National Symposium on Hydrology
Dec.16-18,1987, N.I.H., Roorkee

DETERMINATION OF AQUIFER RECHARGE FOR VARYING RIVER STAGES

G.C. MISHRA[1]

ABSTRACT

A mathematical model has been developed for finding the exchange flow rate between a river and a multiaquifer system for varying river stages. The solution has been obtained by discretising the time parameter and using unit response function coefficients. The analytical solution is tractable for numerical calculation. It is found that the storage coefficient of the lower aquifer controls the recharge from the upper aquifer, besides the aquitard resistance.

1.0 INTRODUCTION

The interaction between surface and groundwater has been examined in some detail in recent years. There are two main aspect of this process:1)the flow of groundwater to support river flow and 2)the flow from river to groundwater. Recharge may occur whenever the stage in a river is above that of the adjacent groundwater table, provided that the bed comprises permeable or semipermeable material. This type of groundwater recharge may be temporary,seasonal or continuous. Also it may be a natural phenomenon or induced by man. Man can induce groundwater recharge from rivers by lowering the water table adjacent to rivers through groundwater abstraction.

In a groundwater basin it is common to identify several aquifers separated either by less permeable or impermeable layers. A river in general penetrates fully or partially the upper aquifer. When the river stage rises during the passage of a flood, the upper aquifer is recharged through the bed and banks of the river. The lower aquifer is recharged through the intervening aquitard. A single aquifer river interaction problem has been studied analytically by several investigators (Morel-Seytoux, 1975, Todd, 1955,Cooper and Rorabaugh, 1963). A digital model of multiaquifer system has been developed by Bredehoeft and Pinder (1970) assuming horizontal flow in the aquifers and vertical flow through the confining layers which separate the aquifers. These assumptions have reduced the mathematical problem to one of solving coupled two dimensional equation for each aquifer in the system. An iterative, alternating-direction-implicit scheme has been used to solve the system of simultaneous, finite difference equations which describe the response of the aquifer system to applied stresses. The quasi three-dimensional model has been developed to simulate a groundwater system having any number of aquifers. In the present paper the interaction among a stream and two aquifers which are separated by an aquitard has been studied for varying river stages using a discrete kernel approach.

2.0 STATEMENT OF THE PROBLEM

A schematic section of a partially penetrating river in a two layered multiaquifer system is shown in Fig.1 . The two aquifers are separated by an aquitard. Each aquifer is homogeneous, isotropic and infite in areal extent. The aquitard's thickness and vertical permeability vary with space. The river and the two aquifers are initially at rest condition. Due to passage

[1] Scientist, National Institute of Hydrology,Roorkee, U.P., India.

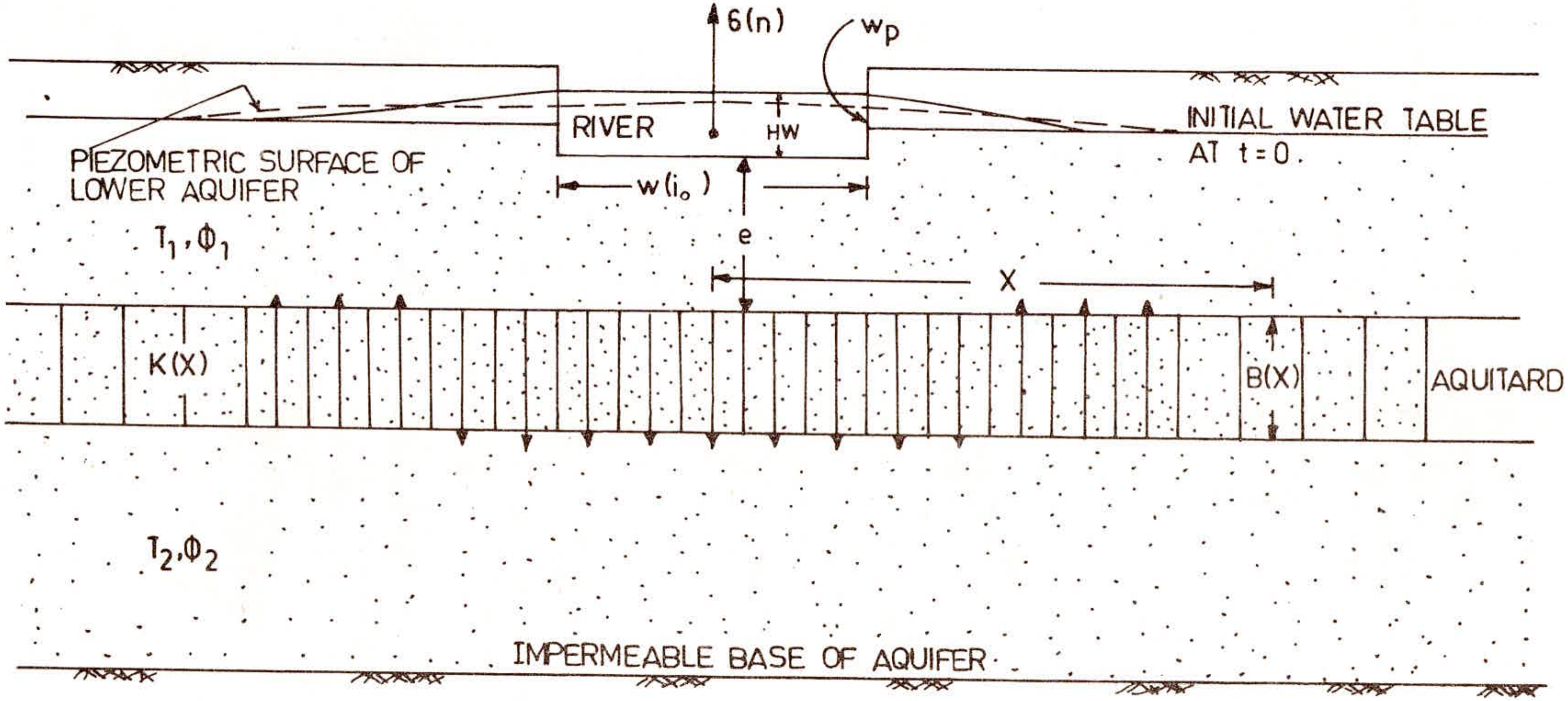

Fig.1. A partially penetrating river and two aquifers separated by an aquitard

of a flood, the river stages change with time. The changes are identical over a long reach of the river. It is required to find the recharge from the river to the top aquifer and the exchange of flow between the two aquifers through the intervening aquitard.

3.0 ASSUMPTIONS

The following assumptions are made for the analysis:

1) The flow in each aquifer is in horizontal direction and one dimensional Boussinesq's equation governs the flow in each aquifer.

2) The flow in the aquitard is vertical and there is no release of water from the aquitard's storage.

3) The variation in aquitard's thickness over a large length parallel to the river is negligible. However, the variation in aquitard's thickness along a line normal to the river axis has been taken care of.

4) The aquifers and the aquitard are divided into identical strips with varying width.

5) At large distance from the river the difference in piezometric surfaces is negligible and therefore the exchange of flow between the aquifers at large distances from the river has been assumed to be negligible.

6) The exchange of flow between the river and the upper aquifer is linearly proportional to the difference in the potentials at the river boundary and in the upper aquifer below the river bed.

4.0 ANALYSIS

The differential equation which governs the flow in the first aquifer is

$$T_1 \frac{\partial^2 s_1}{\partial x^2} = \phi \frac{\partial s_1}{\partial t} - w(x,t) \qquad \ldots(1)$$

in which

s_1 = the water table rise in the first aquifer,

T_1 = transmissivity of the first aquifer,

ϕ_1 = storage coefficient of the first aquifer,

$w(x,t)$ = recharge rate per unit area which is equal to

$$K(x) \frac{S_1(x,t)-S_2(x,t)}{B(x)}$$

$K(x)$ = aquitard's permeability, and

$B(x)$ = aquitard's thickness.

The differential equation that governs the flow in the lower aquifer is

$$T_2 \frac{\partial^2 S_2}{\partial x^2} = \phi_2 \frac{\partial S_2}{\partial t} + w(x,t) \qquad ...(2)$$

in which

T_2 = transmissivity of the lower aquifer

ϕ_2 = storage coefficient of the lower aquifer, and

S_2 = rise in piezometric surface in the lower aquifer.

Solutions to equations (1) and (2) are required to satisfy the following initial and boundary conditions:

If the aquifers and the river were initially at rest, the initial condition to be satisfied is:

$S_1(x,0) = 0$ and $S_2(x,0) = 0$.

The boundary conditions to be satisfied are:

$S_1(\pm\infty,t) = 0$ and $S_2(\pm\infty,t) = 0$.

At the river and upper aquifer interface recharge from the river to the upper aquifer takes place in a manner similar to that from an overlying bed source to an underlying aquifer through an intervening aquitard. The river resistance and the aquitard resistance are analogous. If the river fully penetrates the upper aquifer, then $S_1(r,n) = \sigma(n)$ i.e. the rise in water table height at the river node in the upper aquifer is equal to rise in river stage, $\sigma(n)$. For a partially penetrating river $S_1(r,n) \neq \sigma(n)$ and $S_1(r,n)$ is to be determined as a part of the solution. The unknown recharge which has been assumed to be linearly proportional to the potential difference, $[\sigma(n) - S_1(r,n)]$, is to be incorporated at the river node.

The solution to the problem has been obtained following the principle of superposition.

The upper and the lower aquifer have been divided identically into a number of strips of varying width as shown in Fig. 1 Had the recharge taken place at unit rate per unit area through the ith strip alone, the rise in piezometric surface in the lower aquifer would have been as given below:

$$S_2(x,t) = F(x,T_2,\phi_2,W(i),t) - \frac{[x^2+0.25W^2(i)]}{2T_2}$$

for $|x| < \frac{W(i)}{2}$

$$= F(x,T_2,\phi_2,W(i),t) - \frac{\sqrt{x^2}W(i)}{2T_2}$$

for $|x| > \frac{W(i)}{2}$ (3)

in which

$$F(x,T_2, \phi_2,W(i),t) = \frac{\alpha t}{2\phi_2}[\mathrm{erf}\{\frac{x+0.5W(i)}{\sqrt{(4\alpha_2 t)}}\} - \mathrm{erf}\{\frac{x-0.5W(i)}{\sqrt{(4\alpha_2 t)}}\}]$$

$$+ \frac{1}{4T_2}[\{x+0.5W(i)\}^2 \mathrm{erf}\{\frac{x+0.5W(i)}{\sqrt{(4\alpha_2 t)}}\}$$

$$- \{x-0.5W(i)\ ^2 \mathrm{erf}\{\frac{x-0.5W(i)}{\sqrt{(4\alpha_2 t)}}\}]$$

$$+ \frac{\sqrt{(\alpha_2 t)}}{2T_2\sqrt{\pi}}[\{x+0.5W(i)\}\exp\{-\frac{(x+0.5W(i))^2}{4\alpha_2 t}\}$$

$$- \{x-0.5W(i)\}\ \exp\{-\frac{(x-0.5W(i))^2}{4\alpha_2 t}\}] \quad \ldots(4)$$

α_2 = T_2/ϕ_2

$W(i)$ = width of the i^{th} strip

x = distance measured from the centre of the i^{th} strip to the point of observation

Had the recharge taken place for the first unit time period through the i^{th} strip alone, the rise in piezometric surface at the end of n^{th} unit time step would have been as stated below:

$$S_2(x,1) = F(x,T_2,\phi_2,W(i),1) - \frac{\sqrt{x}^2 W(i)}{2T} \quad \text{for } |x| > \frac{W(i)}{2}$$

$$= F(x,T_2, \phi_2,W(i),1) - \frac{1}{2T_2}[x^2+0.25W^2(i)] \text{for} |x| < \frac{W(i)}{2} \quad \ldots(5)$$

$$S_2(x,n) = F(x,T_2,\phi_2,W(i),n) - F(x,T_2,\phi_2,W(i),n-1)$$

for $n > 2$ and for all x ...(6)

Let the rise in piezometric surface at the j^{th} strip at the end of n^{th} unit time step due to recharge taken place during the first unit time period through the i^{th} strip be designated as $\partial_2(i,j,n)$. Hence,

$$\partial_2(i,j,n) = F(|x(i)-x(j)|\ ,\ T_2,\phi_2,W(i),n)$$

$$-F(|x(i)-x(j)|\ ,\ T_2,\phi_2 W(i),n-1) \quad \ldots(7)$$

for $n > 2$

$$\partial_2(i,j,1) = F(|x(i)-x(j)|,\ T_2,\phi_2,W(i),1)$$

$$-\ \frac{\sqrt{x((i)-x(j))^2}}{2T_2}\ W(i) \qquad \ldots(8)$$

$$\partial_2(i,i,1) = F(0,T_2,\phi_2,W(i),1)-\frac{1}{2T_2}(0.25W^2(i)) \qquad \ldots(9)$$

Dividing the time span into discrete time steps, and assuming that, the recharge per unit area is constant within each time step but varies from step to step, the rise in piezometric surface under jth strip due to time variant recharge through the ith strip alone can be written as

$$S_2(j,n) = \sum_{\gamma=1}^{n} q(i,\gamma)\,\partial_2(i,j,n-\gamma+1) \qquad \ldots(10)$$

in which $q(i,\gamma)$ is the recharge rate per unit area per unit time which is taking place through the ith strip during time step γ. When recharge takes place through all the strips, the resultant rise in piezometric surface can be written as

$$S_2(j,n) = \sum_{\rho=1}^{R}\ \sum_{\gamma=1}^{n} q(\rho,\gamma)\,\partial_2(\rho,j,n-\gamma+1) \qquad \ldots(11)$$

$q(\rho,\gamma)$ are unknown priori. The procedure for determining $q(\rho,\gamma), \gamma=1,2,n$ and $\rho=1,2,\ldots R$ is described below. The recharge which takes place from the upper aquifer to the lower aquifer through ith strip of the aquitard can be expressed as:

$$q(i,n) = \frac{K(i)}{B(i)}[S_1(i,n)-S_2(i,n)] \qquad \ldots(12)$$

Let the bottom width of the river be designated by i_0^{th} strip. The rise in water table height in the upper aquifer, $S_1(i,n)$ is given by

$$S_1(i,n) = \sum_{\gamma=1}^{n} \frac{Q(\gamma)}{W(i_0)}\ \partial_1(i_0,i,n-\gamma+1)$$

$$-\ \sum_{\gamma=1}^{R}\ \sum_{\gamma=1}^{n} q(\rho,\gamma)\,\partial_1(\rho,i,n-\gamma+1) \qquad \ldots(13)$$

in which

$Q(\gamma)$. = river recharge,

$W(i_0)$ = width of river,

$$\partial_1(\rho,i,n) = F(|x(\rho)-x(i)|,\ T_1,\phi_1,W(i),n)$$

$$-F(|x(\rho)-x(i)|,\ T_1,\phi_1,W(i),n-1) \text{ for } n>2 \qquad \ldots(14)$$

$$\partial_1(\rho,i,1) = F(|x(\rho)-x(i)|,\ T_1,\phi_1,W(i),1)$$

$$- \frac{\phi(x(\rho)-x(i))^2 W(i)}{2T_1} \text{ and} \qquad \ldots(15)$$

$$\partial_1(\rho,\rho,1) = F(0,T_1,\phi_1,W(\rho),1) - \frac{1}{2T_1}(0.25W^2(\rho)) \qquad \ldots(16)$$

The first summation in right side of equation (13) represents the rise in water table height due to river recharge taken place upto time step n. The 2nd summation is decline in water table height due to recharge all strips taken place upto time step n.

Similarly the rise in piezometric surface in the lower aquifer is given by

$$S_2(i,n) = \sum_{\rho=1}^{R} \sum_{\gamma=1}^{n} q(\rho,\gamma)\,\partial_2(\rho,i,n-\gamma+1) \qquad \ldots(17)$$

Substituting $S_1(i,n)$ and $S_2(i,n)$ in equation (12),

$$q(i,n) = \frac{K(i)}{B(i)}\Big[\sum_{\gamma=1}^{n} \frac{Q(\gamma)}{W(i_0)}\,\partial_1(i_0,i,n-\gamma+1)$$

$$- \sum_{\rho=1}^{R} \sum_{\gamma=1}^{n} q(\rho,\gamma)\,\partial_1(\rho,i,n-\gamma+1)$$

$$- \sum_{\rho=1}^{R} \sum_{\gamma=1}^{n} q(\rho,\gamma)\,\partial_2(\rho,i,n-\gamma+1)\Big] \qquad \ldots(18)$$

Splitting the temporal summation into two parts and rearranging

$$q(i,n)\,\frac{B(i)}{K(i)} + \sum_{\rho=1}^{R} q(\rho,n)[\partial_1(\rho,i,1)+\partial_2(\rho,i,1)]$$

$$- \frac{Q(n)}{W(i_0)}\,\partial_1(i_0,i,1)$$

$$= \sum_{\gamma=1}^{n-1} \frac{Q(\gamma)}{W(i_0)}\,\partial_1(i_0,i,n-\gamma+1)$$

$$- \sum_{\rho=1}^{R} \sum_{\gamma=1}^{n-1} q(\rho,\gamma)\,\partial_1(\rho,i,n-\gamma+1)$$

$$- \sum_{\rho=1}^{R} \sum_{\gamma=1}^{n-1} q(\rho,\gamma)\,\partial_2(\rho,i,n-\gamma+1) \qquad \ldots(19)$$

or

$$q(i,n)[\frac{B(i)}{K(i)} + \partial_1(i,i,1)+\partial_2(i,i,1)]$$

$$+ \sum_{\substack{\rho=1 \\ \rho\neq 1}}^{R} q(\rho,n)[\partial_1(\rho,i,1) + \partial_2(\rho,i,1)]$$

$$- \frac{Q(n)}{W(i_0)} \partial_1(i_0,i,1)$$

$$= \sum_{\gamma=1}^{n-1} \frac{Q(\gamma)}{W(i_0)} \partial_1(i_0,i,n-\gamma+1)$$

$$\sum_{\rho=1}^{R} \sum_{\gamma=1}^{n-1} q(\rho,\gamma)[\partial_1(\rho,i,n-\gamma+1)+\partial_2(\rho,i,n-\gamma+1)] \quad \ldots(20)$$

'R' number of equations could be written for 'R' number of recharge strips. However, the unknowns at any time 'n' are the 'R' number of unknown $q_i(n)$ and the river recharge Q(n). One more equation could be written as:

$$Q(n) = \Gamma_r[\sigma(n)-S_1(i_0,n)] \quad \ldots(21)$$

in which

Γ_r = reach transmissivity = $\dfrac{\pi T_1}{e \mathrm{Log}_e[(e+H_w)/w_p] + \dfrac{0.5\pi l}{e+H_w}}$ (Bouwer,1969)

$\sigma(n)$ = river stage during time step n,

$S_1(i_0,n)$ = rise in piezometric surface under the river bed,

w_p = wetted perimeter of the river,

l = a distance from the river representing the zone of influence,

H_w = average depth of water in the river, and

e = saturated thickness of the upper aquifer below the river bed.

Substituting for $S_1(i_0,n)$

$$\frac{Q(n)}{\Gamma_r} = \sigma(n)-[\sum_{\gamma=1}^{n} \frac{Q(\gamma)}{W(i_0)} \partial_1(i_0,i_0,n-\gamma+1)$$

$$- \sum_{\rho=1}^{R} \sum_{\gamma=1}^{n} q(\rho,\gamma)\partial_1(\rho,i_0,n-\gamma+1)] \quad \ldots(22)$$

Splitting the temporal summation into two parts and rearranging

$$Q(n)[\frac{1}{\Gamma_r} + \frac{\partial_1(i_0,i_0,1)}{W(i_0)}] - \sum_{\rho=1}^{R} q(\rho,n)\partial_1(\rho,i_0 1) = \sigma(n)$$

$$-[\sum_{\gamma=1}^{n-1} \frac{Q(\gamma)}{W(i_0)} \partial_1(i_0,i_0,n-\gamma+1) - \sum_{\rho=1}^{R} \sum_{\gamma=1}^{n-1} q(\rho,\gamma)\partial_1(\rho,i_0,n-\gamma+1)]$$

Let R be equat to $2i_0-1$. The set of (R+1) equations represented by equations (19) and (22) can be written in the matrix form [A].[B]=[C].Where

$$
[A] = \begin{bmatrix}
\{\frac{B(1)}{K(1)} + \partial_1(1,1,1)+\partial_2(1,1,1)\}, & \{\partial_1(2,1,1)+\partial_2(2,1,1)\} & ,\cdot\ \{\partial_1(2i_0-1.1.1)+\partial_2(2i_0-1,1,)\}, & \{-\frac{\partial_1(i_0,1,1)}{W(i_0)}\} \\
\{\partial_1(1,2,1)+\partial_2(1,2,1)\} & \{\frac{B(2)}{K(2)} + \partial_1(2,2,1)+\partial_2(2,2,1)\},\cdot & \{\partial_1(2i_0-1,2,1)+\partial_2(2i_0-1,2,1)\}, & \{-\frac{\partial_1(i_0,2,1)}{W(i_0)}\} \\
\{\partial_1(1,i_0,1)+\partial_2(i,i_0,1)\} \quad \cdot & \{\partial_1(2,i_0,1)+\partial_2(2,i_0,1)\} & ,\cdot\ \{\partial_1(2i_0-1,i_0,1)+\partial_2(2i_0-1,i_0,1)\}, & \{-\frac{\partial_1(i_0,i_0,1)}{W(i_0)}\} \\
\{\partial_1(1,2i_0-1,1)+\partial_2(1,2i_0-1,1)\}, & \{\partial_1(2,2i_0-1,1)+\partial_2(2,2i_0-1,1)\}, & \{\frac{B(2i_0-1)}{K(2i_0-1)} + \partial_1(2i_0-1,2i_0-1,1) + \partial_2(2i_0-1,2i_0-1,1)\}, & \{-\frac{\partial_1(i_0,2i_0-1,1)}{W(i_0)}\} \\
\{-\partial_1(1,i_0,1)\} & \{-\partial_1(2,i_0,1)\} & ,\cdot\ \{-\partial_1(2i_0-1,i_0,1)\} & ,\{\frac{1}{\Gamma_r} + \frac{\partial_1(i_0,i_0,1)}{W(i_0)}\}
\end{bmatrix}
$$

$$
[B] = [q(1,n),\ q(2,n) \quad q(i_0,n), \quad q(2i_0-1,n),\ Q(n)]^T
$$

and

$$[C] = \begin{bmatrix} \sum_{\gamma=1}^{n-1} \frac{Q(\gamma)}{W(i_0)} \partial_1(i_0,1,n-\gamma+1) - \sum_{\rho=1}^{2i_0-1} \sum_{\gamma=1}^{n-1} q(\rho,\gamma)\{\partial_1(\rho,1,n-\gamma+1)+\partial_2(\rho,1,n-\gamma+1)\} \\ \sum_{\gamma=1}^{n-1} \frac{Q(\gamma)}{W(i_0)} \partial_1(i_0,2,n-\gamma+1) - \sum_{\rho=1}^{2i_0-1} \sum_{\gamma=1}^{n-1} q(\rho,\gamma)\{\partial_1(\rho,2,n-\gamma+1)+\partial_2(\rho,2,n-\gamma+1)\} \\ \sum_{\gamma=1}^{n-1} Q(\gamma)\partial_1(i_0,2i_0-1,n-\gamma+1) - \sum_{\rho=1}^{2i_0-1} \sum_{\gamma=1}^{n-1} q(\rho,\gamma)\{\partial_1(\rho,2i_0-1,n-\gamma+1)+\partial_2(\rho,2i_0-1,n-\gamma+1)\} \\ \sigma(n) - \sum_{\gamma=1}^{n-1} \frac{Q(\gamma)}{W(i_0)} \partial_1(i_0,i_0,n-\gamma+1) + \sum_{\rho=1}^{2i_0-1} \sum_{\gamma=1}^{n-1} q(\rho,\gamma)\,\partial_1(\rho,i_0,n-\gamma+1) \end{bmatrix}$$

Hence $[B] = [A]^{-1}[C]$

For the first time period

$[C] = [0,0,..........,\ (1)]^T$

Thus $q(i,n)$, $i=1,2,.......2i_0-1$ and $Q(n)$ can be solved in succession starting from time step 1.

4.0 RESULTS AND DISCUSSIONS

Assuming that a finite number of strips on both sides of the river takes part in the stream aquifer interaction, the response function coefficients are determined for known set of aquifer parameters T_1, ϕ_1, T_2 and ϕ_2 and for an assumed integer value of i_0 making use of equations (7),(8),(9) and (14),(15),(16). The appropriate width of the strips and their number, $(2i_0-1)$, could be ascertained only after assessing the magnitude of recharge at the farthest strip from the river occurring towards the end of excitation. With known $B(i)$, $K(i)$, $\partial_1(i,j,1)$, and $\partial_2(i,j,1)$, the element of matrix [A] are calculated and inverse of the matrix [A] is found. For known value of the river stage during the first time step, the recharge rates at each recharging strip and the recharge from the river to the upper aquifer are determined for the first time step. Evaluating the element of the matrix [C] in succession, the recharge occurring through each of the $(2i_0-1)$ strips and the recharge from the river to the upper aquifer during other time steps are determined in succession starting from time step 1 for prescribed river stages during each time step.

The variations of $Q(n)$, and $\sum_{\gamma=1}^{n} \sum_{i=1}^{2i_0-1} q(i,\gamma)W(i) / \sum_{\gamma=1}^{n} Q(\gamma)$ with time presented in Figure 2 pertain to a unit step rise in the river stage. The flow characteristics have been evaluated for $T_1=500m^2/day$, $\phi_1 = 0.1$ $T_2=700m^2/day$, $\phi_2=0.01$ width of the river = 500m, and the saturated thickness of the upper aquifer below the river bed = 500m , and wetted perimeter $w_p=510m$.

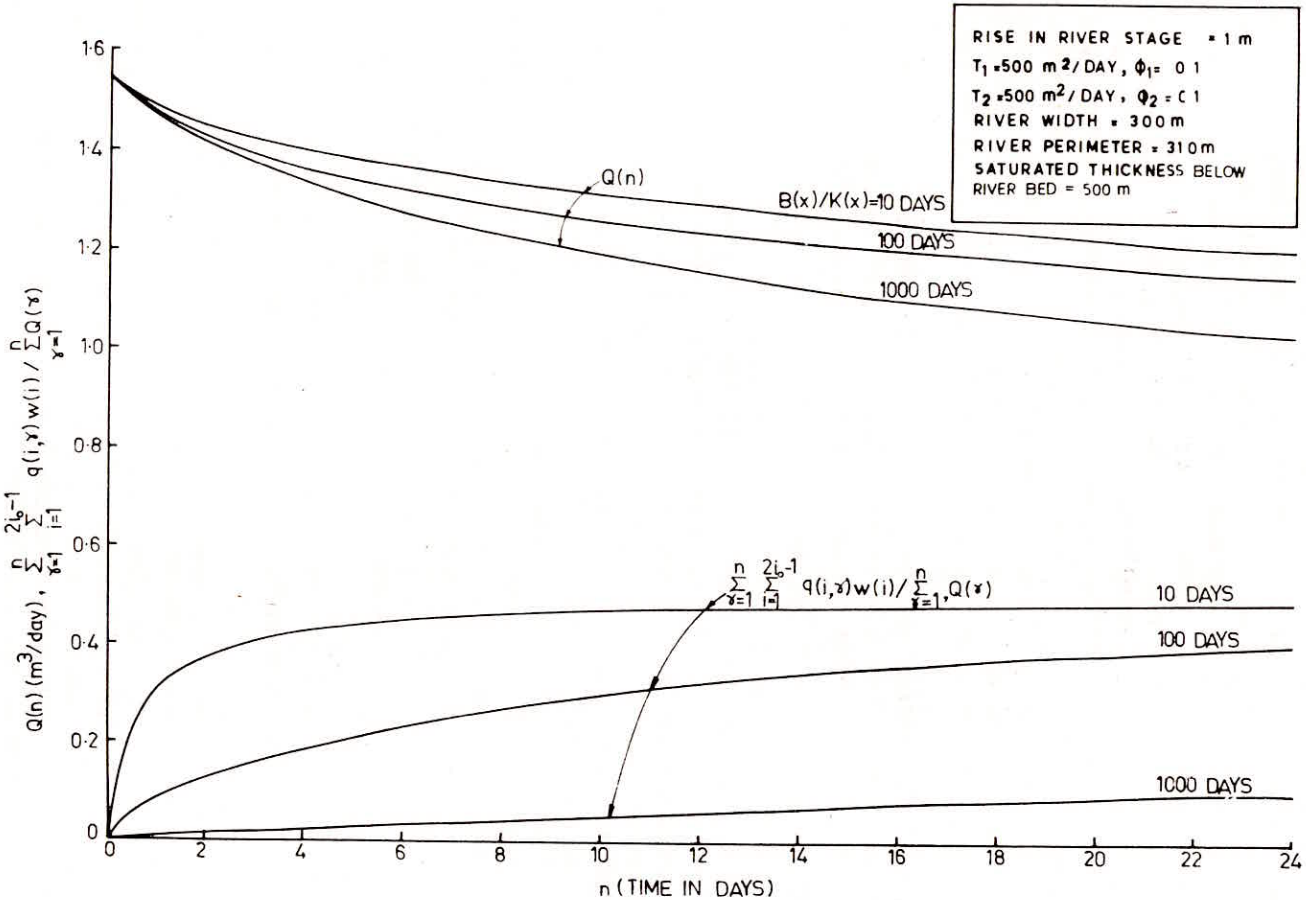

Fig.2 Variations of recharge from the river, to the upper aquifer and ratio of cumulative flows to the lower aquifer to cumulative flows from the river with time for different aquitard resistance

The aquitard has been assumed to be homogeneous. The term $\sum_{\gamma=1}^{n} \sum_{i=1}^{2i_0-1} q(i,\gamma)W(i)$ represents the cumulative recharge taken place upto time step n from the upper to the lower aquifer through the aquitard. It could be seen from the figure that the flow from the river to the upper aquifer decreases with time and the fraction of cumulative river flows entering to the lower aquifer increases with time. The aquitard resistance controls the recharge from the upper aquifer to the lower one, Consequently it also influences the recharge from the river to the upper aquifer. It could be seen that at the end 24th day, 40% of the cumulative flow from the river to the upper aquifer enters the lower aquifer through an aquitard which has a resistance of 100 days. If the aquitard resistance is 1000 day, the cumulative recharge at the end of 24th day is only 10% of the cumulative river flows The increase in aquitard resistance also leads to a decrease in the flow from the river. For assumed aquifer parameters, if the aquitard resistance is changed from 100 day to 1000 day, the flow from the river during the 24th day is reduced by 10%.

The temporal variations of recharge from the river to the upper aquifer and from the upper aquifer to the lower aquifer through the aquitard below the river bed for varying river stages have been evaluated for T_1=500m^2/day, ϕ_1=0.1. T_2=700 m^2/day, ϕ_2=0.01 and are shown in Fig.3(a). The aquitard resistance has been assumed to be 100 day. Results have been presented for two values of river width. As seen from figure if the width of the river is changed from 300 m to 100 m, the flow from the river does not reduce appreciably. The cumulative recharge from the river to the upper aquifer and from the upper aquifer to the lower aquifer at different time for the assumed variations in river stages are presented in Fig.3(b). It could be seen that for the river with a width of 300 m, 37.8 m^3 of water has entered from unit length of the river to the upper aquifer at the end of 24 days. For a river with 100 m the corresponding volume of flow is 36.4 m^3. Thus there is no significant difference between the respective cumulative flow quantities though one of the rivers is three times wider than the other. The ratio of cumulative flows to the lower aquifer to the cumulative flows from the river to the upper aquifer at various nondimensional time for the assumed river stages is shown in Fig.3(c) for ϕ_1/ϕ_2 = 1 and 10 and for T_1/T_2 = 0.5, 1.0 and 2. It could be seen that when ϕ_2 is reduced by 10 times, at the end of non-dimensional time 0.5, the cumulative flows to the lower aquifer is reduced by 26%.

In reality the aquitard thickness and conductivity vary from place to place. The impact of non-homogeneous aquitard on different flow components is presented in Table 1. It could be seen that if the aquitard resistance is 100 day at all strips, the cumulative recharge at the end of 24th day from the upper aquifer to the lower one is 5.79 m^3. If the aquitard resistance is 1000 day every where except below the river bed, where it has a resistance of 100 day, the cumulative recharge at the end of 24th day is found to be 7.05 m^3. The reason for increase in recharge to the lower aquifer in the latter case are as follows: The lower aquifer has higher transmissivity and less storage coefficient in comparison to that of the upper aquifer. In the former case in which the homogenous aquitard has a resistance of 100 day the water which enters the lower aquifer through the aquitard below the river bed comes back to the upper aquifer through adjoining parts of the aquitard owing to higher increment in piezometric head in the lower aquifer. In the latter case the water which enters to the lower aquifer below the river bed is prevented from coming out because of higher aquitard resistance.

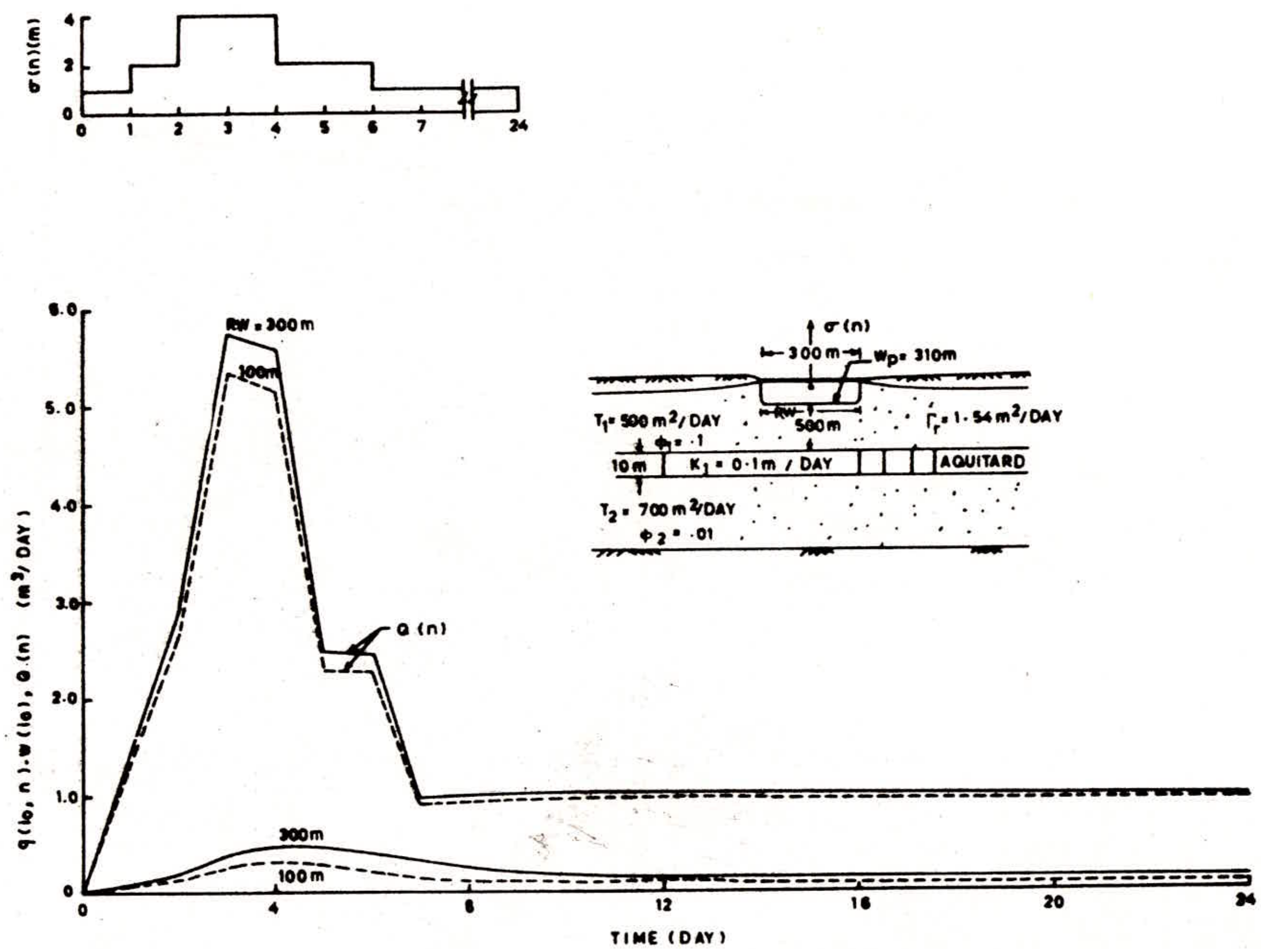

Fig.3(a) Variation of flow from the river to the upper aquifer and flow from the upper aquifer to the lower aquifer through part of the aquitard below rived bed with time for assumed changes in river stages

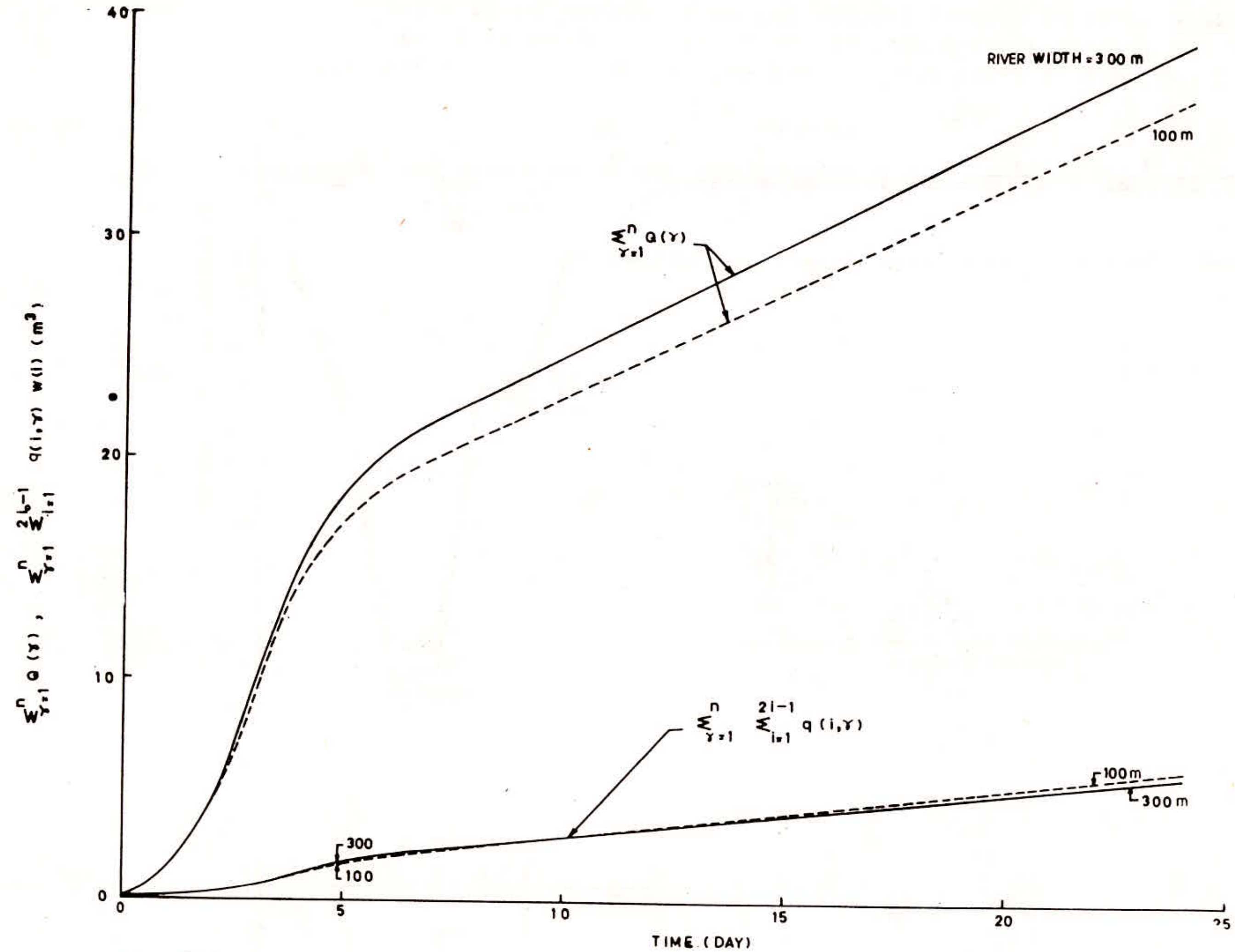

Fig.3(b) Variation of cumulative flows from river to upper aquifer and cumulative recharge from upper aquifer to lower aquifer for different river widths

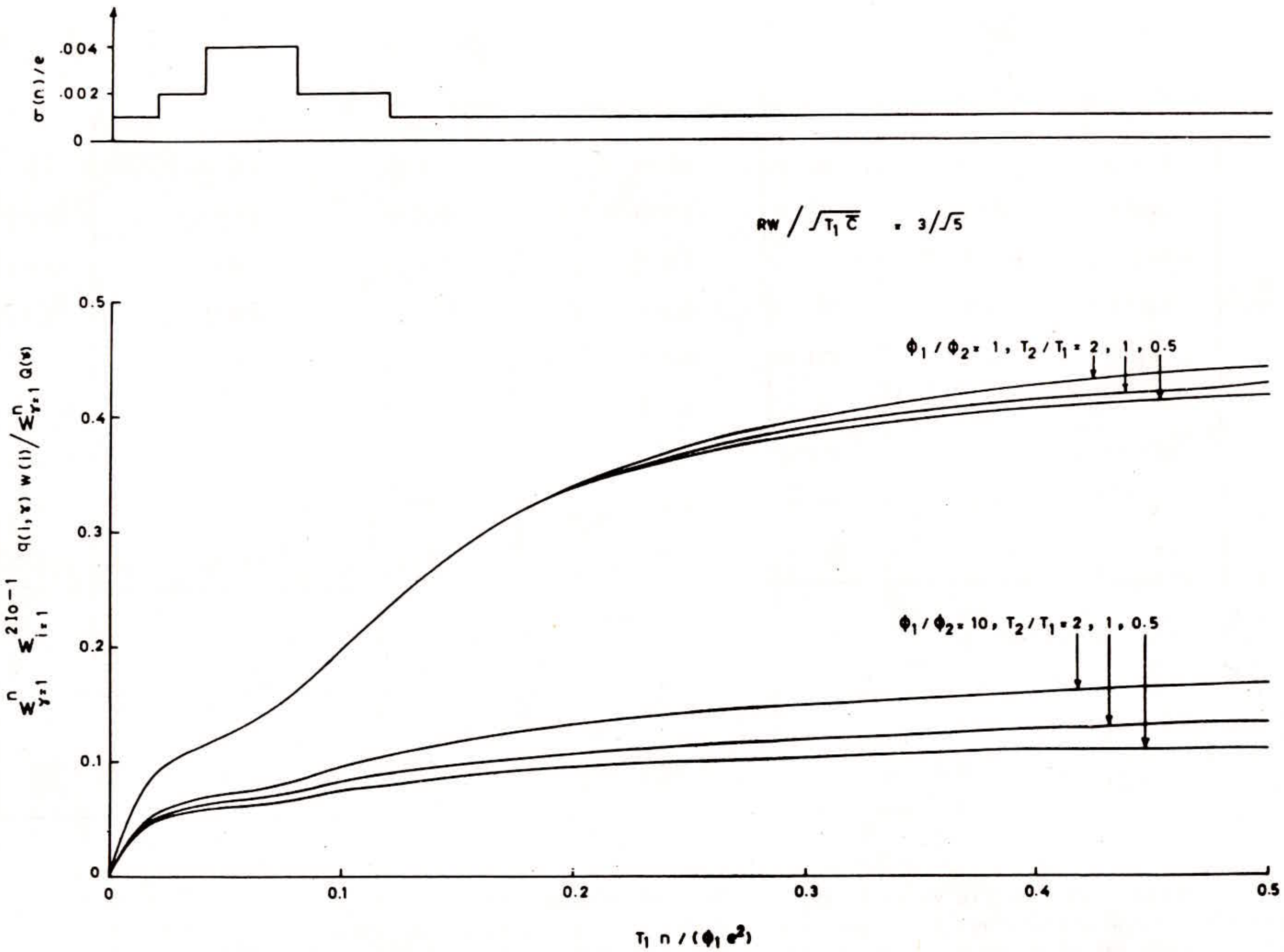

Fig.3(c) Ratio of cumulative flows from upper to lower aquifer to cumulative flows from river to upper aquifer at different time

TABLE 1- Cumulative flows from the river and cumulative recharge quantities from the upper to the lower aquifer at various times evaluated for T_1=500m²/day, ϕ_1=0.10, T_2=700m²/day, ϕ_2=0.01, width of the river = 300m, saturated thickness of the upper aquifer below the river bed = 500m, river stages as indicated in Fig.3(a) and aquitard resistance as indicated in the table

Time day	$\frac{B(x)}{K(x)}$ = 1000 days		$-150 \le x \le 150$, $\frac{B(x)}{K(x)}$ = 10 days; $x > 150$, $x < -150$, $\frac{B(x)}{K(x)}$ = 1000days		$\frac{B(x)}{K(x)}$ = 100 days		$-150 \le x \le$ $\frac{B(x)}{K(x)}$ = 100 days; $x > 150$, $x < -150$, $\frac{B(x)}{K(x)}$ = 1000 days	
n	$\sum_{\gamma=1}^{n} Q(\gamma)$	$\sum_{\gamma=1}^{n}\sum_{i=1}^{2i_0-1} q(i,\gamma)W(i)$	$\sum_{\gamma=1}^{n} Q(\gamma)$	$\sum_{\gamma=1}^{n}\sum_{i=1}^{2i_0-1} q(i,\gamma)W(i)$	$\sum_{\gamma=1}^{n} Q(\gamma)$	$\sum_{\gamma=1}^{n}\sum_{i=1}^{2i_0-1} q(i,\gamma)W(i)$	$\sum_{\gamma=1}^{n} Q(\gamma)$	$\sum_{\gamma=1}^{n}\sum_{i=1}^{2i_0-1} q(i,\gamma)W(i)$
1	1.469	0.015	1.478	0.201	1.472	0.080	1.473	0.087
2	4.354	0.058	4.389	0.666	4.365	0.279	4.368	0.312
3	10.078	0.154	10.177	1.620	10.111	0.695	10.120	0.788
4	15.610	0.296	15.808	2.732	15.681	1.215	15.698	1.409
5	18.047	0.447	18.358	3.496	18.165	1.629	18.192	1.944
10	24.242	1.216	25.124	5.473	24.650	2.931	24.721	3.728
15	29.090	1.963	30.473	6.811	29.780	3.959	29.896	4.979
20	33.869	2.713	35.725	8.105	34.827	4.981	34.991	6.147
24	37.614	3.319	39.835	9.120	38.781	5.789	38.983	7.057

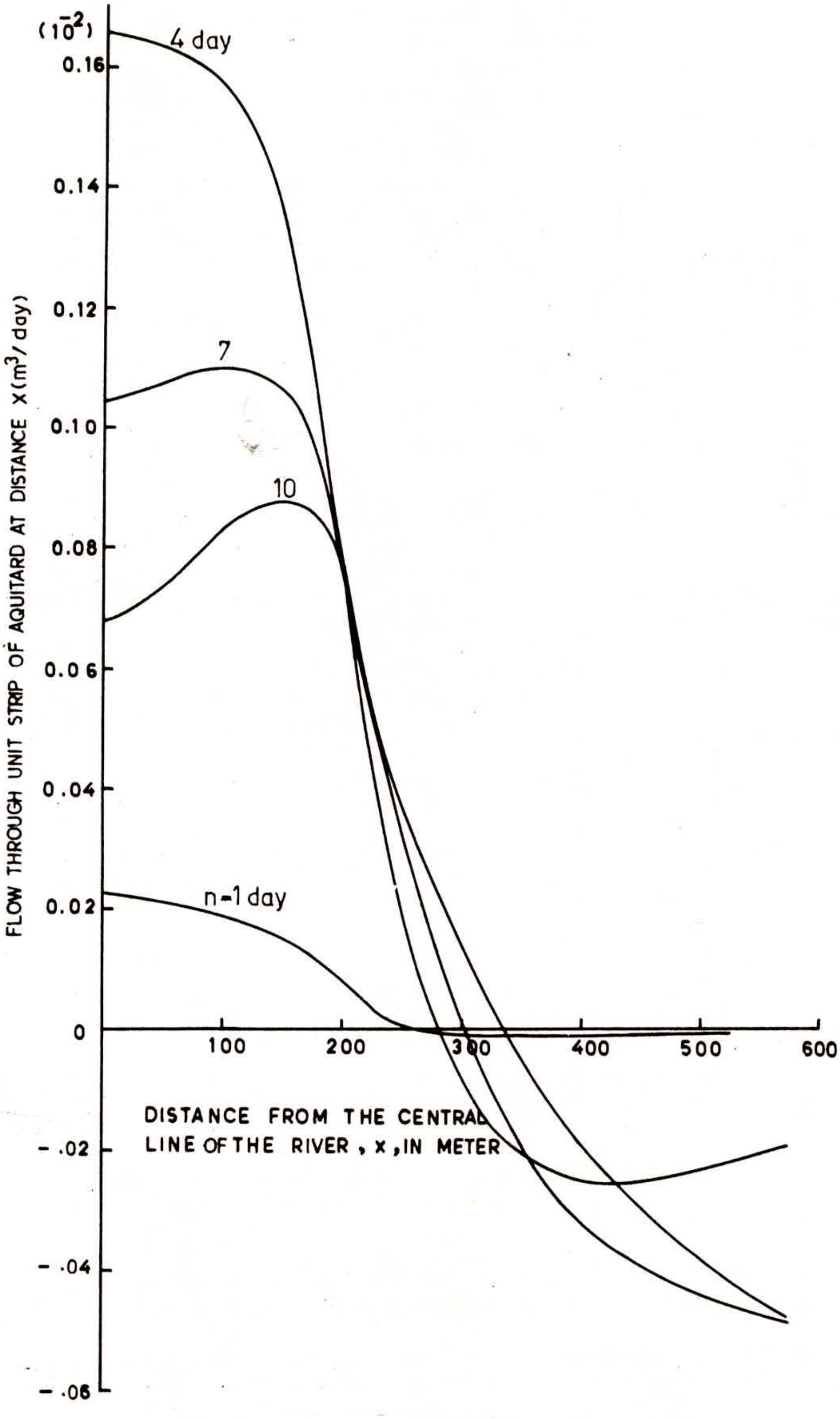

Fig.4 Exchange of flow between the aquifers through the aquitard at different times

The exchange of flow between the two aquifers through the aquitard is governed by the piezometric surfaces in the aquifer. If the two aquifers are identical it is found that, consequent to a rise in river stage recharge from the upper aquifer to the lower aquifer takes place which decreases with distance from the river. For unequal aquifer parameters, recharge always takes place from upper aquifer to the lower aquifer below the river bed due to rise in river stage. If the lower aquifer has less storage coefficient compared to that of the upper aquifer, the water flows from the upper aquifer to the lower aquifer under the river bed. But in region outside the river bed water flows from the lower aquifer to upper aquifer through the aquitard. The distribution of recharge with distance from the river is shown in Fig.4. It could be seen that the water enters from lower aquifer to upper aquifer in regions away from the river.

5.0 CONCLUSIONS

A mathematical model has been developed to study recharge from a river to a multiaquifer system for varying river stages. The analytical solution is tractable for numerical calculation. The solution has been obtained by discretising the time parameter and using unit response function coefficients.

It is found that the storage coefficient of the lower aquifer controls the recharge from the upper aquifer, besides the aquitard resistance. If the storage coefficient of the lower aquifer is assigned valued which differ by an order of magnitude, the fractions of river flows which enters the lower aquifer in some cases would differ by 26%. If the lower aquifer has higher transmissivity and lower storage coefficient, the water which enters the lower aquifer through the aquitard below the river bed flows back to the upper aquifer through the adjoining parts of the aquitard.

ACKNOWLEDGEMENT

The author is grateful to Dr.Satish Chandra, Director, National Institute of Hydrology and Professor H.J. Morel Seytoux, Professor, Colorado State University, for their encouragement and guidance.

REFERENCES

Bouwer, H. 1969. Theory of seepage from open channels. Advances in Hydroscience, edited by V.T. Chow. Academic Press, V. 5, pp.121-172.

Bredehoeft, J.D., and Pinder, G.F., 1970. Digital analysis of areal flow in multiaquifer system: A quasi three-dimensional model. Water Resources Research, Vol.6, No.3, pp.883-888.

Cooper, H.H., Jr., and M.I. Rorabaugh, 1963. Groundwater Movements and Bank Storage Due to Flood Stages in Surface Streams, Geol. Surv. Water Supply Paper 1536-J.

Morel-Seytoux, H.J. and C.J. Daly, 1975. A Discrete Kernel Generator for stream Aquifer Studies. Water Resources Research Journal.

Todd, D.K., 1955. Groundwater flow in relation to a flooding stream, Proc. Amer. Soc. Civil Engineers, Vol.81, Sept. 628, pp.20.