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A Model On Reservoir to Reservoir Routing and Estimation of Net Discharge from Reservoirs

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SYNOPSIS

A series of n-linear reservoirs with different storage co-efficients (k_i 's), at different positions in a mountainous area have been considered and a mathematical expression of run-off model has been presented in the paper. The inflow of water into the particular reservoir is considered as an outflow of water from the preceeding reservoir (situated at higher altitude) as well as the contribution of the rainfall excess (Re) over the reservoir.The net discharge over the area is calculated from the discharge of the nth. reservoir (situated at lowest altitude), after considering outflows of all the reservoirs at different positions within the area. The procedures for finding out the storage co-efficients and the rainfall excesses with infiltrations of all the reservoirs within the area have also been presented in the paper.

1. INTRODUCTION

An estimation of the quantity of water available, as surface run-off is of prime importance and its utilisation is the main aspect in water res o urces engineering. The objectives and implementation of water-allocation policy to different heads like irrigation, hydro-power generation,etc. are dependant on proper utilisation of the available water through water-development projects. The water resources of the mountainous area has much contribution in management of water distribution which involves the socio-economic impact of the country.Resources of water available as a surface run-off from the ' reservoirs at different positions/altitude of a mountainous area are required to be estimated methodicaly with an accuracy. The estimation of net discharge from a series of such reservoirs has been attempted.

Nash (1960) has proposed a model through mathematical equation by considering a series of n-linear reservoirs in a drainage basin, having the same storage coefficient for all the reservoirs. In his model, he has given a mathematical expression in which the outflow of the first reservoir is considered as the inflow to the second and this inflow is used to determine the outflow of the second reservoir by the convolution integral by considering the instanteneous unit hydrograph (IUH). The outflow of the second reservoir is used as an infow to the third reservoir and so on. Rosa & others (1961) have proposed a model through electronic analog of a series of five linear reservoirs with variable storage coefficients. By this analog model, it is possible to fit various outflows on a non-linear basis. Dooge (1959) has proposed by considering a series of alternating linear channels and linear reservoirs in which the drainage basin area is divided into number of isochrones subareas where the outflow from the preceeding subarea serves as the infow to the reservoir. But, this model has a translation effect of flow in a drainage basin which cannot be solved easily in practical cases.

In all the above models, as suggested by different authors, the outflow of the previous reservoir/subarea is considered as the inflow to subsequent reservoir/subarea. But, it is not considered the inflow into the reservoir/subarea as a result of outflow only from the preceeding reservoir/subarea, but also the contribution of the excess rainfall over the reservoir/subarea. Moreover, storage coefficients for all the reservoirs were considered same althrough. But, here the problem has been considered in which the inflow of water into the reservoir is not only the contribution of outflow of the preceeding reservoir, but also the infow contribution due to rainfall excess over the reservoir and the storage coefficients of all the reservoirs are of different values. A series of such reservoirs have been considered here and a mathematical model has been established.

2. THE CONCEPTUAL MODEL

2.1 The theory of outflow

By the theory of linear reservoir, the reservoir storage S is directly proportional to the outflow q and is given by

$$S = K\, q \qquad \ldots \qquad \ldots \qquad (1)$$

where, k is the reservoir constant, called the storage coefficient.

By the continuity equation, the rate of storage $\frac{ds}{dt}$ is equal to the difference between the inflow i to the reservoir and the outflow q of the reservoir and is given by

$$\frac{ds}{dt} = (i - q) \qquad \ldots \qquad \ldots \qquad (2)$$

By equations (1)& (2) and for initial condition q = 0 at t = 0, we have the equation for outflow at time t and is given by

$$q = i\,(1 - e^{-t/k}) \ldots \qquad \ldots \qquad (3)$$

2.2. The net outflow expression

A series of n-linear reservoirs (Fig.1) have been taken into consideration where the net outflow from n-linear reservoirs, situated at different altitudes is possible to calculate by the outflow contribution from the individual reservoir. The outflow contribution of the preceeding reservoir is used as an inflow of the successive reservoir plus the contribution from the rainfall excess over the reservoir.

Thus for an outflow q_1 at time t_1 of the first reservoir, at an altitude h_1, is given by

$$q_1 = Re_0 \left(1 - e^{-t/k_1} \right) \quad \dots \quad \dots \quad (4)$$

where, Re_0 is the inflow, i.e., the rainfall excess received by the first reservoir, k_1 being the storage coefficient of the first reservoir.

Now, the outflow q_1 of the first is used as the inflow to the second reservoir,at an altitude h_2 (lower than h_1) plus the rainfall excess Re_1 received by the second reservoir.

Thus, for an outflow q_2 at time t_2 of the second reservoir is given by

$$q_2 = \left(q_1 + Re_1 \right) \left(1 - e^{-t_2/k_2} \right) \quad \dots \quad \dots \quad (5)$$

where, k_2 is the storage coefficient of the second reservoir.

Now, putting the expression of q_1 from equation (4) in (5) the outflow q_2 becomes,

$$q_2 = Re_0 \left(1 - e^{-t_1/k_1} \right) \left(1 - e^{-t_2/k_2} \right) + Re_1 \left(1 - e^{-t_2/k_2} \right) \quad \dots \quad \dots \quad (6)$$

Preceeding in this way, an outflow q_n at time t_n of the nth. reservoir at an altitude h_n (lower than altitude h_{n-1}), is given by

$$q_n = Re_0 \prod_{i=1}^{n} \left(1 - e^{-t_i/k_i} \right) + Re_1 \prod_{i=2}^{n} \left(1 - e^{-t_i/k_i} \right) + \dots\dots$$

$$\dots\dots + Re_{n-1} \left(1 - e^{-t_n/k_n} \right) \quad \dots \quad \dots \quad (7)$$

where, k_i is the storage coefficient of the i th. reservoir (i = 1,2,..........,n) and the symbol $\prod_i$ represents product upto ith. terms.

The above outflow q_n in the expression (7) is the net outflow from n-linear reservoirs situated at different altitudes with different storage coefficients and this can be found out after obtaining the storage coefficients k_i's of each of the reservoir and the contrinbution of rainfall excess Re_i's over each of the reservoir.

2.3. Calculation of infiltration and rainfall excess

The rainfall excess Re_i over the ith. reservoir can be obtained by subtracting the infiltration $f_{(i)}$ of the reservoir from the rainfall $p_{(i)}$ received over it.

Mathematically,

$$Re_i = P_{(i)} - f_{(i)} \qquad \ldots \qquad \ldots \quad (8)$$

The infiltration f_t at time t is given by

$$f_t = \frac{\left(\frac{ds}{dt}\right)_t - \left(\frac{ds}{dt}\right)_{(t-1)}}{t - (t-1)} \qquad \ldots \quad (9)$$

where, $\left(\frac{ds}{dt}\right)_t$ and $\left(\frac{ds}{dt}\right)_{t-1}$ are the rate of change of storage at time t and (t-1).

The infiltration $f_{(i)}$ of the ith. reservoir can be found out from the Horton's formula (1939) of infiltration f_t at time t,

$$f_t = f_c + (f_o - f_c)\, e^{-m_1 t} \qquad \ldots \quad (10)$$

where, 'm_1' is the constant depending on nature and geometry of the place, f_o & f_c are infiltrations at the beginning and cessation respectively of the rainfall, and can be determined by the method as suggested by Basu (1989)

$$m_1 = \ln \left(\frac{f_t - f_{t-1}}{f_{t+1} - f_t} \right) \qquad \ldots \quad (11)$$

where, f_t, f_{t-1} & f_{t+1} are infiltrations at time t th., (t-1)th. & (t+1)th. respectively.

Thus, the infiltration at time t of the ith. reservoir, $f_{(i)}$ can be calculated from above equations and the rainfall excess Re_i of the ith. reservoir is determined from (8).

Now, the outflow q_n of the expressions (7) becomes,

$$q_n = (p_o - f_o) \prod_{i=1}^{n} (1 - e^{-t_i/k_i}) + (p_1 - f_1) \prod_{i=2}^{n} (1 - e^{-t_i/k_i})$$
$$+ \ldots\ldots\ldots + (p_{n-1} - f_{n-1})(1 - e^{-t_n/k_n}) \quad \ldots (12)$$

2.4 Some ideal cases

Case I.when all the storage coefficients of the reservoirs are equal or nearly equal, i.e. $k_1 = k_2 = \ldots k_n = k$ (say), the expression of outflow q_n from (12) gives

$$q_n = (p_o - f_o) \prod_{i=1}^{n} (1 - e^{-t_i/k}) + (p_1 - f_1) \prod_{i=2}^{n} (1 - e^{-t_i/k}) +$$
$$\ldots\ldots\ldots + (p_{n-1} - f_{n-1})(1 - e^{-t_n/k}) \quad \ldots (13)$$

Case II. In addition to case I and more over when the area under consideration consisting of n-linear reservoirs, is saturated with rain-water (i.e., the rainfall of long duration & after sometimes of the commencement of rainfall), the infiltration becomes very small, practically negligible. The rate of infiltration over all the reservoirs within the area becomes almost zero. In this case, rainfall receive as a whole, resulting totally into a surface runoff, i.e., $f_{(i)} = 0$, for all i, then (8) becomes, $Re_i = p_i$

The expression q_n from (13) becomes,

$$q_n = p_o \prod_{i=1}^{n} (1 - e^{-t_i/k}) + p_1 \prod_{i=2}^{n} (1 - e^{-t_i/k}) + \ldots\ldots$$
$$\ldots\ldots\ldots\ldots + p_{n-1} (1 - e^{-t_n/k}) \quad \ldots (14)$$

Case III. In addition to case I & II above, the average areal rainfall p, spreading throughout the area consisting of n-linear reservoirs is saturated with rain-water. Then $f_{(i)} = 0$, for all i, and $Re_o = Re_1 = \ldots Re_{n-1} = p$ (say).

The expression q_n from (14) becomes,

$$q_n = p \sum_{i=1}^{n} \prod_{i=1}^{n} (1 - e^{-t_i/k_i}) \quad \ldots \quad \ldots \quad (15)$$

In this case, the rainstorm is sufficiently large and covers the whole area.

2.5. Determination of storage coefficient

To determine the storage coefficient (k_i's), the actual discharge from the individual reservoir is required to be found out by the method, as suggested by Basu (1989), in relation with the reservoir levels by the formula.

$$q_j = L\, H_j^N \quad \ldots \quad \ldots \quad (16)$$

where, q_j is the discharge from the reservoir between the reservoir levels h_j & h_{j-1} and $H_j \approx h_j - h_{j-1}$; L & N are constants for the reservoir, j = 1,2, ...m.

After taking logarithm, the equation (16) becomes,

$$\ln q_j = N \ln H_j + \ln L \quad \ldots \quad \ldots \quad (17)$$

Plotting $\ln H_j$ against $\ln q_j$, a straight line will be obtained from which the constants L & N can be evaluated. The actual discharge q_j between the reservoir levels jth. & (j-1)th., is found out after obtaining the values of L & N.

The total actual discharge Q from the reservoir is obtained by summing up discharges of all the water levels of the reservoir, i.e., $Q = \sum q_j$, for all j's.

By using the linear storage-discharge equation (1), we have the storage capacity ΔS_j between the reservoir levels jth. & (j-1)th. of the ith. reservoir and is given by

$$\Delta S_j = k_i\, q_j \quad \ldots \quad \ldots \quad (18)$$

Now, $\Delta S_j \approx S_j - S_{j-1}$ is found out by storage-routing relation by using the fundamental relation between inflow, discharge (outflow) & storage and is given by

$$\Delta S_j \approx S_j - S_{j-1} = \left[\frac{i_j + i_{j-1}}{2}\right] t - \left[\frac{q_j + q_{j-1}}{2}\right] \quad ..(19)$$

where, i & q are the inflow & outflow rates respectively and can be measured. The values of i_{j-1}, q_{j-1} & i_j, q_j refer to the begining of the (j-1)th. level & end of jth. level of the reservoir respectively during the time interval t.

Now, for summing up all the levels of the reservoir, the equation (18) becomes,

$$\sum \Delta S_j = K_i \sum q_j = K_i Q \quad ... \quad ... (20)$$

The storage coefficient k_i of the ith. reservoir is found out from (19) & (20). Similarly, storage coefficients of all the reservoirs (i.e., i = 1,2, ... n) within the mountainous area are calculated by the above method.

3. DISCUSSION

The procedure for determination of net discharge from a series of n-linear reservoirs, as presented, is a generalised mathematical expression. The different possible cases with the associated hydrological and meteorological conditions/situations have also been presented. Moreover, the procedure for finding out the constants in the expression has also been mentioned here.

The storage coefficient 'K' of the individual reservoir is to be pre-determined by the method, as described in 2.5. The rainfall excess (Re) of the individual reservoir during the rainstorm period is to be calculated by 2.3, where, the procedure for determination of infiltration of the reservoir, as discussed, is to be followed, first.

The expressions for net discharge from a series of n-linear reservoirs, under different hydrological and meteorological conditions, as discussed here , may help to evaluate the quantification of discharge during the rainstorm period. Some ideal cases of reservoirs conditions for different situations are also discussed.

4. CONCLUSION

The management of water for long or short term basis from a series of reservoirs, is a major task for planning a project. The prediction of net discharge from a series of linear reservoirs in the mountainous area depends upon both hydrological and meteorological conditions prevailing over the area. Some constant factors of the reservoirs are required to be pre-determined by the methods, as discussed here.

This model of reservoir to reservoir routing for computation of net discharge, having a series of linear reservoirs requires computer simulation for their combining effects of all the reservoirs. This theoretical model for its practical application needs accuracy in calculating the reservoirs' constants and other factors, as discussed in this paper. This model may further be modified by considering a series of linear reservoirs alongwith the sub-area/sub-reaches over the mountainous area by adopting similar procedure with different approach. The rainfall excesses & infiltrations are required to be found out, not only for all the reservoirs, but also for the whole area by the above process.

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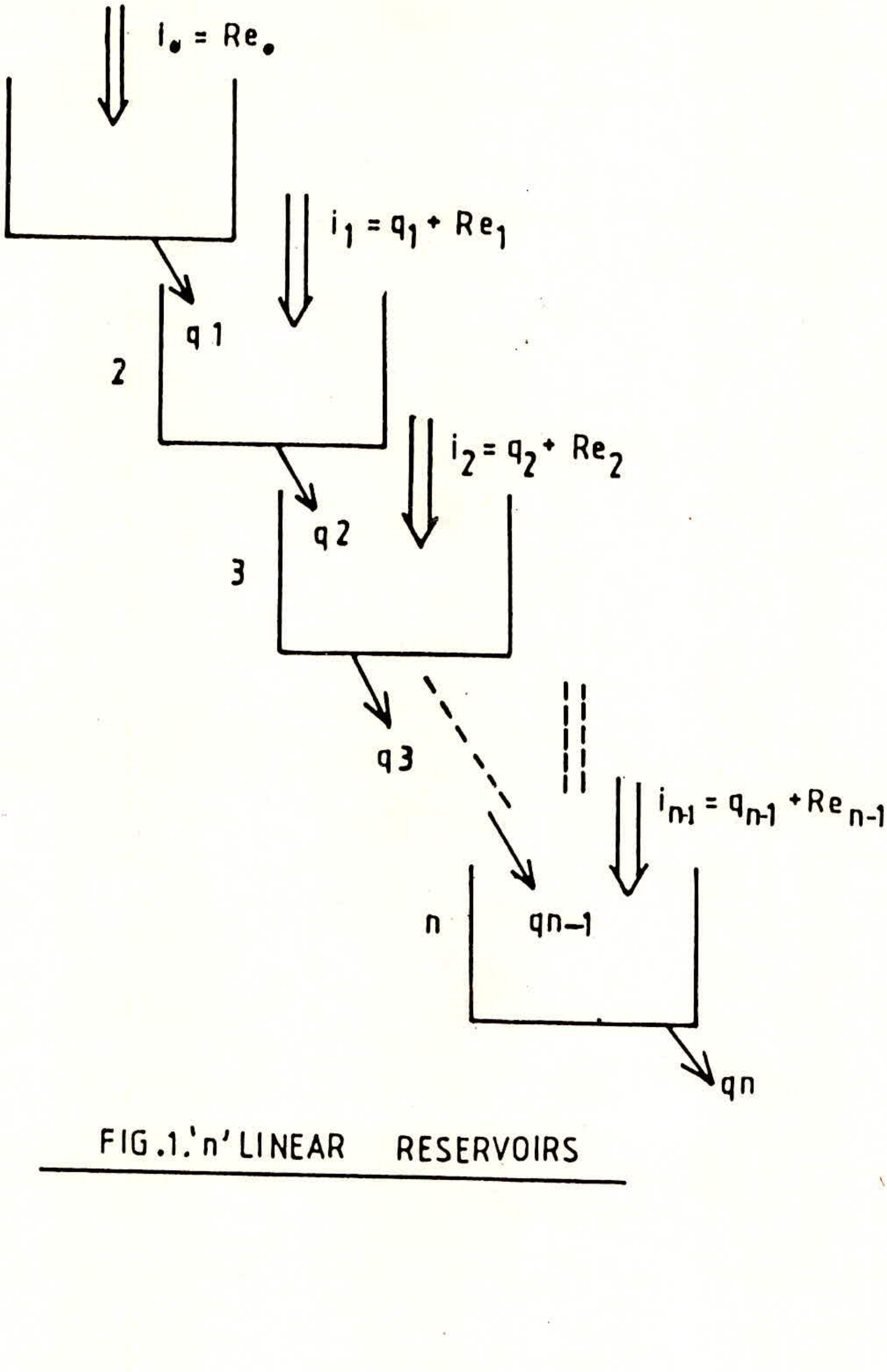

FIG.1.'n' LINEAR RESERVOIRS