APPENDIX-III

THEOREM OF MOMENTS

The Fig. III shows the typical graphs of effective rainfall, the IUH and the storm runoff, where all volumes have been reduced to unity to simplify the equations below.

The following notations are applied :

a, b, c are times from the origin to centre of area of effective rainfall, storm runoff and IUH respectively.

In the transformation from i (t) to q (t) the elementary strip i$d\gamma$ is replaced by an elementary output whose centre of area must be later than γ by c. This applies to every such elementary strip, therefore the centre of area of q (t) must be later than that of i (t) by c i. e. b=a+c.

or $U_1 = Q_1 - I_1$ ...(III 1)

This is the relation between the first moments about the origin. It means that the lag of IUH (U_i) is equal to the lag between the centres of area of effective rainfall and storm runoff even in a complex storm.

The corresponding relations between the nth moments about the centre of area are found as follows :

In the Fig. III. 1, the element of input i (γ) dγ when multiplied by the ordinates of the IUH u (t) produces an element of output (i (γ) dγ) * u (t) which has an nth moment about b given by :

$$dQ_n = \int_{0}^{\infty} [i(\gamma)\, d\gamma]\, u(t)\, (t+\gamma)^n$$

from which by integration :

$$Q_n = \int_{-a}^{\infty} \int_{-c}^{\infty} i(\gamma)\, (d\gamma)\, u(t)\, (t+\gamma^n\, d(t)$$

$$O_n = \int_{-c}^{\infty} \int_{-a}^{\infty} i(\gamma)\, u(t)\, [t^n + n\, t^{n-1}\, \gamma + \frac{n\,(n-1)}{2\,!} t^{n-2}\, \gamma^2 + \ldots\ldots\ldots\ldots\ldots + \gamma^n]\, d\gamma\, dt$$

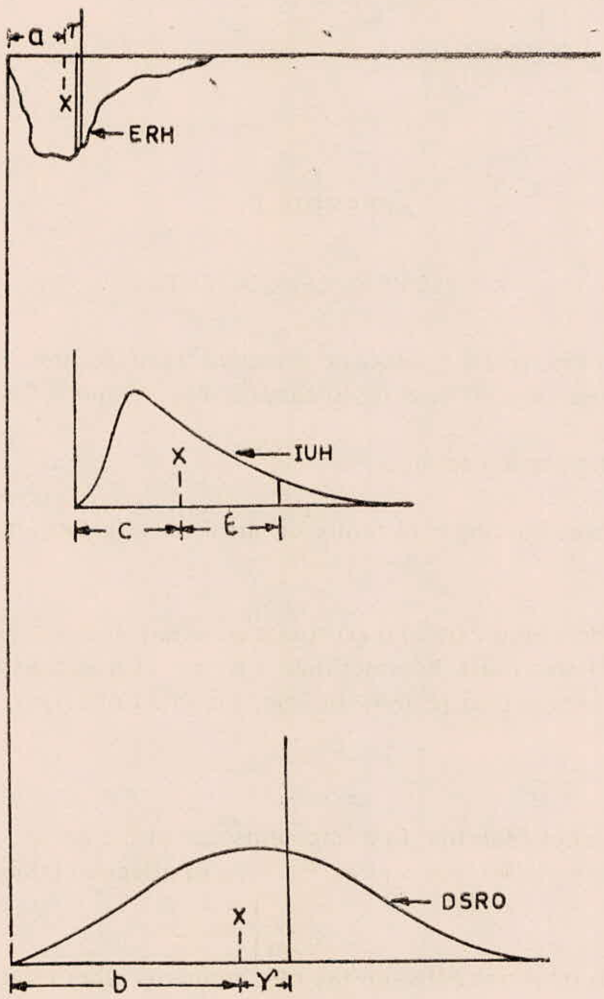

Fig. III.1 Relation Between the Moments

Now $U_n = \int_0^\infty u(t)\, t^n$ and $I_n = \int_0^\infty i(\gamma)\, \gamma^n . dr$

Therefore,

$$Q_n = \int_{-a}^{\infty} i(\gamma)\,(U_n + n\,U_{n-1}.\gamma + \frac{n\,(n-1)}{2!}\,U_{n-2}\,\gamma^2 + \ldots\ldots\ldots\ldots + r^n)\,d\gamma$$

$$Q_n = U_n + n\,U_{n-1}\,I_1 + \frac{n\,(n-1)}{2!}\,U_{n-2}\,.\,I_2\,.\,+\,.\,+\ldots\ldots\ldots\ldots I_n \qquad \text{(III 2)}$$

$$Q_n = (I + U)^n \qquad \ldots\ldots\text{(III. 3)}$$

Where suffixes are written as power indices without of course interpreting them as such except for purpose of binomial expansion :