.19	8.04	8.00	7.93	7.86	7.81
	.9995	11.0	10.6	10.4	10.2	9.94	9.80	9.71	9.53	9.49	9.40	9.32	9.26

n_2	n_1 / p	1	2	3	4	5	6	7	8	9	10	11	12
10	.0005	.0^{4}41	.0^{3}50	.0^{3}49	.015	.028	.043	.057	.071	.085	.097	.108	.119
	.001	.0^{3}17	.0^{3}10	.0^{3}77	.021	.037	.054	.071	.087	.101	.114	.126	.137
	.005	.0^{4}41	.0^{3}50	.023	.048	.073	.098	.119	.139	.156	.171	.185	.197
	.01	.0^{3}17	.010	.037	.069	.100	.127	.151	.172	.190	.206	.220	.233
	.025	.0^{3}10	.025	.069	.113	.151	.183	.210	.233	.252	.269	.283	.296
	.05	.0^{4}41	.052	.114	.168	.211	.246	.275	.299	.319	.336	.351	.363
	.10	.017	.106	.191	.255	.303	.340	.370	.394	.414	.430	.444	.457
	.25	.107	.296	.409	.480	.529	.565	.592	.613	.631	.645	.657	.667
	.50	.490	.743	.845	.899	.932	.954	.971	.983	.992	1.00	1.01	1.01
	.75	1.49	1.60	1.60	1.59	1.59	1.58	1.57	1.56	1.56	1.55	1.55	1.54
	.90	3.28	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.30	2.28
	.95	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91
	.975	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.66	3.62
	.99	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.77	4.71
	.995	12.8	9.43	8.08	7.34	6.87	6.54	6.30	6.12	5.97	5.85	5.75	5.66
	.999	21.0	14.9	12.6	11.3	10.5	9.92	9.52	9.20	8.96	8.75	8.58	8.44
	.9995	25.5	17.9	15.0	13.4	12.4	11.8	11.3	10.9	10.6	10.3	10.1	9.93
11	.0005	.0^{4}41	.0^{3}50	.0^{3}49	.015	.028	.043	.058	.072	.086	.099	.111	.121
	.001	.0^{3}16	.0^{3}10	.0^{3}78	.021	.038	.055	.072	.088	.103	.116	.129	.140
	.005	.0^{3}40	.0^{3}50	.023	.048	.074	.099	.121	.141	.158	.174	.188	.200
	.01	.0^{3}16	.010	.037	.069	.100	.128	.153	.175	.193	.210	.224	.237
	.025	.0^{3}10	.025	.069	.114	.152	.185	.212	.236	.256	.273	.288	.301
	.05	.0^{4}41	.052	.114	.168	.212	.248	.278	.302	.323	.340	.355	.368
	.10	.017	.106	.192	.256	.305	.342	.373	.397	.417	.435	.448	.461
	.25	.107	.295	.408	.481	.529	.565	.592	.614	.633	.645	.658	.667
	.50	.486	.739	.840	.893	.926	.948	.964	.977	.986	.994	1.00	1.01
	.75	1.47	1.58	1.58	1.57	1.56	1.55	1.54	1.53	1.53	1.52	1.52	1.51
	.90	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.23	2.21
	.95	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79
	.975	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.47	3.43
	.99	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.46	4.40
	.995	12.2	8.91	7.60	6.88	6.42	6.10	5.86	5.68	5.54	5.42	5.32	5.24
	.999	19.7	13.8	11.6	10.3	9.58	9.05	8.66	8.35	8.12	7.92	7.76	7.62
	.9995	23.6	16.4	13.6	12.2	11.2	10.6	10.1	9.76	9.48	9.24	9.04	8.88
12	.0005	.0^{4}41	.0^{3}50	.0^{3}49	.015	.028	.044	.058	.073	.087	.101	.113	.124
	.001	.0^{3}16	.0^{3}10	.0^{3}78	.021	.038	.056	.073	.089	.104	.118	.131	.143
	.005	.0^{3}39	.0^{3}50	.023	.048	.075	.100	.122	.143	.161	.177	.191	.204
	.01	.0^{3}16	.010	.037	.070	.101	.130	.155	.176	.196	.212	.227	.241
	.025	.0^{3}10	.025	.070	.114	.153	.186	.214	.238	.259	.276	.292	.305
	.05	.0^{4}41	.052	.114	.169	.214	.250	.280	.305	.325	.343	.358	.372
	.10	.016	.106	.192	.257	.306	.344	.375	.400	.420	.438	.452	.466
	.25	.106	.295	.408	.480	.530	.566	.594	.616	.633	.649	.662	.671
	.50	.484	.735	.835	.888	.921	.943	.959	.972	.981	.989	.995	1.00
	.75	1.46	1.56	1.56	1.55	1.54	1.53	1.52	1.51	1.51	1.50	1.50	1.49
	.90	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.17	2.15
	.95	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69
	.975	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.32	3.28
	.99	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.22	4.16
	.995	11.8	8.51	7.23	6.52	6.07	5.76	5.52	5.35	5.20	5.09	4.99	4.91
	.999	18.6	13.0	10.8	9.63	8.89	8.38	8.00	7.71	7.48	7.29	7.14	7.01
	.9995	22.2	15.3	12.7	11.2	10.4	9.74	9.28	8.94	8.66	8.43	8.24	8.08

n_2	p \ n_1	15	20	24	30	40	50	60	100	120	200	500	∞
10	.0005	.145	.177	.195	.215	.238	.251	.262	.282	.288	.299	.311	.319
	.001	.164	.197	.216	.236	.258	.272	.282	.303	.309	.321	.331	.338
	.005	.226	.260	.279	.299	.321	.334	.344	.365	.370	.380	.391	.397
	.01	.263	.297	.316	.336	.357	.370	.380	.400	.405	.415	.424	.431
	.025	.327	.360	.379	.398	.419	.431	.441	.459	.464	.474	.483	.488
	.05	.393	.426	.444	.462	.481	.493	.502	.518	.523	.532	.541	.546
	.10	.486	.516	.532	.549	.567	.578	.586	.602	.605	.614	.621	.625
	.25	.691	.714	.727	.740	.754	.762	.767	.779	.782	.788	.793	.797
	.50	1.02	1.03	1.04	1.05	1.05	1.06	1.06	1.06	1.06	1.07	1.07	1.07
	.75	1.53	1.52	1.52	1.51	1.51	1.50	1.50	1.49	1.49	1.49	1.48	1.48
	.90	2.24	2.20	2.18	2.16	2.13	2.12	2.11	2.09	2.08	2.07	2.06	2.06
	.95	2.85	2.77	2.74	2.70	2.66	2.64	2.62	2.59	2.58	2.56	2.55	2.54
	.975	3.52	3.42	3.37	3.31	3.26	3.22	3.20	3.15	3.14	3.12	3.09	3.08
	.99	4.56	4.41	4.33	4.25	4.17	4.12	4.08	4.01	4.00	3.96	3.93	3.91
	.995	5.47	5.27	5.17	5.07	4.97	4.90	4.86	4.77	4.75	4.71	4.67	4.64
	.999	8.13	7.80	7.64	7.47	7.30	7.19	7.12	6.98	6.94	6.87	6.81	6.76
	.9995	9.56	9.16	8.96	8.75	8.54	8.42	8.33	8.16	8.12	8.04	7.96	7.90
11	.0005	.148	.182	.201	.222	.246	.261	.271	.293	.299	.312	.324	.331
	.001	.168	.202	.222	.243	.266	.282	.292	.313	.320	.332	.343	.353
	.005	.231	.266	.286	.308	.330	.345	.355	.376	.382	.394	.403	.412
	.01	.268	.304	.324	.344	366	.380	.391	.412	.417	.427	.439	.444
	.025	.332	.368	.386	.407	429	.442	.450	.472	.476	.485	.495	.503
	.05	.398	.433	.452	.469	.490	.503	.513	.529	.535	.543	.552	.559
	.10	.490	.524	.541	.559	.578	.588	.595	.614	.617	.625	.633	.637
	.25	.694	.719	.730	.744	.758	.767	.773	.780	.788	.794	.799	.803
	.50	1.02	1.03	1.03	1.04	1.05	1.05	1.05	1.06	1.06	1.06	1.06	1.06
	.75	1.50	1.49	1.49	1.48	1.47	1.47	1.47	1.46	1.46	1.46	1.45	1.45
	.90	2.17	2.12	2.10	2.08	2.05	2.04	2.03	2.00	2.00	1.99	1.98	1.97
	.95	2.72	2.65	2.61	2.57	2.53	2.51	2.49	2.46	2.45	2.43	2.42	2.40
	.975	3.33	3.23	3.17	3.12	3.06	3.03	3.00	2.96	2.94	2.92	2.90	2.88
	.99	4.25	4.10	4.02	3.94	3.86	3.81	3.78	3.71	3.69	3.66	3.62	3.60
	.995	5.05	4.86	4.76	4.65	4.55	4.49	4.45	4.36	4.34	4.29	4.25	4.23
	.999	7.32	7.01	6.85	6.68	6.52	6.41	6.35	6.21	6.17	6.10	6.04	6.00
	.9995	8.52	8.14	7.94	7.75	7.55	7.43	7.35	7.18	7.14	7.06	6.98	6.93
12	.0005	.152	.186	.206	.228	.253	.269	.280	.305	.311	.323	.337	.345
	.001	.172	.207	.228	.250	.275	.291	.302	.326	.332	.344	.357	.365
	.005	.235	.272	.292	.315	.339	.355	.365	.388	.393	.405	.417	.424
	.01	.273	.310	.330	.352	.375	.391	.401	.422	.428	.441	.450	.458
	.025	.337	.374	.394	.416	.437	.450	.461	.481	.487	.498	.508	.514
	.05	.404	.439	.458	.478	.499	.513	.522	.541	.545	.556	.565	.571
	.10	.496	.528	.546	.564	.583	.595	.604	.621	.625	.633	.641	.647
	.25	.695	.721	.734	.748	.762	.771	.777	.789	.792	.799	.804	.808
	.50	1.01	1.02	1.03	1.03	1.04	1.04	1.05	1.05	1.05	1.05	1.06	1.06
	.75	1.48	1.47	1.46	1.45	1.45	1.44	1.44	1.43	1.43	1.43	1.42	1.42
	.90	2.11	2.06	2.04	2.01	1.99	1.97	1.96	1.94	1.93	1.92	1.91	1.90
	.95	2.62	2.54	2.51	2.47	2.43	2.40	2.38	2.35	2.34	2.32	2.31	2.30
	.975	3.18	3.07	3.02	2.96	2.91	2.87	2.85	2.80	2.79	2.76	2.74	2.72
	.99	4.01	3.86	3.78	3.70	3.62	3.57	3.54	3.47	3.45	3.41	3.38	3.36
	.995	4.72	4.53	4.43	4.33	4.23	4.17	4.12	4.04	4.01	3.97	3.93	3.90
	.999	6.71	6.40	6.25	6.09	5.93	5.83	5.76	5.63	5.59	5.52	5.46	5.42
	.9995	7.74	7.37	7.18	7.00	6.80	6.68	6.61	6.45	6.41	6.33	6.25	6.20

n_2	p \ n_1	1	2	3	4	5	6	7	8	9	10	11	12
15	.0005	.0441	.0350	.0349	.015	.029	.045	.061	.076	.091	.105	.117	.129
	.001	.0416	.0310	.0279	.021	.039	.057	.075	.092	.108	.123	.137	.149
	.005	.0339	.0350	.023	.049	.076	.102	.125	.147	.166	.183	.198	.212
	.01	.0316	.010	.037	.070	.103	.132	.158	181	.202	.219	.235	.249
	.025	.0310	.025	.070	.116	.156	.190	.219	.244	.265	.284	.300	.315
	.05	.0441	.051	.115	.170	.216	.254	.285	.311	.333	.351	.368	.382
	.10	.016	.106	.192	.258	.309	.348	.380	.406	.427	.446	.461	.475
	.25	.105	.293	.407	.480	.531	.568	.596	.618	.637	.652	.667	.676
	.50	.478	.726	.826	.878	.911	.933	.948	.960	.970	.977	.984	.989
	.75	1.43	1.52	1.52	1.51	1.49	1.48	1.47	1.46	1.46	1.45	1.44	1.44
	.90	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.04	2.02
	.95	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48
	.975	6.20	4.76	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	3.01	2.96
	.99	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.73	3.67
	.995	10.8	7.70	6.48	5.80	5.37	5.07	4.85	4.67	4.54	4.42	4.33	4.25
	.999	16.6	11.3	9.34	8.25	7.57	7.09	6.74	6.47	6.26	6.08	5.93	5.81
	.9995	19.5	13.2	10.8	9.48	8.66	8.10	7.68	7.36	7.11	6.91	6.75	6.60
20	.0005	.0440	.0350	.0350	.015	.029	.046	.063	.079	.094	.109	.123	.136
	.001	.0416	.0310	.0279	.022	.039	.058	.077	.095	.112	.128	.143	.156
	.005	.0339	.0350	.023	.050	.077	.104	.129	.151	.171	.190	.206	.221
	.01	.0316	.010	.037	.071	.105	.135	.162	.187	.208	.227	.244	.259
	.025	.0310	.025	.071	.117	.158	.193	.224	.250	.273	.292	.310	.325
	.05	.0440	.051	.115	.172	.219	.258	.290	.318	.340	.360	.377	.393
	.10	.016	.106	.193	.260	.312	.353	.385	.412	.435	.454	.472	.485
	.25	.104	.292	.407	.480	.531	.569	.598	.622	.641	.656	.671	.681
	.50	.472	.718	.816	.868	.900	.922	.938	.950	.959	.966	.972	.977
	.75	1.40	1.49	1.48	1.47	1.45	1.44	1.43	1.42	1.41	1.40	1.39	1.39
	.90	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.91	1.89
	.95	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28
	.975	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.72	2.68
	.99	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.29	3.23
	.995	9.94	6.99	5.82	5.17	4.76	4.47	4.26	4.09	3.96	3.85	3.76	3.68
	.999	14.8	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24	5.08	4.94	4.82
	.9995	17.2	11.4	9.20	8.02	7.28	6.76	6.38	6.08	5.85	5.66	5.51	5.38
24	.0005	.0440	.0350	.0350	015	.030	.046	.064	.080	.096	.112	.126	.139
	.001	.0416	.0310	.0279	.022	.040	.059	.079	.097	.115	.131	.146	.160
	.005	.0440	.0350	.023	.050	.078	.106	.131	.154	.175	.193	.210	.226
	.01	.0316	.010	.038	.072	.106	.137	.165	.189	.211	.231	.249	.264
	.025	.0310	.025	.071	.117	.159	.195	.227	.253	.277	.297	.315	.331
	.05	.0440	.051	.116	.173	.221	.260	.293	.321	.345	.365	.383	.399
	.10	.016	.106	.193	.261	.313	.355	.388	.416	.439	.459	.476	.491
	.25	.104	.291	.406	.480	.532	.570	.600	.623	.643	.659	.671	.684
	.50	.469	.714	.812	.863	.895	.917	.932	.944	.953	.961	.967	.972
	.75	1.39	1.47	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.38	1.37	1.36
	.90	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.85	1.83
	.95	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.21	2.18
	.975	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.59	2.54
	.99	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.09	3.03
	.995	9.55	6.66	5.52	4.89	4.49	4.20	3.99	3.83	3.69	3.59	3.50	3.42
	.999	14.0	9.34	7.55	6.59	5.98	5.55	5.23	4.99	4.80	4.64	4.50	4.39
	.9995	16.2	10.6	8.52	7.39	6.68	6.18	5.82	5.54	5.31	5.13	4.98	4.85

ν_2	ν_1 / p	15	20	24	30	40	50	60	100	120	200	500	∞
15	.0005	.159	.197	.220	.244	.272	.290	.303	.330	.339	.353	.368	.377
	.001	.181	.219	.242	.266	.294	.313	.325	.352	.360	.375	.388	.398
	.005	.246	.286	.308	.333	.360	.377	.389	.415	.422	.435	.448	.457
	.01	.284	.324	.346	.370	.397	.413	.425	.450	.456	.469	.483	.490
	.025	.349	.389	.410	.433	.458	.474	.485	.508	.514	.526	.538	.546
	.05	.416	.454	.474	.496	.519	.535	.545	.565	.571	.581	.592	.600
	.10	.507	.542	.561	.581	.602	.614	.624	.641	.647	.658	.667	.672
	.25	.701	.728	.742	.757	.772	.782	.788	.802	.805	.812	.818	.822
	.50	1.00	1.01	1.02	1.02	1.03	1.03	1.03	1.04	1.04	1.04	1.04	1.05
	.75	1.43	1.41	1.41	1.40	1.39	1.39	1.38	1.38	1.37	1.37	1.36	1.36
	.90	1.97	1.92	1.90	1.87	1.85	1.83	1.82	1.79	1.79	1.77	1.76	1.76
	.95	2.40	2.33	2.39	2.25	2.20	2.18	2.16	2.12	2.11	2.10	2.08	2.07
	.975	2.86	2.76	2.70	2.64	2.59	2.55	2.52	2.47	2.46	2.44	2.41	2.40
	.99	3.52	3.37	3.29	3.21	3.13	3.08	3.05	2.98	2.96	2.92	2.89	2.87
	.995	4.07	3.88	3.79	3.69	3.59	3.52	3.48	3.39	3.37	3.33	3.29	3.26
	.999	5.54	5.25	5.10	4.95	4.80	4.70	4.64	4.51	4.47	4.41	4.35	4.31
	.9995	6.27	5.93	5.75	5.58	5.40	5.29	5.21	5.06	5.02	4.94	4.87	4.83
20	.0005	.169	.211	.235	.263	.295	.316	.331	.364	.375	.391	.408	.422
	.001	.191	.233	.258	.286	.318	.339	.354	.386	.395	.413	.429	.441
	.005	.258	.301	.327	.354	.385	.405	.419	.448	.457	.474	.490	.500
	.01	.297	.340	.365	.392	.422	.441	.455	.483	.491	.508	.521	.532
	.025	.363	.406	.430	.456	.484	.503	.514	.541	.548	.562	.575	.585
	.05	.430	.471	.493	.518	.544	.562	.572	.595	.603	.617	.629	.637
	.10	.520	.557	.578	.600	.623	.637	.648	.671	.675	.685	.694	.704
	.25	.708	.736	.751	.767	.784	.794	.801	.816	.820	.827	.835	.840
	.50	.989	1.00	1.01	1.01	1.02	1.02	1.02	1.03	1.03	1.03	1.03	1.03
	.75	1.37	1.36	1.35	1.34	1.33	1.33	1.32	1.31	1.31	1.30	1.30	1.29
	.90	1.84	1.79	1.77	1.74	1.71	1.69	1.68	1.65	1.64	1.63	1.62	1.61
	.95	2.20	2.12	2.08	2.04	1.99	1.97	1.95	1.91	1.90	1.88	1.86	1.84
	.975	2.57	2.46	2.41	2.35	2.29	2.25	2.22	2.17	2.16	2.13	2.10	2.09
	.99	3.09	2.94	2.86	2.78	2.69	2.64	2.61	2.54	2.52	2.48	2.44	2.42
	.995	3.50	3.32	3.22	3.12	3.02	2.96	2.92	2.83	2.81	2.76	2.72	2.69
	.999	4.56	4.29	4.15	4.01	3.86	3.77	3.70	3.58	3.54	3.48	3.42	3.38
	.9995	5.07	4.75	4.58	4.42	4.24	4.15	4.07	3.93	3.90	3.82	3.75	3.70
24	.0005	.174	.218	.244	.274	.309	.331	.349	.384	.395	.416	.434	.449
	.001	.196	.241	.268	.298	.332	.354	.371	.405	.417	.437	.455	.469
	.005	.264	.310	.337	.367	.400	.422	.437	.469	.479	.498	.515	.527
	.01	.304	.350	.376	.405	.437	.459	.473	.505	.513	.529	.546	.558
	.025	.370	.415	.441	.468	.498	.518	.531	.562	.568	.585	.599	.610
	.05	.437	.480	.504	.530	.558	.575	.588	.613	.622	.637	.649	.659
	.10	.527	.566	.588	.611	.635	.651	.662	.685	.691	.704	.715	.723
	.25	.712	.741	.757	.773	.791	.802	.809	.825	.829	.837	.844	.850
	.50	.983	.994	1.00	1.01	1.01	1.02	1.02	1.02	1.02	1.02	1.03	1.03
	.75	1.35	1.33	1.32	1.31	1.30	1.29	1.29	1.28	1.28	1.27	1.27	1.26
	.90	1.78	1.73	1.70	1.67	1.64	1.62	1.61	1.58	1.57	1.56	1.54	1.53
	.95	2.11	2.03	1.98	1.94	1.89	1.86	1.84	1.80	1.79	1.77	1.75	1.73
	.975	2.44	2.33	2.27	2.21	2.15	2.11	2.08	2.02	2.01	1.98	1.95	1.94
	.99	2.89	2.74	2.66	2.58	2.49	2.44	2.40	2.33	2.31	2.27	2.24	2.21
	.995	3.25	3.06	2.97	2.87	2.77	2.70	2.66	2.57	2.55	2.50	2.46	2.43
	.999	4.14	3.87	3.74	3.59	3.45	3.35	3.29	3.16	3.14	3.07	3.01	2.97
	.9995	4.55	4.25	4.09	3.93	3.76	3.66	3.59	3.44	3.41	3.33	3.27	3.22

n_2	n_1 / p	1	2	3	4	5	6	7	8	9	10	11	12
30	.0005	.0^{4}40	.0^{3}50	.0^{3}50	.015	.030	.047	.065	.082	.098	.114	.129	.143
	.001	.0^{3}16	.0^{3}10	.0^{3}80	.022	.040	.060	.080	.099	.117	.134	.150	.164
	.005	.0^{4}40	.0^{3}50	.024	.050	.079	.107	.133	.156	.178	.197	.215	.231
	.01	.0^{3}16	.010	.038	.072	.107	.138	.167	.192	.215	.235	.254	.270
	.025	.0^{3}10	.025	.071	.118	.161	.197	.229	.257	.281	.302	.321	.337
	.05	.0^{4}40	.051	.116	.174	222	.263	.296	.325	.349	.370	.389	.406
	.10	.016	.106	.193	.262	.315	.357	.391	.420	.443	.466	.481	.497
	.25	.103	.290	.406	.480	.532	.571	.601	.625	.645	.661	.676	.688
	.50	.466	.709	.807	.858	.890	.912	.927	.939	.948	.955	.961	.966
	.75	1.38	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.35	1.34
	.90	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.79	1.77
	.95	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09
	.975	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.46	2.41
	.99	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.91	2.84
	.995	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45	3.34	3.25	3.18
	.999	13.3	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39	4.24	4.11	4.00
	.9995	15.2	9.90	7.90	6.82	6.14	5.66	5.31	5.04	4.82	4.65	4.51	4.38
40	.0005	.0^{4}40	.0^{3}50	.0^{3}50	.016	.030	.048	.066	.084	.100	.117	.132	.147
	.001	.0^{3}16	.0^{3}10	.0^{3}80	.022	.042	.061	.081	.101	.119	.137	.153	.169
	.005	.0^{4}40	.0^{3}50	.024	.051	.080	.108	.135	.159	.181	.201	.220	.237
	.01	.0^{3}16	.010	.038	.073	.108	.140	.169	.195	.219	.240	.259	.276
	.025	.0^{3}99	.025	.071	.119	.162	.199	.232	.260	.285	.307	.327	.344
	.05	.0^{4}40	.051	.116	.175	.224	.265	.299	.329	.354	.376	.395	.412
	.10	.016	.106	.194	.263	.317	.360	.394	.424	.448	.469	.488	.504
	.25	.103	.290	.405	.480	.533	.572	.603	.627	.647	.664	.680	.691
	.50	.463	.705	.802	.854	.885	.907	.922	.934	.943	.950	.956	.961
	.75	1.36	1.44	1.42	1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.31
	.90	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.73	1.71
	.95	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00
	.975	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.33	2.29
	.99	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.73	2.66
	.995	8.83	6.07	4.98	4.37	3.99	3.71	3.51	3.35	3.22	3.12	3.03	2.95
	.999	12.6	8.25	6.60	5.70	5.13	4.73	4.44	4.21	4.02	3.87	3.75	3.64
	.9995	14.4	9.25	7.33	6.30	5.64	5.19	4.85	4.59	4.38	4.21	4.07	3.95
60	.0005	.0^{4}40	.0^{3}50	.0^{3}51	.016	.031	.048	.067	.085	.103	.120	.136	.152
	.001	.0^{3}16	.0^{3}10	.0^{3}80	.022	.041	.062	.083	.103	.122	.140	.157	.174
	.005	.0^{4}40	.0^{3}50	.024	.051	.081	.110	.137	.162	.185	.206	.225	.243
	.01	.0^{3}16	.010	.038	.073	.109	.142	.172	.199	.223	.245	.265	.283
	.025	.0^{3}99	.025	.071	.120	.163	.202	.235	264	.290	.313	.333	.351
	.05	.0^{4}40	.051	.116	.176	.226	.267	.303	.333	.359	.382	.402	.419
	.10	.016	.106	.194	.264	.318	.362	.398	.428	.453	.475	.493	.510
	.25	.102	.289	.405	.480	.534	.573	.604	.629	.650	.667	.680	.695
	.50	.461	.701	.798	.849	.880	.901	.917	.928	.937	.945	.951	.956
	.75	1.35	1.42	1.41	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.29
	.90	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.68	1.66
	.95	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92
	.975	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.22	2.17
	.99	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56	2.50
	.995	8.49	5.80	4.73	4.14	3.76	3.49	3.29	3.13	3.01	2.90	2.82	2.74
	.999	12.0	7.76	6.17	5.31	4.76	4.37	4.09	3.87	3.69	3.54	3.43	3.31
	.9995	13.6	8.65	6.81	5.82	5.20	4.76	4.44	4.18	3.98	3.82	3.69	3.57

ν_2	ν_1 / p	15	20	24	30	40	50	60	100	120	200	500	∞
30	.0005	.179	.226	.254	.287	.325	.350	.369	.410	.420	.444	.467	.483
	.001	.202	.250	.278	.311	.348	.373	.391	.431	.442	.465	.488	.503
	.005	.271	.320	.349	.381	.416	.441	.457	.495	.504	.524	.543	.559
	.01	.311	.360	.388	.419	.454	.476	.493	.529	.538	.559	.575	.590
	.025	.378	.426	.453	.482	.515	.535	.551	.585	.592	.610	.625	.639
	.05	.445	.490	.516	.543	.573	.592	.606	.637	.644	.658	.676	.685
	.10	.534	.575	.598	.623	.649	.667	.678	.704	.710	.725	.735	.746
	.25	.716	.746	.763	.780	.798	.810	.818	.833	.839	.848	.856	.862
	.50	.978	.989	.994	1.00	1.01	1.01	1.01	1.02	1.02	1.02	1.02	1.02
	.75	1.32	1.30	1.29	1.28	1.27	1.26	1.26	1.25	1.24	1.24	1.23	1.23
	.90	1.72	1.67	1.64	1.61	1.57	1.55	1.54	1.51	1.50	1.48	1.47	1.46
	.95	2.01	1.93	1.89	1.84	1.79	1.76	1.74	1.70	1.68	1.66	1.64	1.62
	.975	2.31	2.20	2.14	2.07	2.01	1.97	1.94	1.88	1.87	1.84	1.81	1.79
	.99	2.70	2.55	2.47	2.39	2.30	2.25	2.21	2.13	2.11	2.07	2.03	2.01
	.995	3.01	2.82	2.73	2.63	2.52	2.46	2.42	2.32	2.30	2.25	2.21	2.18
	.999	3.75	3.49	3.36	3.22	3.07	2.98	2.92	2.79	2.76	2.69	2.63	2.59
	.9995	4.10	3.80	3.65	3.48	3.32	3.22	3.15	3.00	2.97	2.89	2.82	2.78
40	.0005	.185	.236	.266	.301	.343	.373	.393	.441	.453	.480	.504	.525
	.001	.209	.259	.290	.326	.367	.396	.415	.461	.473	.500	.524	.545
	.005	.279	.331	.362	.396	.436	.463	.481	.524	.534	.559	.581	.599
	.01	.319	.371	.401	.435	.473	.498	.516	.556	.567	.592	.613	.628
	.025	.387	.437	.466	.498	.533	.556	.573	.610	.620	.641	.662	.674
	.05	.454	.502	.529	.558	.591	.613	.627	.658	.669	.685	.704	.717
	.10	.542	.585	.609	.636	.664	.683	.696	.724	.731	.747	.762	.772
	.25	.720	.752	.769	.787	.806	.819	.828	.846	.851	.861	.870	.877
	.50	.972	.983	.989	.994	1.00	1.00	1.01	1.01	1.01	1.01	1.02	1.02
	.75	1.30	1.28	1.26	1.25	1.24	1.23	1.22	1.21	1.21	1.20	1.19	1.19
	.90	1.66	1.61	1.57	1.54	1.51	1.48	1.47	1.43	1.42	1.41	1.39	1.38
	.95	1.92	1.84	1.79	1.74	1.69	1.66	1.64	1.59	1.58	1.55	1.53	1.51
	.975	2.18	2.07	2.01	1.94	1.88	1.83	1.80	1.74	1.72	1.69	1.66	1.64
	.99	2.52	2.37	2.29	2.20	2.11	2.06	2.02	1.94	1.92	1.87	1.83	1.80
	.995	2.78	2.60	2.50	2.40	2.30	2.23	2.18	2.09	2.06	2.01	1.96	1.93
	.999	3.40	3.15	3.01	2.87	2.73	2.64	2.57	2.44	2.41	2.34	2.28	2.23
	.9995	3.68	3.39	3.24	3.08	2.92	2.82	2.74	2.60	2.57	2.49	2.41	2.37
60	.0005	.192	.246	.278	.318	.365	.398	.421	.478	.493	.527	.561	.585
	.001	.216	.270	.304	.343	.389	.421	.444	.497	.512	.545	.579	.602
	.005	.287	.343	.376	.414	.458	.488	.510	.559	.572	.602	.633	.652
	.01	.328	.383	.416	.453	.495	.524	.545	.592	.604	.633	.658	.679
	.025	.396	.450	.481	.515	.555	.581	.600	.641	.654	.680	.704	.720
	.05	.463	.514	.543	.575	.611	.633	.652	.690	.700	.719	.746	.759
	.10	.550	.596	.622	.650	.682	.703	.717	.750	.758	.776	.793	.806
	.25	.725	.758	.776	.796	.816	.830	.840	.860	.865	.877	.888	.896
	.50	.967	.978	.983	.989	.994	.998	1.00	1.00	1.01	1.01	1.01	1.01
	.75	1.27	1.25	1.24	1.22	1.21	1.20	1.19	1.17	1.17	1.16	1.15	1.15
	.90	1.60	1.54	1.51	1.48	1.44	1.41	1.40	1.36	1.35	1.33	1.31	1.29
	.95	1.84	1.75	1.70	1.65	1.59	1.56	1.53	1.48	1.47	1.44	1.41	1.39
	.975	2.06	1.94	1.88	1.82	1.74	1.70	1.67	1.60	1.58	1.54	1.51	1.48
	.99	2.35	2.20	2.12	2.03	1.94	1.88	1.84	1.75	1.73	1.68	1.63	1.60
	.995	2.57	2.39	2.29	2.19	2.08	2.01	1.96	1.86	1.83	1.78	1.73	1.69
	.999	3.08	2.83	2.69	2.56	2.41	2.31	2.25	2.11	2.09	2.01	1.93	1.89
	.9995	3.30	3.02	2.87	2.71	2.55	2.45	2.38	2.23	2.19	2.11	2.03	1.98

n_2	p \ n_1	1	2	3	4	5	6	7	8	9	10	11	12
120	.0005	.0^{4}40	.0^{4}50	.0^{4}51	.016	.031	.049	.067	.087	.105	.123	.140	.156
	.001	.0^{3}16	.0^{3}10	.0^{3}81	.023	.042	.063	.084	.105	.125	.144	.162	.179
	.005	.0^{4}39	.0^{4}50	.024	.051	.081	.111	.139	.165	.189	.211	.230	.249
	.01	.0^{3}16	.010	.038	.074	.110	.143	.174	.202	.227	.250	.271	.290
	.025	.0^{3}99	.025	.072	.120	.165	.204	.238	.268	.295	.318	.340	.359
	.05	.0^{2}39	.051	.117	.177	.227	.270	.306	.337	.364	.388	.408	.427
	.10	.016	.105	.194	.265	.320	.365	.401	.432	.458	.480	.500	.518
	.25	.102	.288	.405	.481	.534	.574	.606	.631	.652	.670	.685	.699
	.50	.458	.697	.793	.844	.875	.896	.912	.923	.932	.939	.945	.950
	.75	1.34	1.40	1.39	1.37	1.35	1.33	1.31	1.30	1.29	1.28	1.27	1.26
	.90	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.62	1.60
	.95	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.87	1.83
	.975	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.10	2.05
	.99	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.40	2.34
	.995	8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.81	2.71	2.62	2.54
	.999	11.4	7.32	5.79	4.95	4.42	4.04	3.77	3.55	3.38	3.24	3.12	3.02
	.9995	12.8	8.10	6.34	5.39	4.79	4.37	4.07	3.82	3.63	3.47	3.34	3.22
∞	.0005	.0^{4}39	.0^{4}50	.0^{4}51	.016	.032	.050	.069	.088	.108	.127	.144	.161
	.001	.0^{3}16	.0^{3}10	.0^{3}81	.023	.042	.063	.085	.107	.128	.148	.167	.185
	.005	.0^{4}39	.0^{4}50	.024	.052	.082	.113	.141	.168	.193	.216	.236	.256
	.01	.0^{3}16	.010	.038	.074	.111	.145	.177	.206	.232	.256	.278	.298
	.025	.0^{3}98	.025	.072	.121	.166	.206	.241	.272	.300	.325	.347	.367
	.05	.0^{2}39	.051	.117	.178	.229	.273	.310	.342	.369	.394	.417	.436
	.10	.016	.105	.195	.266	.322	.367	.405	.436	.463	.487	.508	.525
	.25	.102	.288	.404	.481	.535	.576	.608	.634	.655	.674	.690	.703
	.50	.455	.693	.789	.839	.870	.891	.907	.918	.927	.934	.939	.945
	.75	1.32	1.39	1.37	1.35	1.33	1.31	1.29	1.28	1.27	1.25	1.24	1.24
	.90	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.57	1.55
	.95	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.79	1.75
	.975	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.99	1.94
	.99	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.25	2.18
	.995	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62	2.52	2.43	2.36
	.999	10.8	6.91	5.42	4.62	4.10	3.74	3.47	3.27	3.10	2.96	2.84	2.74
	.9995	12.1	7.60	5.91	5.00	4.42	4.02	3.72	3.48	3.30	3.14	3.02	2.90

ν_2	ν_1 / p	15	20	24	30	40	50	60	100	120	200	500	∞
120	.0005	.199	.256	.293	.338	.390	.429	.458	.524	.543	.578	.614	.676
	.001	.223	.282	.319	.363	.415	.453	.480	.542	.568	.595	.631	.691
	.005	.297	.356	.393	.434	.484	.520	.545	.605	.623	.661	.702	.733
	.01	.338	.397	.433	.474	.522	.556	.579	.636	.652	.688	.725	.755
	.025	.406	.464	.498	.536	.580	.611	.633	.684	.698	.729	.762	.789
	.05	.473	.527	.559	.594	.634	.661	.682	.727	.740	.767	.785	.819
	.10	.560	.609	.636	.667	.702	.726	.742	.781	.791	.815	.838	.85
	.25	.730	.765	.784	.805	.828	.843	.853	.877	.884	.897	.911	.92
	.50	.961	.972	.978	.983	.989	.992	.994	1.00	1.00	1.00	1.01	1.0
	.75	1.24	1.22	1.21	1.19	1.18	1.17	1.16	1.14	1.13	1.12	1.11	1.10
	.90	1.55	1.48	1.45	1.41	1.37	1.34	1.32	1.27	1.26	1.24	1.21	1.19
	.95	1.75	1.66	1.61	1.55	1.50	1.46	1.43	1.37	1.35	1.32	1.28	1.25
	.975	1.95	1.82	1.76	1.69	1.61	1.56	1.53	1.45	1.43	1.39	1.34	1.31
	.99	2.19	2.03	1.95	1.86	1.76	1.70	1.66	1.56	1.53	1.48	1.42	1.38
	.995	2.37	2.19	2.09	1.98	1.87	1.80	1.75	1.64	1.61	1.54	1.48	1.43
	.999	2.78	2.53	2.40	2.26	2.11	2.02	1.95	1.82	1.76	1.70	1.62	1.54
	.9995	2.96	2.67	2.53	2.38	2.21	2.11	2.01	1.88	1.84	1.75	1.67	1.60
∞	.0005	.207	.270	.311	.360	.422	.469	.505	.599	.624	.704	.804	1.00
	.001	.232	.296	.338	.386	.448	.493	.527	.617	.649	.719	.819	1.00
	.005	.307	.372	.412	.460	.518	.559	.592	.671	.699	.762	.843	1.00
	.01	.349	.413	.452	.499	.554	.595	.625	.699	.724	.782	.858	1.00
	.025	.418	.480	.517	.560	.611	.645	.675	.741	.763	.813	.878	1.00
	.05	.484	.543	.577	.617	.663	.694	.720	.781	.797	.840	.896	1.00
	.10	.570	.622	.652	.687	.726	.752	.774	.826	.838	.877	.919	1.00
	.25	.736	.773	.793	.816	.842	.860	.872	.901	.910	.932	.957	1.00
	.50	.956	.967	.972	.978	.983	.987	.989	.993	.994	.997	.999	1.00
	.75	1.22	1.19	1.18	1.16	1.14	1.13	1.12	1.09	1.08	1.07	1.04	1.00
	.90	1.49	1.42	1.38	1.34	1.30	1.26	1.24	1.18	1.17	1.13	1.08	1.00
	.95	1.67	1.57	1.52	1.46	1.39	1.35	1.32	1.24	1.22	1.17	1.11	1.00
	.975	1.83	1.71	1.64	1.57	1.48	1.43	1.39	1.30	1.27	1.21	1.13	1.00
	.99	2.04	1.88	1.79	1.70	1.59	1.52	1.47	1.36	1.32	1.25	1.15	1.00
	.995	2.19	2.00	1.90	1.79	1.67	1.59	1.53	1.40	1.36	1.28	1.17	1.00
	.999	2.51	2.27	2.13	1.99	1.84	1.73	1.66	1.49	1.45	1.34	1.21	1.00
	.9995	2.65	2.37	2.22	2.07	1.91	1.79	1.71	1.53	1.48	1.36	1.22	1.00

APPENDIX 1

SOURCE PROGRAMME

```
C       MASTER MULTIPLE REGRESSION
        DIMENSION XBAR(40),STD(40),D(40),RY(40),ISAVE(40),B(40),
       1SB(40),T(40),W(40),AW(40)
        DIMENSION RX(1600),R(820),ANS(10),TITLE(80)
        OPEN(UNIT=5,FILE='MREG.DAT',STATUS='OLD')
        OPEN(UNIT=6,FILE='MREG.OUT',STATUS='NEW')
        OPEN(UNIT=13,FILE='R.DAT',STATUS='NEW')
 1      FORMAT(80A1)
 2      FORMAT(5X,'MULTIPLE REGRESSION.....'//,6X,
       1'SELECTION.....',I2//)
 3      FORMAT(1X,'VARIABLE',3X,'MEAN',3X,'STANDARD',3X,'CORRELATION',
       11X,'REGRESSION',1X,'STD. ERROR',2X,'COMPUTED'/6H   NO.,13X,
       2'DEVIATION',4X,6HX VS Y,4X,'COEFFIENT',1X,'OF REG.COFF.',
       31X,7HT VALUE)
 4      FORMAT(1H ,I4,6F11.3)
 5      FORMAT(10H DEPENDENT)
 6      FORMAT(1H0/10H INTERCEPT,10X,F16.5//' MULTIPLE CORRELATION  ',
       1F13.5//' STD. ERROR OF ESTIMATE',F13.5//)
 7      FORMAT(1H0,21X,' ANALYSIS OF VARIANCE FOR REGRESSION'//5X,
       1'SOURCE OF VARIATION',7X,'DEGREES',7X,'SUM OF',10X,'MEAN',
       26X,'F VALUE'/30X,'OF FREEDOM',4X,'SQUARES',9X,'SQUARES')
 8      FORMAT(' ATTRIBUTABLE TO REGRESSION  ',I6,2F16.3,F11.3/' DEVIA
       1TION FROM REGRESSION   ',I6,2F16.3)
 9      FORMAT(1H ,5X,5HTOTAL,19X,I6,F15.3)
 10     FORMAT(36I2)
 11     FORMAT(1H ,15X,'TABLE OF RESIDUALS'//' CASE NO.',5X,7HY VALUE,
       15X,10HY ESTIMATE,6X,8HRESIDUAL)
 12     FORMAT(1H ,I6,F15.5,2F14.5)
 13     FORMAT(' NUMBER OF SELECTION NOT SPECIFIED, JOB TERMINATED')
 14     FORMAT(' THIS MATRIX IS SINGULAR, THIS SELECTION IS SKIPPED')
 17     FORMAT(13F10.5)
C       READ  PROBLEM PARAMETER CARD
 100    READ(5,1)TITLE
        WRITE(6,1)TITLE
        READ(5,*)N,M,NS
        IO=0
        X=0.0
        CALL CORRE (N,M,IO,X,XBAR,STD,RX,R,D,B,T)
        REWIND 13
C       TEST NUMBER OF SELECTIONS
        IF(NS)108,108,109
 108    WRITE(6,13)
        GO TO 300
 109    DO 200 I=1,NS
        WRITE(6,2)I
        READ(5,10) NRESI,NDEP,K,(ISAVE(J),J=1,K)
C       NRESI.....OPTION CODE FOR TABLE RESIDUALS
C                 0 IF TABLE IS NOT REQUIRED
C                 1 IF TABLE IS NOT REQUIRED
C       NDEP..... DEPENDENT VARIABLE
```

```
C       K........ NO OF INDEPENDENT VARIABLES INCLUDED
C       ISAVE.... A VECTOR CONTAINING THE INDEPENDENT VARIABLES INCLUDED
        CALL ORDER (M,R,NDEP,K,ISAVE,RX,RY)
        CALL MINV(RX,K,DET,B,T)
C       TEST SINGULARITY OF MATRIX INVERTER
        IF(DET) 112,110,112
110     WRITE(6,14)
        GO TO 200
 112    CALL MULTR(N,K,XBAR,STD,B,RX,RY,ISAVE,R,SB,T,ANS)
C       PRINT MEANS,STANDARD DEVIATIONS,INTERCORRELATIONS BETWEEN
C       X AND Y,REGRESSION COEFFICIENTS,STANDARD DEVIATIONS OF
C       REGRESSION COEFFICIENTS,AND COMPUTED T-VALUES
        MM=K+1
        WRITE(6,3)
        DO 115 J=1,K
        L=ISAVE(J)
 115    WRITE(6,4) L,XBAR(L),STD(L),RY(J),B(J),SB(J),T(J)
        WRITE(6,5)
        L=ISAVE(MM)
        WRITE(6,4) L,XBAR(L),STD(L)
C       PRINT INTERCEPT,MULTIPLE CORRELATION COEFFICIENT,AND
C       STANDARD ERROR OF ESTIMATE
        WRITE(6,6)ANS(1),ANS(2),ANS(3)
C       PRINT ANALYSIS OF VARIANCE FOR REGRESSION
        WRITE(6,7)
        L=ANS(8)
        WRITE(6,8) K,ANS(4),ANS(6),ANS(10),L,ANS(7),ANS(9)
        L=N-1
        SUM=ANS(4)+ANS(7)
        WRITE(6,9) L,SUM
        IF(NRESI) 200,200,120
C       PRINT TABLE OF RESIDUALS
 120    WRITE(6,2)I
        WRITE(6,11)
        MM=ISAVE(K+1)
        DO 140 II=1,N
        READ (13,17) (W(J),J=1,M)
        SUM=ANS(1)
        DO 130 J=1,K
        L=ISAVE(J)
 130    SUM=SUM+W(L)*B(J)
        RESI=W(MM)-SUM
 140    WRITE(6,12) II,W(MM),SUM,RESI
        REWIND 13
 200    CONTINUE
 300    CONTINUE
        STOP
        END
        SUBROUTINE DATA(M,D)
        DIMENSION D(1)
```

```
  1     FORMAT(3F10.3)
C       THIS SUBROUTINE IS CALLED BY SUBROUTINE CORRE
        READ(5,*)(D(I),I=1,M)
        WRITE(13,1)(D(I),I=1,M)
        RETURN
        END
        SUBROUTINE MINV(A,N,D,L,M)
        DIMENSION A(1),L(1),M(1)
C       SEARCH FOR LARGEST ELEMENT
        D=1.0
        NK=-N
        DO 80 K=1,N
        NK=NK+N
        L(K)=K
        M(K)=K
        KK=NK+K
        BIGA=A(KK)
        DO 20 J=K,N
        IZ=N*(J-1)
        DO 20 I=K,N
        IJ=IZ+I
10      IF(ABS(BIGA)-ABS(A(IJ))) 15,20,20
15      BIGA=A(IJ)
        L(K)=I
        M(K)=J
20      CONTINUE
C       INTERCHANGE ROWS
        J=L(K)
        IF(J-K) 35,35,25
25      KI=K-N
        DO 30 I=1,N
        KI=KI+N
        HOLD=-A(KI)
        JI=KI-K+J
        A(KI)=A(JI)
30      A(JI)=HOLD
C       INTERCHANGE COLUMNS
35      I=M(K)
        IF(I-K)45,45,38
38      JP=N*(I-1)
        DO 40 J=1,N
        JK=NK+J
        JI=JP+J
        HOLD=-A(JK)
        A(JK)=A(JI)
40      A(JI)=HOLD
C       DIVIDE COLUMNS BY MINUS PIVOT
45      IF(BIGA)48,46,48
46      D=0.0
        RETURN
```

```
48      DO 55 I=1,N
        IF(I-K)50,55,50
50      IK=NK+I
        A(IK)=A(IK)/(-BIGA)
55      CONTINUE
C       REDUCE MATRIX
        DO 65 I=1,N
        IK=NK+I
        HOLD=A(IK)
        IJ=I-N
        DO 65 J=1,N
        IJ=IJ+N
        IF(I-K)60,65,60
60      IF(J-K)62,65,62
62      KJ=IJ-I+K
        A(IJ)=HOLD*A(KJ)+A(IJ)
65      CONTINUE
C       DIVIDE ROW BY PIVOT
        KJ=K-N
        DO 75 J=1,N
        KJ=KJ+N
        IF(J-K)70,75,70
70      A(KJ)=A(KJ)/BIGA
75      CONTINUE
C       PRODUCT OF PIVOTS
        D=D*BIGA
C       REPLACE PIVOT BY RECIPROCAL
        A(KK)=1.0/BIGA
80      CONTINUE
C       FINAL ROW AND COLUMN INTERCHANGE
        K=N
100     K=(K-1)
        IF(K)150,150,105
105     I=L(K)
        IF(I-K)120,120,108
108     JQ=N*(K-1)
        JR=N*(I-1)
        DO 110 J=1,N
        JK=JQ+J
        HOLD=A(JK)
        JI=JR+J
        A(JK)=-A(JI)
110     A(JI)=HOLD
120     J=M(K)
        IF(J-K)100,100,125
125     KI=K-N
        DO 130 I=1,N
        KI=KI+N
        HOLD=A(KI)
        JI=KI-K+J
```

```
      A(KI)=-A(JI)
130   A(JI)=HOLD
      GO TO 100
150   RETURN
      END
      SUBROUTINE CORRE(N,M,IO,X,XBAR,STD,RX,R,B,D,T)
      DIMENSION X(1),XBAR(1),STD(1),RX(1),R(1),B(1),
     1D(1),T(1)
C     INITIALISATION
      DO 100 J=1,M
      B(J)=0.0
100   T(J)=0.0
      K=(M*M+M)/2
      DO 102 I=1,K
102   R(I)=0.0
      FN=N
      L=0
      IF(IO) 105,127,105
105   DO 108 J=1,M
      DO 107 I=1,N
      L=L+1
107   T(J)=T(J)+X(L)
      XBAR(J)=T(J)
108   T(J)=T(J)/FN
      DO 115 I=1,N
      JK=0
      L=I-N
      DO 110 J=1,M
      L=L+N
      D(J)=X(L)-T(J)
110   B(J)=B(J)+D(J)
      DO 115 J=1,M
      DO 115 K=1,J
      JK=JK+1
115   R(JK)=R(JK)+D(J)*D(K)
      GO TO 205
127   IF(N-M) 130,130,135
130   KK=N
      GO TO 137
135   KK=M
137   DO 140 I=1,KK
      CALL DATA(M,D)
      DO 140 J=1,M
      T(J)=T(J)+D(J)
      L=L+1
140   RX(L)=D(J)
      FKK=KK
      DO 150 J=1,M
      XBAR(J)=T(J)
150   T(J)=T(J)/FKK
```

```
      L=0
      DO 180 I=1,KK
      JK=0
      DO 170 J=1,M
      L=L+1
  170 D(J)=RX(L)-T(J)
      DO 180 J=1,M
      R(J)=R(J)+D(J)
      DO 180 K=1,J
      JK=JK+1
  180 R(JK)=R(JK)+D(J)*D(K)
      IF(N-K) 205,205,185
  185 KK=N-KK
      DO 200 I=1,KK
      JK=0
      CALL DATA(M,D)
      DO 190 J=1,M
      XBAR(J)=XBAR(J)+D(J)
      D(J)=D(J)-T(J)
  190 R(J)=R(J)+D(J)
      DO 200 J=1,M
      DO 200 K=1,J
      JK=JK+1
  200 R(JK)=R(JK)+D(J)*D(K)
  205 JK=0
      DO 210 J=1,M
      XBAR(J)=XBAR(J)/FN
      DO 210 K=1,J
      JK=JK+1
  210 R(JK)=R(JK)-R(J)*R(K)/FN
      JK=0
      DO 220 J=1,M
      JK=JK+J
  220 STD(J)=SQRT(ABS(R(JK)))
      DO 230 J=1,M
      DO 230 K=J,M
      JK=J+(K*K-K)/2
      L=M*(J-1)+K
      RX(L)=R(JK)
      L=M*(K-1)+J
      RX(L)=R(JK)
      IF(STD(J)*STD(K))225,222,225
  222 R(JK)=0.0
      GO TO 230
  225 R(JK)=R(JK)/(STD(J)*STD(K))
  230 CONTINUE
      FN=SQRT(FN-1.0)
      DO 240 J=1,M
  240 STD(J)=STD(J)/FN
      L=-M
```

```
      DO 250 I=1,M
      L=L+M+1
250   B(I)=RX(L)
      RETURN
      END
C
      SUBROUTINE ORDER(M,R,NDEP,K,ISAVE,RX,RY)
      DIMENSION R(1),ISAVE(1),RX(1),RY(1)
      MM=0
      NN=1
50    FORMAT(/30X,'MATRIX AND VECTOR SELECTED BY ORDER'/)
      DO 140 J=1,K
      L2=ISAVE(J)
      IF(NDEP-L2)122,123,123
122   L=NDEP+(L2*L2-L2)/2
      GO TO 125
123   L=L2+(NDEP*NDEP-NDEP)/2
125   RY(J)=R(L)
      DO 130 I=1,K
      L1=ISAVE(I)
      IF(L1-L2)127,128,128
127   L=L1+(L2*L2-L2)/2
      GO TO 129
128   L=L2+(L1*L1-L1)/2
129   MM=MM+1
130   RX(MM)=R(L)
120   FORMAT(10F12.6)
      NN=NN+K
140   CONTINUE
150   FORMAT(//10F12.6)
      ISAVE(K+1)=NDEP
      RETURN
      END
C
      SUBROUTINE MULTR(N,K,XBAR,STD,R,RX,RY,ISAVE,B,SB,T,ANS)
      DIMENSION XBAR(1),STD(1),RX(1),RY(1),B(1),ISAVE(1),
     1B(1),SB(1),T(1),ANS(1)
      MM=K+1
C     BETA WEIGHTS
      DO 100 J=1,K
100   B(J)=0.0
      DO 110 J=1,K
      L1=K*(J-1)
      DO 110 I=1,K
      L=L1+I
110   B(J)=B(J)+RY(I)*RX(L)
      RM=0.0
      BQ=0.0
      L1=ISAVE(MM)
C     COEFFICIENTS OF DETERMINATION
```

```
      DO 120 I=1,K
      RM=RM+B(I)*RY(I)
C     REGRESSION COEFFICIENTS
      L=ISAVE(I)
      B(I)=R(I)*(STD(L1)/STD(L))
C     INTERCEPT
120   BO=BO+B(I)*XBAR(L)
      BO=XBAR(L1)-BO
C     SUM OF SQUARES ATTRIBUTABLE TO REGRESSION
      SSAR=RM*D(L1)
C     MULTIPLE CORRELATION COEFFICIENT
122   RM=SQRT(ABS(RM))
C     SUM OF SQUARES OF DEVIATIONS FROM REGRESSION
      SSDR=D(L1)-SSAR
C     VARIANCE OF ESSTIMATE
      FN=N-K-1
      SY=SSDR/FN
C     STANDARD DEVIATIONS OF REGRESSION COEFFICIENTS
      DO 130 J=1,K
      L1=K*(J-1)+J
      L=ISAVE(J)
125   SB(J)=SQRT(ABS((RX(L1)/D(L))*SY))
C     COMPUTED T-VALUES
130   T(J)=R(J)/SB(J)
C     STANDARD ERROR OF ESTIMATE
135   SY=SQRT(ABS(SY))
C     F VALUE
      FK=K
      SSARM=SSAR/FK
      SSDRM=SSDR/FN
      F=SSARM/SSDRM
      ANS(1)=BO
      ANS(2)=RM
      ANS(3)=SY
      ANS(4)=SSAR
      ANS(5)=FK
      ANS(6)=SSARM
      ANS(7)=SSDR
      ANS(8)=FN
      ANS(9)=SSDRM
      ANS(10)=F
      RETURN
      END
```

APPENDIX II

INPUT SPECIFICATIONS

Data file contains control cards, data cards and selection cards:

(i) Control Cards:

Card	Variable	Description	Format
First	TITLE	Title of the problem	A
Second	N	Number of observations	Free
	M	Number of variables	
	NS	Number of selections	

(ii) Data Cards:

Since input data are read into the computer one observation at a time, each row of the variables (dependent and independent) is given in one line in free format. The input subroutine DATA is written by the user depending upon the type of problem.

(iii) Selection Cards:

The selection card is used to specify a dependent variable and a set of independent variables in a multiple linear regression analysis. The selection cards are given after data cards. The selection card is prepared as follows:

Variable	Description	Format
NRESI	Option code for table of residuals	12
	00 if table is not required	
	01 if table is required	
NDEP	Dependent variable designated for the forthcoming regression analysis	12
K	Number of independent variables included in the forthcoming regression analysis	12
ISAVE	Vector containing independent variables(numbers of independent variables are given in 12 format)	

APPENDIX III

OUTPUT DESCRIPTION

The output of the multiple linear regression analysis programme includes the following statistics:

(a) Means

(b) Standard deviations

(c) Correlation coefficients between the independent variables and dependent variable.

(d) Regression coefficients

(e) Standard error of regression coefficients

(f) Computed T values

(g) Multiple correlation coefficient

(h) Analysis of variance for the multiple regression

(i) Standard error of estimate

APPENDIX IV

TEST DATA

The following data has been used to illustrate the use of programme. The data has been taken from 'Statistical Methods in Hydrology' by C.T. Haan page 201.

SN.	Q (1000 cfs)	Q (1000 Mile^2)	I (inches)
1.	15.50	1.250	1.7
2.	8.50	0.871	2.1
3.	85.00	5.690	1.9
4.	105.00	8.270	1.9
5	24.80	1.620	2.1
6	3.80	0.175	2.4
7	1.76	0.148	3.2
8.	18.00	1.400	2.7
9.	8.75	0.297	2.9
10.	8.25	0.322	2.9
11.	3.56	0.178	2.8
12.	1.90	0.148	2.7
13.	16.50	0.872	2.1
14.	2.80	0.091	2.9

Here Q is discharge in thousand cfs, A is area in thousand square miles, I is 24 hour rainfall depth in inches. Q is a dependent variable while A and I are independent variables.

APPENDIX V

TEST INPUT

```
RELATIONSHIP BETWEEN Q AND AREA AND INTENSITY OF RAINFALL
14,3,1
15.50  1.250  1.70
 8.50  0.871  2.10
85.00  5.690  1.90
105.00 8.270  1.90
24.80  1.620  2.10
 3.80  0.175  2.40
 1.76  0.148  3.20
18.00  1.400  2.70
 8.75  0.297  2.90
 8.25  0.322  2.90
 3.56  0.178  2.80
 1.90  0.148  2.70
16.50  0.872  2.10
 2.80  0.091  2.90
0101020203
```

APPENDIX VI

TEST OUTPUT

RELATIONSHIP BETWEEN Q AND AREA AND INTENSITY OF RAINFALL
MULTIPLE REGRESSION.....

SELECTION..... 1

VARIABLE NO.	MEAN	STANDARD DEVIATION	CORRELATION X VS Y	REGRESSION COEFFIENT OF REG.COEF.	STD. ERROR	COMPUTED T VALUE
2	1.524	2.422	0.994	13.151	0.562	23.414
3	2.450	0.477	-0.390	0.011	2.853	0.004
DEPENDENT						
1	21.723	32.050				

0

INTERCEPT 1.65700

MULTIPLE CORRELATION 0.99357

STD. ERROR OF ESTIMATE 3.94391

0 ANALYSIS OF VARIANCE FOR REGRESSION

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARES	F VALUE
ATTRIBUTABLE TO REGRESSION	2	13182.623	6591.312	423.758
DEVIATION FROM REGRESSION	11	171.099	15.554	
TOTAL	13	13353.722		

MULTIPLE REGRESSION.....

SELECTION..... 1

TABLE OF RESIDUALS

CASE NO.	Y VALUE	Y ESTIMATE	RESIDUAL
1	15.50000	18.11483	-2.61483
2	8.50000	13.13506	-4.63506
3	85.00000	76.50771	8.49229
4	105.00000	110.13742	-5.13742
5	24.80000	22.98520	1.81480
6	3.80000	3.98529	-0.18529
7	1.76000	3.63917	-1.87917
8	18.00000	20.09868	-2.09868
9	8.75000	5.59531	3.15469
10	8.25000	5.92409	2.32591
11	3.56000	4.02922	-0.46922
12	1.90000	3.63357	-1.73357
13	16.50000	13.14821	3.35179
14	2.80000	2.88620	-0.08620