LECTURE - 4

LAG AND ROUTE METHOD OF FLOOD ROUTING

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OBJECTIVES:

This lecture described the methodology for Lag and Route method of flood routing which is a linear storage routing method. The parameters estimation technique based on method of moment is also described.

1.0 INTRODUCTION

Most of the hydraulic methods of flood routing are storage routing methods wherein the continuity and storage equations are solved to provide the governing equation for the flood routing. Different forms of storage equation used by many investigators lead to number of hydrologic methods of flood routing. The storage routing methods may deal with linear, quasilinear and non-linear flood routing problems. In linear routing the storage is considered to be a linear function of inflow and/or outflow and the parameters of the model are kept constant throughout the touting operation. On the other hand in quasi-linear routing methods some or all the parameters of the model change from one time step to another even though the storage equation may be kept in linear form. Lag and Route method which is discussed in this lecture is basically a linear storage routing method which considers the storage as a linear function of out flow.

2.0 LAG AND ROUTE METHOD

The Lag and Route model is a two parameter model based on the following input-storage-output relationships:

$$\frac{ds}{dt} = I - Q \qquad \text{...(1)}$$

$$S = K\,Q(t+\tau) \qquad \text{...(2)}$$

Where, τ is the travel time of the leading edge of the flood wave to arrive at the outlet section and K, the storage constant of the linear reservoir. The concept of Lag and Route model was based on intuition (Meyer, 1941) and thus it has been considered for a long time as an empirical model (Dooge, 1973). This model attempts to simulate the complex action of a channel by a simple combination of a linear channel and a linear reservoir in series.

The impulse response function of this particular system is given as:

$$u(o,t) = \frac{1}{K} e^{-(t-\tau)/K} \qquad \text{for } t>\tau$$

$$u(o,t) = 0 \qquad \text{for } t<\tau \qquad \text{...(3)}$$

Dooge (1973) proposed a n-multiple single linear reservoir in series alongwith a linear channel instead of using single linear reservoir (SLR) in the Lag and Route method. Such a model has the flexibility of applying it to long river reaches and it IUH form is given as:

$$u(o,t) = \frac{1}{K\,n}\,\frac{(t-\tau)}{K}^{n-1}\,e^{-(t-\tau)/K} \qquad \text{for } t>\tau \qquad \text{...(4)}$$

Equation (4) reveals that the model is a combination of n-linear reservoirs in series alongwith a linear channel whose characteristic is to translate the flow to downstream end of the reach without any modification. Equation (4) represents the three parameter Gamma distribution. The parameters of this model can be estimated from the inflow and outflow flood records of a given reach.

For practical consideration the single linear reservoir based Lag and Route model can be discretised in the finite difference form and the following relationships can be obtained:

$$Q_{i+NLAG} = C_1 I_i + C_2 I_{i-1} + C_3 Q_{i+NLAG-1} \qquad \text{...(5)}$$

Where,

$$C_1 = \frac{0.5\ \Delta t}{K+0.5\ \Delta t} \qquad ...(6)$$

$$C_2 = C_1 \qquad ...(7)$$

$$C_3 = (1-C_1-C_2) \qquad ...(8)$$

K and NLAG($=-\tau$) are the two parameters of the SLR based Lag and Route model.

3.0 ESTIMATION OF PARAMETERS

The parameters of the Lag and Route model can be estimated using one of the available methods which include (i) least square method, (ii) method of moments, (iii) method of comulants and (iv) direct optimisation method. However, the inclusion of method of moments, and method of cumulants as separate methods of parameter estimation is not correct, as the method of cumulants has to be reduced to method of moments before computing the parameters and thus they would give the same parameter values (Dooge, 1973). In this lecture only method of moments is described for the estimation of the parameters. Since only two parameters are required to be estimated, it leads to the estimation of first and second moments of direct inflow and direct outflow hydrographs for a given river reach. The first two moments (normalised) of the direct inflow and direct outflow hydrographs about the origin are given as:

$$XM10 = \sum_{i=1}^{n} \frac{I_i+I_{i+1}}{2}\,(t) / \sum_{i=1}^{n} \frac{I_i+I_{i+1}}{2} \qquad ...(9)$$

$$XM20 = \sum_{i=1}^{n} \frac{I_i+I_{i+1}}{2}\,(t)^2 / \sum_{i=1}^{n} \frac{I_i+I_{i+1}}{2} \qquad ...(10)$$

$$XM10 = \sum_{i=1}^{n} \frac{Q_i+Q_{i+1}}{2}\,(t) / \sum_{i=1}^{n} \frac{Q_i+Q_{i+1}}{2} \qquad ...(11)$$

$$XM20 = \sum_{i=1}^{n} \frac{Q_i+Q_{i+1}}{2} (t)^2 / \sum_{i=1}^{n} \frac{Q_i+Q_{i+1}}{2} \quad ...(12)$$

Where, XM10 and XM20 are the first and second moments of direct inflow hydrograph about the origin. YM10 and YM20 are the first and second moments of direct outflow hydrograph about the origin. t is the time interval in hours from the origin to mid point of each time interval. The parameters NLAG and K are related with the moments of inflow and outflow hydrographs as follows:

$$K = (YM2C - XM2C)^{1/2} \quad ...(13)$$

$$NLAG = -K + (YM10 - XM10) \quad ...(14)$$

Where,

YM2C = Second central moment of direct outflow hydrograph

$$= YM20 - (YM10)^2 \quad ...(15)$$

XM2C = Second Central moment of direct inflow hydrograph

$$= XM20 - (XM10)^2 \quad ...(16)$$

4.0 METHODOLOGY

The Step by step procedures involved in SLR based Lag & Route method of Flood Routing are given in this section.

4.1 Parameter Estimation using Method of Moments

The following steps are followed:

(i) Compute the direct inflow hydrograph and direct outflow hydrograph after subtracting the baseflow from the inflow and outflow hydrographs respectively. Check the volume under the direct inflow hydrograph and the direct outflow hydrograph which are required to be the same in order to satisfy the requirement of continuity equation. In case both the volumes are not same either due consideration must be given to account for the lateral inflows in between the reaches, if any, or appropriate adjustments may be made in the ordinates of direct inflow and outflow hydrographs in order to satisfy the requirement of continuity.

(ii) Compute the first and second moments of the direct inflow hydrograph about the origin using Equation (9) and (10) respectively. This step will provide the estimates for XM10 and XM20.

(iii) Compute the first and second moments of the direct outflow hydrograph about the origin using Equation (11) and (12). This step will provide the estimates for YM10 and YM20.

(iv) Estimate the second moment of direct inflow hydrograph and direct outflow hydrograph about the centroid using the Equation (15) and (16). This step will provide XM2C and YM2C.

(v) Compute the parameters NLAG and K substituting the values of XM10, XM2C, YM10 and YM2C in Equation (13) and (14).

4.2 Flood Routing using Lag and Route Method

The observed direct inflow hydrograph can be routed through a calibrated river reach for Lag and Route model to obtain the direct outflow hydrograph. The computational steps are given below:

(i) Compute the routing co-efficients C_1, C_2 and C_3 using Equation (6), (7) and (8) respectively from given Δt and estimated parameter K.

(ii) Estimate the direct outflow hydrograph ordinates substituting the routing co-efficients C_1 C_2 and C_3 in Equation (5) which is a recursive form of equation. Estimated value of NLAG may be used in the equation. While computing the direct outflow hydrograph ordinates.

5.0 EXAMPLE

The data given below represent observed direct inflow and direct outflow flood hydrograph for a channel reach.

A. Estimate the parameters NLAG and K for the Lag and Route Method using method of moments.

B. Route the given direct inflow hydrograph through the reach using the estimated parameters of Lag & Route method. Plot the computed direct outflow hydrograph and the observed direct outflow hydrograph.

Time (hrs)	Direct Inflow hydrograph (I) (m^3/s)	Direct outflow hydrograph (Q) (m^3/s)
0	0	0
1	200	18.2
2	400	201.66
3	600	400.15
4	800	600.01
5	1000	800.00
6	800	963.60
7	600	796.69
8	400	599.70
9	200	399.97
10	0	200.00
11	0	18.20
12	0	1.66
13	0	0.16
14	0	0

SOLUTION:

A. Parameter Estimation using method of moments

The computations are performed as follows:

(i) Estimate the first and second moments of direct inflow and direct outflow hydrographs. The calculations are performed in Table 1. The computed values of the moments are:

XM10 = 25000/5000 = 5.0
XM20 = 146250/5000 = 29.25
YM10 = 29999.97/5000 = 6.0
YM20 = 202250.4/5000 = 40.45

(ii) Compute the second central moments of direct outflow hydrograph and direct inflow hydrograph using Equation (15) and (16) respectively.

$$\begin{aligned} \text{YM2C} &= \text{YM20} - (\text{YM10})^2 \\ &= 40.45 - 6^2 \\ &= 4.45 \\ \text{XM2C} &= \text{XM20} - (\text{XM10})^2 \\ &= 29.25 - 5^2 \\ &= 4.25 \end{aligned}$$

(iii) Compute the parameters NLAG and K substituting the values of XM10, XM2C, YM10 and YM2C in Equation (13) and (14).

$$\begin{aligned} \text{K} &= (4.45 - 4.25)^2 \\ &= 0.447 \text{ hours (Here } K < 0.5\Delta t,\ \Delta t = 1 \text{ hour).} \\ \text{NLAG} &= -0.447 + (6-5) \\ &= 0.553 \text{ hours} \\ &= 0.6 \text{ hours.} \end{aligned}$$

B. Routing of the observed direct inflow hydrograph using Lag & Route Method

The computational steps are as follows:

(i) Compute the routing co-efficient C_1, C_2 and C_3 using the following equations:

$$C_1 = \frac{0.5\Delta t}{K+0.5\Delta t} = \frac{0.5 \times 1}{0.447+0.5 \times 1} = 0.528$$

$$C_2 = C_1 = 0.528$$

$$\begin{aligned} C_3 &= 1 - (C_1 + C_2) \\ &= 1 - (0.528 + 0.528) \\ &= -0.056 \end{aligned}$$

Table 1: Computations of Moments of Direct Inflow and Direct outflow hydrographs about the origin(Δt = 1 hr.)

t (hrs.)	I_i	$\frac{I_i+I_{i+1}}{2}$ (m³/s)	$\frac{I_i+I_{i+1}}{2} t$ (m³/s-hour)	$\frac{I_i+I_{i+1}}{2} t^2$ (m³/s-hour²)	Q_i (m³/s)	$\frac{Q_i+Q_{i+1}}{2}$ (m³/s)	$\frac{Q_i+Q_{i+1}}{2} t$ (m³/s-hour)	$\frac{Q_i+Q_{i+1}}{2} t^2$ (m³/s-hour²)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0	0				0			
0.5		100	50	25		9.1	4.55	2.275
1.0	200				18.2			
1.5		300	450	675		109.93	164.895	247.3425
2.0	400				201.66			
2.5		500	1250	3125		300.905	752.2625	1880.6562
3.0	600				400.15			
3.5		700	2450	8575		500.08	1750.280	6125.980
4.0	800				600.01			
4.5		900	4050	18225		700.005	3150.0225	14175.101
5.0	1000				800.00			
5.5		900	4950	27225		881.80	4849.90	26674.450
6.0	800				963.60			
6.5		700	4550	29575		880.145	5720.9425	37186.126
7.0	600				796.69			
7.5		500	3750	28125		698.195	5236.4625	39273.468
8.0	400				599.70			
8.5		300	2550	21675		499.835	4248.5975	36113.078
9.0	200				399.97			
9.5		100	950	9025		299.985	2849.8575	27073.646
10.0	0				200.00			
10.5		0	0	0		109.10	1145.550	12028.275
11.0	0				18.20			
11.5		0	0	0		9.93	114.195	1313.2425
12.0	0				1.66			
12.5		0	0	0		0.91	11.375	142.1875
13.0	0				0.16			
13.5		0	0	0		0.08	1.080	14.580
14.0	0				0			
TOTAL	5000	5000	25000	146250	5000.00	5000.00	29999.97	202250.40

(ii) Compute Q values using the following recursive equation

$$Q_{i+NLAG} = C_1 I_i + C_2 I_{i-1} + C_3 Q_{i+NLAG-1}$$

$$Q_{i+0.6} = 0.528I_i + 0.528I_{i-1} - 0.056Q_{i+0.6-1}$$

$$Q_{i+0.6} = 0.528I_i + 0.528I_{i-1} - 0.56Q_{i-04}$$

The computations are given in the Table 2.

(iii) The observed and computed direct outflow hydrograph are plotted as shown in Figure 1.

Table 2: Computation of outflow hydrograph using Lag & Route Method

Δt = 1 hrs. $C_1 = 0.528$, $C_2 = 0.528$, $C_3 = -0.056$

Time (hrs.)	I_i (m^3/s)	I_{i-1} (m^3/s)	$Q_{i-0.4}$ (m^3/s)	$C_1 I_i$ (m^3/s)	$C_2 I_{i-1}$ $(m^3\ s)$	$C_3 Q_{i-0.4}$ (m^3/s)	$Q_{i+0.6}$ (m^3/s)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0	0	-	0	0	0	0	0
0.6			0	0	0	0	0
1.0	200	0		105.6	0	0	
1.6			105.6			-5.9136	105.6
2.0	400	200		211.2	105.6		
2.6			310.89			-17.4098	310.89
3.0	600	400		316.8	211.2		
3.6			519.59			-28.593	510.59
4.0	800	600		422.4	316.8		
4.6			710.607			-39.793	710.607
5.0	1000	800		528.0	422.4		
5.6			910.607			-50.994	910.607
6.0	800	1000		422.4	528.0		
6.6			899.406			-50.367	899.406
7.0	600	800		316.8	422.4		
7.6			688.833			-38.575	688.833
8.0	400	600		211.2	316.8		
8.6			489.425			-27.408	489.425
9.0	200	400		105.6	211.2		
9.6			289.392			-16.206	289.392
10.0	0	200		0	105.6		
10.6			89.394			-5.006	89.394
11.0	0	0		0	0		
11.6			-5.006			+0.2803	-5.006
12.0	0	0		0	0		
12.6			0.2803			-0.0157	0.2803
13.0	0	0		0	0		
13.6			-0.0157			0.00088	-0.0157
14.0	0	0		0	0		
14.6			0.000888				0.00088

Note: This illustrative example also indicates the unrealistic outflow ordinates due to K being less than 0.5Δt i.e. C_3 = -ive.

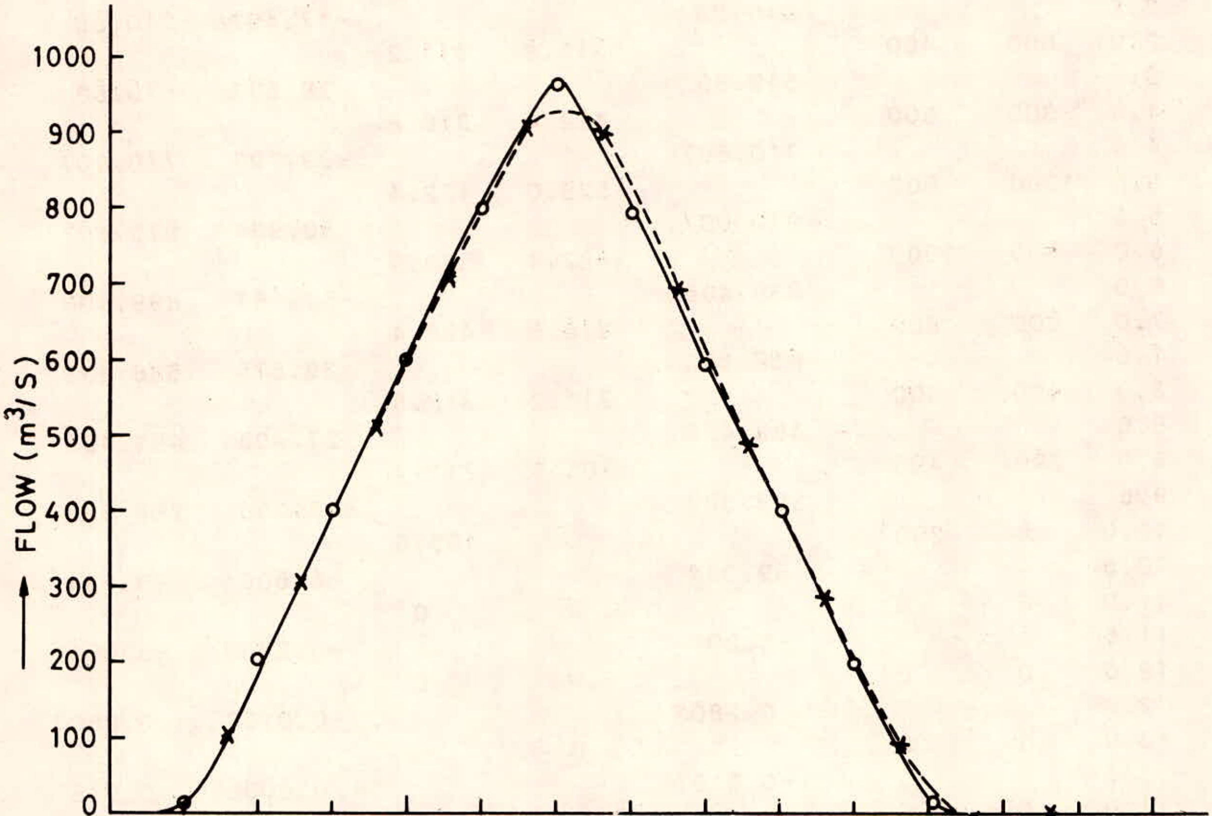

FIG. 1- COMPARISON OF OBSERVED AND COMPUTED DIRECT OUTFLOW HYDROGRAPHS

5.0 LIMITATIONS

The Lag and Route method of flood routing has the following limitations:

(i) The parameters of the Lag and Route model are estimated using the available records of inflow and outflow flood hydrographs leading to the different sets of parameter values from different flood events. As such this method can not be applied for channel routing in ungauged streams.

(ii) The parameters of the Lag and Route model are considered as reach averages and are constant at each time step for a flood event.

(iii) The routing co-efficients C_1, C_2 and C_3 are the function of the parameter K and time interval Δt. When $K > 0.5\Delta t$ the routing co-efficients are positive. In case $K < 0.5\Delta t$, the routing coefficient C_3 is negative and C_1 & C_2 are positive resulting in unrealistic shape of outflow flood hydrograph particularly after the cesation of the inflow flood hydrograph.

While applying Lag and Route method, it is desirable to choose Δt value such that it satisfies the requirement of $K > 0.5\Delta t$. Where inflow ordinates are given at larger Δt, interpolated values should be used at shorter Δt.

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