International Symposium on
Hydrology of Mountainous Areas
Shimla (INDIA) May 28-30, 1992

Hydrologic Study of Springs

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Abstract

Springs have been used as a dependable source of water supply since ancient time. They are part and parcel of the groundwater. There is enough scope for hydrologic study of springs including the mathematical modelling of their flow behaviour. A review has been made in this paper regarding the work done in the area of hydrologic modelling of springflow. A mathematical model has been suggested for predicting springflow. Method of image and response function coefficients have been used for the modelling.

Introduction

Spring had aroused curiosity in the mind of the people in the past because of the fact that it flows freely with no obvious source. In the middle of the 17th century, French scientist Pierre Perrault conducted systematic study on spring in the Seine river basin and established that springs are the natural outlets of groundwater flow and the ultimate source of springwater is precipitation.

Spring is a ready source of water, a place of natural beauty, and a recreational spot. Springs generally provide clean water. They are found in the Himalayas, in the Western Ghats and in other places in india where it is logistically difficult to create storage for water. As such, study of springflow has relevance to the water supply to rural areas, specially in the hilly region. However, there is no systematic study of the springs so far and there is enough scope and need to study spring, particularly in respect of mathematical modelling of springflow.

Development of Mathematical Models for Springflow

Discharge of a spring does not remain constant with time. Fluctuation of spring discharge are due to the variations in rate of recharge and the prevailing hydrologic and geologic conditions. The discharge of a spring depends on the difference between the elevations of water table (or piezometric head) in the aquifer in the vicinity of the spring, and the elevation of the spring threshold. A typical portion of a spring hydrograph is shown in Fig.1. The shape of the springflow hydrograph is manifestation of the recharge into the spring's flow domain and it will vary from spring to spring. Significant information about the dynamic storage in the spring's flow domain can be ascertained by analysing the recession portion of the springflow hydrograph.

Existing Hydrologic Springflow Models

i)Model based on linear discharge-storage relationship:

Most of the existing springflow models are based on the assumption that the discharge of a spring is linearly proportional to the dynamic storage in the spring flow domain. Based on this assumption the springflow equation has been derived as:

$$Q(t) = Q_o \exp(-t/t_o) \qquad \ldots(1)$$

where t_o is known as the depletion time which is a parameter of the spring. It is the time that will be taken to empty the live storage of the spring at the present flow rate, i.e., the dynamic storage at any time t is equal to $Q(t)\ t_o$. The variation of spring discharge with time can be plotted in a semilog paper (discharge being in log scale). The slope of such a plot provides the depletion time, t_o. It is likely that the slope of the discharge time plot may vary from season to season . Any change in the slope of the line from year to year or within a year is an indication of interference in the groundwater system. A progressive flattening of the slope indicates the replenishment of the aquifer storage in the supposedly dry season (probably due to return flow of irrigation/urban effluent or seepage from reservoirs) and steepening of the slope indicates groundwater abstraction from the aquifer or reduction in recharge. Occurrence of earthquake can have effect on spring discharge and on the slope of the time discharge line considerably. The models suggested by Maillet (1905 vide Singh, 1989) Mero (1963), Bear (1979) and Mandel and Shiftan (1981) are based on the linearity assumptions mentioned

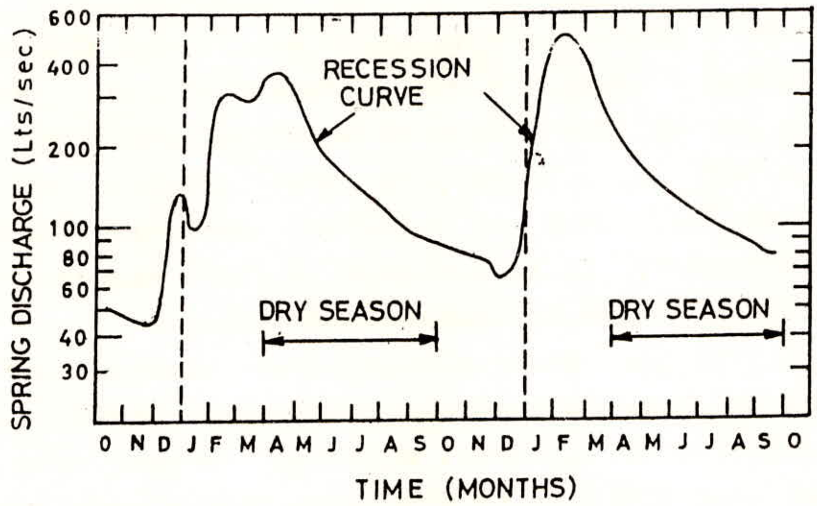

FIG. 1-A TYPICAL SPRING HYDROGRAM WITH SEASONAL FLUCTUATION (after Bear, 1979)

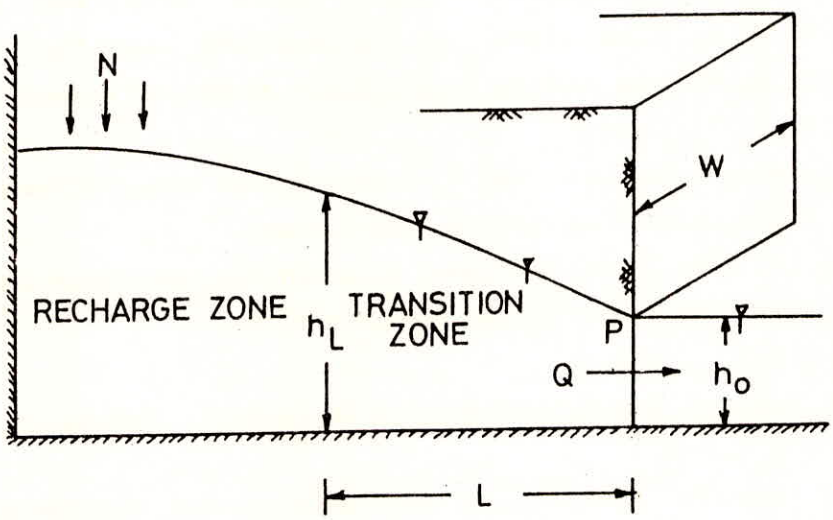

FIG. 2-MODEL OF A SPRING (after Bear, 1979)

above and provide equations similar to Eq.(1) for springflow for a lumped recharge. Bear (1979) suggested a hydrological decomposition of the flow domain of a spring into two zones, i) recharge zone and ii) transition zone (Fig.2). He expressed depletion time in terms of aquifer geometry and aquifer parameters. Therefore, it is possible to find aquifer properties from the analysis of the springflow hydrograph .

The Eq.(1) is the most widely used equation of baseflow recession and has been derived by Boussinesq (1877 vide Singh, 1989) as a solution to diffusion equation. Werner and Sundquist (1951) showed that diminishing discharge from a confined aquifer could be expressed by this equation. Equation (1) has been rewritten as:

$$Q\ (t) = Q_0 k^{t} \qquad \ldots(2)$$

where $k = \exp(-1/t_0)$. For observations of discharge made at regular interval equation(2) has been recasted as:

$$Q\ (t) = k\ Q_{(t-1)} \qquad \ldots(3)$$

A plot of Q_{t-1} vs. Q_t on a simple graph will result in a straight line passing through the origin and its slope will specify k.

ii)Model based on non-linear discharge-storage relationship

Boussinesq (1904 vide Singh, 1989) developed the following equation for flow from an unconfined aquifer to a fully penetrating stream having negligible depth of water in it (Fig.3).

$$Q(t) = Q_0/(1+ct)^2 \qquad \ldots(4)$$

where c is a constant. This equation has been extensively used in Europe for estimating spring discharge.

Eq.(4) has been derived by solving the equation ds/dt=-Q for a non-linear storage-discharge relation

$$Q = a\ s^{n}\ ;\ n = 1 \qquad \ldots(5)$$

with an initial condition $Q(o) = Q_0$. This yields

$$Q = Q_0 (1+ct)^{n(1-n)} \qquad \ldots(6)$$

where $c = (1-a)a^{1/n}/Q_0^{\ n(1-n)}$, a constant. n = 2 yields equation (4).

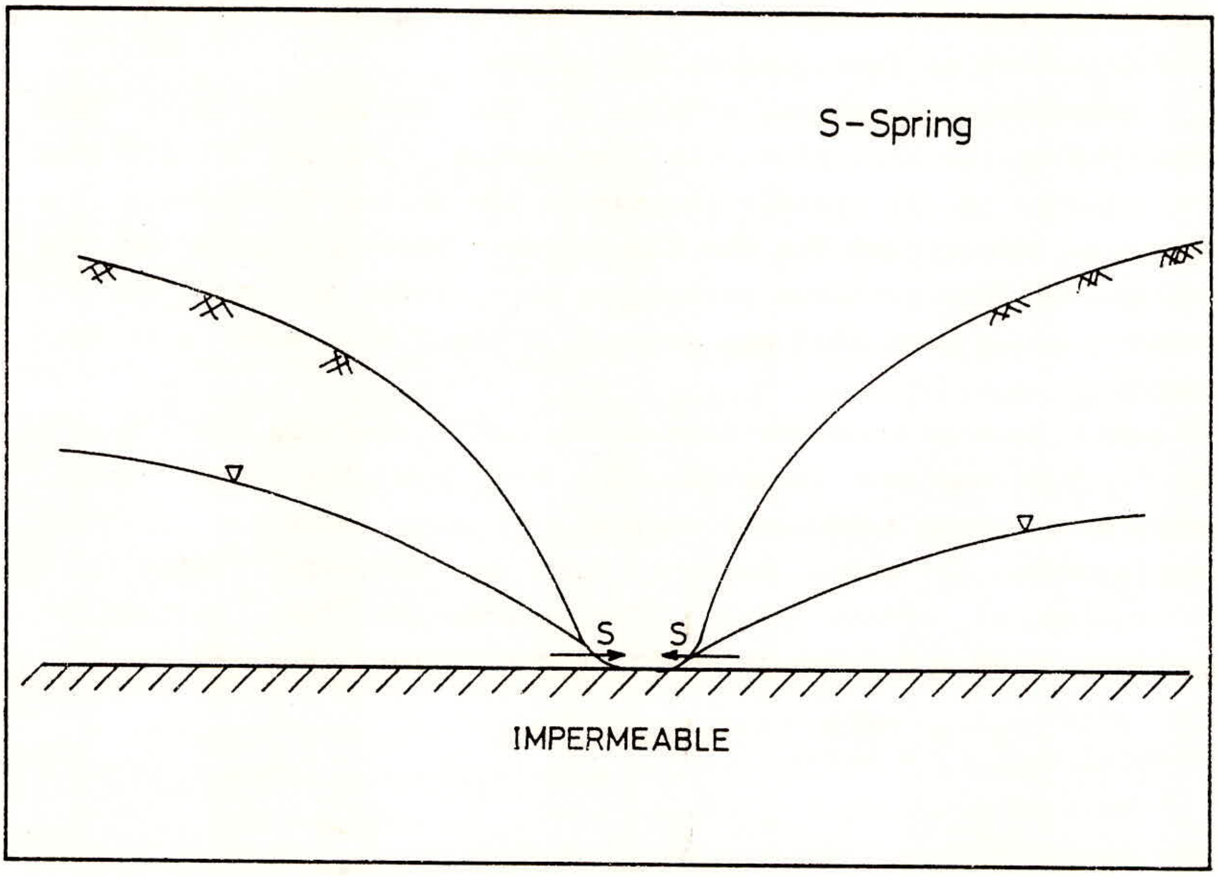

FIG. 3-MODEL CONFIGURATION FOR BOUSSINESQ EQUATION (1904)

It has been observed that hydrograph of a karstic spring exhibits non-linear behaviour. A hyperbola provided by equation with an exponent n gives a better fit of the springflow from karstic rock. The exponent n usually lies in the range of 0.5 to 2.

Recent Studies on Modelling of Springflow

Existing mathematical models of the springflow have been developed on the assumption that the spring outflow is linearly proportional to the dynamic storage in the spring flow domain. The models do not account for the time variant recharge. Moreover, the groundwater flow problems pertaining to a spring for the various boundary conditions that may prevail in the field have not been solved

At NIH; a groundwater flow model (1989) starting with Bear's model (1979) has been adapted (Fig.4) and the time variant recharge has been taken into account by using response function coefficients. The model assumes that an unsteady state is a succession of steady state. The expression for springflow, depletion time and response function coefficient obtained are:

$$\text{(i)}\ Q(t) = \frac{WT}{AL} [\frac{R}{4} e^{\frac{-WTt}{AL\phi}}] \qquad \ldots(7)$$

$$\text{(ii)}\ t_o = \frac{AL\phi}{T} \qquad \ldots(8)$$

$$\text{(iii)}\ \delta(n) = e^{-n/t_o}(e^{1/t_o} - 1) \qquad \ldots(9)$$

where

Q(t) = springflow

W = width of the spring's opening

A = Area of recharge zone

L = Length of the transition zone

R = total recharge over the recharge zone

T, ϕ = aquifer parameters

t = time since the instantaneous recharge commenced

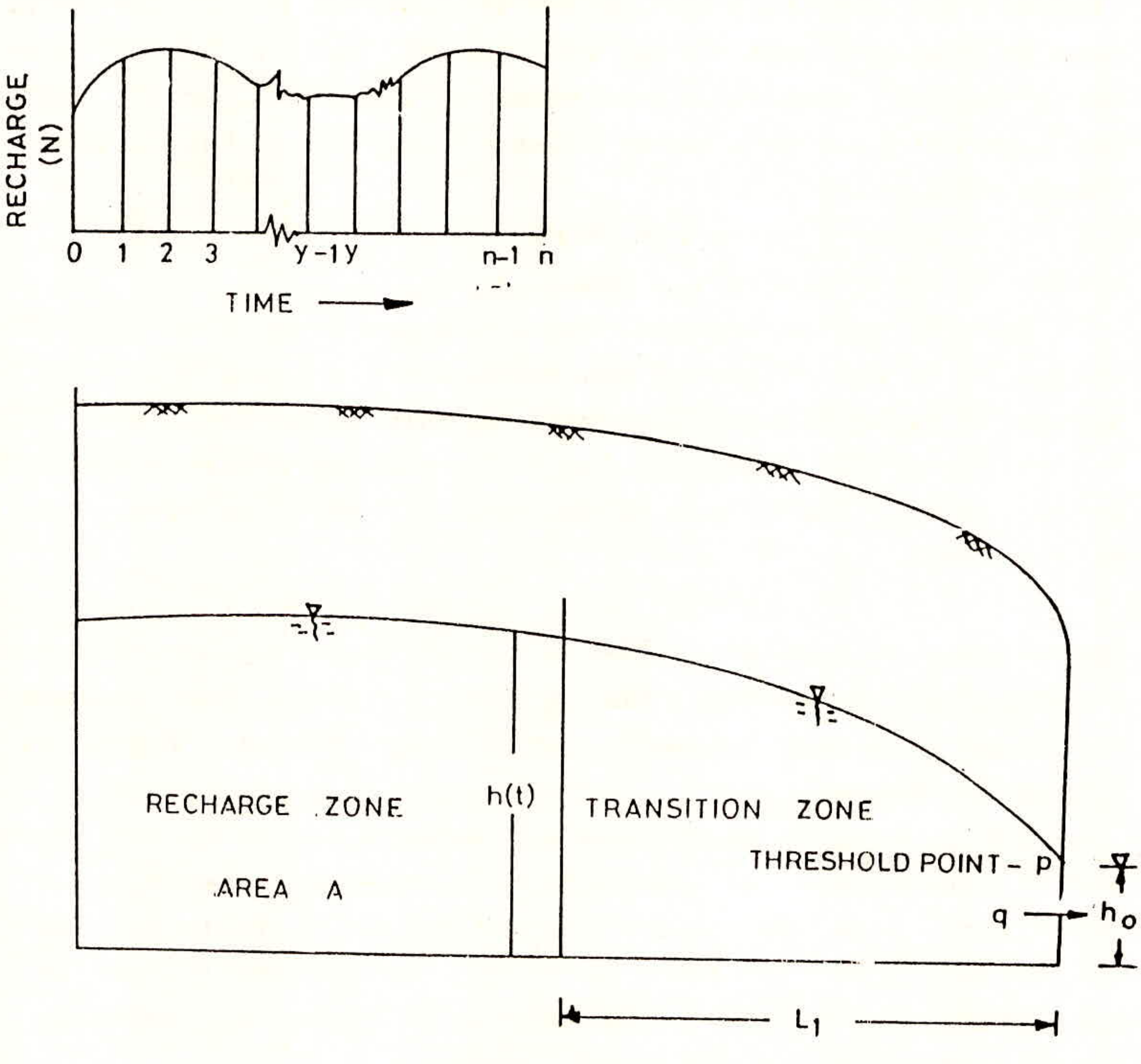

FIG. 4 - PROPOSED MODEL CONFIGURATION

t_o = depletion time

$\delta(.)$ = response function coefficient for a unit pulse excitation.

The model has been calibrated and verified to simulate the discharge from Parda spring, Nainital. The result showing the observed and simulated springflow is given in Fig.5. The model has been used to evaluate annual recharge as an inverse problem by using the following equations which tallied well with estimated recharge obtained from water balance approach of the spring's flow domain (Table-1).

$$q(n)=\sum_{\gamma=1}^{n} \delta\ (n-\gamma+1)\ N(\gamma) \qquad (10)$$

where $\delta(n)= \exp(-n/t_o)\ [\exp(1/t_o-1)]$

Eq.(10) can be written in a generalised form ,[A][B]=[C] where [A] is the matrix of nxn dimension involving $\delta(.)$,[B] is the matrix of nx1 dimension involving monthly recharges,R,and [C] is the matrix of nx1 dimension involving monthly observed springflows after removing the effect of recharge of previous year. Solving for recharge ,

$$[B]=[A^{-1}][C] \qquad (11)$$

The dynamic storage available in the spring after the cessation of rainfall was estimated. The dynamic storage can sustain the discharge of a deep tubewell continuously between beginning and end of dry season.

But in reality there is a gap between the onset of recharge and its appearance as springflow discharge especially if the transition zone is long and hydraulic diffusivity of the transition zone is low. As such, groundwater flow in the transition zone of the spring need to be taken as unsteady flow instead of succession of steady states

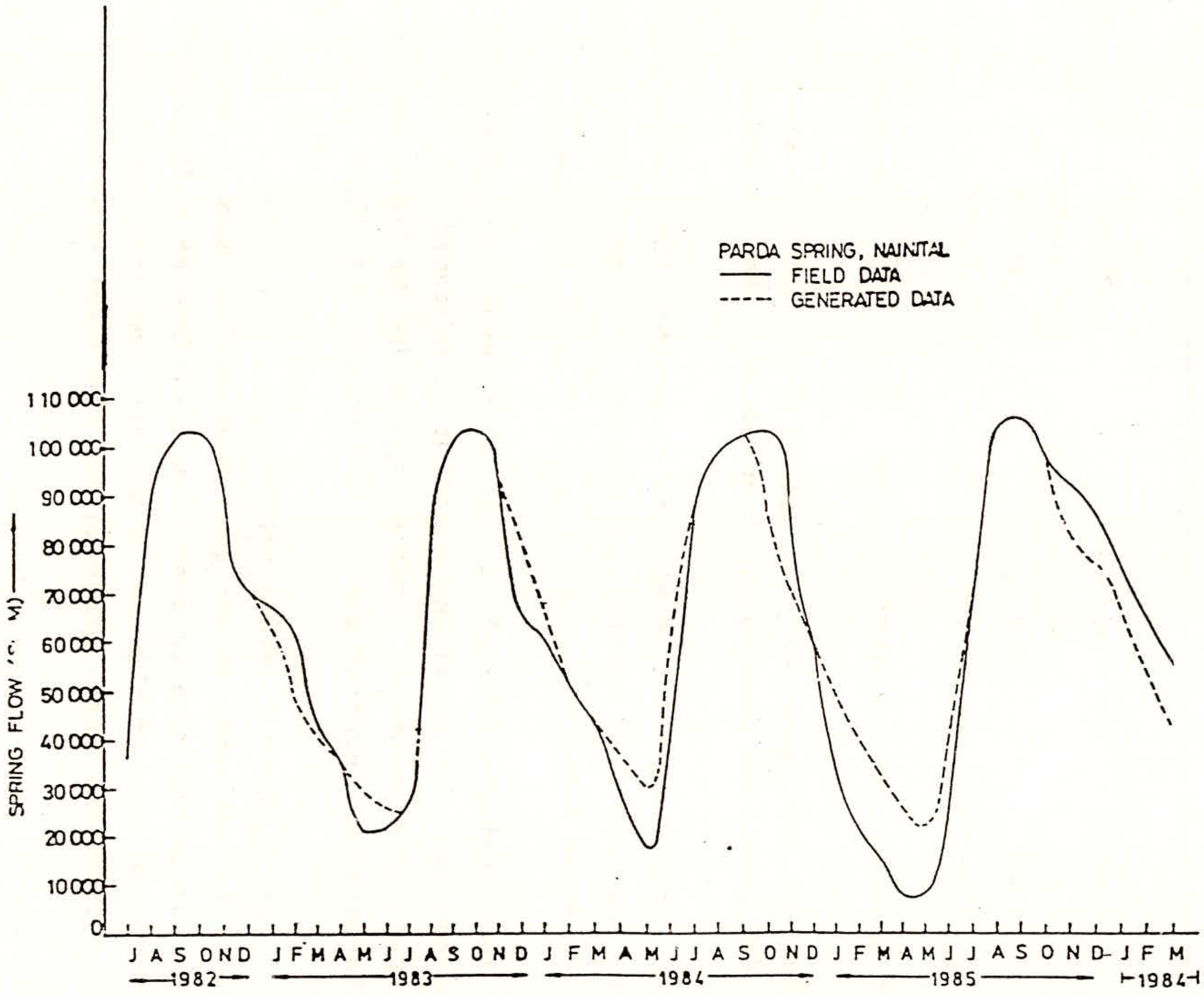

FIG. 5. PLOT OF GENERATED AND FIELD SPRINGFLOW DATA FOR PARDA SPRING

Table 1. Calculated Recharge to aquifer in Parda spring, Nainital

Year	Depletion time (month)	Estimated Recharge($x10^6$ cum) Water Balance Approach	Discrete kernel Approach
1982-83	5.6	0.924	0.960
1983-84	5.2	0.866	1.330
1984-85	3.9	0.826	0.880
1985-86	6.5	0.972	0.840

A mathematical model for springflow

Statement of the Problem

A schematic configuration of a spring flow domain is shown in Fig.6(a). The idealised flow domain adopted for the analysis is shown in Fig.6(b).

The method of image can be applied to convert the finite flow domain into infinite flow domain .The system of image and real spring is shown in Fig.7.

Analysis

Let the spring's threshold be considered as datum for measuring drawdown. The drawdown at spring's threshold is always zero. The rise in piezometric surface, $s_1(n)$, due to time variant recharge through the recharge zone in the equivalent flow domain is given by:

$$s_1(n)=\sum_{\gamma=1}^{n} R(\gamma)\ \delta(2W,L,l,n-\gamma+1) \qquad ...(12)$$

in which $R(\gamma)$ is the recharge per unit area during time step γ, and $\delta(2W,L,l,m)$ is the rise in piezometric surface at a distance l from the recharge zone at the end of the mth time step due to unit recharge per unit area taken place during the first time step.

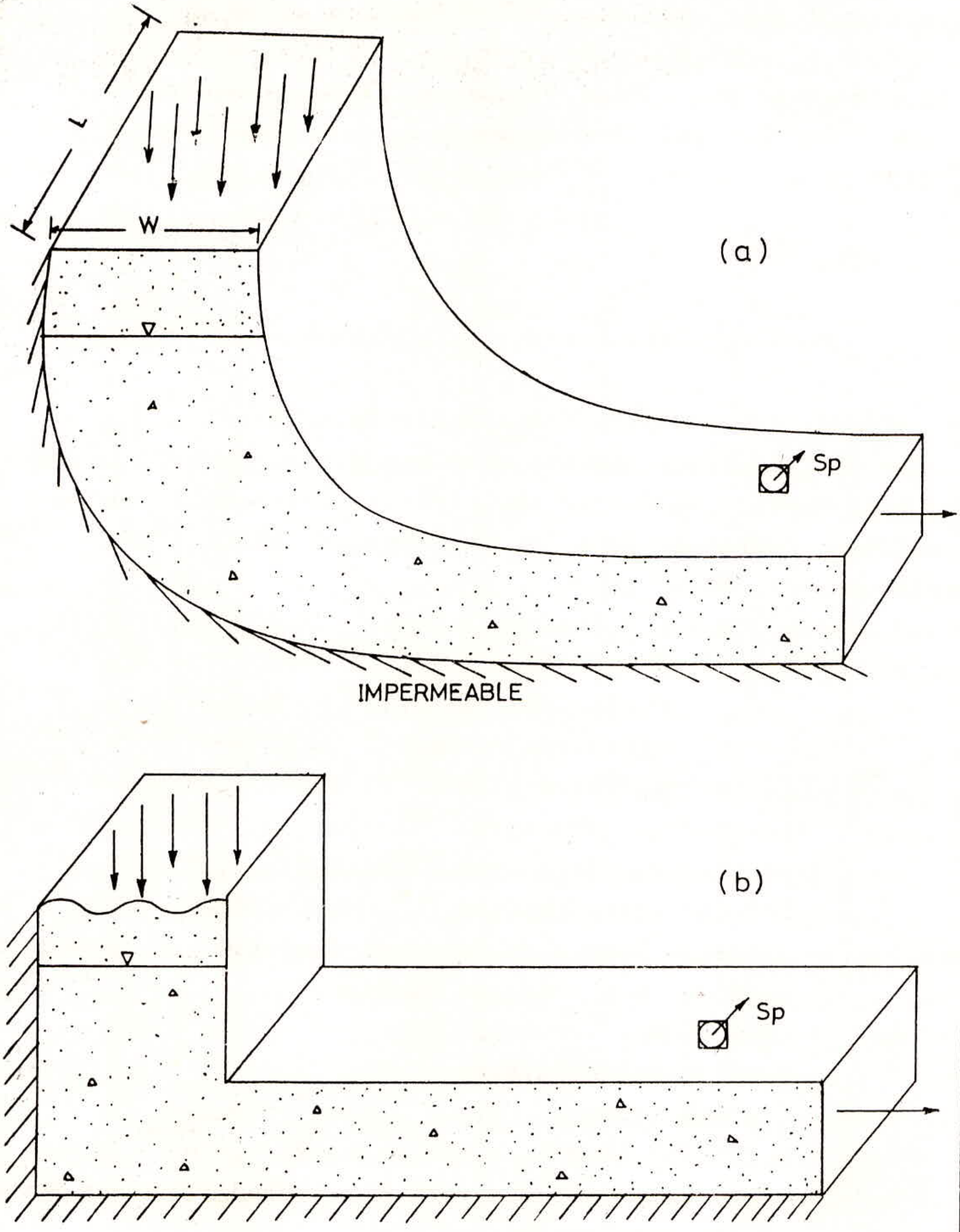

FIG. 6 a & b – SCHEMATIC AND IDEALISED FLOW DOMAIN OF A SPRING

The fall in piezometric surface, $s_2(n)$, due to springflow from the system of real and imaginary springs is given by:

$$s_2(n)=\sum_{\gamma=1}^{n} q(n)\{\delta(a,a,\emptyset,n-\gamma+1) + \delta(a,a,2l,n-\gamma+1)\} \quad ...(13)$$

in which $q(\gamma)$ is the spring discharge during time step γ.

Equating the resultant drawdown to zero, we have,

$$[\sum_{\gamma=1}^{n} R(\gamma)\delta(2W,L,l,n-\gamma+1) - \sum_{\gamma=1}^{n-1} q(\gamma)\{\delta(a,a,\emptyset,n-\gamma+1)+\delta(a,a,2l,n-\gamma+1)\}$$
$$- q(n)\{ \delta(a,a,\emptyset,1)+ \delta(a,a,2l,1)\}]=\emptyset \quad ...(14)$$

Solving,

$$q(n)= [\sum_{\gamma=1}^{n} R(\gamma)\delta(2W,L,l,n-\gamma+1)- \sum_{\gamma=1}^{n-1} q(\gamma)\{\delta(a,a,\emptyset,n-\gamma+1)$$

$$+\delta(a,a,2l,n-\gamma+1)\}]/ [\delta(a,a,\emptyset,1)+\delta(a,a,2l,1)] \quad (15)$$

The $\delta(.)$ coefficients are obtained from Hantush's solution (1967 vide Bouwer, 1978) for the rise of piezometric surface due to uniform recharge from a rectangular basin (fig.8). The expression for $\delta(.)$ is:

$$\delta(A,B,x,m)=\emptyset.25m/\phi\ [F\{((A/2+x)/(4Tm/\phi)^{0.5}),((B/2+y)/(4Tm/\phi)^{0.5})\}$$
$$+F\{((A/2+x)/(4Tm/\phi)^{0.5}),((B/2-y)/(4Tm/\phi)^{0.5})\}$$
$$+F\{((A/2-x)/(4Tm/\phi)^{0.5}),((B/2+y)/(4Tm/\phi)^{0.5})\}$$
$$+F\{((A/2-x)/(4Tm/\phi)^{0.5}),((B/2-y)/(4Tm/\phi)^{0.5})\}]$$
$$-\frac{(m-1)}{4\phi}[F\{((A/2+x)/(4T(m-1)/\phi)^{0.5}),((B/2+y)/(4T(m-1)/\phi)^{0.5})\}$$
$$+F\{((A/2+x)/(4T(m-1)/\phi)^{0.5}),((B/2-y)/(4T(m-1)/\phi)^{0.5})\}$$
$$+F\{((A/2-x)/(4T(m-1)/\phi)^{0.5}),((B/2+y)/(4T(m-1)/\phi)^{0.5})\}$$
$$+F\{((A/2-x)/(4T(m-1)/\phi)^{0.5}),((B/2-y)/(4T(m-1)/\phi)^{0.5})\}]$$

where δ (.)= discrete kernel coefficient for rise in piezometric surface in a confined aquifer,

m = time step ,

ϕ = coefficient of storage,

T = trasmissivity,

A,B = width and length of the recharge basin,

x,y = coordinates of observation measured from the centre of the recharge basin.

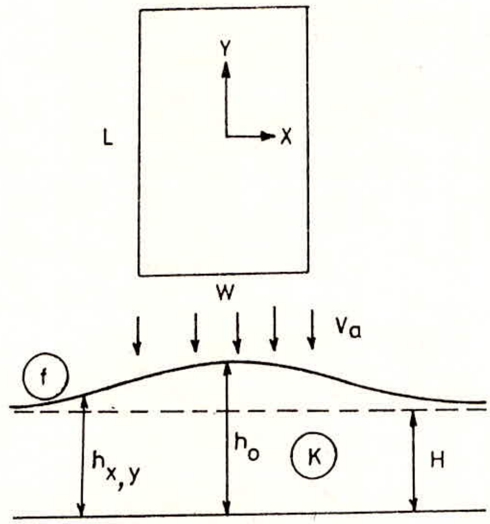

FIG.8 - INFILTRATION AREA AND GROUNDWATER MOUND

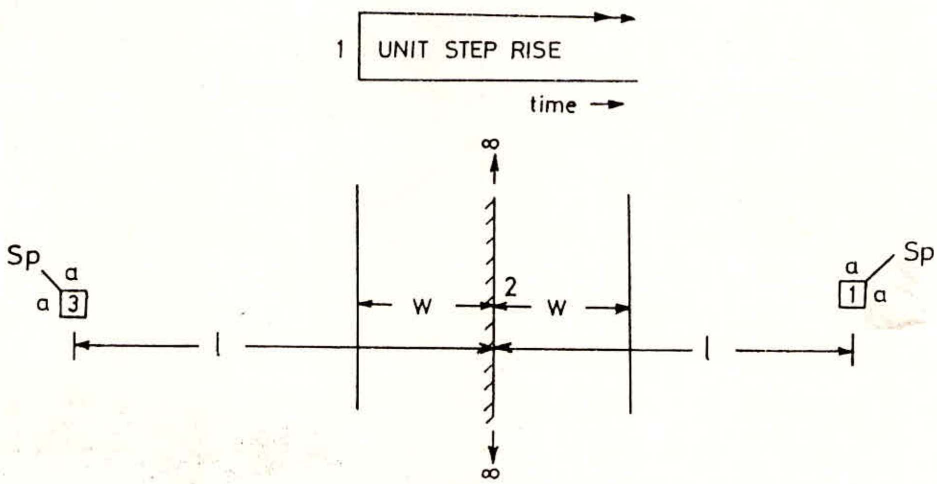

FIG. 7 - FLOW DOMAIN OF THE PROPOSED MODEL BASED ON THEORY OF IMAGE

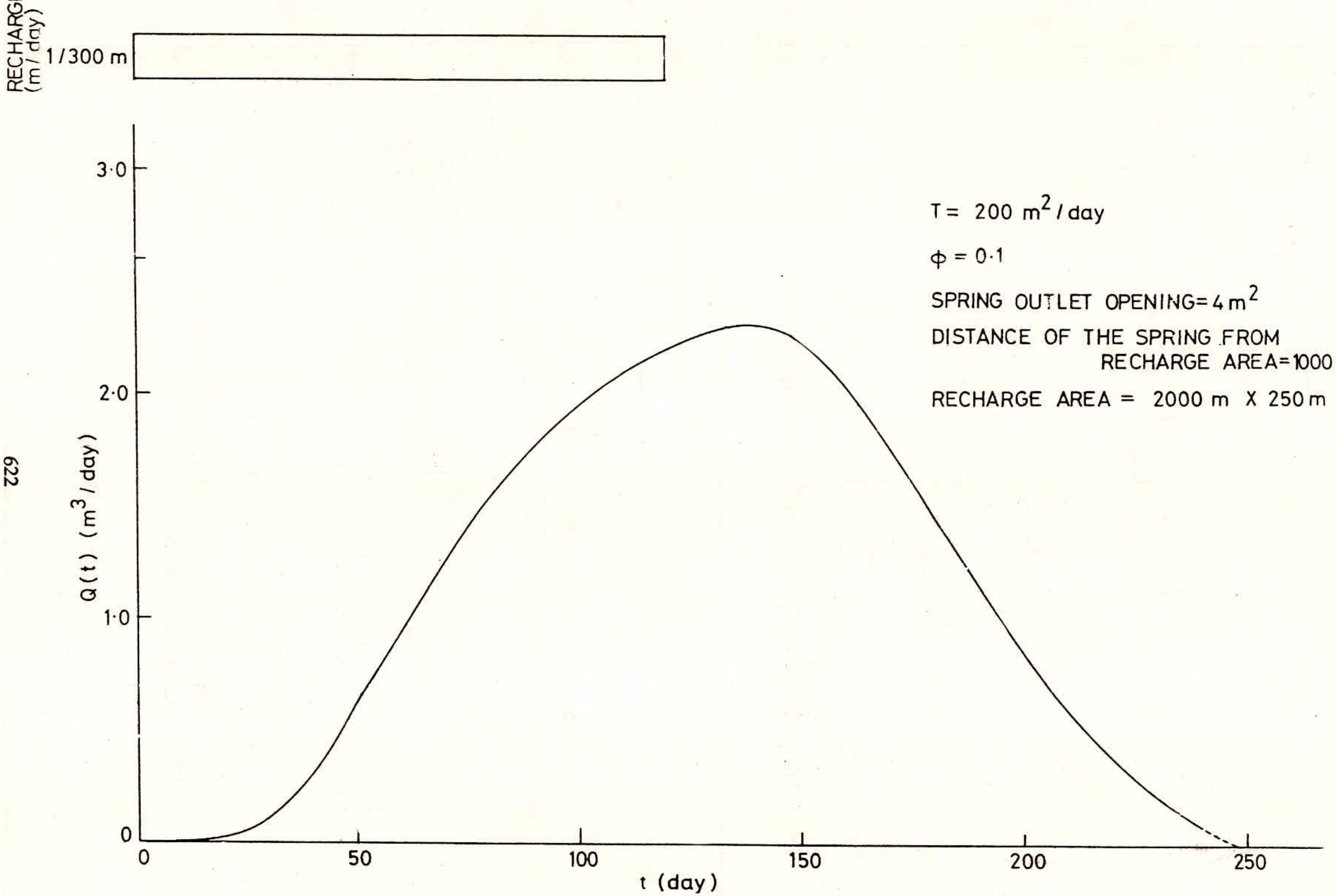

FIG. 9 – VARIATION OF SPRING FLOW WITH TIME

Discharge of a spring computed using the above model is presented in figure 9. A recharge of 40 cm is assumed to occur in a span of 120 days continuously at a uniform rate of 1/300 m per day. As seen from the figure the springflow during recession does not strictly follow an exponential decay curve. Out of 2×10^5 m^3 of the total recharge to the aquifer 283 m^3 of water appears as springflow.

Conclusion

A mathematical model for describing springflow has been developed. The recession of the springflow hydrograph does not follow an exponential decay curve. A very small quantity of the total recharge to an aquifer appears as springflow.

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DISCUSSION

A. R. BHANDARI : Natural springs/streams are either drying up or their discharge is fast decreasing in the entire sub-Himalayan region where a sizable population of the country lives and is dependent on these sources for their water needs. Not only this, but the contribution of these sources to river water is very important. Therefore, will it not be appropriate for this August House to take note of the situation and send a message to the appropriate quarters about the need to take up the work on devising the methods and techniques about recharging of these natural springs.

AUTHOR(S) : The authors agrees with the need which the honourable delegate rightly pointed out. Authors are using various fora of NIH and other agencies to stress upon the need. A paper emphasising the interlinking the development of land (in the recharge area) and water resources (springflow) without adversely affecting each other ,has been published by the first author in IAH Journal in January 91 issue.

It is requested that the chairman of the session may kindly recommend the point raised by Dr. Bhandari for inclusion in the recommendation of the symposium.

H. P. SHARMA : In hilly areas the secondary porosity control the movement of ground water and this movement is along undefined conduits at depth. The relation of storage and spring discharge in such hydrological conditions need elaborated.

AUTHOR(S) : The kastic terrain often possesses both primary and secondary porosity. The flow in the pores and small fissure follows Darcy's law and the flow in the channels is turbulent. The underground flow can be described as flow in a surface stream with a roof.