LECTURE 7

CONCEPTUAL MODELLING

OBJECTIVES

The main objective of this lecture is to provide necessary background to the participants about the conceptual modelling of hydrological process.

7.1 INTRODUCTION

A MODEL IS AN ABSTRACTION FROM REALITY

The model is just a simplification of the real world and the basis of formulation of model is the real world situation. In case of catchment Models, the formulation of model is based on process of real natural catchment. The development of a working mathematical model requires the consideration of the following steps—

Step (1) :

As a first step, a conceptual model is formulated consisting of various components which represents to some degree the various elements and the system of the real catchment. In formulation of model components use is made of known information and various theories concerning different elements of the system and their interrelationships. However, the conceptualization and various theories, and assumptions regarding real catchment processes are not general, since these have to consider the available data in a particular case. The use is made of most pertinent and accurate data available in creating the conceptual model. As additional data or information becomes available the conceptual model is improved by incorporating necessary modifications, so as to more closely approximate real catchment within the constraint of data availability.

Step (2) :

The second step involves description of conceptual model in terms of mathematical formulations which can be programmed on a computer. This will involve further modifications in the conceptual model to make it working mathematical computer model. Various processes and relationships of natural catchment as identified by the conceptual model are thus further simplified. The resulting working model will thus be a somewhat gross representation of real catchment.

The loss of information, first between the real catchment and the conceptual model, and second between the conceptual model and computer model is comparable to filtering processes. The real catchment is viewed through various kinds of data which are gathered about the system. More the information available about the real catchment, in terms of data, better will be conceptualization of various processes and hence conceptual model will be improved. The improvement in conceptual model will provide a basis for improvement in the working computer model.

The output from computer model are compared with the corresponding output functions from the real catchment. The extent of discripencies between these two outputs will indicate need for modifications in both the conceptual models and the working computer models. The models are tested by and are dependent upon data. In the absence of adequate data understanding is not necessarily gained by the manipulation of a complex model. The various steps in the development and application of a model are briefly discussed.

7.2 SYSTEM IDENTIFICATION AND MODEL FORMULATION

The basis of identification of real catchment system is the conceptual model of the real catchment developed through various kinds of data which are gathered about various processes. The points at which the real system is monitored may be regarded as 'windows'. Through these windows, the dynamic processes of real catchment are observed in space and time. The spacing of these observations in time and space will largely determine the refinement of the conceptual model in terms of the processes of the real catchment.

7.2.1 Evaluation and Analysis of Available Data

The data available in a particular situation is crucial in model evaluation. The data provides an understanding of the processes in real catchment and it also forms a basis for evaluating the model performance. The accuracy and reliability of the predictions from a particular model are governed to a large extent by the quality and adequacy of information on which the model is based and also the accuracy of the data which are input to the model to provide the predicted output functions.

7.2.2 Model Formulation

Due to practical data limitations and problem constraints, increments of time and space have to be considered in model design. The complexity of a model designed to represent a particular hydrologic system depends to a great extent upon the magnitudes of time and space increments used in its design. For large increments of time and space, the effect of phenomena which change over relatively small increments of space and time is considered negligible, e.g. in a model using monthly time increments, interception rates may be neglected. If annual time increments are considered, the storage changes within a hydrologic system may be assumed insignificant; which is however not true for monthly time increments.

As time and space increments decrease, the hydrologic process has to be defined more elaborately. No longer short term transient effects or appreciable variations in spatial properties could be more and more complex with corresponding increase in data requirements and computer facilities required.

7.3 REQUIREMENT IN A MODEL

Nash and Suchliffe (1970) have discussed this aspect of the model formulation. If a model were required solely to forecast the flow from a particular basin, it would be adequate to specify the model structure and values of parameters so that computed flows give sufficiently close reproduction of the observed flows. But if the model is to be used for basins without any records by establishing relations between the model parameters and catchment characteristics; then it is necessary to study the relative significance of the model parts and accuracy of the parameter values evaluated. Although many simplifications of real catchment are assumed in the model particularly with reference to spatial variability, it is desirable that the model reflects the physical reality as closely as possible. The more the departure of model operation from physical reality, the more complex will be the relationship between model parameters and physical characteristics. The model becomes more and more complex in order to be nearer to physical reality and it becomes difficult to evaluate model parameters using any optimization procedure. The problem is more severe if the model parameters are interdependent, which is particularly the case with hydrologic models.

Nash et al (1970) point out that there should not be unneccessary proliferation of parameters to be optimised and model parts with similar effects should not be combined. The requirement of versatality should be added to those of simplicity and lack of duplication. Each additional part of a model must substantially extend the range of application of the whole model.

7.4 MODEL CALIBRATION AND TESTING

A general hydrologic model is applied to a particular catchment through a verification procedure using the available data; and the values of model parameters are established for the particular prototype system as represented by data. Verification of a simulation model is done in two steps (1) Calibration or parameter estimation (2) Testing the model. For both these steps data from the real catchment is required.

1 (a) For model calibration : The parameters of the model have to be adjusted until a close agreement is achieved between observed and computed output functions. It is obvious that the accuracy of the model cannot exceed that provided by the historical data from the real catchment system. Optimization procedure is generally adopted for estimating the model parameters. **Optimisation** is the collective process of finding the set of conditions required to achieve the best result from a given situation. When some restrictions or constraints are present on the parameter values, the optimum solution is obtained relative to them.

In case of catchment models the input-output data generally consists of

INPUT :—Values of rainfall 'R', and potential evaporation 'E' recorded over a particular length of time.

OUTPUT :—Values of runoff 'Q' recorded over a particular length of time.

—This data may consist of hourly, three hourly or daily values. Using rainfall 'R' and potential evaporation E for a particular time interval as input data, a particular catchment model with its parameters X_1 , X_2 , X_3 , ... , X_n (where N is the total no. of parameters) is used to obtain the output values for the calculated runoff Q_c for that time interval. The objective criterion for optimisation generally adopted is the minimisation of sum of squares of differences between observed runoff Q and calculated runoff Q_c . The objective in such a case is to reduce the objective function F^2 to zero.

$$F^2 = \sum_{I=1}^{M} [\, Q\,(I) - Q_c\,(I)]^2 \qquad (7.1)$$

where,

M = No. of data points, and

F^2 = is analogous to residual variance of a regression analysis.

The problem thus becomes a search of a set of values of model parameters $X_1, X_3, \ldots X_n$, for a particular model structure which will minimise and reduce the function F^2 to zero, for given data points. The optimization techniques for multivariable search can be of analytical or numerical type. For catchment models, due to their particular structures, numerical search techniques have been used. Rosenbrock (1960) technique is one such technique which has been extensively used for catchment models.

(b) For judging model efficiency, Nash et al (1970) proposed following expression, for efficiency,

$$R^2 = (F_g^2 - F^2) / F_o^2 \qquad (7.2)$$

$$F_o^2 = \sum_{I=1}^{M} [Q(I) - \overline{Q}]^2 \qquad (7.3)$$

Where F_o^2 is initial variance which may be defined as no model value of F^2, and $\overline{Q}$ is the mean of observed Q values. The efficiency of a separable model part may be judged by the change in R^2 which follows insertion of the part or by the proportion of the residual variance accounted for by its insertion.

$$r^2 = (F_1^2 - F_2^2) / F_1^2 = (R_2^2 - R_1^2) / (1 - R_1^2) \qquad (7,4)$$

where suffixes (1) and (2) denote before and after insertion of the model part under consideration. If F^2 contained only one minimum in parameter space, steepest descent methods could find it quickly. In case of multiple optima, the whole space must be searched and the advantages of such methods are reduced. The shape of the F^2 surface in the vicinity of optimum point may be used as an indication stability (the inverse of sampling variance) of the optimum value of parameters. Greater the radius of curvature at x (i.e, $\partial (F^2)/\partial x$), the less well is x defined. The higher the angle of intersection ($\partial^2 (F^2)/\partial x^2$) at Xopt, the better is the definition of Xopt.

7.5 SEARCH STRATEGY FOR MULTIVARIABLE NUMERICAL OPTIMISATION

The general strategy of the search for an extreme value (maximum or minimum) can be summarised as a sequential search involving the successive calculation of the objective function, and the comparison of its value with the best value which has been so far obtained for the particular criterion (maximization or minimisation). The general sequence of optimization procedure for a particular problem may be as follows :

(i) Obtain a set of valid starting parameters values (i.e. satisfying the constraints) and find the value of the objective function at the starting point.

(ii) Try a set of exploratory steps in the vicinity of this starting or base point for information about the direction of movement.

(iii) The next feature is distance of movement in the selected direction. It can be a single 'step' or a series of steps in which the objective function improves (i.e. goes on reducing for minimisation). Different optimisation methods employ different techniques for accelerating the search by increasing the step size for each success. This is repeated for each search direction, and the final location reached is made new base point. Total number of search directions is equal to total number of parameters to be optimised.

(iv) Based upon progress made in the previous cycle of search the value of the objective function improves or at least remains constant, and eventually no further improvement has been obtained. That means convergence has occured and the corresponding parameter values are called the 'optimised parameter values'.

Basically, any optimisation technique is developed with some particular assumptions regarding the response surface. When it is applied to a conceptual catchment model, it is presented with a response surface, which depends on model structure, its validity, accuracy and adequacy of data, objective criteria etc. There is need for development of optimisation techniques to suit typical problems of conceptual catchment models.

The optimal fitting procedure is based on the supposition that the model is contained in the data. They are thus designed for establishing best representatives of specific sets of data. In view of this, these techniques may be adequate for limited problem areas, but may fail to produce logical models for transfer of information to other types of situations.

(2) Model testing involves using a second and **independent set of data** from the same catchment, and using optimised parameter values to find out the level of agreement between the observed and predicted output functions.

In order to calibrate and test a model it is desirable to divide or split the given data set into two parts. Then use first part to obtain optimised parameter values and use second part to test the performance of the model in predicting output with given 'Optimised parameter values'.

7.6 MODEL RESULTS AND INTERPRETATIONS

The model operation during calibration and testing procedure will indicate if some adjuetments are necessary, either in the data on which the model is based or in the structure of the model itself. When the model has suitably been verified, it should be tested by sensitivity analysis.

This involves changing of one system variable while holding the remaining variables constant and noting the changes in the model output function as reflected in value of objective function F^2 or otherwise. If small changes in a particular parameter produce large changes in the output or response functions, the system is said to be sensitive to that parameter. Such study helps in establishing the relative importance of various parameters with reference to system response for given input data.

7.7 CORRELATION OF MODEL PARAMETERS AND CATCHMENT CHARACTERISTICS

As discussed earlier. a conceptual model with particular structure is calibrated and testsd for its performance with independent data for different storms/events/seasons/years as the case may be. Similar tests are performed for data from other catchments in the same hydrologically homogeneous region. If model with particular structure performs satisfactorily for all the catchments, its performance is tested for some independent catchments in the same region. If necessary model structure is suitably modified and entire exercise is repeated. This procedure finally leads to selection of a suitable model structure which would be expected to perform satisfactorily even for ungauged basins in the same region.

However, the application to ungauged basins, suitable relationships have to be developed between model parameters X_1, X_2 .. ...X_{NP} etc. with basin and climatic characteristics. Basin characteristics could include area of catchment, length of main stream, circularity ratio, elongation ratio, mean slope, land use & land cover etc. The climatic characteristics could include total amount of rainfall excess, duration of rainfall excess, intensity of rainfall excess, shape factor of rainfall excess hyetograph etc. Functional relationships between model parameters and catchment charactetistics for the homogeneous region are obtained using calibrated parameters values and related data for all gauged catchments used in calibration, using multiple regression analysis procedure.

For any ungauged basin in the region, model parameters could be determined by using appropriate values of catchment and climatic characteristics. With these parameter values, the model can then be used to simulate the rainfall-runoff process with given input data of rainfall and potential evapotranspiration. Such models can be used to simulate storm events or time series of runoff depending upon specific features of their structure.

For example, Nash (1960) made a unit hydrograph study with particular reference to British catchments, and related parameters n and K to catchment characteristics.

$$nK = 27.6\, A^{0.3}\, (OLS)^{-0.3} \qquad (7.5)$$

$$n = (1/0.41)\, L^{0.1}$$

where, A = catchment area m Sq. miles, L = length of longest stream to catchment boundary in (miles), OLS = overland slope in parts per 10000. These relationships can be used to determine n and K for ungauged catchments, knowing only physiographic characteristics of the catchment. for region where these relationships are applicable.

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