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Accuracy of Hydrodynamic Models of Free-Surface Flows

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ABSTRACT

Hydrodynamic models of overland flow and channel flow are based on the shallow water-wave theory that is described by the St. Venant (SV) equations. These models are derived from either the kinematic-wave (KW) approximation, the diffusion-wave (DW) approximation or the dynamic-wave (DYW) representation of the SW equations. In the studies reported to date, different types of criteria have been established to evaluate the adequacy of the KW and DW approximations, but no explicit relations either in time or in space between these criteria and the errors resulting from these approximations have been derived yet. Furthermore, when doing hydrologic modeling, it is not evident if the KW and DW approximations are valid for the entire hydrograph or a portion thereof. In other words, most of these criteria take on fixed point-values for a given rainfall-runoff event. This paper attempts to derive, under simplified conditions, error equations for the KW or DW approximation for space-independent flows, which provide a continuous description of error in the flow-discharge hydrograph. A dimensionless parameter γ is defined which reflects the effect of initial depth of flow, channel-bed slope, lateral inflow, and channel roughness. The kinematic wave, diffusion wave and dynamic wave solutions are parameterized through γ. By comparing the kinematic wave and diffusion wave solutions with the dynamic wave solution, equations are derived in terms of γ for the error in the kinematic wave and diffusion wave approximations.

INTRODUCTION

Physically-based models of overland flow, channel flow, surface irrigation, and many other phenomena involving unsteady, free surface open channel flows are based on the shallow water wave (SWW) theory. These models are based either on the kinematic wave (KW) approximation (Lighthill and Whitham, 1955), diffusion wave approximation (DW) or diffusion analogy (DA) or dynamic wave (DYW) representation. Lighthill and Whitham (1955) showed that at the Froude numbers less than one (appropriate to flood waves) the dynamic waves are rapidly attenuated and the kinematic waves become dominant. Using a dimensionless form of the St. Venant (SV) equations, Woolhiser and Liggett (1967) obtained what is now referred to as the kinematic wave number, K, as a criterion for evaluating the adequacy of the KW approximation. For K greater than 20, the KW approximation was considered to be an accurate representation of the SV equations in modeling of overland flow. However, no relation between K and the error in the KW approximation was suggested. Morris and Woolhiser (1980) modified the above criterion with an explicit inclusion of Froude number, F_0, and showed, based on numerical experimentation, that $F_0^2K \geq 5$ was a better indicator of the adequacy of the KW approximation. A relation between this criterion and the error resulting from the KW approximation was not derived, however.

Using a linear perturbation analysis, Ponce and Simons (1977) derived properties of the KW and DW approximations as well as DYW representations in modeling of open channel flows. They derived a spectrum showing the regions of the validity of the KW and DW approximations. Menendez and Norscini (1982) extended the work of Ponce and Simons by including the phase lag between the depth and velocity of flow. Their results were, however, similar to those of Ponce and Simons (1977). In another but similar study, Ponce, et al. (1978), based on propagation characteristics of sinusoidal perturbation, derived criteria to evaluate the adequacy of the KW and DW approximations. Daluz Viera (1983) compared solutions of the SV equations with those of the KW and DW approximations for a range of F_0 and K, and defined the regions of validity of these approximations in the K-F_0 space.

Fread (1985) developed criteria for defining the range of application of the KW and DW approximations. These were based on an analysis of the magnitude of the normalized errors in the momentum equation due to omission of certain terms. In a comprehensive study, Ferrick (1985) defined a group of dimensionless scaling parameters to establish the spectrum of river waves, with continuous transitions between wave types and subtypes. With the aid of these parameters he was able to discern when the KW and DW approximations would be valid.

In most of these studies, different types of criteria have clearly been established to evaluate the adequacy of the KW and/or DW approximations, but no explicit relations either in time or space between these criteria and the errors resulting from these approximations have been derived yet. Furthermore, when doing hydrologic modeling it is not evident if the KW and DW approximations are valid for the entire hydrograph or a portion thereof. In other words, most of these criteria take on fixed point values for a given event. The objective of this study is to derive, under simplified conditions, error equations for the KW and DW approximations for space-independent flows, which specify errors as a function of time.

SHALLOW WATER-WAVE (SWW) THEORY

The SWW theory can be described by some form of the SV equations. For flow over an infiltrating plane subject to uniform rainfall, these equations can be written in one-dimensional form on a unit width basis as:

Continuity equation,

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(uh) = q - f \tag{1}$$

Momentum equation,

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(\frac{1}{2}u^2 + gh\right) = g(S_0 - S_f) - \frac{qu}{h} \tag{2}$$

where h is the depth of flow (L), u is local mean velocity (L/T), q is uniform rainfall intensity (L/T), f is uniform infiltration rate (L/T), g is acceleration due to gravity, x is space coordinate in the direction of flow (L), t is time (T), S_0 is bed slope, and S_f is frictional slope.

Note $Q = uh$ is discharge (L^3/TL) per unit width. S_f can be approximated as

$$S_f = \beta \frac{u^2}{h} \tag{3}$$

where β is some resistance parameter. If the Chezy relation is used for representing the friction then $\beta = g/C^2$, where C is Chezy's resistance parameter.

The DYW representation employs the full form of equations (1) and (2). The KW approximation is based on equation (1) and equation (2) with the left side omitted,

$$g(S_0 - S_f) - \frac{qu}{h} = 0 \tag{4}$$

The DW approximation uses equation (1) and equation (2) with local and convective acceleration deleted,

$$g \frac{\partial h}{\partial x} = g(S_0 - S_f) - \frac{qu}{h} \tag{5}$$

Analytical solutions of the SV equations or their variants in the KW and DW approximations are tractable only for simple cases. To that end, the space independent case is considered in this study. In this case, the water surface is flat.

Space-Independent Flows

For space-independent (or uniform) flows, equation (1) takes the form

$$\frac{dh}{dt} = q - f \tag{6}$$

and equation (2) becomes

$$\frac{dh}{dt} = g(S_0 - S_f) - \frac{qu}{h} \tag{7}$$

Equations (6) and (7) are the governing equations for the DYW representation for spatially uniform flows. The KW approximation is based on equation (6) and equation (7) with the left side dropped,

$$g(S_0 - S_f) - \frac{qu}{h} = 0 \tag{8}$$

The DW approximation uses equation (6) as well as equation (8). Therefore, for the spatially uniform flows, the KW approximation is identical to the DW approximation. Equation (8) can also be approximated by neglecting the momentum exchange between lateral inflow and longitudinal channel flow as

$$S_0 = S_f \tag{9}$$

which can be expressed as equation (3). Similarly, equation (7) can be written as

$$\frac{dh}{dt} = g(S_0 - S_f) \tag{10}$$

Depending upon the presence or absence of f, equation (6) can also be simplified. If $f = 0$, then

$$\frac{dh}{dt} = q \tag{11}$$

Types of Scenarios

Depending upon the presence of lateral inflow and infiltration, four different scenarios can be considered:

1. $f = 0$, $q = q_0$ = constant
   This includes the case $q = 0$
2. $q = q_0$ = constant
   $f = f_0$ = constant
   This includes the case $q = f = 0$
3. $q - f = 0$, $q = q_0$ = constant
   This includes the case $q = f = 0$
4. $q = 0$, $f = f_0$ = constant.
   This includes the case $f = 0$

It may be noted that the scenario with $q = 0$ in equation (11) or $(q - f) = 0$ in equation (6) applies to the recession hydrograph. The same applies if $(q - f) < 0$.

Initial Conditions

Two types of initial conditions can be assumed:

(1) $h(0) = h_0$, $u(0) = u_0$, (12)

(2) $u(0) = 0,\ h(0) = 0$ (13)

Scenarios for Determination of Error

Error equations have been derived for the KW and DW approximations under the above-mentioned conditions for four different scenarios (Singh, 1992a, 1992b). To summarize, the following cases were treated and are summarized in Table 1:

(1) Scenario 1: Equations (11) and (8) are the governing equations for the KW approximation, and equations (11) and (7) for the DYW representation, with the initial condition given by equation (12).

(2) Scenario 1: Equations (11) and (8) are the governing equations for the KW approximation, and equations (11) and (7) for the DYW representation, with the initial condition given by equation (13).

(3) Scenario 1: Equations (11) and (9) are the governing equations for the KW approximation, and equations (11) and (7) for the DYW representation, with the initial condition given by equation (12).

(4) Scenario 1: Equations (11) and (9) are the governing equations for the KW approximation, and equations (11) and (7) for the DYW representation, with the initial condition given by equation (13).

(5) Scenario 1: Equations (11) and (9) are the governing equations for the KW approximation, and equations (11) and (10) for the DYW representation, with the initial condition given by equation (12).

(6) Scenario 1: Equations (11) and (9) are the governing equations for the KW approximation, and equations (11) and (10) for the DYW representation, with the initial condition given by equation (13).

(7) Scenario 2: Equations (6) and (8) are the governing equations for the KW approximation, and equations (6) and (7) for the DYW representation, with the initial condition given by equation (12).

(8) Scenario 2: Equations (6) and (8) are the governing equations for the KW approximation, and equations (6) and (7) for the DYW representation, with the initial condition given by equation (13).

Table 1 List of cases for error analysis.

Case No.	Scenario No.	Governing Equations		Initial Conditions	Lateral Inflow q	Lateral Outflow f
		KW/DW Approximations	DYW Representation			
1	1	Equations (11) and (8)	Equations (11) and (7)	Equation (12)	q = constant	$f = 0$
2	1	Equations (11) and (8)	Equations (11) and (7)	Equation (13)	q = constant	$f = 0$
3	1	Equations (11) and (9)	Equations (11) and (7)	Equation (12)	q = constant	$f = 0$
4	1	Equations (11) and (9)	Equations (11) and (7)	Equation (13)	q = constant	$f = 0$
5	1	Equations (11) and (9)	Equations (11) and (10)	Equation (12)	q = constant	$f = 0$
6	1	Equations (11) and (9)	Equations (11) and (10)	Equation (13)	q = constant	$f = 0$
7	2	Equations (6) and (8)	Equations (6) and (7)	Equation (12)	q = constant	f = constant
8	2	Equations (6) and (8)	Equations (6) and (7)	Equation (13)	q = constant	f = constant
9	2	Equations (6) and (9)	Equations (6) and (7)	Equation (12)	q = constant	f = constant
10	2	Equations (6) and (9)	Equations (6) and (7)	Equation (13)	q = constant	f = constant
11	2	Equations (6) and (9)	Equations (6) and (10)	Equation (12)	q = constant	f = constant
12	2	Equations (6) and (9)	Equations (6) and (10)	Equation (13)	q = constant	f = constant
13	3	Equations (11) with $q = 0$ and (8)	Equations (11) with $q = 0$ and (7)	Equation (12)	$q = 0$	$f = 0$

Table 1 (continued)

Case No.	Scenario No.	Governing Equations		Initial Conditions	Lateral Inflow q	Lateral Outflow f
		KW/DW Approximations	DYW Representation			
14	3	Equations (11) with $q = 0$ and (9)	Equations (11) with $q = 0$ and (7)	Equation (12)	$q = 0$	$f = 0$
15	3	Equations (11) with $q = 0$ and (9)	Equations (11) with $q = 0$ and (10)	Equation (12)	$q = 0$	$f = 0$
16	4	Equations (6) with $q = 0$ and (8)	Equations (6) with $q = 0$ and (7)	Equation (12)	$q = 0$	$f =$ constant
17	4	Equations (6) with $q = 0$ and (9)	Equations (6) with $q = 0$ and (9)	Equation (12)	$q = 0$	$f =$ constant
18	4	Equations (6) with $q = 0$ and (9)	Equations (6) with $q = 0$ and (10)	Equation (12)	$q = 0$	$f =$ constant

(9) Scenario 2: Equations (6) and (9) are the governing equations for the KW approximation, and equations (6) and (7) for the DYW representation, with the initial condition given by equation (12).

(10) Scenario 2: Equations (6) and (9) are the governing equations for the KW approximation, and equations (6) and (7) for the DYW representation, with the initial condition given by equation (13).

(11) Scenario 2: Equations (6) and (9) are the governing equations for the KW approximation, and equations (6) and (10) for the DYW representation, with the initial condition given by equation (12).

(12) Scenario 2: Equations (6) and (9) are the governing equations for the KW approximation, and equations (6) and (10) for the DYW representation, with the initial condition given by equation (13).

(13) Scenario 3: Equation (11) with $q = 0$ and equation (8) are the governing equations for the KW approximation, and equations (11) and (7) for the DYW representation, with the initial condition given by equation (12).

(14) Scenario 3: Equation (11) with $q = 0$ and equation (9) are the governing equations for the KW approximation, and equations (11) and (7) for the DYW representation, with the initial condition given by equation (12).

(15) Scenario 3: Equation (11) with $q = 0$ and equation (9) are the governing equations for the KW approximation, and equations (11) and (10) for the DYW approximation, with the initial condition given by equation (12).

(16) Scenario 4: Equations (6) with $q = 0$ and equation (8) are the governing equations for the KW approximation, and equation (6), with $q = 0$ and equation (7) for the DYW approximation with the initial condition given by equation (12).

(17) Scenario 4: Equation (6) with $q = 0$ and equation (9) are the governing equations for the KW approximation, and equation (6) with $q = 0$ and equation (7) for the DYW approximation, with the initial condition given by equation (12).

(18) Scenario 4: Equation (6) with $q = 0$ and equation (9) are the governing equations for the KW approximation, and equation (6) with $q = 0$ and equation (10) for the DYW approximation, with the initial condition given by equation (12).

We consider cases 6 and 7 for derivation of error equations, where $f = 0$, and $q = q_0$.

ERROR EQUATIONS: USE OF NONZERO INITIAL CONDITIONS

Kinematic Wave and Diffusion Wave Solution

Equation (11), subject to equation (12), has the solution:

$$h = h_0 + q_0 t \tag{14}$$

From the kinematic wave approximation,

$$u = \left(\frac{S_0}{\beta}\right)^{0.5} h^{0.5} \tag{15}$$

It is convenient to define a dimensionless time τ as

$$\tau = \frac{h}{h_0} = \frac{h_0 + q_0 t}{h_0}, \quad h_0 \neq 0, \quad \tau \geq 1 \tag{16}$$

Equation (14) can be expressed as

$$h = h_0 \tau$$

In dimensionless form, the flow depth is

$$h_* = \frac{h}{h_0} = \tau \tag{17}$$

Equation (15) can be expressed as

$$u = \left(\frac{S_0 h_0}{\beta}\right)^{0.5} \tau^{0.5} \tag{18}$$

In dimensionless form

$$v = \frac{u}{U} = \tau^{0.5} \tag{19}$$

where

$$U = \left(\frac{S_0 h_0}{\beta}\right)^{0.5} \tag{20}$$

In terms of discharge Q,

$$Q(t) = \left(\frac{S_0}{\beta}\right)^{0.5} (h_0 + q_0 t)^{1.5} \tag{21}$$

Equation (21) can be expressed in terms of τ as

$$Q(\tau) = (\frac{S_0}{\beta})^{0.5} h_0^{1.5} \tau^{1.5} \tag{22}$$

In terms of dimensionless discharge Q_*,

$$Q_* = \frac{Q}{Q_0} = \tau^{1.5} \tag{23}$$

where

$$Q_0 = (\frac{S_0}{\beta}) h_0^{1.5} \tag{24}$$

The kinematic wave (KW) and diffusion wave (DW) solution is given by equations (14) and (15). The dimensionless velocity and dimensionless discharge are plotted against τ, as shown in Figures 1 and 2, respectively. Evidently, the velocity increases parabolically with τ.

Dynamic Wave Solution

Equation (11) has the solution given by equation (14). Equation (2) reduces to

$$\frac{du}{dt} = gS_0 - g\beta \frac{u^2}{h} \tag{25}$$

This can be expressed in terms of τ as

$$\frac{du}{d\tau} = \frac{gS_0h_0}{q_0} - g\beta \frac{u^2}{q_0\tau} \tag{26}$$

In dimensionless terms,

$$\frac{dv}{d\tau} = \frac{g}{q_0} (\beta S_0 h_0)^{0.5} - \frac{g}{q_0} (\beta S_0 h_0)^{0.5} \frac{v^2}{\tau} \tag{27}$$

If

$$\gamma = \frac{4g^2 \beta S_0 h_0}{q_0^2}, \text{ then}$$

$$\frac{dv}{d\tau} = \frac{\gamma^{0.5}}{2} - \frac{\gamma^{0.5}}{2} \frac{v^2}{\tau} = \frac{\Gamma}{2} - \frac{\Gamma v^2}{2\tau}, \quad \Gamma = \gamma^{0.5} \tag{28}$$

In terms of discharge, equation (25) can be expressed as

$$\frac{dQ}{dt} = \frac{q_0}{h_0 + q_0 t} Q + gS_0 (h_0 + q_0 t) - g\beta \frac{Q^2}{(h_0 + q_0 t)} \tag{29}$$

Equation (29) can be expressed in terms of τ as

$$\frac{dQ}{d\tau} = \frac{Q}{\tau} + \frac{gS_0 h_0^2}{q_0} \tau - \frac{g\beta}{h_0 q_0 \tau^2} Q^2 \tag{30}$$

In terms of dimensionless discharge, equation (30) becomes

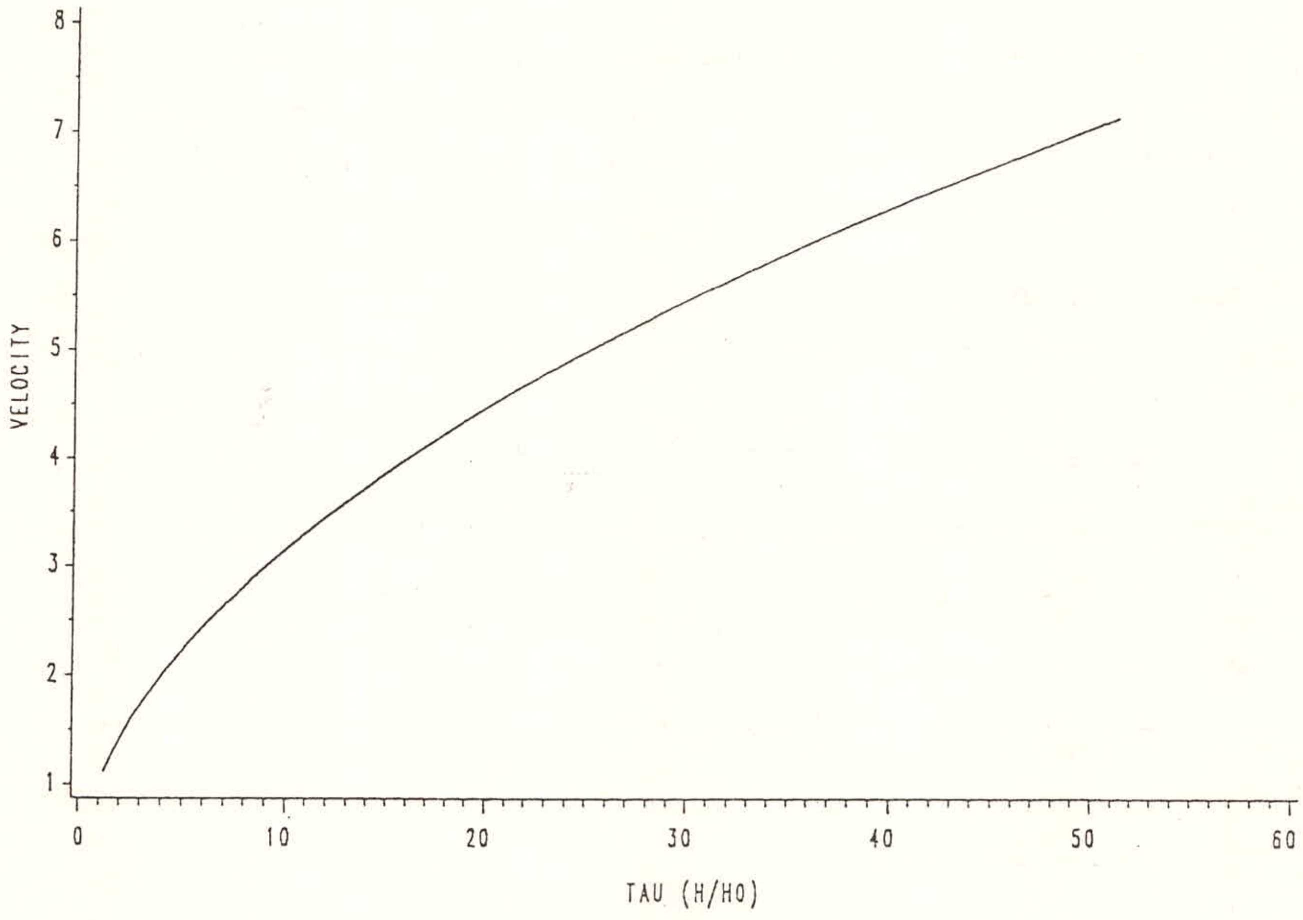

Figure 1. Dimensionless kinematic wave velocity as a function of dimensionless time.

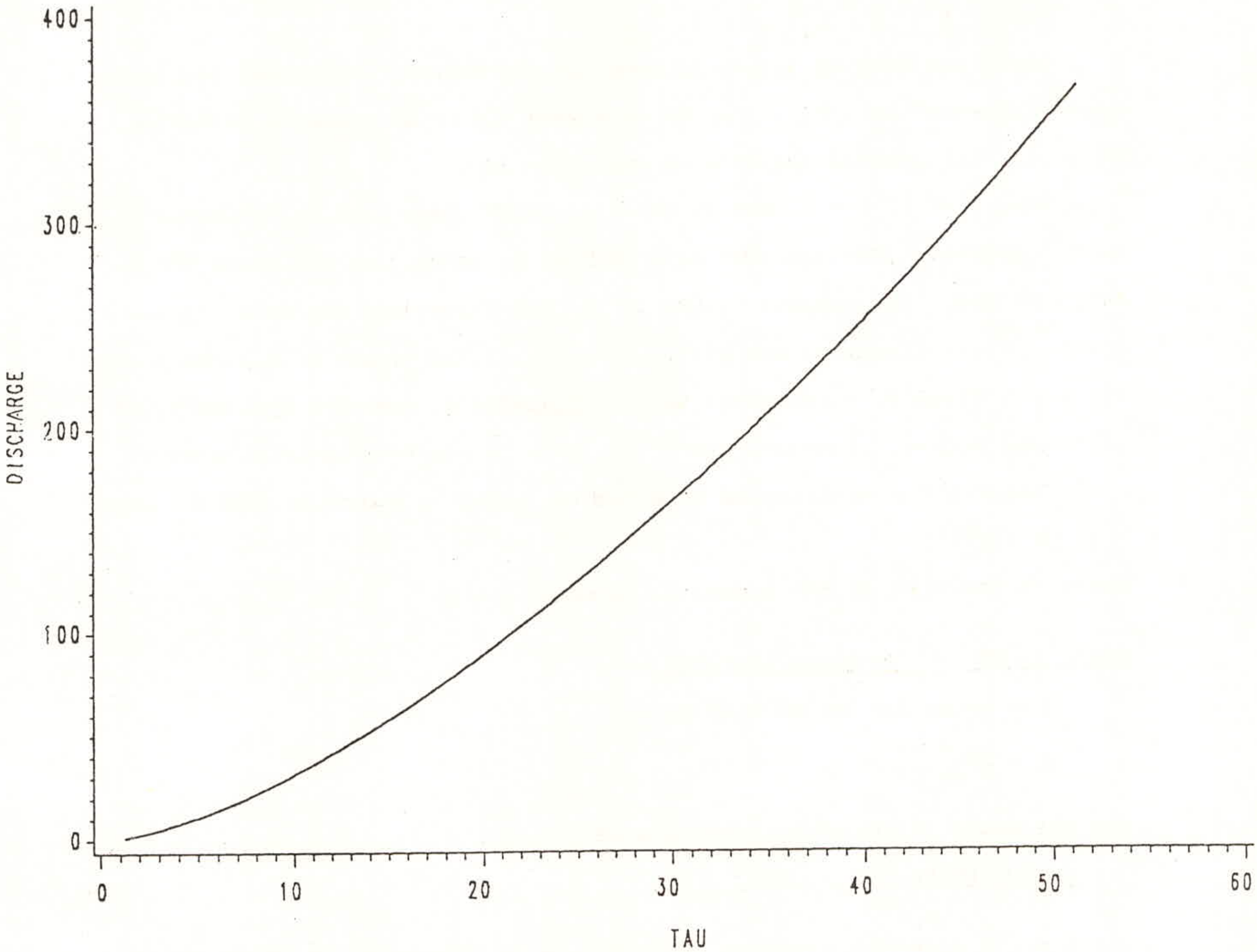

Figure 2. Dimensionless kinematic wave discharge as a function of dimensionless time.

$$\frac{dQ_*}{d\tau} = \frac{Q_*}{\tau} + \frac{g}{q_0} (S_0\beta h_0)^{0.5} \tau - \frac{g}{q_0} (S_0\beta h_0)^{0.5} \frac{Q_*^2}{\tau^2}$$

or

$$\frac{dQ_*}{d\tau} = \frac{Q_*}{\tau} + \frac{\Gamma}{2} \tau - \frac{\Gamma}{2} \frac{Q_*^2}{\tau^2} \tag{31}$$

Equation (28) is a special case of the Riccati equation, and so is true with equation (31). The dynamic wave (DYW) solution is given by equation (14) and the solution of equation (28).

When $\tau = 1$, $v = 1$, and $dv/d\tau = 0$. With these initial conditions in hand, equations (28) and (31) were solved by using the 4th order Runge-Kutta method. For various values of γ, the dimensionless velocity and dimensionless discharge are plotted against τ, as shown in Figures 3 and 4. For a fixed γ, v increases with increasing τ, and for a fixed τ, it increases with γ. However, for $\gamma \geq 1.5$, v is not very sensitive to γ.

The maximum velocity is obtained by equating equation (28) to zero:

$$v = \tau^{0.5} \tag{32}$$

where is the same as the kinematic wave velocity.

Error in KW and DW Approximations

The error can be defined as

$$E = \frac{v_K - v_D}{vD} \tag{33}$$

and the error differential equation as

$$\frac{dE}{d\tau} = \frac{(E+1)}{v_K} \frac{dv_K}{d\tau} - \frac{(E+1)^2}{v_K} \frac{dv_K}{d\tau} \tag{34}$$

where v_k is given by equation (19), and v_D by the solution of equation (28). To that end,

$$\frac{dv_K}{d\tau} = \frac{1}{2\tau^{0.5}}, \quad v_D = \frac{v_K}{E+1} \tag{35}$$

By inserting these terms, and equations (19) and (28) into equation (34), one obtains

$$\frac{dE}{d\tau} = C_0(\tau) + C_1(\gamma,\tau)\, E + C_2(\gamma,\tau)\, E^2, \quad E(1) = 0, \quad \tau \geq 1 \tag{36}$$

where

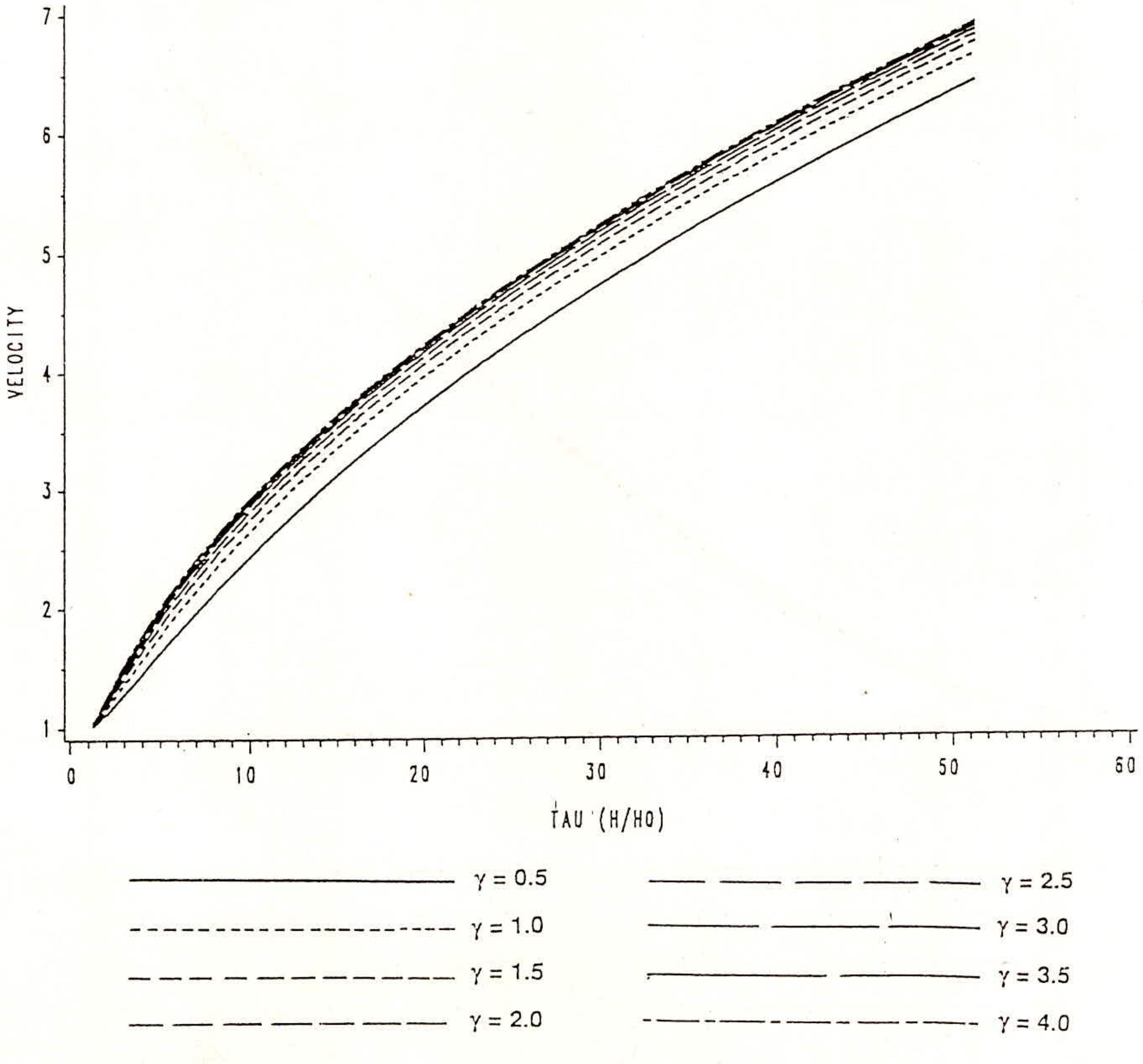

Figure 3. Dimensionless dynamic wave velocity as a function of dimensionless time.

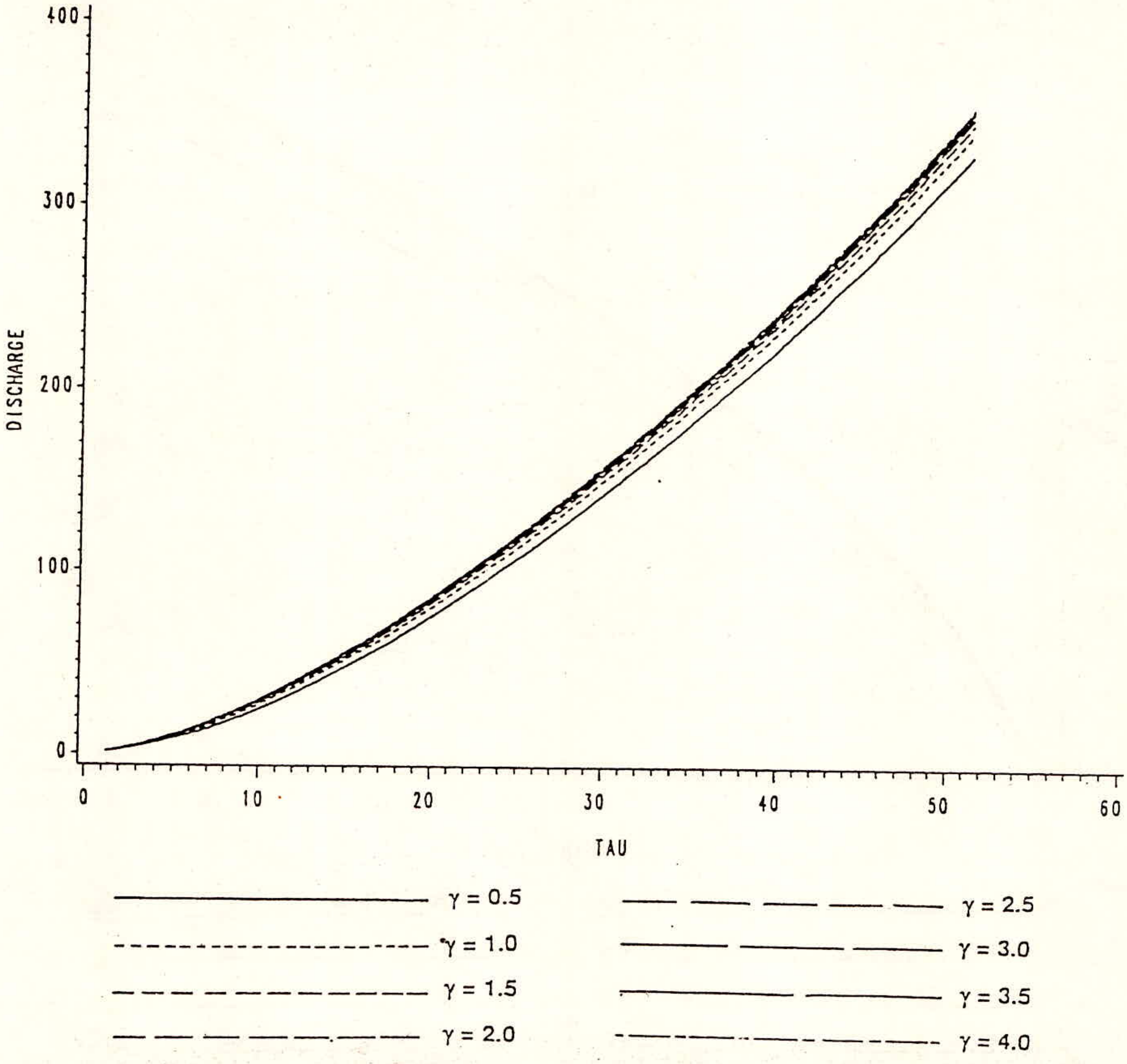

Figure 4. Dimensionless dynamic wave discharge as a function of dimensionless time.

$$C_0(\tau) = \frac{1}{2\tau} \tag{37}$$

$$C_1(\gamma,\tau) = \frac{1}{2\tau}\,(1 - 2\,\Gamma\,\tau^{0.5}) \tag{38}$$

$$C_2(\gamma,\tau) = -\,\frac{\Gamma}{2\tau^{0.5}} \tag{39}$$

Equation (36) is a Riccati equation and has to be solved numerically. Equation (6.21) also holds for error in discharge. This equation was solved by using the 4th order Runge-Kutta method. At $\tau = 1$, $E(1) = 0$, the error derivative $dE/d\tau$ was obtained by use of forward differencing. The distribution of error in τ is shown in Figure 5. The distribution is highly skewed, with a sharp rise and gradual decline over an extended range of τ. For a fixed τ, error increases with decreasing γ, and for a fixed γ, it follows the distribution of Figure 5. Except for a small range of τ, $1 \le \tau \le 25$, the error derivative is almost independent of γ. It has the highest value in the beginning, declines sharply, then rises a little bit, and tends to asymptotically approach a constant value. C_0 is independent of γ, and is inversely proportional to τ and is always positive. Both C_1 and C_2 are quite sensitive to τ, and are negative for all values of τ. Both coefficients have the lowest value in the beginning and increase with increasing τ. For a fixed τ, both have higher values with lower values of γ and vice versa.

ERROR EQUATIONS; USE OF ZERO INITIAL CONDITIONS

Kinematic Wave and Diffusion Wave Solution

Equation (11), subject to equation (13), has the solution

$$h = q_0 t \tag{40}$$

Equation (2) takes the form

$$u = \left(\frac{S_0}{\beta}\right)^{0.5} h^{0.5} \tag{41}$$

It is convenient to define a dimensionless parameter τ as

$$\tau = \frac{tg}{q_0}, \quad \tau \ge 0 \tag{42}$$

In terms of τ, equation (40) becomes

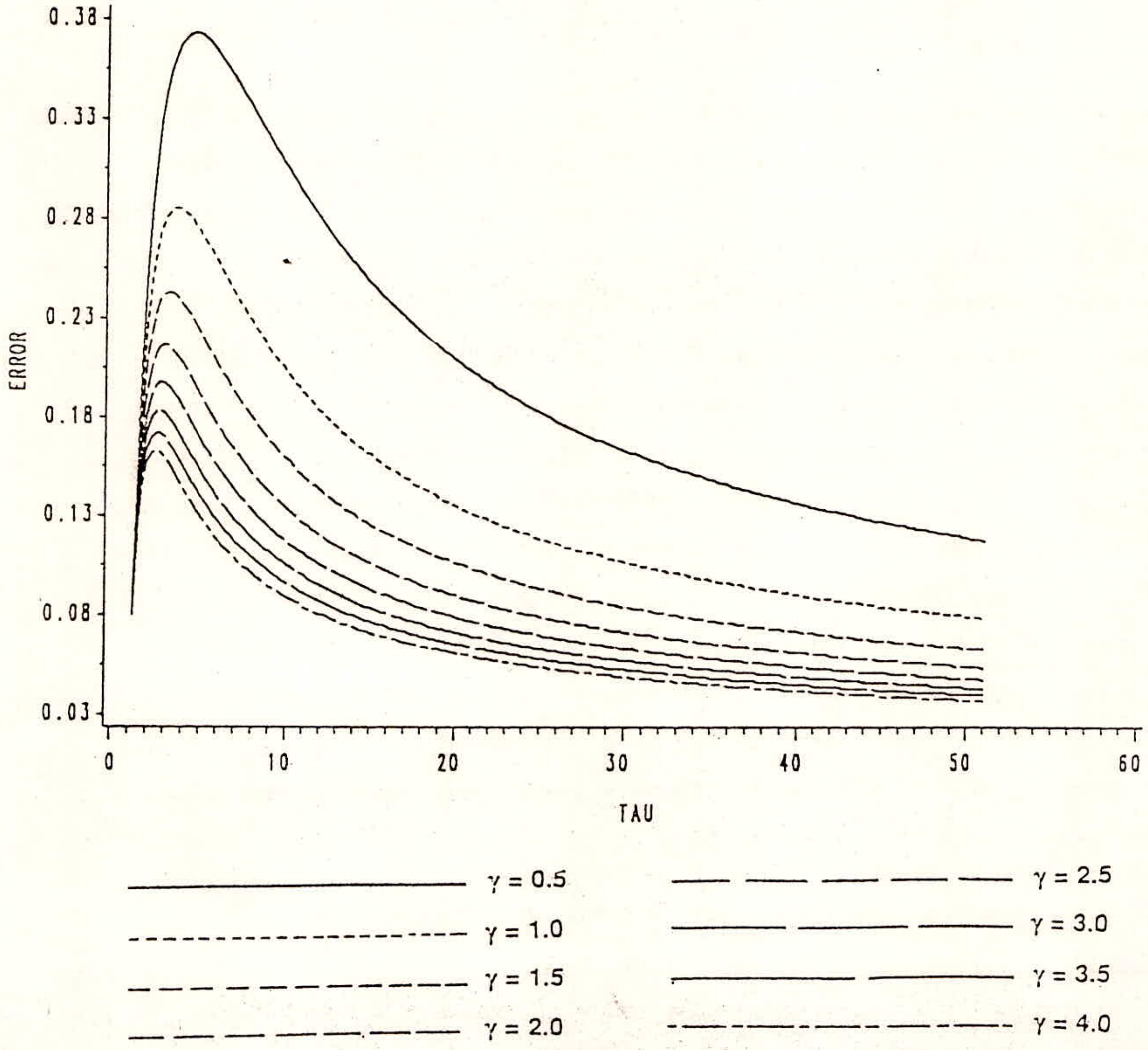

Figure 5. Error in the KW or DW approximation as a function of dimensionless time.

$$h = \frac{q_0^2}{g} \tau \quad (43)$$

In dimensionless terms h_*,

$$h_* = \frac{h}{h_0} = \tau_* \quad (44)$$

where

$$h_0 = \frac{q_0^2}{g} \quad (45)$$

and equation (41) becomes

$$u(\tau) = q_0 \left(\frac{S_0}{g\beta}\right)^{0.5} \tau^{0.5} \quad (46)$$

or

$$v = \frac{u}{U} = \tau^{0.5} \quad (47)$$

where

$$U = q_0 \left(\frac{S_0}{g\beta}\right)^{0.5} \quad (48)$$

In terms of discharge Q,

$$Q(t) = \left(\frac{S_0}{\beta}\right)^{0.5} q_0^{1.5} t^{1.5} \quad (49)$$

Equation (49) can be expressed in terms of τ as

$$Q(\tau) = \left(\frac{S_0}{\beta}\right)^{0.5} \frac{q_0^3}{g^{1.5}} \tau^{1.5} \quad (50)$$

In terms of dimensionless discharge $Q_* = Q/Q_0$, equation (49) becomes

$$Q_* = \tau^{1.5} \quad (51)$$

where

$$Q_0 = \left(\frac{S_0}{\beta}\right)^{0.5} \frac{q_0^3}{g^{1.5}} \quad (52)$$

The kinematic wave (KW) and diffusion wave (DW) solution is given by equation (43) and (47). The dimensionless velocity and dimensionless discharge against τ follow the curves shown in Figures 1 and 2, respectively. The velocity increases parabolically with τ.

Dynamic Wave Solution

Equation (11) has its solution given by equation (39). Equation (2) takes the form given by equation (25). Therefore,

$$\frac{dv}{d\tau} = \frac{\Gamma}{2} - \frac{\Gamma}{2\tau} v^2 \tag{53}$$

where

$$\Gamma = 2(S_0 g\beta)^{0.5}, \quad \gamma = 4S_0 g\beta = \Gamma^{0.5} \tag{54}$$

The dimensionless discharge Q_* can be expressed as

$$\frac{dQ_*}{d\tau} = \frac{Q_*}{\tau} + \frac{\Gamma}{2}\tau - \frac{\Gamma}{2}\frac{Q_*^2}{\tau^2} \tag{55}$$

The dynamic wave (DYW) solution is given by equation (43) and the solution of equation (53).

Equation (53) was solved by using the 4th order Runge-Kutta method. At $\tau = 0$, $v = 0$ and the derivative $dv/d\tau$ has a singularity. The derivative at $\tau = 0$ was estimated by using the forward differencing. Dimensionless velocity and dimensionless discharge against τ for various values of γ will be similar to those shown in Figures 3 and 4. For a fixed τ, the velocity is higher for higher values of γ and vice versa. For $\gamma \geq 1.5$, it is virtually insensitive to γ. The maximum velocity is, again, the kinematic wave velocity.

Error in KW and DW Approximations

Because the dimensionless solutions in this case are of the same form as in the preceding case, the error equation, by analogy, should follow

$$\frac{dE}{d\tau} = C_0(\tau) + C_1(\gamma,\tau)\,E + C_2(\gamma,\tau)\,E^2, \quad E(0) = 0, \quad \tau \geq 0 \tag{56}$$

where

$$C_0(\tau) = \frac{1}{2\tau} \tag{57}$$

$$C_1(\gamma,\tau) = \frac{1}{2\tau}(1 - 2\Gamma\,\tau^{0.5}) \tag{58}$$

$$C_2(\gamma,\tau) = -\frac{\Gamma}{2\tau^{0.5}} \tag{59}$$

Equation (56) is a Riccati equation, and was solved by using the 4th order Runge-Kutta method. This equation also describes error in discharge. The error derivative has a discontinuity at $\tau = 0$. The value of the derivative can be estimated by using finite differences in a manner similar to that of the previous section.

The distribution of error in the KW or DW solution is shown in Figure 6 for various values of γ. For a ~~fixed~~ τ, the error increases with decreasing γ. For a fixed γ, the error distribution is highly skewed, with a sharp rise to a peak and then recession over an extended τ, approaching almost a constant value. The error derivative the highest value in the beginning, declines sharply for $0 \le \tau \le 3$, and approaches a constant value. For $\tau \ge 15$, the error derivative is virtually independent of γ, and for $\tau < 15$, it is only marginally sensitive to γ. C_0 is independent of γ, is always positive, and is inversely proportional to τ. C_1 assumes both negative and positive values, depending upon γ and τ. Except for $\gamma = 0.5$, C_1 is always negative, with the smallest value in the beginning and approaching almost a constant value as τ becomes large. C_2 is always negative, has the lowest value in the beginning and rises to almost a constant value as τ increases. It is inversely proportional to the square root of τ.

CONCLUSIONS

For space-independent flows, the kinematic-wave and diffusion-wave approximations are sufficiently accurate when the dimensionless parameter $\gamma \ge 3$. This parameter reflects the effect of initial flow depth, bed slope, lateral inflow, and channel roughness. The error of these approximations declines exponentially when the dimensionless time exceeds 5. The dimensionless time is obtained with the use of initial depth.

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