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Modelling the Seepage from A Lake Using 3D Groundwater Flow Model

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Abstract

The analysis of unsteady seepage from a lake in a homogeneous and anisotropic aquifer has been done using a three dimensional ground water flow model. Hypothetical setting of boundaries have been assumed, i.e., rivers on first two parallel sides of the lake and no flow boundaries on the other two parallel sides. A family of type curves have been developed for different sizes of the lake using which the transient recharge from the lake can be determined knowing the head changes in an observation well situated within the influence area of water body. Ideal location for such an observation well has also been suggested.

Introduction

Large water bodies are very common features on land surface and are important in respect of various water uses. In spite of early recognition of their water management studies, very few attempts were made to understand the interaction of large water bodies with aquifer. The assessment of recharge from lake has drawn the attention of many investigator during recent past because often the recharge from a lake forms a significant component of ground water balance. Among the early investigators who stressed upon the groundwater flow pattern around a lake, are Mayboon(1967) and Janquest(1976). Bergstrom and Hansen(1962), Skinner and Borman(1973) and Cartwrite et al.(1979) have computed the ground water flow from the lake Michigan. Bergstrom and Hansen(1962) and Skinner and Borman(1973) estimated the groundwater component as the residual of their water balance equation but cartwrite et al.(1979) made direct measurement of hydraulic gradients . The wide variations between their estimates in seepage rates indicate the need of refined methods for determining the groundwater flux from a large lake.

Some insight into the groundwater regime of discharge estimates was provided by Winter(1976, 1978), who used two and three dimensional steady state models applied to hypothetical groundwater lake systems and has listed the hydrological factors that control the interaction of lakes and groundwater along the entire lake bottom for a wide variety of hypothetical settings. He showed that the movement of groundwater to and from a lake depends on the continuity of the boundary separating the local groundwater flow system associated with lake, from intermediate and regional flow systems passing at depth beneath the lake. Based on his simulation study, he suggested the field methods and new approaches to the study of the interaction of lakes and groundwater along with critique of commonly used approaches. Studies in which one or many wells are placed near a lake to determine the interaction of lakes and groundwater, must be scrutinized carefully, because placement and construction wells are critical to a proper understanding of the interrelationship of lakes and groundwater.

McBride and Pfannkuch(1975) used a numerical model to evaluate the vertical component of groundwater flow into one side of a lake for a number of hypothetical settings. Munter and Anderson(1981) showed that two and three dimensional groundwater flow models provide flexible and effective means of calculating flow rates in well defined but complex natural flow systems around lakes. Bhar and Mishra(1988) developed a mathematical model using discrete-kernel generator and method of superposition to study the interaction between depression storage and a shallow water table aquifer, when the depressions are filled up and is allowed to deplete while recharging to groundwater.

From the critical review of the past studies it is observed that in most of the water balance studies, the quantification of seepage from large water bodies have been done as a residual of water balance equation, thus leading to erroneous values because the uncertainties in the estimation of other components get incorporated into seepage component. In most of the model studies the interference of the various variables involved, on the recharge rate from water bodies have not been clearly analyzed and thus, no guidelines or methods have been evolved for the assessment of recharge from a lake. recently Singh and Seethapathi(1988) have analyzed the problem of lake aquifer river interaction for a hypothetical lake of square cross-section in a homogeneous and isotropic aquifer. They have presented a type curve which enables the assessment of transient recharge from the lake. The purpose of the present study is to quantify the recharge from a lake in a homogeneous and anisotropic aquifer.

Statement of the Problem

The schematic plan view and cross-sectional view of the lake and the fully penetrating river are shown in fig. 1 and 2 respectively. The lake was assumed at the centre of the aquifer in plan and the aquifer extends to a distance of 3 km from the centre of the water body in both positive and negative x and y directions. The boundaries of the aquifer parallel to one side of lake are fully penetrating constant head boundaries(rivers) and boundaries of the aquifer parallel to the other side of the lake are taken as no flow boundaries. the lake was assumed to be of square cross-section. two different sizes (viz., 270mx270m and 340mx340m) of the lake was taken. The uniform depth of the lake was taken as 9m. The total depth of aquifer for simulation up to the impermeable boundary was taken as 50m. The aquifer was assumed to be homogeneous and anisotropic with different value of K_h/K_v

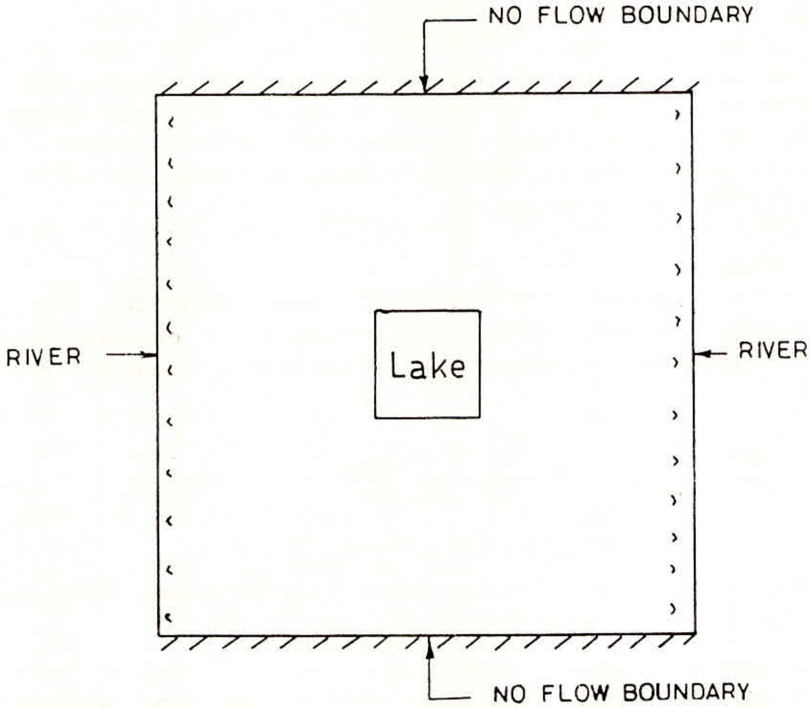

Fig.1-PLAN VIEW OF THE PROBLEM

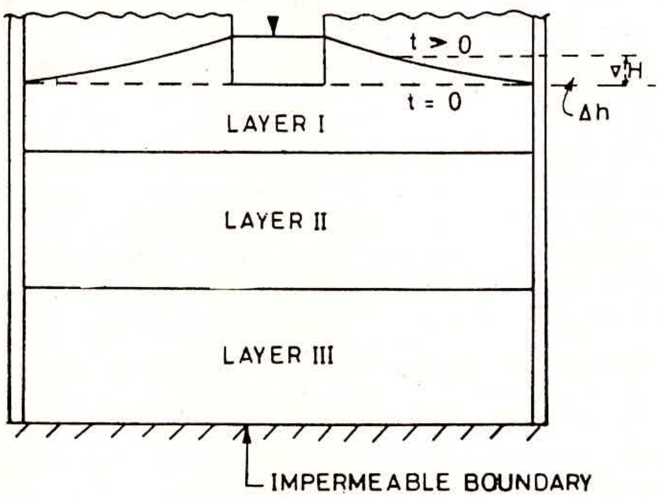

Fig.2-CROSS-SECTIONAL VIEW OF THE PROBLEM

,i.e., 1, 10, 100, 250, 500 and 750. The initial head in the aquifer was taken to be equal to the water level in the rivers. The lake water level above the water level in the rivers(ΔH) was varied from 3m to 9m (i.e., different values of ΔH were taken as 3m, 5m, 7m, 9m). A three dimensional ground water flow model(Macdonald and Harbaugh,1984) was used for the simulation of the lake-aquifer-river interaction.

Mathematical Model

The governing partial differential flow equation for the three dimensional unsteady (transient) movement of incompressible groundwater through heterogeneous and anisotropic medium may be described as

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right) - w = S_s\frac{\partial h}{\partial t} \quad \ldots(1)$$

where,

- x,y,z are the cartesian coordinates aligned along the major axes of conductivity K_{xx}, K_{yy}, K_{zz};
- h is the piezometric head (L);
- w is the volumetric flux per unit volume and represents source and/or sinks (t^{-1});
- S_s is the specific storage of the porous material of the aquifer (L^{-1}); and
- t is time (t)

This equation when combined with boundary conditions (flow and/or head conditions at the boundaries of the aquifer system) and initial conditions (in case of transient flow, specification of head conditions at t=0), constitute the a mathematical model of transient groundwater flow.

The aquifer system has been discretized into a mesh of points termed nodes, forming 32 rows 32 columns and 3 layers. Finer grids were taken close to the lake and coarser grids away from the lake. The spatial discretization for the quarter portion of the aquifer and the lake is shown in fig.3. The width of cells along rows is designated as Δr_j for the j^{th} column; the width of cells along columns are designated as Δc_i for i^{th} row; and the thickness of layers in vertical are designated as Δv_k for the k^{th} layer. Block-centered formulation was adopted, with the nodes are at the centre of the cells. The flow into the cell i,j,k in row direction from cell i,j-1,k is given by

$$q_{i,j-1/2,k} = CR_{i,j-1/2,k}(h_{i,j-1,k} - h_{i,j,k}) \quad \ldots(2)$$

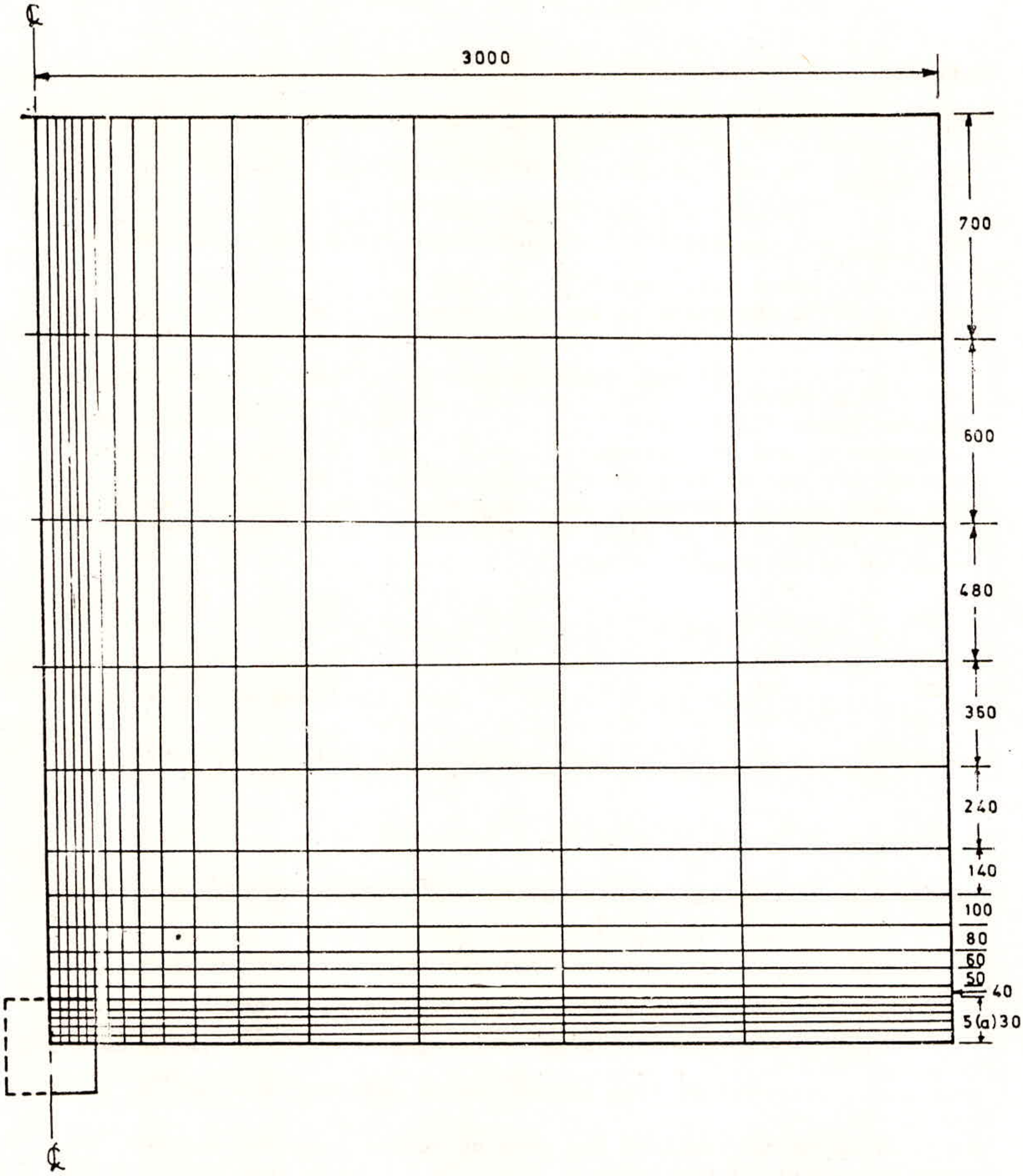

Fig.3-SPATIAL DISCRETIZATION FOR A QUARTER PORTION OF THE LAKE AND THE AQUIFER

where,

$CR_{i,j-1/2,k} = KR_{i,j-1/2,k} \Delta C_i \Delta V_k / \Delta r_{j-1/2}$

$q_{i,j-1/2,k}$ is the volumetric flow discharge through the face between the cells i,j,k and i,j-1,k (L^3t^{-1});

$KR_{i,j-1/2,k}$ is the hydraulic conductivity along the row between nodes i,j,k and i,j-1,k; and

$\Delta r_{j-1/2}$ is the distance between nodes i,j,k and i,j-1,k (L)

$CR_{i,j-1/2,k}$ is the conductance in i^{th} row and k^{th} layer between nodes i,j-1,k and i,j,k $[L^2t^{-1}]$. Here, C represents the conductance and R represents for row direction. Similar expressions can be written approximating the flows into or out of the cell i,j,k through the remaining five faces. Applying continuity equation of flow to the cell i,j,k ;the following equation is obtained.

$$CR_{i,j-1/2,k}(h_{i,j-1,k} - h_{i,j,k}) + CR_{i,j+1/2,k}(h_{i,j+1,k} - h_{i,j,k}) +$$

$$CC_{i-1/2,j,k}(h_{i-1,j,k} - h_{i,j,k}) + CC_{i+1/2,j,k}(h_{i+1,j,k} - h_{i,j,k}) +$$

$$CV_{i,j,k-1/2}(h_{i,j,k-1} - h_{i,j,k}) + CV_{i,j,k+1/2}(h_{i,j,k+1} - h_{i,j,k}) \pm$$

$$QS_{i,j,k} = SS_{i,j,k}(\Delta r_j \Delta C_i \Delta V_k).(\Delta h_{i,j,k}/\Delta t) \qquad ...(3)$$

where, $\Delta h_{i,j,k}/\Delta t$ is a finite difference approximation for head change with respect to time $[LT^{-1}]$

$SS_{i,j,k}$ is the specific storage of cell i,j,k $[L^{-1}]$

$\Delta r_j \Delta C_i \Delta V_k$ is the volume of cell i,j,k $[L^3]$; and

$QS_{i,j,k}$ is the recharge from the $(i,j,k)^{th}$ rectangular cell of lake bottom (as an external source$[L^3T^{-1}]$

In order to simulate the interaction of lake and aquifer, the term representing the leakage ,i.e., $QS_{i,j,k}$ has been added to the equation 3. due to the discretization , the bed of the lake consists of a number of rectangular cells. The recharge from one of such rectangular cell has been expressed as

$$QS_{i,j,k} = CL_{i,j,k}(H_{i,j,k} - h_{i,j,k}) \qquad ...(4)$$

where,

K,L,and W are the hydraulic conductivity, length, and width of the bed material of lake contained in the cell i,j,k ;

D is the thickness of the bed material of the lake contained in the cell i,j,k ;

$H_{i,j,k}$ is the water level in the lake in the cell i,j,k.

$CL_{i,j,k} = \frac{K.L.W}{D}$ = conductance of the lake bed contained in the cell i,j,k

After substituting equation 4 in equation 3 ,the equation 3 is written in backward difference form by specifying flow term at t_m, the end of the time interval, and approximating the time derivative of head over the interval t_{m-1} to t_m ; for each of the 'n' cells in the system; and, since there is only one unknown head for each cell, we are left with a system 'n' equations and 'n' unknowns. For the solution of these equations strongly implicit procedure has been adopted.

Result and Discussion

The head distribution in the aquifer system at discrete nodes at each time step over a period of time was obtained. The simulation was carried out for different value of ΔH keeping the head in the rivers constant. In each simulation the difference between the head at discrete points (nodes) and the river water level were computed for each time-step and was designated as Δh. $(\Delta h)_s$ is the value of Δh when steady state has reached.

Variation of $((\Delta h)_s - (\Delta h))/\Delta H$ with X/L and t was observed for all values of t for each simulation (ΔH = 3.0m,5.0m,7.0m and 9.0m; Here, X is the perpendicular distance from the lake to the observation well and L is the perpendicular distance between centre of the water body to the constant head boundary). The variation of $((\Delta h)_s - (\Delta h))/\Delta H$ with X/L and t was found to be similar for different value of ΔH and for different sizes of lake. Analysis of the above plots show that with increasing time, the maxima of the curve shift in a positive X-direction and shift of the maxima for all value of t is ranged between X/L = 0.15 to 0.25 (this range of X/L was found to be the same for different values of ΔH and for different sizes of lake). Fig. 4 shows such a plot for K_h/K_v = 100 (lake size=270mx270m). Same range of X/L, i.e., 0.15 to 0.25 has also been reported by Singh and Seethapathi(1988). Therefore, if an observation well is located within the above range, it will observe a comparatively rapid change of head and thus it will be more sensitive and the observation thus obtained will be less liable to errors.

The further analysis of results showed that the parameters $X^2S/T.t$ and $T.\Delta h/Q_R$ are uniquely related for a fixed value of K_h/K_v and the relation was found to be the same for different

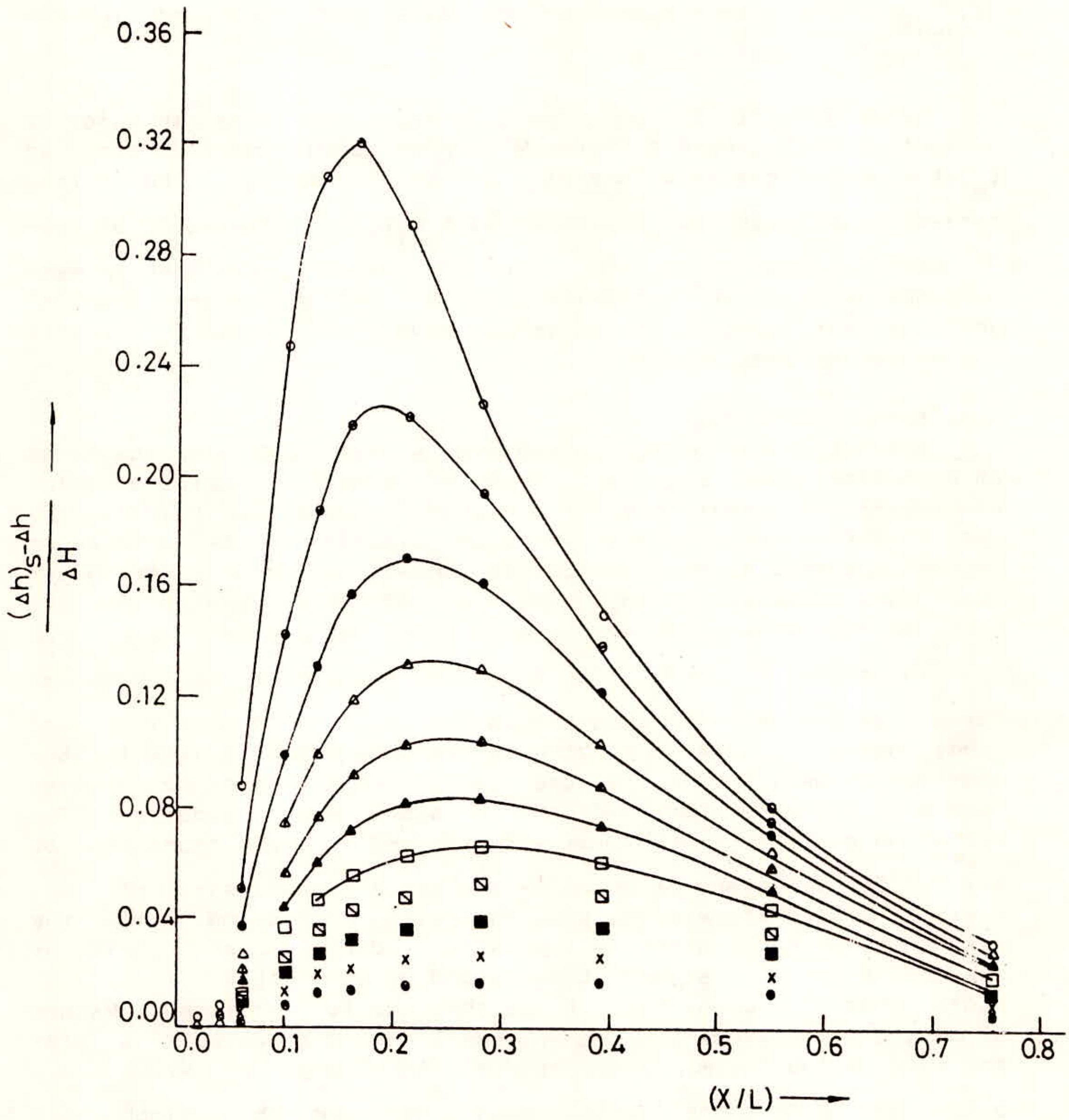

Fig.4-VARIATION OF $((\Delta h)_s-\Delta h)/\Delta H$ WITH X/L FOR $K_h/K_v=100$.

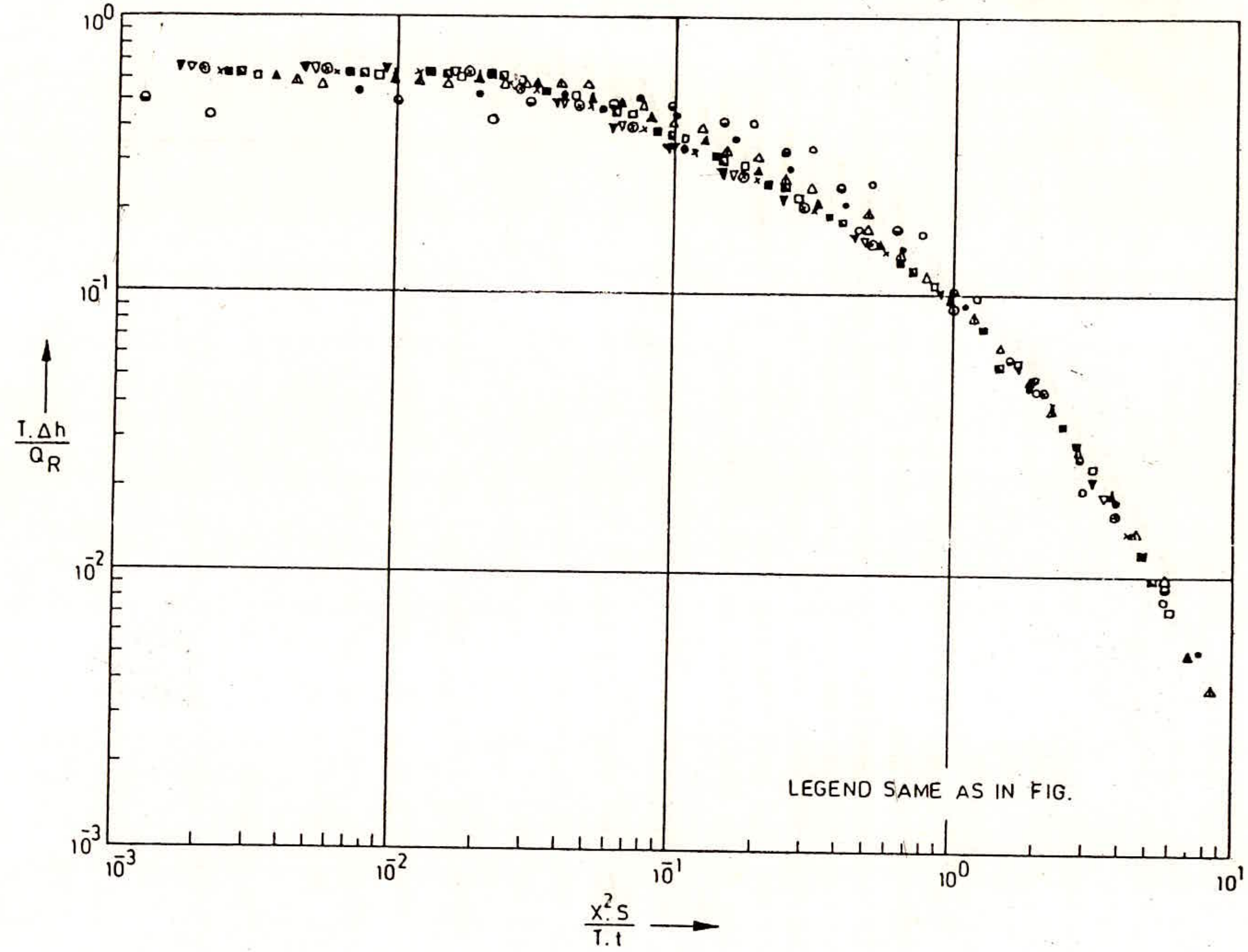

Fig.5-VARIATION OF $T.\Delta h/Q_R$ WITH $X^2S/T.t$ for K_h/K_v=100

values of ΔH (similar observation were reported by the author,1988), but the relation is different for different sizes of lake (Here, T is transmissivity of the aquifer, Q_R is the rate of seepage from the water body and t is the time since the start of simulation). Fig. 5 and 6 show such relationship for K_h/K_v = 100 for lake size=270mx270m and 340mx340m respectively. These figures show that the type curves are the same for the two lake sizes considered.If the head changes in an observation well(situated within the influence area of the lake)Δh, and the aquifer parameters are known, the recharge from the lake can be determined using fig 5 or fig. 6 within the range of parameters considered.

Conclusions

A three dimensional model study of the seepage from lake of square cross sections and uniform depths in a homogeneous and anisotropic aquifer with constant head boundaries on one sides and no flow boundaries on the other sides of the water body each at a distance of 3km. from the centre of the water body, has been carried out and the results have been analyzed. The conclusions drawn from the study are as given below.

1. The parameters ($X^2S/T.t$) and $T.\Delta h/Q_R$ are found to be uniquely related for every value of K_h/K_v irrespective of the value of ΔH and lake size. This relation has been expressed in the form of type curve.

2. With the help of the type curves developed , the recharge from a lake, i.e., Q_R can be determined provided the aquifer parameters and the water level fluctuations in the observation well is known.

3. The proper location of an observation well to record the rapid change of head has also been suggested, i.e.,X/L=0.15 to 0.25.

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