

**RESERVOIR OPERATING RULE FOR NARMADASAGAR -  
DYNAMIC PROGRAMMING APPROACH**

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**ABSTRACT**

This work relates to application of dynamic programming model to Narmadasagar reservoir across river Narmada in the state of Madhya Pradesh. The model is formulated to determine the operating rule for maximizing power after meeting irrigation. The irrigation demand pertains to the development scenario 1991-92 while the full development of irrigation utilisation takes place only during 2023-24 (final scenario). The inflow into Narmadasagar for the historic record of 32 years, for the 1991-92 scenario of development is obtained from a simulation model (I.I.M., Bangalore, June, 1985). Eventhough, the reservoir is to be built to a height of 262.13 m (12212 Mm<sup>3</sup>) with a live storage of 10855 Mm<sup>3</sup>, the model brings out the performance for FRLs namely 256.0 m, 257.6 m, 259.0 m, 260.67 m and 262.13 m. The power generation for installed capacities namely 875 MW, 1000 MW, 1125 MW and 1250 MW are also discussed.

In Dynamic programming problems, the accuracy of the results depends on the fineness of the state variables. As the number of states increase, the computer time increases and vice versa. The effect of variation of number of states on the performance and on the CPU time required are analysed. Number of states considered are 9, 15, 20, 25 and 40. The CPU time needed for a run with 40 states is about 20 times that of a run with 9 states. Benefit from irrigation is taken as 8.8 paise per cubic metre of water and the power benefit at bus bar is taken as 30 paise/KW hr. Efficiency of power generation is taken as 85 percent.

Depending upon whether the inflow is less than 4600 Mm<sup>3</sup>, 4600-16250 Mm<sup>3</sup>, 16250 Mm<sup>3</sup> - 28750 Mm<sup>3</sup> more than 28750 Mm<sup>3</sup> a year is classified as very dry, dry, normal or wet for the purpose of arriving at an optimal operating policy. The model runs were taken using IBM 370/155 system at I.I.T., Madras. The results obtained are discussed.

**INTRODUCTION**

One of the major concerns of many developing countries including that of India for improving its food production is to provide assured irrigation, maximize power production and minimize the effects of floods and droughts by operating the water resources systems in an optimal way. For large scale water resource systems consisting of several multi-purpose projects the operating rules are complex and are often arrived at using simulation techniques, as otherwise the mathematical formulation of the problem becomes complex in algorithm logic. One such problem faced by the Government of Madhya Pradesh, India is to arrive at an operating policy for Narmadasagar which is one of the 30 dams contemplated on the river Narmada by the States of Madhya Pradesh and Gujarat.

The river Narmada with a length of 1312 Km is the longest west flowing river and the fifth largest river in India, draining an area

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of 98,800 sq.km., and traversing through the states of Madhya Pradesh, Gujarat and Maharashtra (Figure 1). The dispute among the participating states regarding sharing of the Narmada waters was referred to the Narmada Water Dispute Tribunal (NWDT) in October 1969 which has passed its award in December 1979. As per the NWDT award, the 75 percent dependable yield at Sardar Sarovar was fixed at 28 MAF (34537 MCM) and the proportion of sharing among the riparian states was fixed as follows.

#### SHARES OF NARMADA WATERS

State	Proportional share	Share of water if Utilisation is 34537 MCM (28 MAF)	
		MAF	MCM
Madhya Pradesh	73	18.25	22511
Gujarat	36	9.00	11101
Rajasthan	2	0.50	617
Maharashtra	1	0.25	308

The tribunal has stipulated that the construction of Narmadasagar dam with FRL 262.13m in Madhya Pradesh must be completed concurrently with or earlier than Sardar Sarovar dam in Gujarat. The former is the largest of the reservoirs to be built in Madhya Pradesh. It stores about 40 percent of the utilisable share of the Government of Madhya Pradesh and forms the Kingpin of the Narmadasagar complex consisting of Narmadasagar, Omkareshwar and Maheshwar, all in Madhya Pradesh. Their location is also shown in Figure 1.

The contemplated live storage of Narmadasagar works out to 9745MCM corresponding to FRL 262.13m and MDDL of 243.23m (2467 MCM). The planning of the complex is being carried out by the Government of Madhya Pradesh and the World Bank assistance is being sought for the construction of the complex. A detailed simulation study of the complex for the scenarios of development 1991-92, 2003-04, 2013-14, and 2023-24 was carried out for an assumed set of operating policies and results such as firm power generation capacity, allocation to irrigation and other industrial requirements, and secondary power generation were obtained (IIM, 1985).

The present investigation aims specifically to study the application of dynamic programming to Narmadasagar reservoir to arrive at an optimal policy pertaining to 1991-92 scenario of development. This technique is specifically chosen as it can consider sequential decision taking and any kind of non-linearity or discontinuity of the objective function. The operation model thus achieved on the annual basis would be obtained for all the years for which the historical record is available for planning. One could attempt to study any optimal pattern of operation that could be deduced from the annual operation policies.

In the dynamic programming model developed, the month is taken as the unit of stage and the states are varied in number from 9 to 40. The volume contents of the live storage is divided into equal parts and each part corresponds to a state. This type of state discretization has shown added advantages (Jayarama Reddy, 1978). The inflows are authenticated as they are taken from the simulation runs. The downstream commitments are honoured first. The irrigation requirements are quiet small in the order of magnitude compared with the storage. Once, the downstream commitment is satisfied, further power releases are taken up only after these requirements are met. So, there was no need for

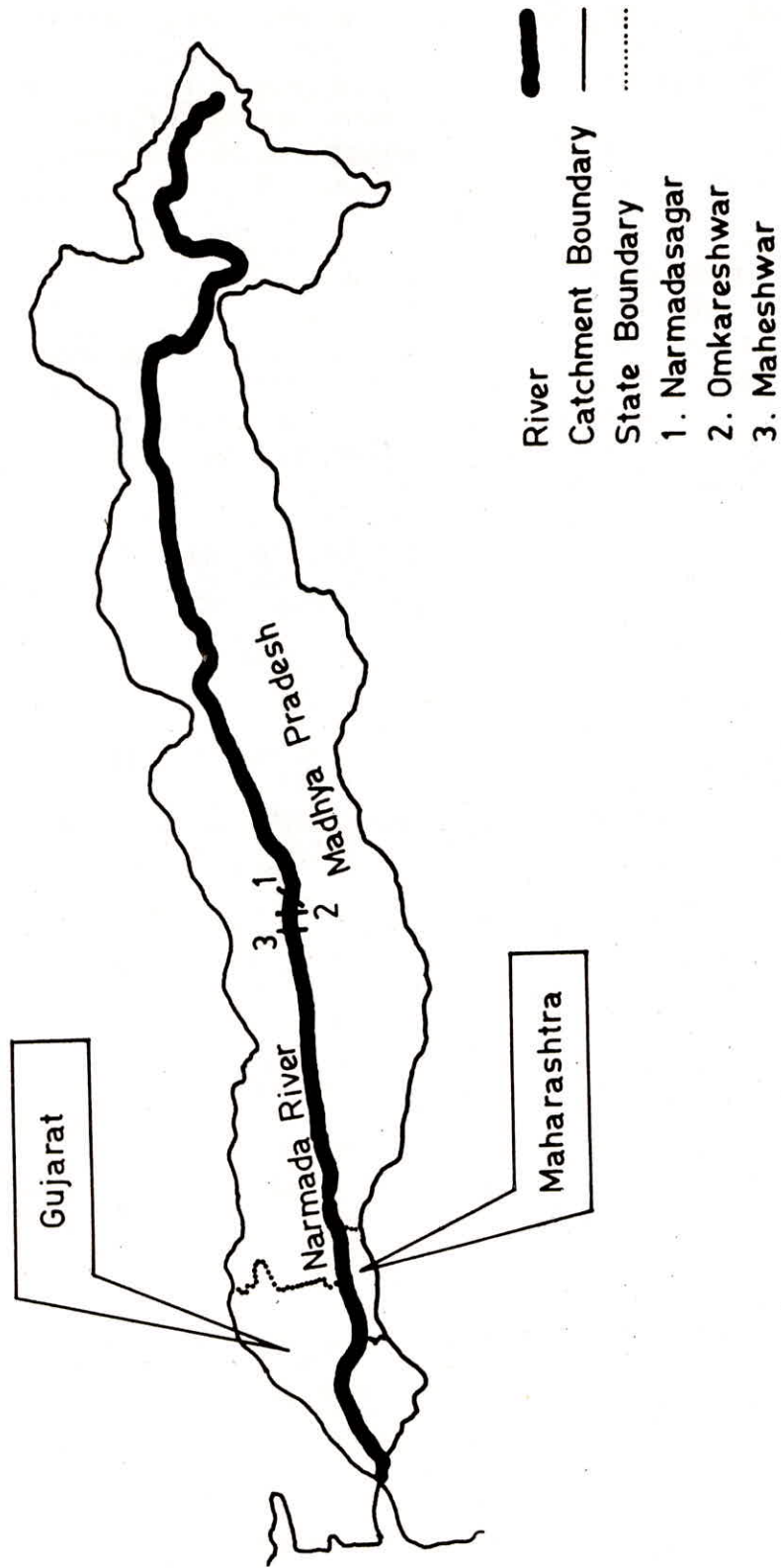


Fig.1. Map of Narmada Basin

considering any penalty. The power tariffs and load factors are assumed. The water year starts from 1st July. The monsoon is well defined and lasts for four months from July to October.

The level of discretization has been chosen by and large by practical experience and limited by the computer capability. The formulation has to limit the number of states for the state variable just enough to yield appropriate accuracy of the policy thereby saving the computer storage as well as the computation time and cost. But this decision is better arrived at through a systematic study. The present study has investigated this point also as one of the objectives. An attempt has been made to prepare an operation procedure for the planning purposes using the optimal policies obtained for individual years of the record.

### FORMULATION OF THE ALGORITHM

The procedure uses the backward recursive equations for the various stages. The state variable is  $q$ , a part of the live storage  $Q$  which is discretized into convenient number of states. In the formulation, months counted from June are taken as stages. The other variables of the formulation are

$X_{inf}$	=	monthly inflows (MCM)
$X_I$	=	irrigation release (MCM)
$X_p$	=	power release (MCM)
$E$	=	evaporation (MCM)
$V_i, V_p$	=	irrigation and power benefit functions

It is assumed that there will be no over year storage to start with. The recursive equation for the first stage will be

$$f_1(q_1) = \text{Max} \left\{ V_i(X_{I,1}) + V_p(X_{p,1}) \right\}$$

$$\text{subject to } 0 \leq q_1 < Q$$

$$X_1 = X_{I,1} + X_{p,1}$$

$$X_1 \geq X_{I,1} \text{ minimum for the concerned month}$$

$$0 \leq X_{p,1} \leq X \text{ power plant capacity}$$

The continuity equation that has to be respected as a constraint is

$$q_1 + X_{inf,1} - E_1 - X_1 = 0$$

For the second stage

$$f_2(q_2) = \text{Max} \left\{ V_i(X_{I,2}) + V_p(X_{p,2}) + f_1(q_1) \right\}$$

with constraints as for the first stage

$$0 \leq q_2 \leq Q$$

$$X_2 = X_{I,2} + X_{p,2}$$

$$X_2 \geq X_{I,2} \text{ minimum for the concerned month}$$

$$0 \leq X_{p,2} \leq X \text{ power plant capacity}$$

$$q_2 + X_{inf,2} - E_2 - X_2 = q_1 \text{ (Equation of continuity)}$$

For the last stage

$$f_{12}(q_{12}) = \text{Max} \left\{ (V_i (X_{I,12}) + V_p (X_{P,12}) + f_{11} (q_{11})) \right\}$$

With constraints similar to those above with appropriate values.

#### OTHER SALIENT DATA USED IN THE ALGORITHM

- i. Planned irrigation utilisation as furnished in Table 1
- ii. Yearly inflow into Narmadasagar as shown in Figure 2
- iii. Typical set of storages and the corresponding state for the case of 25 states, shown in Table 2
- iv. Tailwater level for Narmadasagar power house 196.60m
- v. Benefit from irrigation 8.8 paise per cubic meter of water
- vi. Power benefit at bus bar Rs.0.30/KWh
- vii. Efficiency of power generation 85 percent

#### DISCUSSION OF RESULTS

##### Effect of number of States on the benefit function

For a given set of parameters, namely FRL of Narmada Sagar at 262.13m (12212MCM) installed capacity of 1000MW, the effect of number of states of the state variable on the benefit function is studied. The number of states considered are 9, 15, 20, 25, 40. The curve relating number of states versus total benefit is shown in Figure 3. It can be seen that the total benefit does not vary much beyond 15 states. For this study IBM 370/155 with 512K at IIT, Madras was used. As the states increase, the computation time required also increases. Table 3 shows the CPU time for the various states studied. Even though fifteen states is good enough, 25 states were used in this study.

##### Effect of sizing of Narmada Sagar Reservoir (NWDT) on benefits

Eventhough Narmada Water Disputes Tribunal has stipulated that the FRL of Narmada Sagar reservoir should be 262.13m (12212MCM) the effect of decreasing the FRL on the total benefit accruing out of Narmada Sagar was investigated. The FRL considered are 256.0 M, 257.6 M, 259.0 M, 260.67 M and 262.13 M respectively. For different FRL's the installed capacity of power installation is assumed to be 1000 MW. The total benefit as a function of full reservoir capacity is shown in Figure 4. There is a steep rise in benefits from FRL of 260.67 m to 262.13 m. The maximum benefit being at the maximum storage of 12212 MCM (262.13 M).

##### Effect of installed capacity

Installed capacities of 875, 1000, 1125 and 1250 MW were tried at Narmada Sagar. A plot of installed capacity vs total benefits in Figure 5 shows that the total benefit increases steeply from 875 MW to 1000 MW as compared to installed capacities beyond 1000 MW. A typical benefit stream for installed capacity of 1000 MW is shown in Figure 6.

##### Operating policy

For deriving the operating policy, FRL is taken as 262.13M (12212 MCM) and installed capacity as 1000 MW. Depending upon the inflow during monsoon, an year can be classified as very dry, dry, normal or wet depending upon whether the inflow is less than 4600. MCM

**TABLE 1. PLANNED IRRIGATION UTILISATION -  
1991-92 SCENARIO**

Month	Irrigation demand in MCM
July	2.747
August	2.324
September	14.057
October	17.785
November	15.441
December	9.187
January	16.725
February	17.855
March	9.057
April	7.523
May	4.677
June	8.622

**TABLE 2. DETAILS OF STATE FORMULATION**

State	Elevation in metre	Capacity in MCM
1	243.20	2467
2	244.60	2873
3	246.00	3279
4	247.30	3685
5	248.40	4091
6	249.51	4497
7	250.61	4903
8	251.69	5309
9	252.78	5715
10	253.52	6121
11	254.17	6527
12	254.83	6933
13	255.58	7339
14	256.13	7745
15	256.75	8151
16	257.37	8557
17	257.99	8963
18	258.61	9369
19	259.18	9775
20	259.67	10181
21	260.15	10587
22	260.64	10993
23	261.05	11399
24	261.61	11805
25	262.13	12212

**TABLE 3. CPU TIME FOR VARIOUS STATES**

Number of States	CPU Time
9	40 Sec.
15	1 min. 39 Sec.
20	2 min. 50 Sec.
25	4 min. 36 Sec.
40	13 min. 12 Sec.

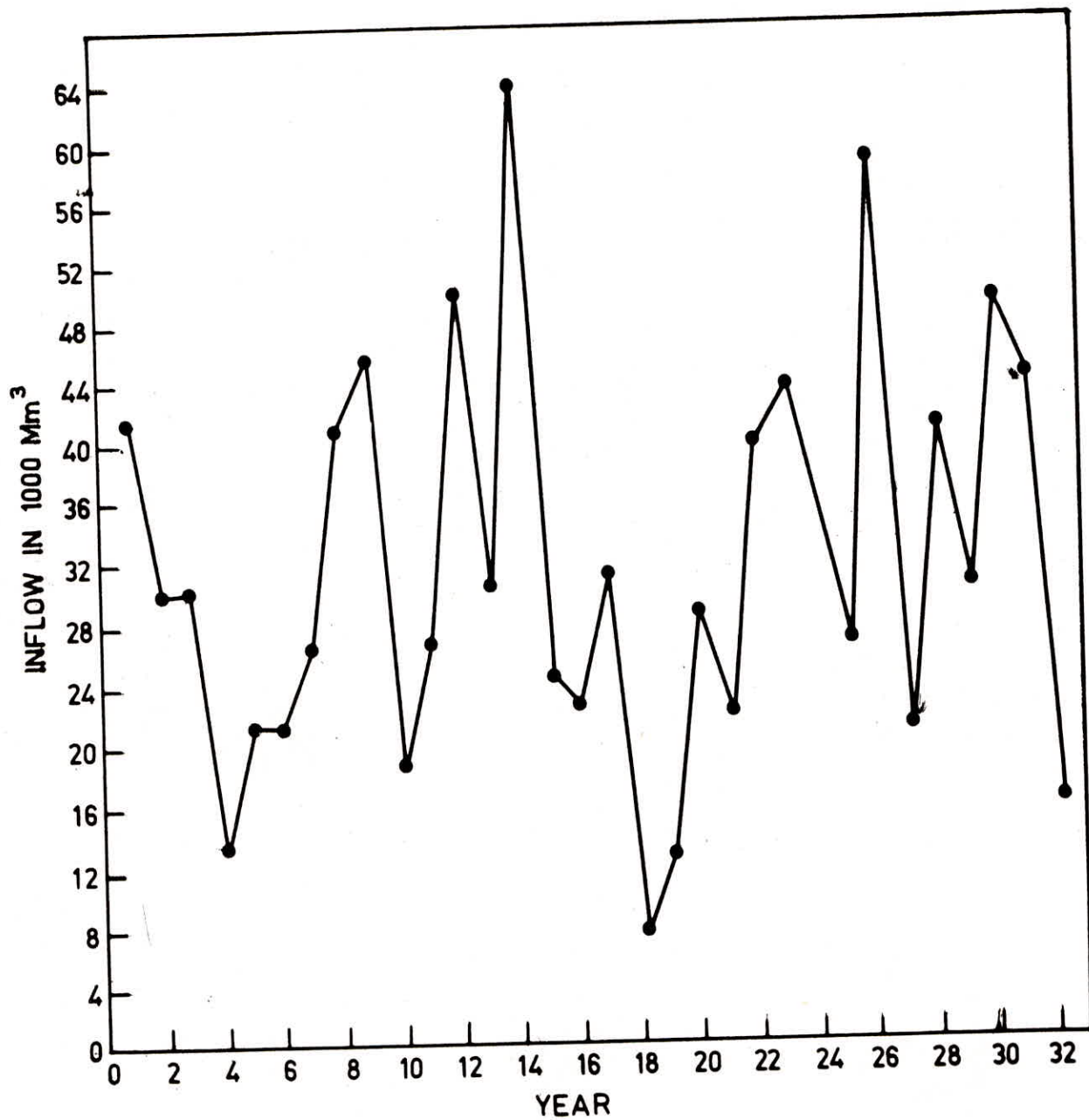


FIG.2. YEARLY INFLOW TO NARMADA SAGAR

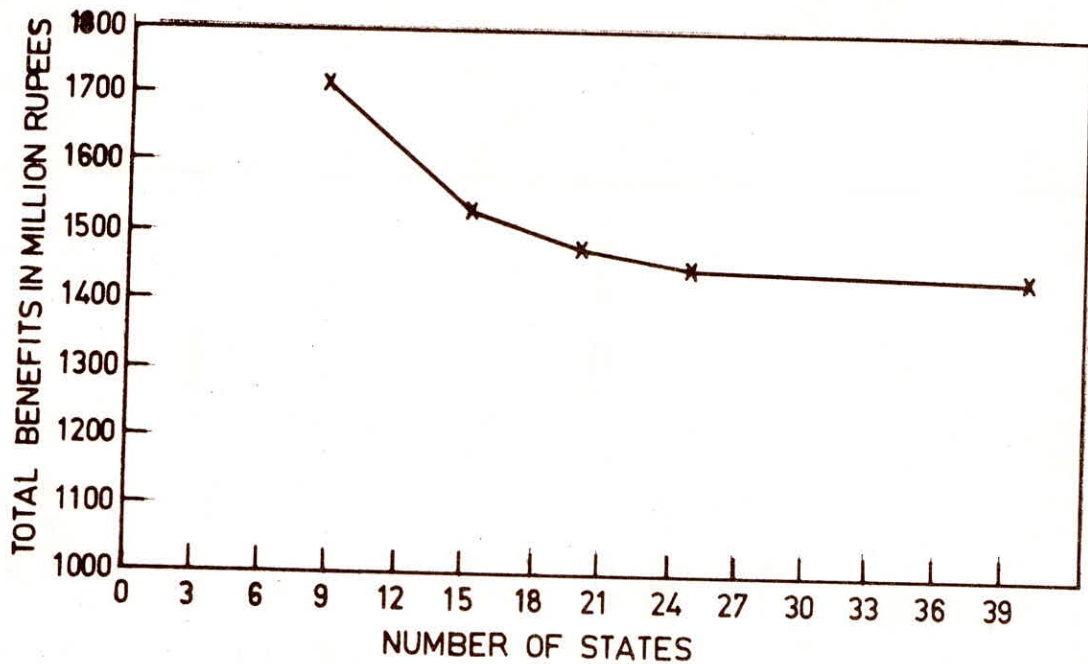


FIG.3.VARIATION OF TOTAL BENEFITS WITH NUMBER OF STATES

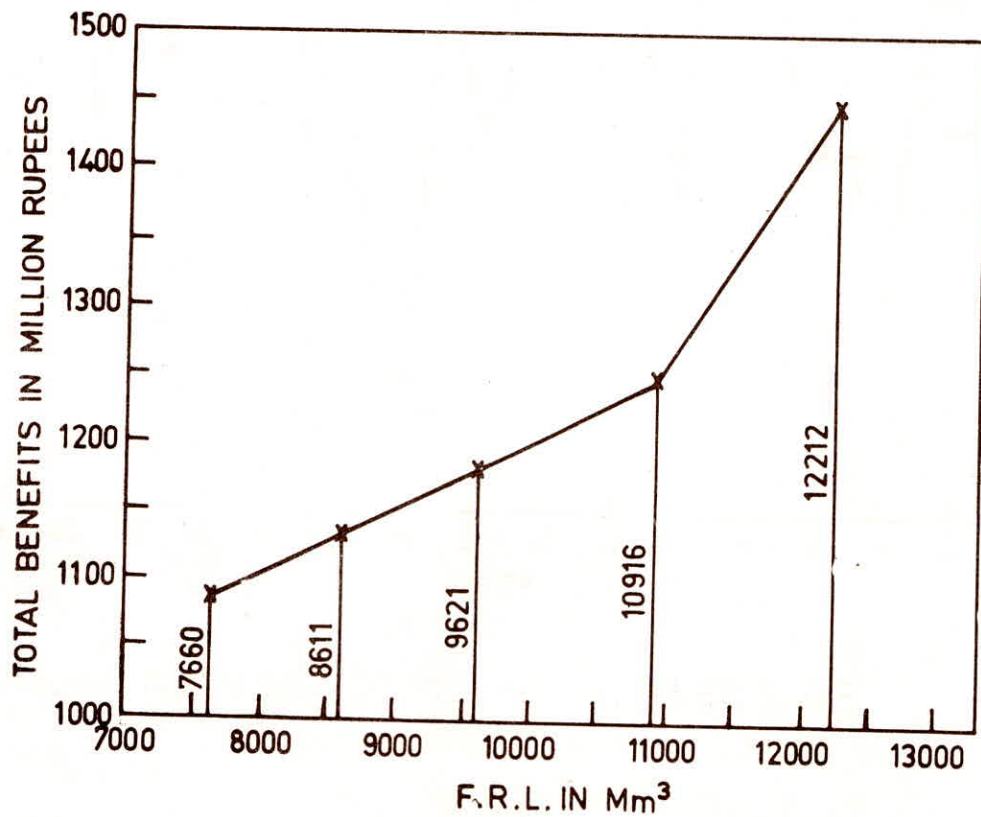


FIG.4.VARIATION OF TOTAL BENEFITS WITH RESERVOIR CAPACITY



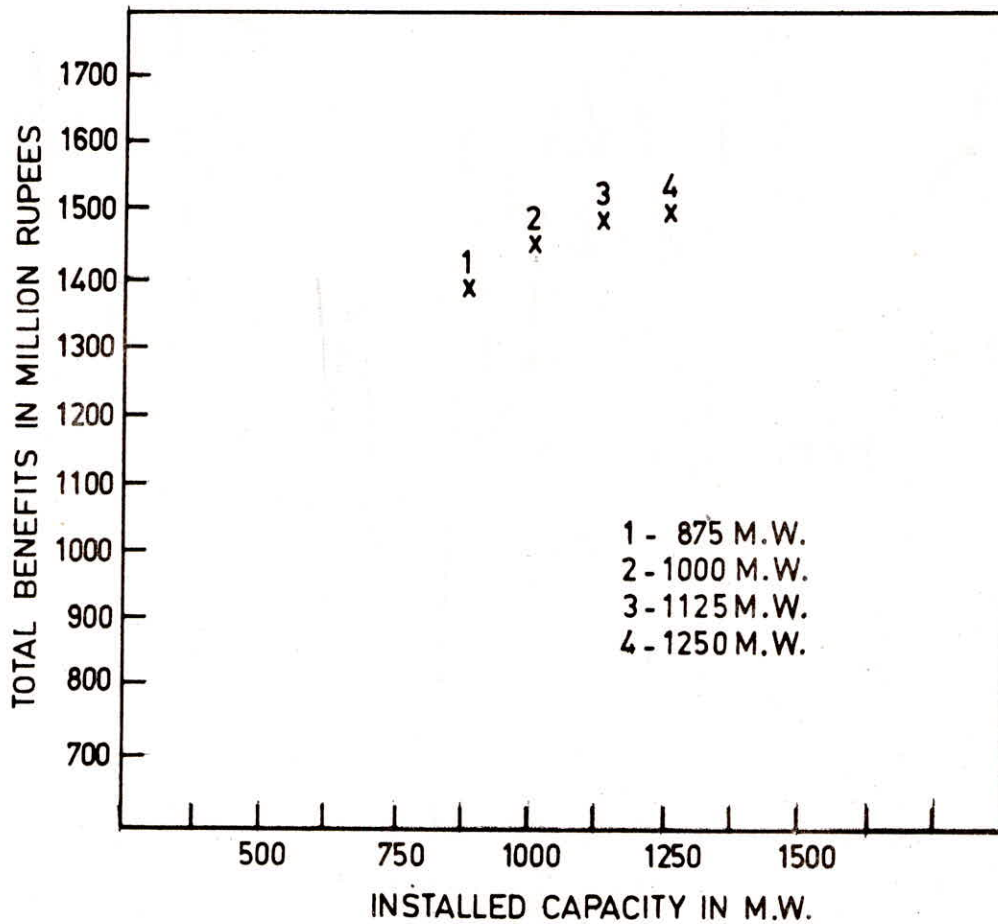


FIG.5. VARIATION OF TOTAL BENEFITS WITH INSTALLED CAPACITY

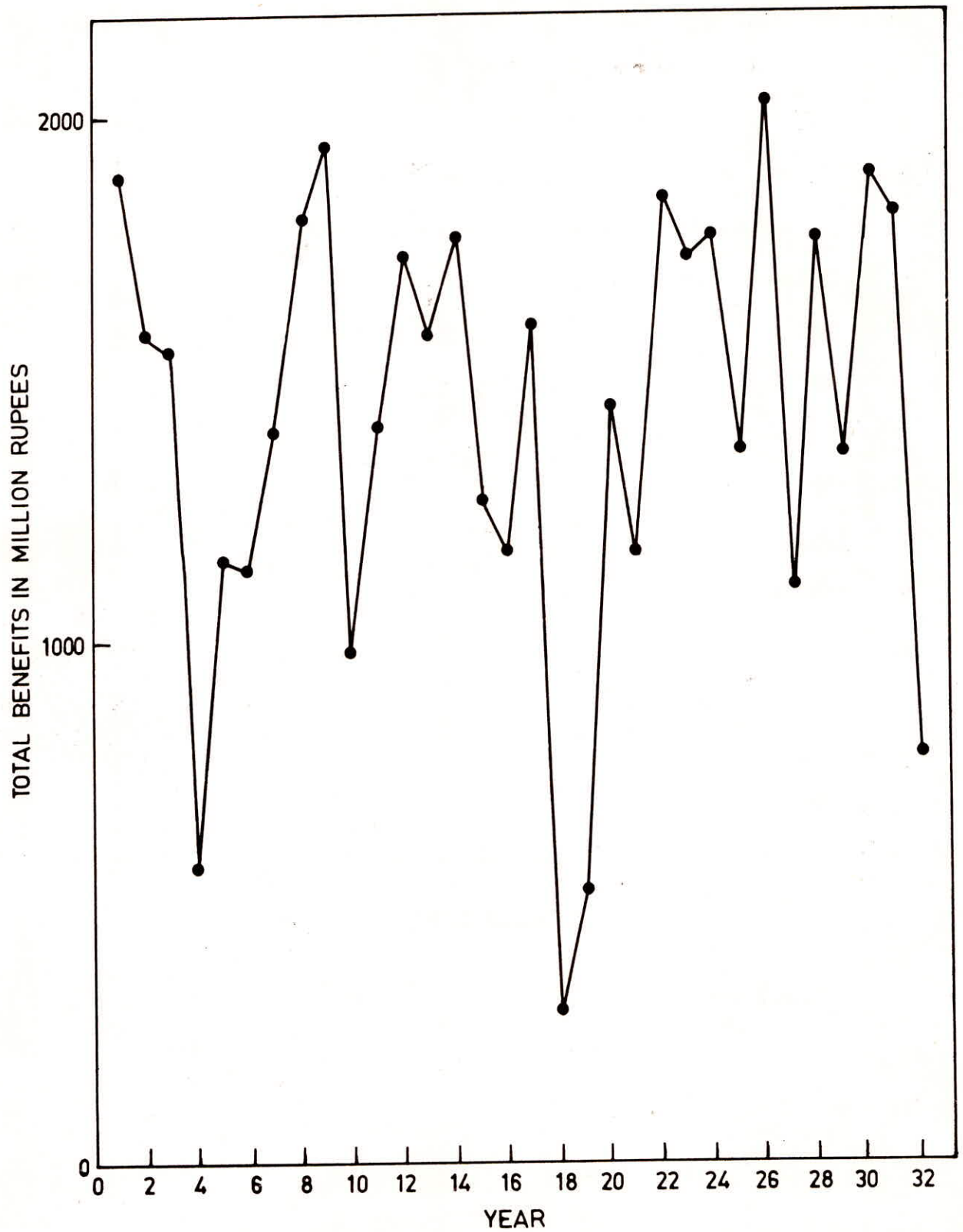


FIG.6.BENEFIT STREAM FOR INSTALLED CAPACITY OF 1000 M.W

4600 MCM-16250 MCM, 16250 MCM-28750 MCM, more than 28750 MCM respectively. The reservoir operating rule for each of the above categories is shown in Figure 7. Briefly the following steps are followed to arrive at the operational policy.

1. After the July flows are realised, the expected monsoon flow are estimated from the graph July flow Vs monsoon flow (Figure 8). Using the estimated monsoon flow the appropriate policy is used for operation in the month of August.
2. Again, the monsoon flows are re-estimated from the graph drawn between the sum of July and August flows Vs Monsoon flows (Figure 9). The new operation policy, if change is indicated, is followed from September onwards till the end of June.
3. The regression of monsoon flows between the successive years is not definitive to start with a suitable operation policy from July itself.

### CONCLUSIONS

Using Dynamic programming approach, an operating rule for Narmada Sagar reservoir is suggested. The full reservoir level (262.13 M) of Narmada Sagar as stipulated by Narmada Water Dispute Tribunal gives higher benefits compared to the other lower FRL'S. An installed capacity of 1000 MW appears to be an optimum. For obtaining improved operation of policies, the stochastic nature of inputs has to be considered.

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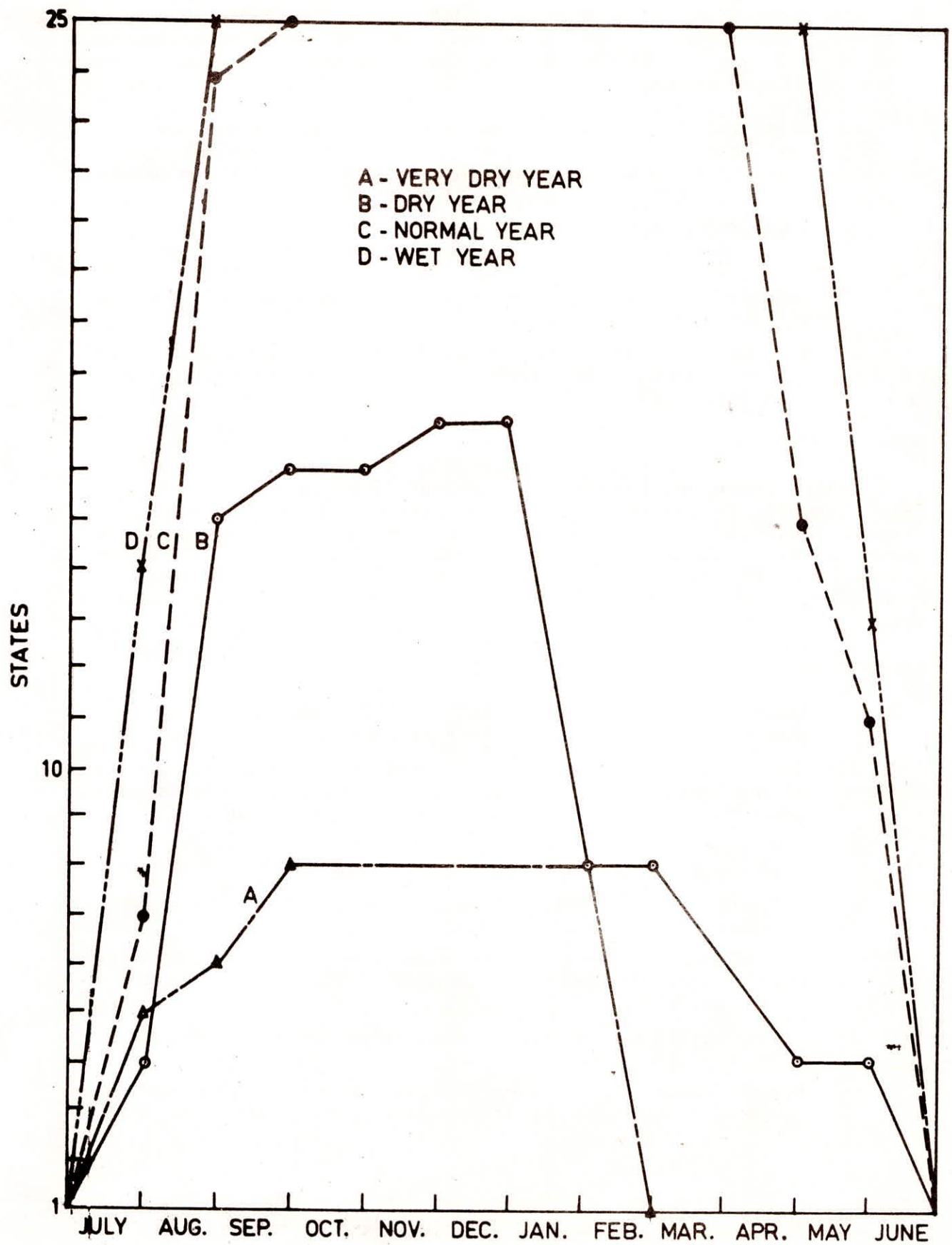


FIG. 7 RESERVOIR OPERATION RULE

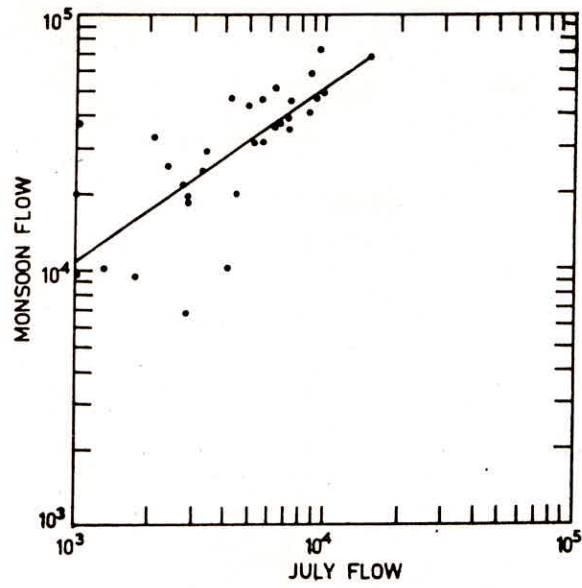


FIG 8 JULY FLOW Vs MONSOON FLOW ( $Mm^3$ )

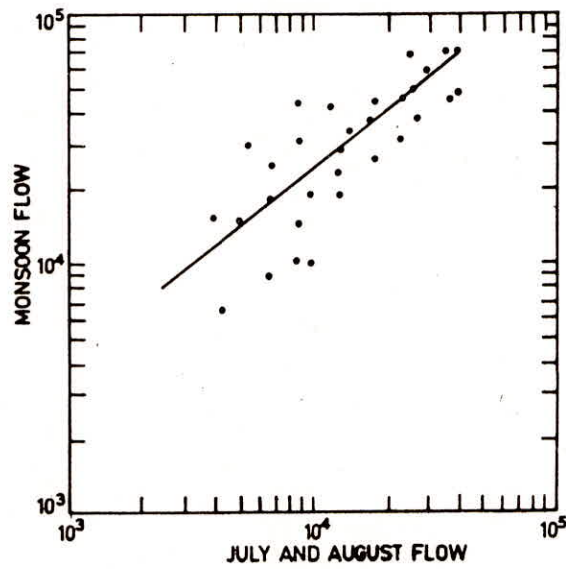


FIG 9 JULY AND AUGUST FLOW Vs MONSOON FLOW ( $Mm^3$ )