RN-34

CONJUNCTIVE USE OF SURFACE AND GROUNDWATER

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CONTENTS

LIST OF FIGURES

ABSTRACT

In a general sense, factors that must be considered in conjunctive use analyses have been identified and discussed. The factors have been distilled from the work of many authors who have examined numerous conjunctive systems. The methods adopted by several researchers in optimal management of conjunctive use system are critically discussed. A strategy for conjunctive use of surface & ground water based on linear programming and discrete kernel approach is proposed whose solution is very easy to obtain on a computer and an optimal cropping pattern in a reservoir command can be obtained. In this approach, rain water was also taken into account, while calculating crop water requirement. Hence, in effect, this methodology can broadly be identified as conjunctive utilization of rain-surface-ground water. This strategy will also be useful to check water-logging near canal reaches through one of the constraints.

1.0 INTRODUCTION

When multiple sources of water with different characteristics are available, it may be possible to develop an operating strategy that exploits the difference of sources. This exploitive strategy has become known as the conjunctive management of different sources of water or "Conjunctive Use".

Conjunctive Use can be defined as the coordinated and planned utilisation of two or more sources of water. The concept of conjunctive use is a way of thinking about water utilisation. For most of the places,future demands for water can not be met entirely from new surface reservoirs, because economically feasible storage sites are limited. Optimum beneficial use of water can be obtained by conjunctive use. Water is available on earth in different forms and at differtent positions. Several types of sources of water on earth include :

(i) Surface fresh water in streams, lakes, reservoirs, estuaries, ponds and swamps;

(ii) Fresh ground waters in water table conditions, artesian aquifers, coastal aquifers, fractured rocks, karst and lava aquifers, etc;

(iii) Precipitation from atmosphere in the form of rain, snow, ice, water vapour etc.;

(iv) Soil moisture;

(v) Surface or Subsurface brackish waters with varying nature of salinity;

(vi) Sea water, mixed estuarine water or desalinated water

; and

(vii) Effluent waters which may be partially, fully or non treated. Depending on the availability of demands, one generally resorts to one or more of these sources. In case the interactions between the several sources are exploited in the efficient use of the above sources then water is said to be used conjunctively. Conjunctive use implies not only the use of several different sources of water but also their exploitation through efficient use in technoeconomic terms.

Most attention has been given in the past to two combinations of conjunctive water use: (i) Surface and subsurface sources of water, and (ii) Effluent urban and surface sources of water. The former case has been studied in detail by many groups around the world. The latter has been somewhat classical from the time of first recycling of effluent waters, mixing them with the surface waters. Recently, the new cases are being well investigated, namely (iii) Subsurface and effluent sources of waters, and (iv) Surface, subsurface and effluent sources of waters. Adding the unconventional sources of water, which will become conventional with time, a large number of combinations results.

1.1 Various Schemes of Conjunctive Water Use

Yevjevich (1978) described several typical schemes for conjunctive water uses. For simplicity, only two sources of water, say A and B, are dealt though the schemes may be easily extended to three or more sources of water.

1.1.1 Source and user separated schemes

This type of scheme, as given in Fig.1, has a source A supplying a well defined part of the demand and the user's side of the scheme. The system may well permit a flexibility on the user's side, namely that in case of deficits or surplus of water in either of two sources, the portions of user's demand may be allocated for a larger or a smaller delivery by each source. In other words, there should be interconnections between the two parts of the water supply area or grid, so that this flexibility may be practiced whenever needed or economical.

Water flows from the two sources only in the direction of the water demand. The example of this type of scheme is a twin city, each with its own water supply (say one with surface reservoirs ,and the other with a large aquifer water supply), with connections for interchange of water surpluses or alleviating particular deficit in one of the parts.

1.1.2 Source separated but user integrated schemes

This type of scheme, as given in Fig.2, has both sources supplying water in a well integrated water supply network. The integrated users draw the water from either of the sources in a unidirectional manner, so that the water can flow only from the sources of water towards the users, for an

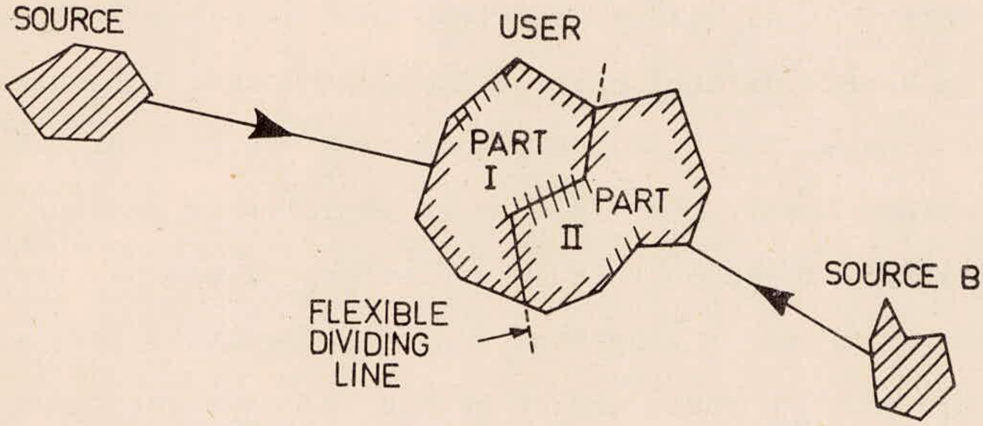

Fig.1. Conjunctive water use of source and user separated schemes

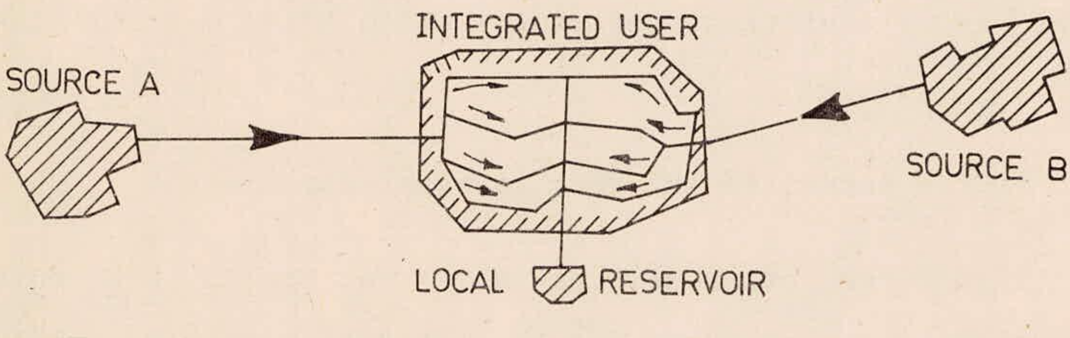

Fig.2. Conjunctive water use of separated sources of water, but integrated user schemes

additional flexibility of satisfying its water demand, the users may install an additional water storage capacity.

The conjunctive use of two (or more) sources of water is then made, and integrated, indirectly by the user's system of water distribution network or by a similar distribution.

A typical example is an industrial complex, with a distribution system of pipelines within this complex, receiving water from two different sources of water, mixed only either in the distribution system or prior to it in a small or large water storage capacity.

1.1.3 Both, source and user integrated schemes

This type of scheme enables not only a supply of water to the user from any of the sources of water, but also enables a partial or a full interchange of water between the sources, for the major purpose of a better storage of water surplus or a change in water quality, by shifting water between the sources. These schemes may have four alternatives depending on the unidirectional or bidirectional (or multidirectional) shifts of water and whether the shifts occur through the user's network, through special connection, or through both.

(i) Unidirectional shift through user network is represented by the case of Fig.2, except that the surplus water from a source, say source B, may flow through the network to source A, and there be stored, because of no or limited storage at source B.

(ii) Bidirectional shift through the user network is also represented by Fig.2, except that water can flow to each source from the other source, for any particular purposes (say surplus surface water at source A is shifted for groundwater recharge at source B, and groundwater of source B is shifted to source A to improve quality of stored water)

(iii) Unidirectional shifts through a special connection between the sources, as well as through the user network is shown in Fig.3, from source B to source A. This may be necessary because the use of the long lines from source B to the user, with the supply of water to the user, and then the long backflow from the user to source A, may be uneconomical.

(iv) Bi-directional shifts between two sources, with storage of any water surplus at any storage facility, and water mixing for quality purposes are represented by Fig.4.

1.2 Conjunctive Use of Surface and Groundwater Reservoirs

The concept of conjunctive use of surface water and ground water is predicated on surface reservoirs impounding streamflow, which is then transferred at an optimum rate to ground water storage. Surface storage in reservoirs behind dams supplies most annual water requirement. While the groundwater storage can be retained for cyclic storage to cover years of subnormal precipitation. During periods of above normal precipitation , surface water is utilised to the maximum extent

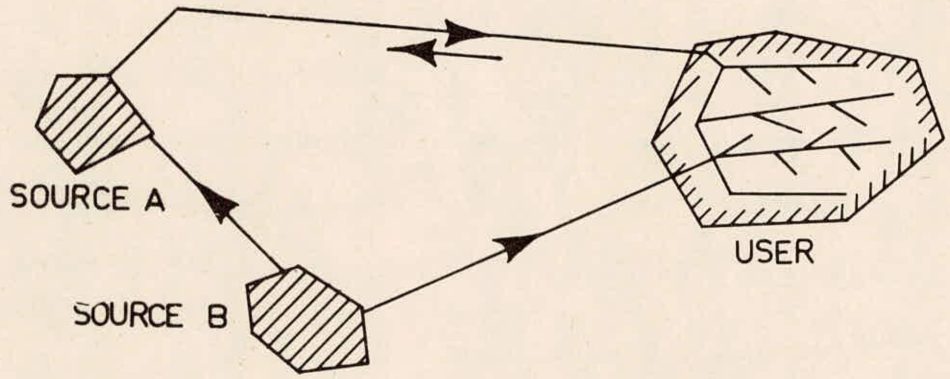

Fig.3. Uni-directional shifts of water from source B to source A, both by the direct connection and the user's network

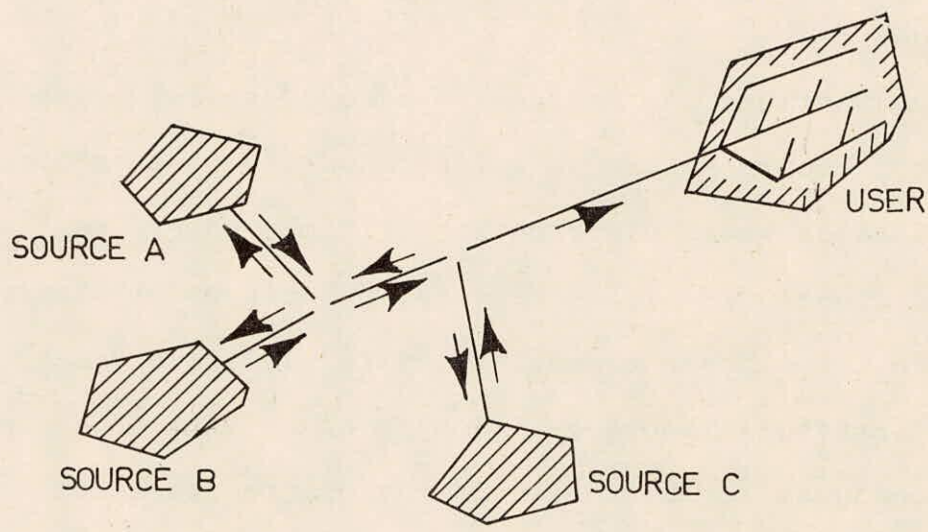

Fig.4. Bi-directional water interchanges between sources of water, before the water supply to user, for purposes of better meeting demand, storage requirement, and water quality control (through mixing of waters of different qualities)

possible and also artificially recharged into the ground to augment groundwater storage and raise water table. Conversely, during drought periods, limited surface water resources are supplemented by pumping ground water, thereby lowering groundwater table. The feasibility of the conjunctive use approach depends on operating a ground water basin over a range of water levels; that is, there must be space to store recharge water, and, in addition, there must be water in a storage for pumping when needed.

Management by conjunctive use requires physical facilities for water distribution, for artificial recharge and for pumping. The procedure does require careful planning to optimize use of available surface water and ground resources. Such operations require competent personnel, detailed knowledge of hrdrogeology of the basin, records of pumping and recharge rates, and continually updated information on ground water levels and quality.

A conjunctive use management study requires data on surface water resources, ground water resources, and geologic conditions; data on water distribution systems, water use, and waste water disposal are also necessary. A basin model simulate the responses of a basin to variations in variables such as natural and artificial recharge and pumping so that the best operating procedures for basin management can be practiced.

1.3 Advantages

Todd (1980) has discussed merits and demerits of conjunctive use of surface and ground water. Following are the advantages of conjunctive use systems.

(1) Greater Water Conservation- operation of both surface and ground water reservoirs provides for large water storage.

(2) Smaller Surface Storage- ground water storage can provide for water requirements during a series of dry years.

(3) Smaller Surface Distribution System- greater utilisation of ground water from widely distributed wells.

(4) Smaller Drainage System- pumping from wells aids in controlling the water table.

(5) Reduced Canal Lining- seepage from canals is an asset because it provides artificial recharge to ground water.

(6) Greater Flood Control- release of stored surface waters for artificial recharge requires less control reservation and furnishes both water conservation and flood control.

(7) Ready Integration with Existing Development- generally conjunctive operations occurs after extensive basin development, but integration can be made to increase water supplies without loss of investment in existing pumping plants.

(8) Stage Development Facilities- final compltion of projects may require 20 to 40 yrs., hence development by stages is desirable as it reduces the idle

potential of the project; stage construction of surface reservoirs is costly, but can be minimised with smaller reservoirs.

(9) Smaller Evapotranspiration Losses- greater underground water storage with lowered ground water levels reduces losses.

(10) Greater Control Over Outflow- surface waste and subsurface outflow are reduced by conjunctive use, thereby providing greater water conservation.

(11) Improvement of Power Load and Pumping Plant Use Factors- in areas which can be served by either surface or ground water, surface water can be released for irrigation during peak power demand periods to effect a saving in power costs.

(12) Less Danger from Dam Failure- should failure ever occur, the smaller the dam and reservoir storage, the smaller the damage.

(13) Reduction in Weed Seed Distribution- with a smaller surface distribution system there is less opportunity for spread of noxious weed seeds.

(14) Better Timing of Water Distribution- an irrigation prefers to have water available when he wants it, as from a pump, than to take water on schedule from surface conduits.

1.4 Disadvantages

(1) Less Hydroelectric Power- smaller surface reservoirs generate less energy and conjunctive use operation

provides less from power.

(2) Greater Power Consumption- more pumping and from greater depths.

(3) Decreased Pumping Efficiency- large fluctuations in ground water levels reduce pumping efficiency.

(4) Greater Water Salination- natural and artificially recharged ground waters contain more dissolved solids than surface water does.

(5) More Complex Project Operation- greater supervision of project operation is required and artificial recharge works need careful management.

(6) More Difficult Cost Allocation- varying water supplies from two different sources require analysis to fix equitable water rates.

(7) Artificial Recharge is required- this is costly to operate, difficult to accomplish on land containing relatively impermeable sub soil, and occupies land otherwise available for agricultural purposes.

2.0 REVIEW OF LITERATURE

The complexities of the problem of conjunctive operation of ground and surface water facilities and the advantage of conjunctive utilization of these two forms were formally recognised nearly four decades ago. Since then, several analytical approaches for conjunctive utilization have been developed.

Authors who have dealt with the problem of conjunctive use of ground and surface water systems such as Clenderen (1954), Thomas (1957), Macksoud (1961), and others, have discussed the economic advantages of such combination and have pointed out its effectiveness in the conservation of sizeable volumes of water. When these authors have dealt with the problems of economic optimization, the methods of analysis are based upon investigation of a limited number of alternatives and selection of the best one according to the benefit-cost ratio during the economic life of the project. The work of these authors, however has been concerned mainly with the engineering problems on the design and operation of the conjunctive use system.

Todd (1959) indicated positive economic factors in conjunctive use, including greater water conservation, smaller surface storage and distribution system, better flood control, ready integration with existing development, less danger from dam failure, and better timing of availability of water for distribution.

Fowler (1964) has suggested that solving the engineering problems associated with the development of a conjunctive use system requires a thorough understanding and investigations of the geology of the ground water basin, of the hydrology of surface and ground waters, of the existing surface and ground water facilities including storage and transmission characteristics, and of existing and expected water demands and the economics associated with meeting these demands. Fowler states that where ground water basins can be operated in a fully integrated fashion with surface water supplies, then optimum use of water resources can be achieved. However, in order to achieve this integrated operation, new methods and institutions must be devised to coordinate and manage the operation.

Saunders (1967), states that in order to assess the value of planned conjunctive use in relation to a particular area or basin, it is necessary to look at the economic, hydrologic, and legal system as a whole. A planning procedure is then presented to enable a planninng agency to determine, at minimum cost, the feasibility of planned conjunctive use. The procedure consists of determining system characteristics.

The various analytical approaches that have been followed so far towards optimization of conjunctive use of water resources may broadly be grouped into three categories. The first of these considers the problem from a resources allocation view point and makes use of mathematical programming techniques for optimization. The second approach is based on groundwater basin simulation. Various feasible alternative plans of surface and groundwater use are evaluated in terms of a groundwater basin operation. The optimum combination is then selected according to the criteria of economic optimization. The third approach is a combination of the first two.

2.0.1 Mathematical Programming Approach

The techniques of dynamic programming and linear programming were commonly used among the studies of this category. Castle and Lindeborg (1961) formulated a linear programming model to allocate water from two sources (ground water and surface water) to two agricultural areas. Buras (1963) adopted the dynamic programming technique to optimize the conjunctive operation of water released from two storage sources for irrigation in two agricultural areas. The optimization problem involved the solution of three problems (1) determination of design criteria for the surface storage and recharge facilities (2) determination of the extent of the system service area, and (3) determination of the operating policy specifying the reservoir releases and aquifer pumpage. The operating policy was developed for a number of seasons using the logic of dynamic programming. Burt (1964) used dynamic programming to derive decision rules for the optimal allocation of water resources. The decision rules were based on the volume of water pumped in each season which was a function of the storage available at the beginning of the season. The optimum policy was determined based on the maximum present worth of net benefits. Dracup (1965) used a parametric linear programming model to optimize the ground-water-surface water system. The optimal policy minimized the costs of water importation, storage, boostage and artificial recharge. Five sources of water were used to satisfy three requirements. The analysis extended over a 30 year period and three different decision rules were analysed. Aron (1969) used dynamic programming to optimize the conservation and use of a groundwater-surfacewater system involving several streams, reservoirs, recharge facilities, distribution pipelines and aquifers. The complex system was sub-divided into smaller subsystem and wherever the interdependence between these subsystems was relatively small, they were optimised independently. Milligan (1970) developed several linear programming approach models for economic optimization of the use of surface and groundwaters. The models were formulated for one hypothetical basin and two real basins. Cochran and Butcher (1970) used a dynamic programming model to determine the optimal allocation of existing water with possible augmentation from imported water to Las Vegas Vally. Nev.Longenbaugh (1970) developed a linear programmimg model allowing for a constant interaction between an aquifer and connected stream. This interaction was assumed to be unaffected by pumping of the aquifer. Although this poses practical limitations on its use, the model was applied to the Arkansas River Valley in Colorado.

Yu and Haimes (1974) have developed a multi-level optimization technique for conjunctive use of water for complex systems emphasizing hierarchical decision making in a general sense. The basin was divided into several subregions and each subregion was optimized separately. They conclude that the aquifer is the key element in optimal operation of conjunctive use systems. Chaudhary et al (1974) used a decomposition and multilevel optimization technique for optimal conjunctive use of water in the Indus basin in Pakistan. The submodel was to minimize the cost of supplying water to meet given irrigation water requirment. Maddock (1974) developed operating procedure and rules for conjunctive use when both demand and supply of water are stochastic. Jonch-Clausen (1979) used iterative quadratic programming to optimize the allocation of water resources considering economic and hydrologic characteristics of river basin.The basic element in the planning model was a single period, single objective allocation model. Other works of significance are Haimes (1973), Moody (1976) and Boster and Martin (1977).

2.0.2 Simulation Approach

Tyson and Weber (1964), working on the Los Angles basin, have developed a groundwater model to understand better the mechanisms which comprise the groundwater resources and to predict what might happen under various future conditions. The groundwater basin was divided into small polygonal zones, the size of which was dependent on the variations in replenishments, extractions, transmission, storage and water level data. The non-linear partial differential equation of groundwater flow was approximated by a set of finite difference equations at each node point and these equations were solved by the Gauss-Siedel iterative procedure. The basin parameters were obtained by simulation on an analog computer and comparison with historical data was made. The values so obtained were used in the model for performing operational studies on the basin. This model was used by Chun, Mitchel and Mido (1964) for determining the most optimal plan for conjuctive operation of the surface water ground water system. Alternative plans were presented in terms of a groundwater basin operation as a combination of four decision variables: (1) areal pattern of groundwater extraction; (2) methods of prevention of sea water intrusion; (3) schedules of artificial recharge at specific locations; and (4) pumping schedule at fixed locations. For each alternative, analysis was carried out separately for the surface and subsurface systems. The final optimum alternative combination of surface and groundwater facilities was selected according to the criterion of minimizing the annual costs. The approacn was essentially one of trial and error. Also, a cost minimizing procedure is not necessarily the most economical nor it is the proper measure of objective for all situations.

Since the work of Tyson and Weber, a host of groundwater models have appeared in literature which can be used to solve the problem of economic optimization of the surface water groundwater system using the trial and error technique outlined by Chun, Mitchel & Mido above. A review of the various groundwater modelling technique was given by Prickett (1976).

A classical example of the application of simulation models for conjunctive use of surface and groundwater is that developed for the Indus Valley in Pakistan to find a solution to the mounting problems of salinity and waterlogging in the irrigated areas of the basin (Thomas and Burden, 1965) Similar studies have been performed for the Indo-Gangetic plain (Revelle and Lakshminarayana, 1975) for more efficient use of the monsoon flow in the river system.

2.0.3 Combination of Simulation and Programming Approaches

A recent development has been the integration of the two approaches outlined above, that of groundwater modelling and mathematical programming for economic optimization. Aguado (1976) used a technique for incorporating numerical groundwater flow models in linear groundwater management models. In these models, the finite difference forms of the groundwater flow equations were introduced into the linear programming model as a set of constraints. The groundwater variables were included directly as decision variables in the linear programming formulation. Young and Bredehoeft (1972) developed a basin-planning simulation that incorporated time and space relationships of a stream aquifer system. The stochastic properties of surface flow, and the response of water users to hydrologic, economic and institutional conditions. A planning stage linear programming model that determined the type of crop and the acreage planted on the basis of estimates of water availability and a monthly operating model that determined actual irrigation procedures, were used as economic models.

Dimensionality problems and computability changed analysis direction as more sophisticated representations of reality were attempted. Maddock (1974) introduced a quadratic programming model for operating a stream aquifer system under stochastic demand and supply. A linear groundwater model (response matrix) was used. The impact of pumping at one place upon another location was determined in advance to avoid costly repetitive groundwater dynamics computations. Groundwater pumping costs were modeled as quadratic functions of the total lift (drawdown plus initial lift) and the quantity of water pumped. DeRidder and Erez (1977) used linear programming technique to determine optimum solutions for irrigation water supply for surface and underground sources. The optimization was done to minimize the costs of supply, the surface water being limited in that area. The groundwater model was then used to test the impact these solutions might have on the watertable, and only those solutions were permitted which did not have any undesirable effects on the watertable.

2.1 Management Strategies for Crop Water Use

The development of optimal cropping pattern is a function of management strategies available for crop water use. The important factors which determine the management strategy for crop water use include the rate of water consumption by the crop, its tolerance to moisture stress, the physiological stage of its growth, the soil moisture conditions and the timing and availability of other inputs. Strategies for management of irrigation water would be different for different objectives. For example, a strategy which manages soil misture content for maximum crop yields would be different from that which maximizes profits. A management strategy constitutes an optimal irrigation policy which determines the amount and frequency of irrigation that meets a specified objective.

2.1.1 Irrigation Management Programmes

Development of mathematical models to generate irrigation management programme has received the attention of many researchers. Flinn and Musgrave (1967) presented a dynamic programming model having one state variable, namely, the quantity of irrigation water available for application over the remainder of the season. Hall and Butcher (1968) proposed a dynamic programming model with two state variables, the quantity of water available for application over the remainder of the season and the soil moisture condition at the beginning of the season. In this approach, the growing season of a crop was divided into a number of stages determined by its physiology. The information on the response of the crop to different deficits at such stage, in terms of the final yield recorded, was basea on field experiments. The optimal schedule was then determined as that which specified the amount of irrigation water to be applied at each stage, when a given total quantity of water is available at the beginning of the season. Dudley et al (1971) considered the problem from a stochastic point of view but made many unrealistic assumption to simplify the structure (e.g., crop growth at any stage is independent of the previous growth pattern). All of these procedure suffer from the dimensionality problem of the dynamic programming approach, in that they become unmanageable when a large number of state variables are involved.

The USDA-ARS model developed by Jensen et al (1970) describes a computer program for scheduling irrigation by estimating soil moisture depeltion based on climate, crop and soil data. Stewart and Hagan (1975) and Stewart et al (1974) developed an optimal irrigation program based on evapotranspiration and a linear water production function. The model generates irrigation management programmes that take into account evapotranspirational deficits at critical stages of crop growth. Some studies describe the computation of evapotranspiration and soil water depletion (e.g., Wright and Jensen, 1978) while others deal with simulation of crop growth under moisture stress(e.g., Childs et al 1977) which may be incorporated in the irrigation scheduling programmes (Jensen and Wright, 1976).

A comparatively novel approach is that of Fogel et al (1974,1976) who drew an analogy between the farmer's problem of determining an optimal irrigation policy and the businessman's problem of determining an optimal ordering policy. In both cases, that of the farmer as well as the businessman, the state of the system is examined periodically to determine the optimal qunatity and frequency of replenishment in relation to demand. The procedure employs the use of existing solutions of inventory control problems as found in operations research literature. Both deterministic and stochastic approaches were considered.

Development of Cropping Pattern: Models described above generate irrigation management programmes for each crop. Information from these can be used to determine cropping patterns which will maximize economic returns for a given water supply and land area.

The Ralph M Parson's Co (1970) report on efficient water use and farm management in India uses the Hall and Butcher (1968) model for irrigation programming and suggests a linear programming model to be used for selecting optimum crop patterns at the district planning level. The cropping pattern is generally decided on the basis of available water, other inputs and some basic data on climate, soil etc. Anderson and Mass (1971) developed a digital computer simulation model which can be used in determining how best to allocate irrigation water among crops and among farms when supply is limited. The effects of various water supply restrictions and rules for water delivery on cropping patterns, crop productions and farm incomes could be examined with the use of this model.

3.0 MODELS FOR CONJUNCTIVE USE

Physical, social, legal and economic factors determine the operation of conjunctive ground-surface-rain water systems. The relative importance of the interacting parts of the total system causes different levels of complexity in different systems. Of the many interacting parts of a system, the physical characteristics are often relatively well understood - economic and legal aspects less so.

In most analyses, the legal characteristics of the system have been applied as a set of constraints. The major difficulty lies in transferring laws and regulations into quantitative measures. In some cases, legal restrictions may overwhelm the other characteristics of the system and simply dictate the policy for conjunctive use operation. However, by studying system sensitivity to legal constraints, their impact on the overall operation and their cost can be determined. Some of the models used in literature for conjunctive use operation are described here.

3.1 Optimisation Models

These models have been very popular among water resources planners. Whereas a simulation model seeks to reproduce the dynamics of a system, an optimization model seeks to design the best system. Linear Programming, Dynamic Programming, and Nonlinear Programming are the main tools of optimization, which are discussed in brief below.

3.1.1 Linear Programming

Linear programming is a method of system optimization in which all the operations can be approximated by linear equations (straight lines). The two parts of a linear optimization model are the objective function and the constraints: that is,

$$\text{Max (or Min)} \sum_{j=1}^{n} c_j x_j \qquad (3.1)$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \qquad i = 1 \ldots m \qquad (3.2)$$

$$\underline{x} \geq 0 \qquad (3.3)$$

This type of problem is solved with the help of simplex algorithm. The simplex algorithm for a straight forward linear optimization model is carried out as follows.

STEP 1: Convert the objective functions to maximize and all the constraints to less then inequalities.

STEP 2: Add slack variables to all the constraints. This, in effect, makes all the variables zero, and places one at the origin for initial extremal point. These slacks are called the basis.

STEP 3: Check the objective function to see if there is a nonbasic (zero valued) variable that would increase the value of the objective function. If so, it should be brought into the basis; If not, one has reached the optimal solution and so can stop.

STEP 4: Calculate how large the new basic variable from step 3 be made without becoming infeasible and which of the old basic variables it will replace in the basis. This variable and the respective constraint comprise the pivot element.

STEP 5: Pivot; i.e., by algebraic manipulation (like Gaussian Elimination), drive to zero all non pivot elements in the column of the new basic variable and make the pivot element 1.0 by dividing that constraint by the original pivot element (coefficient).

STEP 6: Go back to step 3.

3.1.2 Dynamic Programming

Dynamic programming (DP) is specially suited to the class of problems which require sequences of optimal decisions. The sequences may be over time or over space. In DP , one usually starts from the end and work back to the beginning - a procedure called recurrasion. Each sequence is called a stage, and the situation at each stage is called the state.

Dynamic programming has the advantage of allowing almost any kind of objective function. The objective function may be non-linear or even discontinuous. However, there are severe restrictions on the number of decision variables. Normally, there is only one decision variable, but some problems and computers may be able to handle upto four.

3.1.3 Non Linear Programming

Classical nonlinear programming can be divided into constrained or unconstrained models. For each division, the objective function may be convex (concave) or nonconvex. When the objective function is nonconvex, one may not be able to guarantee the solution.

In the unconstrained case, the necessary conditions for an optimal solution are

$$\frac{\partial F}{\partial x_i} = 0 \qquad i = 1....n \tag{3.4}$$

If the objective function is nonconvex, this may only be a local stationary point; consequently, it is helpful to determine the convexity of nonlinear objective functions. To do this, one evaluates the determinates of the Hessian D_i The Hessian H is the matrix of second partial derivatives. If all the D_i´s are nonnegative, the objective function is convex; if the old ones are nonpositive and even ones are negative, the function is concave. If neither of these conditions holds, the function is nonconvex (saddle point).

For constrained nonlinear optimization, the Lagrangian equation is the classical approach.

$$L(\underline{x},\underline{\lambda}) = F(\underline{x}) + \underline{\lambda}[g(x)] \tag{3.5}$$

This may also be formulated as

$$\text{Max } F(\underline{x}) \tag{3.6}$$

subject to

$$g_i(\underline{x}) = 0 \qquad \text{for all } i \tag{3.7}$$

The λ values like the dual variables in linear programming, are shadow costs (or prices). The values are 'dummy parameters invoked to assist in the solution. The necessary conditions for a solution are

$$\frac{\partial L}{\partial \underline{x}} = 0, \quad \frac{\partial L}{\partial \underline{\lambda}} = 0 \tag{3.8}$$

3.1.4 Optimization Techniques Used in Conjunctive Use

Rogers and Smith (1970) formulated a linear programming model to aid in the planning of irrigation projects. The model, focuses on the interactions of a surface water-groundwater system within the economic context of irrigation management. The objective of the irrigation project is assumed to be to maximize the annual net revenues considering crop returns and project costs. The following capacity variables were selected by the program.

(i) S is the total delivery capacity of the tubewells installed for the purposes of supplying irrigation water and of removing subsurface drainage water.

(ii) W is the area to be included in the irrigation project.

(iii) Y is the delivery capacity of the canal system conveying water from the surface water supply to the field channels.

(iv) Z is the total removal capacity of the surface drainage works whose construction is necessitated by the project under consideration.

The system is operated in such a way as to most effectively contribute to the goal of maximising net revenues while satisfying certain continuity capacity, agronomic and land constraints.

The operating variables are as follows:

(i) A_l is the area under l^{th} crop.

(ii) s_k is the amount of tubewell water pumped during the k^{th} month and delivered to the field channels.

(iii) t_k is the amount of tubewell water pumped during the k^{th} month and delivered to the surface drainage channels.

(iv) y_k is the amount of water diverted from the river to the canal system during the k^{th} month.

To define the system, the following exogenously given parameters and variables are assumed known:

(i) M is the largest amount of land deemed suitable for irrigation development in the project.

(ii) R is the net horizontal aquifer recharge.

(iii) x_k is the supply of surface water during the month.

(iv) α is the fraction of the river supply that is available for irrigation diversion.

(v) γ_i is the fraction of the irrigation water delivered to the i^{th} point in the system that becomes recharge of the aquifer. The points where recharge is assumed to occur are: i=1, canal system; i=2, non rice field channels; i=3, non rice farms; i=4, rice field channels; i=5, rice farms; and i=6, non irrigated farms.

(vi) δ_{kl} is the water requirement of the l^{th} crop in the k^{th} month.

(vii) η is the annual evaporation from the surface of the ground water.

(viii) λ_{kl} is the land use coefficient for the l^{th} crop during k^{th} month.

(ix) μ is the rate at which the ground water may be mined.

(x) ξ_k is the subsurface return flow to the river during the k^{th} month.

(xi) ρ_i is the fraction of irrigation water delivered to the i^{th} point in the system that become non-beneficial evapotranspiration fraction of precipitation.

(xii) σ_i is the fraction of the irrigation water delivered to the i^{th} point in the system that becomes surface runoff. A prime denotes the surface runoff fraction of precipitation.

(xiii) ϕ_k is the monthly precipitation over the irrigation district.

(xiv) ψ_k is the monthly deep percolation loss from rice fields.

The cost of the project is composed of eight elements. These and the crop prices make up the economic parameters.

(i) C_o^s is the sum of the unit capital and maintenance costs of tubewells and associated structures.

(ii) C_o^y is the sum of the unit capital and maintenance costs of the project's surface irrigation works, where the canal configuration is fixed according to the project topography.

(iii) C_o^z is the sum of the unit capital and maintenance costs of the project's surface drainage facilities.

(iv) C^s is the unit operating cost of tubewells.

(v) C^y is the unit operating cost of surface irrigation works.

(vi) C^z is the unit operating cost of surface drainage works.

(vii) e is the sum of the unit capital and maintenance costs of land preparation and field channel installation.

(viii) f is the sum of the unit capital and maintenance costs of flood protection.

(ix) p_l is the revenue of the crop net of all but project costs.

(x) $\hat{p}$ is the revenue per unit area with no project, assuming projeted cropping pattern and yields.

Some additional notation permits a clearer description of the model.

(i) g is the number of non rice crops

(ii) h is the number of rice crops

(iii) z is an intermediate variable denoting the diversion to field channels. A prime indicates diversion to rice field channels.

(iv) ζ_k is the total surface runoff during k^{th} month

(v) $\theta_i = 1 - \rho_i - \sigma_i - \gamma_i$.

(vi) ν is a conversion factor.

The project objective is to maximize:

$$(\sum_l p_l A_l - \hat{p}W \quad - \quad C_o^s S - C_o^y Y - C_o^z Z \quad - \quad fW$$

additional revenues from irrigation — Capital costs of irrigation works — Capital cost of flood protection

$$- eW \quad - \quad \sum_k [C^s(s_k + t_k) + C^y y_k + C^z \zeta_k]) \qquad (3.9)$$

capital cost of land protection — operating costs of irrigation works

This revenue maximization is affected subject to the following constraints.

(i) Canal diversions can not exceed 100 per cent of the discharge in the river

$$y_k \leq \alpha x_k \qquad \text{for all } k \qquad (3.10)$$

(ii) The water requirements of the crops must be met exactly

$$\theta_1 y + s = \frac{\nu}{\theta_2 \theta_3} \sum_{l=1}^{g} \delta_l A_l + \frac{\nu}{\theta_4} [\sum_{l=g+1}^{g+h} A_l(\delta_l + \psi)] \qquad (3.11)$$

for all months

(iii) The total available land may not be exceeded

$$\sum_l \lambda_{kl} A_l \leq W \qquad \text{for all } k \qquad (3.12)$$

and $W \leq M$

(iv) The net amount of ground water removed during a year must not exceed the mining allowance nor may be water table show a net rise after a lapse of two months

$$0 \leq \sum_{k} [\xi_k + s_k + t_k + \eta - R - \gamma_1 y_k - \gamma_2 \frac{\nu}{\theta_2 \theta_3} \sum_{l=1}^{g} \delta_{kl} A_l - \nu \sum_{l=1}^{g} (\frac{\gamma_3}{\theta_3} \delta_{kl} A_l + \gamma_3' \phi_k \lambda_{kl} A_l)$$

$$- \nu \sum_{l=g+1}^{g+h} \{ A_l \frac{\gamma_4}{\theta_4} (\delta_{kl} + \psi_k)\} - \nu \gamma_6' \phi_k (M - \sum_{l=1}^{g+h} \lambda_l A_l)] \leq \mu \qquad (3.13)$$

(v) In no month may the amount of groundwater pumped exceed the capacity of the wells

$$s_k + t_k \leq S \qquad \text{for all } k \qquad (3.14)$$

(vi) Irrigation diversion may not exceed the capacity of the canals

$$y_k \leq Y \qquad \text{for all } k \qquad (3.15)$$

(vii) Surface drainage may not exceed the capacity of the surface drainage works

$$\zeta_k \leq Z \qquad \text{for all } k \qquad (3.16)$$

(viii) The production of certain crops must be limited in the absence of the explicit incusion of the demand functions in the objective function. Thus, constraints on the maximum amounts of fruits, vegetables, sugar cane, and certain interplanted oilseeds are necessary.

(ix) In order to contribute towards the goal of self sufficiency in food grains, a constraint is included to ensure at least a minimum amount of rice plus wheat.

This model with linear objective function and linear constraints, may be solved by a linear programming algorithm to find the capacity and operating variables. Apart from the solution itself, the analysis of the sensitivity of the solution to changes in the data is of equal importance.

Parametric programming, an extension of linear programming, will indicate how the optimal pattern of irrigation development should change with change in relative prices. If the original optimal solution were to indicate no ground water utilization it would be useful to decision-making to know how much the cost of tubewell construction, maintenance and operation would need to be reduced relative to the cost of surface irrigation works before ground water development was indicated. If ground water development were indicated by the original solution, it would be useful to know by how much its relative cost must increase before it would leave the solution. These questions may be answered by parametric programming on the cost coefficients of the objective function.

A similar procedure is possible on the right hand sides of requirement vector. In this case, the sensitivity of the solution may be tested to changes in water availability. One might thus determine the effect on system capacities of using, for example the 90 per cent dry year instead of mean rainfall or streamflow.

Kaushal and Khepar (1980) developed a linear programming model for the conjunctive utilisation of poor quality ground water with canal water The model was developed for the south-west region of Punjab where the unscientific use of poor quality water for irrigation, has posed problems of soil salinization. The solution to this problem could be proper leaching for optimal salt balance in the soil root zone. Canal water and poor quality ground water need to be used in conjunction with each other to achieve maximum benefit through optimal cropping pattern. Mathematical model developed is described below.

Objective function

$$\text{Max } Z_i = \sum_{j=1}^{2} \sum_{w=1}^{W} \sum_{k=1}^{K} NR_{ijwk} X_{ijwk} - \sum_{j=1}^{2} CS_j SW_j - \sum_{j=1}^{2} CT_{ij}(1+LR_i)TW_j \quad (3.17)$$

where i is water mix index, i=1,2,...4; j is growing season, j=1 winter season, j=2 monsoon season; w is level of irrigation w=1,2...W; k is crop, k=1,2...K; NR_{ijwk} is net returns in Rs. per ha above all variable production costs excluding the cost of irrigation water for crop k, grown with water mix index i, with level of irrigation w in season j; Z_i is total net returns over variable costs for the command area using water mix index i; X_{ijwk} is area allocated in ha to crop k, grown with water mix index i with level of irrigation in season j; SW_j is surface water allocated in ha cm in season j; CS_j is cost of applying one ha cm. of surface water in season j; CT_{ij} is cost of applying one ha cm of ground water in season j for water mix index i; TW_j is ground water allocated in ha cm excluding leaching requirement in season j and LR_i is leaching requirement in fraction using water mix index i. This objective function is subject to following constraints.

(i) Water allocation constraint:

$$\sum_{w=1}^{W} \sum_{k=1}^{K} X_{ijwk} W_{ijwk} - SW_j - (1+LR_i)TW_j = 0 \qquad (3.18)$$

where W_{ijwk} is irrigation water required in ha cm for crop k, grown with water mix index i, with level of irrigation w in season j

(ii) Land area constraint:

$$\sum_{w=1}^{W} \sum_{k=1}^{K} ALFA_j X_{ijwk} \leq TA_j \qquad \text{for all j} \qquad (3.19)$$

where $ALFA_j$ is land area occupying coefficient for crop activity, $ALFA_j=1$ if the crop is grown in season j, otherwise zero and TA_j is crop land available in ha in season j.

(iii) Surface water constraints

$$SW_j \leq ASW_j \qquad \text{for all j} \qquad (3.20)$$

where ASW_j is surface water available in ha in season j, after allowing for all losses.

(iv) Tubewell water pumped

$$(1+LR_i)TW_j \leq ATW_j \qquad \text{for all j} \qquad (3.21)$$

where ATW_j is tubewell water available in ha cm in season j and water mix index i, after allowing for all losses.

(v) Minimum area required

$$\sum_{w=1}^{W} X_{ijwk} \geq MIAR_{jk} \qquad (3.22)$$

where $MIAR_{jk}$ is minimum area required in ha for crop k, grown in season j.

(vi) Maximum allowable area

$$\sum_{w=1}^{W} X_{ijwk} \leq MAAL_{jk} \qquad (3.23)$$

where $MAAL_{jk}$ is maximum area in ha allowable to crop k, grown in season j.

(vii) Ground water and canal water mix

$$PTW.SW_j - (10-PTW)(1+LR_i)TW_j = 0 \quad \text{for all j} \qquad (3.24)$$

where PTW is 1/10th of per cent ground water mixed.

(viii) Non negativity constraint

$$X_{ijwk} \geq 0; \; SW_j \geq 0; \; TW_j \geq 0 \qquad (3.25)$$

Optimal cropping patterns were found using different ratio of ground water and canal water and the net retuns from each of the alternatives were compared for the area in consideration. It was concluded that for maximum net returns, 30% ground water and 70% canal water should be used in an optimal cropping pattern.

Keeping in mind the unconsistent nature of rainfall in our country and to utilise rain water conjunctively with ground water and surface water, Tyagi and Dhruva Narayana (1980) have formulated a chance constrained linear programming model with rainfall as stochastic input. The model is based on water balance representing the realistic hydrology of Jundla command area, a typical unit of salt affected soils, in Western Jamuna Canal Command.

The system is composed of two limited sources of water namely surface water from canal system and the ground water supply from within the area for distribution among various crop activities and the augmentation programme. The main components of water resources system under study include hydrologic and economic sub-systems which are linked together by production function of crops (Fig.5).

The system is operated under the main constraints of soil alkalinity and ground water mining. The other constraints include;

(i) water availability from two sources,

(ii) irrigation system capacity,

(iii) crop irrigation requirements,

(iv) available culturable area and the area under certain specific crops.

It is assumed that the objective of the programme is to maximize the expected value of the total income from water use in irrigation crops, and the export (augmentation supply) The objective function is written as:

$$Z = \max \sum_{j=1}^{m} P_j A_{ij} + \sum_{i=1}^{n} P^{DT}\, DT_i - \sum_{i=1}^{n} C^{CW}\, CW_i - \sum_{i=1}^{n} C^{ST}\, ST_i \qquad (3.26)$$

where

P_j = income from crop j per unit area

A_{ij} = area under crop j

P^{DT} = income from water export per unit volume

DT_i = volume of water exported during period i

C^{CW} = sale price of canal water per unit volume

CW_i = Volume of canal water released during period i

C^{ST} = sale price of shallow tubewell water

ST_i = volume of water pumped by shallow tubewells during period i

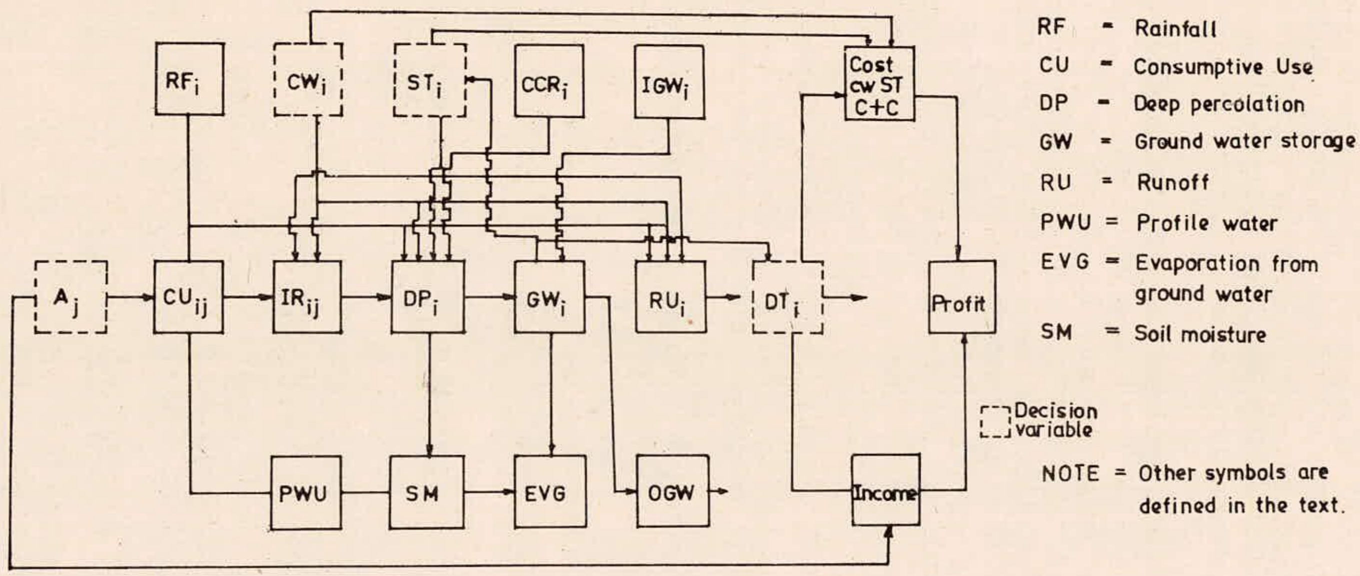

Fig.5. Schematic Diagram of the Model Showing Interaction of Agricultural, Hydrologic and Economic Parameters (Tyagi & Dhruva Narayana, 1980)

Assuming of rainfall as stochastic input, has the effect of making irrigation requirements and the ground water recharge as uncertain quantities. It requires that the irrigation requirements during each decision period must be met at least in 10 B years out of 10. Such probability statements require that sufficient shallow tubewell capacity be provided to supply water with 100 B per cent reliability. It is also required that the ground water be so managed that the average depletion does not exceed the mining allowance; and the average annual accretion to ground water is zero.

With the introduction of chance constraint in irrigation requirements, it can be written as:

$$Pr \sum_{i=1}^{n} (CE_C CW_i + CE_T ST_i) = \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{IR_{ij} A_{ij}}{AE_j} \quad B_i^W \qquad (3.27)$$

where

CE_C = conveyance efficiency of canal system

CE_T = conveyance efficiency of shallow tubewell system

IR_{ij} = irrigation requirement of crop j during period i

AE_j = field water application efficiency of crop j

B_i^W = percentile value indicating the level probability

If F_{ij} (ERF_{ij}) and T_{ij} (IR_{ij}) are the distribution functions of rainfall and irrigation requirements during period i then,

$$T_{ij}(IR_{ij}) = \frac{1 - F_{ij}(CU_{ij} - ERF_{ij})}{AE_j} \qquad (3.28)$$

The selection of B_i^W requires that the canals and shallow requirements determined by $T_{ij}^{-1}(B_i^w)$.

Therefore, the probabilistic constraints become:

$$(CE_C\ CW_i + CE_T\ ST_i) \geq T_{ij}^{-1}(B_i^w) \tag{3.29}$$

where, $T_{ij}^{-1}(.)$ is the distribution function of:

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \frac{IR_{ij}\ A_{ij}}{AE_j} \tag{3.30}$$

The percentile distribution of $T_{ij}^{-1}\ (B_{ij}^w)$ can be determined from the distribution function of effective rainfall by noting that:

$$T_{ij}^{-1}(B_i^w) = \frac{CU_{ij} - F_{ij}^{-1}(1-B_i^w)}{AE_j} \tag{3.31}$$

Since the mining allowance for the problem being analysed is zero, the expected change in ground water storage is also zero. Mathematically,

$$\sum_{i=1}^{n} (ST_i + DT_i + OGW_i + EGW_i) - \sum_{i=1}^{n} \sum_{j=1}^{m} EXP[CPR_{ij}]$$

$$- \sum_{i=1}^{n} (CWR_i + STR_i + CCR_i + IGW_i) \geq MGW \tag{3.32}$$

where,

OGW = outflow from ground water storage

EGW = evaporation from ground water storage

CPR = recharge from crop land

CWR = recharge from canal system

STR = recharge from shallow tubewell system

CCR = recharge from carrier channels and drains

IGW = ground water inflow

MGW = mining allowance

The results obtained from the model were presented in terms of cropping pattern, income and shadow prices, and ground water recharge. The author concluded that at low risk levels, the overall water supply was inadequate to permit full utilization of land resource. In order to undertake the alkali land reclamation programme, the water supply for use in the project area would have to be increased either by reducing the quantity of water export or by increasing the canal water releases.

A distributed conjunctive use model was applied by Kashyap and Chandra(1982) to Krishni-Hindon Inter-basin which deals with optimal conjunctive use policy for predefined pattern of surface water availability incorporating spatially and temporally distributed ground water withdrawals and spatially distributed cropping pattern.

The area of interest can be divided into a number of zones of near uniform surface water supplies, net irrigation requirements and hydrological conditions. Similarly, the time domain can be discretized by finite number of periods. The decision variables which are to be considered in a distributed model are:

(i) areas under different crops (A_{jl}),

(ii) ground water withdrawal (W_{kl})

The subscripts j,k,l denote the crop number, time and the zone number respectively.

The net benefits from the agricultural activity can be expresed as:

$$Y = \sum_j (B_j \sum_l A_{jl}) - C_g \sum_l \sum_k W_{kl} - C_S \quad (3.33)$$

where B_j is the return per unit area under the j^{th} crop, adjusted for the cost of all the inputs other than water, C_g is the cost of a unit volume of ground water and C_S is the total cost of surface water supplies. The above objective function is to be maximized subject to the following constraints:

(i) Crop water requirement constraints

In order to ensure that the net irrigation requirement is satisfied the following constraint is to be imposed for each zone during each time period:

$$\eta_g W_{kl} + \eta_s S_{kl} \geq \sum_j \delta_{jkl} A_{jl} \quad (3.34)$$

η_g and η_s being the irrigation efficiencies of ground water and surface water resepectively. δ_{jkl} is the net irrigation requirement of the j^{th} crop during k^{th} period in the l^{th} zone. S_{kl} is surface water availability in the l^{th} zone during k^{th} period.

(ii) Land availability constraints

The total land allocated to different crops in a zone at any time cannot exceed the total available land in that zone. The land available constraint can be written as:

$$\sum_j \lambda_{jkl} \leq 1 \qquad \text{for all k and l} \quad (3.35)$$

where

$$\lambda_{jkl} = \frac{\text{Area under } j^{th} \text{ crop in } l^{th} \text{ zone in } k^{th} \text{ period}}{\text{Cultivable Command Area in the } l^{th} \text{ zone}}$$

(iii) Maximum and minimum areas under each crop

There should be certain minimum and maximum cropping area allocation for each crop in the whole area of interest which is ensured by the following constraints:

$$A_{Max,j} > \sum_{1} A_{jl} > A_{Min,j} \tag{3.36}$$

(iv) Maximum and minimum depths to the water table

Water table in a region assumes a state of dynamic equilibrium for a given annual recharge-discharge pattern and boundary conditions. This state of dynamic equilibrium is disturbed with any change in recharge-discharge pattern caused by some natural phenomena or human-activities. However, after some time water table in the region regains a new state of dynamic equilibrium characterised by a different pattern of annual fluctuations. For example, an increase in ground water recharge in the areas caused by the implementation of a surface water scheme, will result in a changed pattern of water table levels displaying conspicuously higher water table as well as an increase of sub-surface flow towards adjoining water bodies like rivers, lakes etc. Similarly an increase in rate of ground water withdrawals results in lowering of water table and decrease of sub-surface flows towards the adjoining water bodies. Thus any change in the recharge-discharge conditions changes not only the water table pattern in the area but also the discharge pattern in the adjoining rivers, which may be hydraulically connected with the aquifer.

While deciding an optimal ground water withdrawal policy, the planners are governed by the following constaints:

(i) The peak of the temporal fluctuation of the water table corresponding to the changed state of dynamic equilibrium should be below the root zone of the plants, so as to avoid water logging and other related problems.

(ii) The decision regarding the permissible extent of maximum lowering of water table, i.e., trough level of the fluctuation pattern, is more difficult to be made and may involve many subjective decisions. Lowering is associated with the following technological and economic manifestations:

(1) Some of the shallow wells may dry up completely.

(2) Cost of pumping a unit volume of water increases for all time to come.

(3) Discharges in the river hydraulically connected with the aquifer will decrease for all times to come.

A discretized function relating the extent of lowering of water table with tangible losses associated with first two effects, i.e.(1)&(2) as listed above can be arrived at, and included in the objective function. However, from social point of view it may not be permissible to lower the water table to the extent that shallow wells go dry since most of the shallow wells are owned by poor cultivators having small holdings. If possible water table should not be lowered below the river bed and should also maintain at least 2m water depth in the majority of the shallow wells in the area.

(iii) The discharge in hydraulically connected rivers is altered with change in ground water withdrawal. With the present state of knowledge of ground water hydraulics, it is possible to estimate the change in the dishcarge corresponding to a known change in pumping pattern. Thus the withdrawal pattern should be adopted such that the river discharge remains within permissible limits. However, the pattern can be designed from the point of view of a limited flood control measure as well, if the reduction in discharge occurs during the period of peak flood.

The constraints relating to the maximum and minimum permissible depths to the water table can be imposed by expressing the depths to the water table, under the conditions of dynamic equilibrium, as functions of the zonal withdrawal pattern (W_{kl}), boundary conditions, aquifer parameters, recharge and the ground elevations. Out of these, only (W_{kl}) are the decision variables and the rest are the input data. The depth to water table can be evaluated at discrete space points for various alternatives of withdrawal pattern, using a ground water simulation model. The procedure consists of the numerical solution of the Boussinesq's equation

$$\frac{\partial}{\partial x}(T_{xx}\partial h/\partial x) + \frac{\partial}{\partial y}(T_{yy}\partial h/\partial y) + Q = S\,\partial h/\partial t \qquad (3.37)$$

for given initial and boundary conditions, where h is the water table elevation, x and y are the coordinates along two orthogonal directions co-linear with directions of principal permeabilities, T_{xx} and T_{yy} are the transmissivities (LT^{-2}) in directions x and y respectively. Q is the net vertical accretion rate (LT^{-1}), S is the specific yield and t is the time. W_{kl} values are to be regarded as negative of Q and simulation is to be carried out till the state of dynamic equilibrium is established.

Boussinesq's equation can be solved numerically by discretizing the entire area of interest by a finite number of nodes. This discretization is necessary for numerical evaluation of aquifer response to the known trial patterns of the zonal pumpings. Thus, if the space is discretized by a number of nodes and h_{ik} and G_i are the water table elevations at the i^{th} node in the k^{th} time period and the ground elevation of the i^{th} node respectively, then,

$$\max_{i,k}(G_i - h_{ik}) < d_{max} \qquad (3.38)$$

$$\min_{i,k}(G_i - h_{ik}) > d_{min}$$

where d_{min} and d_{max} are the permissible minimum and maximum depths to the water table respectively. These depths define the permissible range of water table fluctuations and are derived from engineering and/or socio-economic considerations. The minimum permissible depth, d_{min} , can be governed by considerations of waterlogging or salt accumulation in the case of canal irrigated areas. Similarly d_{max} can be determined by the depths of existing shallow wells, economics of pumping, the minimum permissible baseflow in hydraulically connected rivers.

In the proposed conjunctive use model, cropping patterns and ground water withdrawals have been distributed in space by dividing the area into a number of zones. Each zone has uniform cropping pattern, rainfall, surface water supplies and hence uniform ground water withdrawals. The zones should also display near uniform drawdown/discharge characteristics. This can be ensured by selecting zones which are as homogeneous as possible in terms of geology, hydrology and boundary effects. The size and the number of zones are primarily governed by the necessity of restricting the number of decision variables and constraints to a manageable limit. Apart from this, the number of zones should not be too large to permit the implementation of the evolved policy.

The discretization of an area by a finite number of nodes for obtaining a numerical solution of Boussinesq's equation is governed by an entirely different set of conditions relating mainly to the properties of the algorithm adopted for the numerical solutions. The finite difference algorithms require the nodes to be positioned along two orthogonal directions coinciding with the major and minor permeability directions. The spacing of nodes is governed by the consideration of stability, convergence and truncation errors.

For restricting truncation errors, a much smaller nodal spacing (as compared to the size of a zone) will be required.

While calcuating the aquifer response, the pumpages at all the nodes lying in a given zone are governed by the current trial ground water withdrawal pattern of the zone. Thus, the feasibility with respect to the constraints given in (3.38) is checked at the nodal level by studying the nodal response of the aquifer to the zonal ground water withdrawal pattern.

Maximization of objective function

The maximization of Y [eqn.(3.33)] with respect to the decision variables (A_{jl}) and (W_{kl}), subject to the constraints given in inequalities (3.34), (3.35), (3.36) and (3.38) cannotbe carried out by linear programming since the constraints given in inequality (3.38) are nonlinear implicit functions of the decision variables (W_{kl}). This points towards the necessity of employing nonlinear optimization methods such as sequential unconstrained minimization technique (McCormick and Fiacco, 1968). These methods permit implicit evaluations of the objective function, the constraints and their derivatives.

A linear programming model with multiperiod and multi-irrigation level is developed recently by Asthana & Gupta, for conjunctive use of surface and groundwater. The objective function is taken as the maximization of the annual net returns from the crop products subject to resource constraints. It comprises the terms for the sum of net returns from each crop process (a crop at a certain level of irrigation is considered a seperate crop process) excluding the cost of water and the terms from each of operation and maintenance cost of surface and groundwater. It is written as

$$\text{Max } Z = \underbrace{\sum_{i=1}^{M} \sum_{j=1}^{N} R_{ij} A_{ij}}_{\text{Return from crops}} - \underbrace{C_s \sum_{k=1}^{K} D_k}_{\text{Operational \& maintenance cost of S.W.}} - \underbrace{C_g \sum_{k=1}^{K} G_k}_{\text{Operational \& maintenance cost of G.W.}} \quad (3.39)$$

where, A_{ij} is the area under i^{th} crop at j^{th} level of irrigation, R is the net return from the crop less the cost of water; C_s is the operation and maintenance cost of surface water diversion D_k in k^{th} time period; C is operation and maintenance cost of groundwater G_k in k^{th} period.

Subject to the constraints:

(i) Surface water diversion in any time period (D_k) is less than the availability (X_k) and is limited to the canal capacity (C^c)

$$D_k \leq X_k \qquad \text{for all } k \qquad (3.40)$$

$$D_k \leq C^c \qquad \text{for all } k \qquad (3.41)$$

(ii) The draft in any time period (G_k) is limited to a fraction (α) of the annual draft (T_p). The annual draft is also limited to the safe yield (S_y).

$$\sum_{k=1}^{K} G_k \leq T_p \qquad (3.42)$$

$$G_k \leq T_p \qquad \text{for all } k \qquad (3.43)$$

$$0 \leq T_p \leq S_y \qquad (3.44)$$

(iii) The total surface and ground water use is equal to the crop requirements in each time period

$$\eta_s D_k + \eta_g G_k = \sum_{i=1}^{M} \sum_{j=1}^{N} A_{ijk} W_{ijk} \qquad \text{for all } k \qquad (3.45)$$

where η_s and η_g are surface and ground water efficiencies; and W_{ijk} is net irrigation requirement of i^{th} crop at j^{th} level of irrigation in k^{th} time period.

(iv) The total cropped land in any time period is less than total cultivated land (A_c)

$$\sum_{i=1}^{M} \sum_{j=1}^{N} A_{ijk} \le A_c \qquad \text{for all } k \qquad (3.46)$$

(v) Total area under any crop (i) is limited to a fraction (β_i) of total cultivated land

$$\sum_{j=1}^{N} A_{ij} \le \beta_i A_c \qquad \text{for all } i \qquad (3.47)$$

This model was run for Sarda Sahayak Canal Command to work out the possible changes in irrigated crop pattern, benefits, and required increase in pumping capacity, for the use of additional groundwater proposed to be pumped to avoid water logging.

3.2 Simulation Models

Simulation model also play important role in conjunctive use planning and management. Morel-Seytoux(1975) described an efficient yet accurate model of a stream-aquifer interactive behaviour. The model combines the classical finite difference method with the efficient systematic generation of solutions by the Green's function approach. The optimal rules of operation are deduced from well structured Mathematical Programming formulation for which efficient solution algorithms exist. This model can be termed as a combination of simulation and programming techniques.

3.2.1 Traditional Simulation Approach

The basic saturated flow equation (using the Dupuit's assumption and a few other) describing the evolution of an isotropic water table aquifer (without stream) is the Boussinesq's equation

$$\phi \frac{\partial s}{\partial t} - \frac{\partial}{\partial x} (T\frac{\partial s}{\partial x}) - \frac{\partial}{\partial y} (T\frac{\partial s}{\partial y}) = Q_p \delta_p \qquad (3.48)$$

where, ϕ is effective porosity, s is the drawdown measured positive downward from a horizontal datum located at distance H above the datum for the water table elevation, t is time, x and y are the horizontal cartesian coordinates, T is the transmissivity, Q_p is the instantaneous pumping volume of well p, and δ_p is a Dirac delta function, singular at the point of coordinates ξ_p , η_p and τ (where, ξ_p and η_p are the x,y coordinates of well p, τ is time).

Assuming a homogeneous aquifer of finite extent and no previous development, then it is well known that the drawdown at point w at time t due to pumping at well p at a rate $Q(\tau)$ is :

$$s_{wp}(t) = \int_o^t Q_p(\tau) \frac{e^{-\frac{R^2_{wp}}{4T(t-\tau)}}}{4\pi T} \frac{d\tau}{t-\tau} \qquad (3.49)$$

where, R_{wp} is the distance between point w and well p. If the pumping rates vary from time to time but constant within that period , then

$$s_{wp}(n) = \sum_{\nu=1}^{n} Q_p(\nu) \int_{\nu-1}^{\nu} \frac{e^{-\frac{\phi R_{wp}^2}{4T(\nu-\tau)}}}{4\pi T(\nu-\tau)} d\tau \qquad (3.50)$$

3.2.2 The Discrete kernel approach

Let the pumping kernel be defined as

$$k_{wp}(u) = \frac{e^{-\frac{\phi R_{wp}^2}{4Tu}}}{4\pi T u} \qquad (3.51)$$

then Eqn.(3.50) can be rewritten as

$$s_{wp}(n) = \sum_{\nu=1}^{n} \delta_{wp}(n-\nu+1) Q_p(\nu) \qquad (3.52)$$

where the discrete kernel coefficients are defined as

$$\delta_{wp}(\nu) = \frac{1}{4\pi T} \int_0^1 e^{-\frac{\phi R_{wp}^2}{4T(\nu-\tau)}} \frac{d\tau}{\nu-\tau} \qquad (3.53)$$

If the effect of different pumping patterns (say 100 of them) on the drawdowns is to be investigated, the operation with traditional simulation approach will have to be repeated 100 times, where as here, once the $\delta_{wp}(\nu)$ coefficients have been calaculated and saved, then the generation of 100 sets of drawdowns corresponding to 100 different pumping patterns is easily obtained numerically from Eqn.(3.52).

3.2.3 General integral equation

It has been shown (Morel-Seytoux and Daly,1975) that the return flows to reaches of the stream are the solution of a system of R integral equations, namely

$$Q_r(t) + \Gamma_r \sum_{\rho=1}^{R} \int_0^t Q(\tau)k_{r\rho}(t-\tau)d\tau$$

$$= \Gamma_r[\sigma_r(t)-v_r(t)- \sum_{p=1}^{P} \int_0^t Q_p(\tau)k_{rp}(t-\tau)d\tau] \quad r=1,2,..R \qquad (3.54)$$

where, R is the total number of reaches of the river, p is the total number of wells, Γ_r is the transmissivity of reach r, $k_{r\rho}(.)$ is the reach kernel, σ_r is the drawdown to the stream surface water level, $v_r(t)$ is the natural aquifer drawdown under reach r, and k_{rp} is the pumping kernel. The reach kernel for homogeneous aquifer of infinite extent is known analytically as

$$k_{rr}(t-\tau) = \frac{1}{\phi ab} \operatorname{erf}\{\frac{a}{2}[\frac{\phi}{4T(t-\tau)}]^{\frac{1}{2}}\}. \operatorname{erf}\{\frac{b}{2}[\frac{\phi}{4T(t-\tau)}]^{\frac{1}{2}}\} \qquad (3.55)$$

and

$$k_{r\rho}(t-\tau) = \frac{e^{-\frac{\phi R_{r\rho}^2}{4T(t-\tau)}}}{4\pi T(t-\tau)} \qquad \text{for } \rho \neq r \qquad (3.56)$$

where, a and b are length and width of the reach. Once these coefficients are known, for example, the last integral on the right hand side of Eqn.(3.54) takes the discrete form

$$\sum_{\nu=1}^{n} \delta_{wp}(n-\nu+1)Q_p(\nu) \qquad (3.57)$$

Ultimately, the objective is to solve a discretized form of Eqn.(3.54) for the return flows at discrete time intervals, $Q_r(1), Q_r(2), ...Q_r(n)$, where $Q_r(n)$ is the average return flow rate during period n. If the terms in $\sigma_r - v_r$ in Eqn.(3.54) are set equal to zero, the solution of this special system of Eqn.(3.54) provides precisely the seepage flows from the river due to pumping and not caused by other factors such as rise in stage levels. If there is only one pervious reach and only one pumping well, then Eqn.(3.54) takes the discrete form

$$Q_r(n) + \Gamma_r \sum_{\nu=1}^{n} \delta_{rr}(n-\nu+1)Q_r(\nu)$$
$$+ \Gamma_r \sum_{\nu=1}^{n} \delta_{rp}(n-\nu+1)Q_p(\nu) = 0 \qquad (3.58)$$

which being linear has a solution of the form

$$Q_r(n) = \sum_{\nu=1}^{n} \varepsilon(n-\nu+1)Q_p(\nu) \qquad (3.59)$$

where, the ε coefficients are solutions of the linear system of equations

$$\varepsilon(n-\nu+1)+\Gamma_r \sum_{m=\nu}^{n} \varepsilon(m-\nu+1)\delta_{rr}(n-m+1) = \Gamma_r\delta_{rp}(n-\nu+1) \qquad (3.60)$$

For a homogeneous aquifer of infinite extent, the δ_{rp} & δ_{rr} can be calculated by the discrete kernel generator or analytically according to the formulae

$$\delta_{rp}(\nu) = \frac{1}{4\pi T} [E_i\{ \frac{-\phi R^2}{4T(\nu-1)} \} - E_i \{ \frac{-\phi R^2}{4T\nu} \}] \qquad (3.61)$$

where, $E_i(.)$ is the exponential integral function, and

$$\delta_{rr}(\nu) = \frac{1}{\phi ab} \int_0^1 \mathrm{erf}\{\frac{a}{2} [\frac{\phi}{4T(\nu-\tau)}]^{\frac{1}{2}}\}.\mathrm{erf}\{ \frac{b}{2}[\frac{\phi}{4T(\nu-\tau)}]^{\frac{1}{2}}\} d\tau \qquad (3.62)$$

The strategy for conjunctive use depends on the user's need and planning. The different stratagies that may be followed were studied by Morel-Seytoux (1975) for an area in Colorado.

Downstream from a reach in hydraulic connection with an alluvial aquifer (Fig.6), a farmer is entitled by a decree dating back to 1875 to divert a flow of 700 m /week throughout the warm season to irrigate his fields. In 1974, a Developer has built a subdivision which is located upstream from the point of diversion. In late 1974, the Developer has petitioned for the right to drill a well to supply the domestic needs of the subdivision and in early 1975, a decree is granted. The state engineer, however, is concerned by the proximity of the well to the stream (100 m) and its potential detrimental effect on the river flow. The aquifer is homogeneous with $T=10,000\ m^3$ /week. Seepage transmissivity of pervious reach is known as $T=4,000\ m^3$ /week. The length and width of pervious reach is 10 m. The flows in the river have been gauged by USGS. Expected weekly flows for the 16 weeks of the irrigation season are given. The weekly demand is given as 200 m^3

3.2.4 Maximum withdrawal strategy

A plausible stategy is to permit pumping rates during each period to maximize the total withdrawal during the season subject to the downstream constraint that the residual runoff past the diversion point must always exceed $700 m^3$ /week. Mathematically,

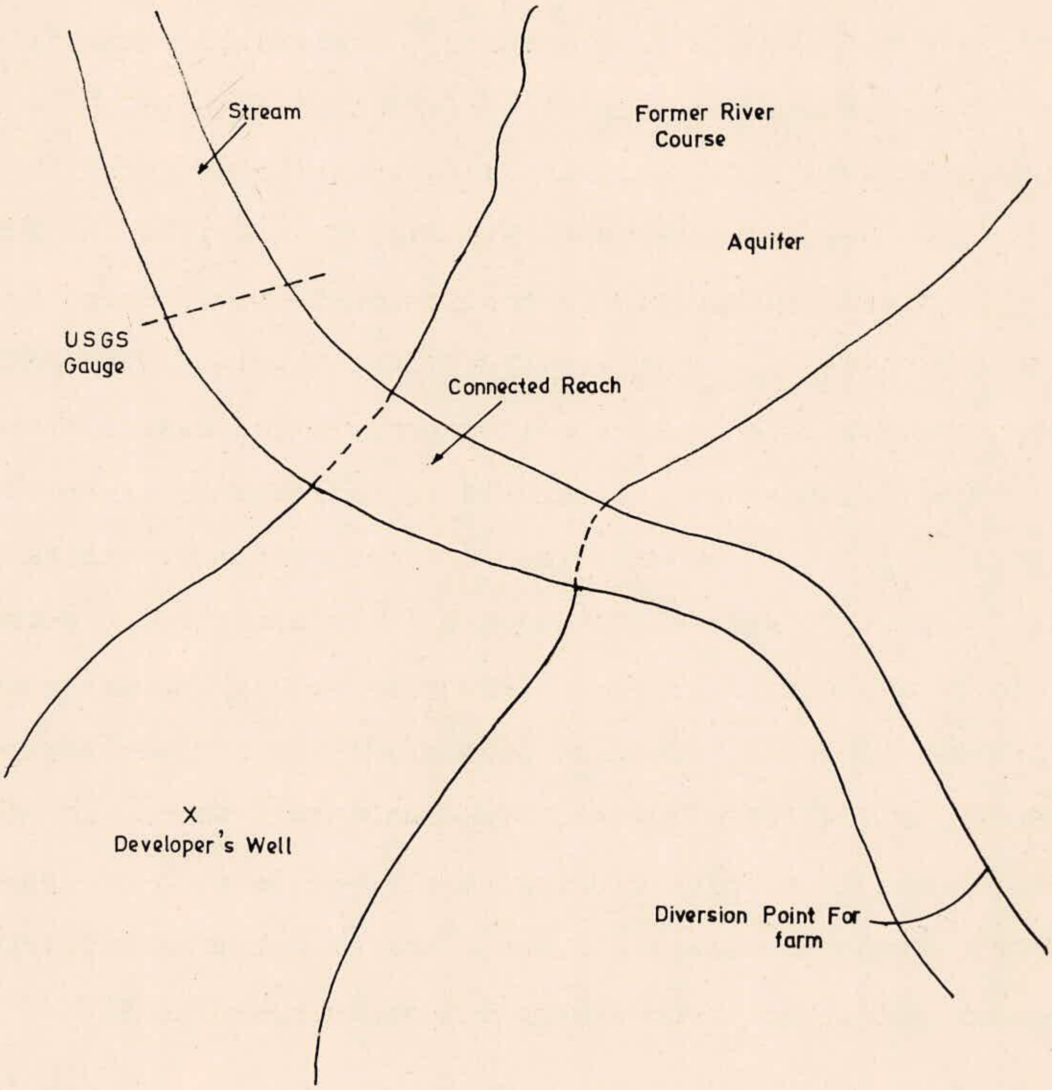

Fig. 6. Geometry of stream-aquifer system (Morel-Seytoux, 1975)

$$\text{Max}\ \{\sum_{\nu=1}^{16} Q_p(\nu)\} \tag{3.63}$$

Subject to non negativity constraints

$$Q_p(\nu) \geqq 0 \qquad \nu = 1,2....16 \tag{3.64}$$

and the legal constraints

$$R(n) + \sum_{\nu=1}^{n} \varepsilon(n-\nu+1)Q_p(\nu) \geqq 700 \qquad n=1,2,....16 \tag{3.65}$$

where R(n) are the weekly flow values. For the solution of this LP problem, the Developer should have a storage capacity of $2600m^3$.

3.2.5 Constant pumping strategy

For constraint pumping strategy, it suffices to add to the constraints of Eqn.(3.65), i.e., the legal constraints, the regularity constraints that all $Q_p(\nu)$ be equal to a common value Q.

The new strategy is

$$\text{Max}[Q] \tag{3.66}$$

with respect to Q, subject to

$$Q \geq 0 \tag{3.67}$$

and the legal constraints

$$R(n) + [\sum_{\nu=1}^{n} \varepsilon(n-\nu+1)]Q \geq 700 \qquad n=1,2,....16 \tag{3.68}$$

The solution to this problem is obtained simply namely

$$Q = \min_{n} \frac{R(n) - 700}{\sum_{\nu=1}^{n} |\varepsilon(n-\nu+1)|} \tag{3.69}$$

3.2.6 Minimum storage strategy

The constant pumping rate by the Developer would have resulted in a water delivery to the farmer below his right. The best strategy could then be the minimum need of storage. This would lead to the problem

$$\mathrm{Min}[S] \tag{3.70}$$

subject to non negativity conditions

$$Q_p(\nu) \geq 0 \quad \nu=1,2....16 \text{ and } S > 0 \tag{3.71}$$

the legal constraints of Eqn.(3.65), the demand constraints

$$\sum_{\nu=1}^{n} Q_p(\nu) \geq 200\, n \quad n = 1,2....16 \tag{3.72}$$

and the feasibility constraints

$$\sum_{\nu=1}^{n} Q_p(\nu) - 200n \leq S \quad n = 1,2....16 \tag{3.73}$$

The solution of this LP problem lead to a required storage capacity of 75 m^3 , a vast improvement over the earlier value of 2600 m^3

Hence, it is shown that with the use of discrete kernel approach, it is possible to solve problems of optimal management through the efficient techniques of Mathematical Programming rather than through the use of successive trial and error by simulation.

4.0 RECOMMENDATIONS

The above discussions trace a history of analytical techniques for optimal management that were applied as sufficient computing ability became available. At the present time, analyses of conjunctive use systems already in the literature must be evaluated critically as to what variables were modeled and what could and should have been modeled. An important question is - were appropriate constraints used and adequately represented ?

A simple model incorpotaring the important features of conjunctive use mangement for optimal cropping pattern is proposed in the following paragraphs.

4.1 The Proposed Model

After the review of existing models in literature, a suitable model, which is a combination of simulation and optimization approach, is being presented here, which can easily be solved on a computer system.

The area of interest may be a command area of a reservoir scheme where several canals and distributories are running. One can divide the area in a number of zones of near uniform surface water supplies, net irrigation requirements and hydrological conditions.

To maximize the benefits from the crop yield, the objective function is defined as,

$$\text{Max } Z = \sum_k \sum_i A_{ik} Y_{ik} P_i - \sum_k \sum_j C_s S_{jk} - \sum_k \sum_j C_g G_{jk} - \sum_k \sum_i A_{ik} C_{ik} \qquad (4.1)$$

where,

i = crop index

j = time index

k = zone index

A_{ik} = area under i^{th} crop in k^{th} zone

Y_{ik} = yield per unit area of i^{th} crop in k^{th} zone

P_i = unit selling price of i^{th} produce

C_s = unit cost of surface water

C_g = unit cost of ground water

S_{jk} = amount of surface water diverted in canals in k^{th} zone during j^{th} period

G_{jk} = amount of ground water withdrawn in k^{th} zone during j^{th} period

C_{ik} = cost of other inputs for i^{th} crop in k^{th} zone (as, fertilizers, labour cost etc.)

This function may be subject to the following constraints:

(i) The total surface water diverted to all the zones during j^{th} period should not be more than the reservoir release (RR_j) in j^{th} period.

$$\sum_k S_{jk} \leq RR_j \qquad \text{for all } j \qquad (4.2)$$

(ii) Surface water diverted to k^{th} zone during j^{th} period must be less than or at most equal to the canal capacity (CC_k) in k^{th} zone.

$$S_{jk} \leq CC_k \qquad \text{for all } j \text{ and for all } k \qquad (4.3)$$

(iii) Net water requirement

The crop water requirement for all the crops in any area during all time periods should be met from surface water diverted and ground water withdrawal in that area during the particular time period.

$$\sum_i A_{ik} b_{ij} = S_{kj} + G_{kj} \quad \text{for all k and for all j} \tag{4.4}$$

where,

$G_{kj} = \sum_l Q_p(l,k,j)$ for all j

b_{ij} = net crop water requirement for i^{th} crop in j^{th} period

$Q_p(l,k,j)$ = quantity of ground water withdrawal from l^{th} well in k^{th} zone during j^{th} period

These $Q_p(l,k,j)$ can be determined using discrete kernel approach for linear systems with the constraints on drawdown on water table. The rise in water table should not be so large as to cause water logging and it should not be lowered so much as to leave shallow wells dry. The procedure for implementing these constraints is discussed below. Let the flow conditions follow Dupuit Forchheimer assumptions then it can be shown that the drawdown at well m at time n due to combined effect of pumping at all wells and recharge through all canal reaches is given by

$$S(l,n) = \sum_{\gamma}\sum_{l} Q_p(l,k,\gamma)\delta(l,m,n-\gamma+1) - \sum_{\gamma}\sum_{r} Q_R(r,k,\gamma)\delta(r,m,n-\gamma+1) \quad (4.5)$$

where

$Q_p(l,k,\gamma)$ = quantity of witharawal from l^{th} well in k^{th} zone during unit time-step γ

$Q_R(l,k,\gamma)$ = quantity of recharge from l^{th} canal reach in k^{th} zone during unit time-step γ

For a homogeneous, isotropic, infinite aquifer,

$$\delta(l,m,n-\gamma+1) = \frac{1}{4\pi T}\int_0^1 \frac{e^{-\frac{\phi R_{lm}^2}{4T(n-\tau)}}}{(n-\tau)}\,d\tau \quad (4.6)$$

and

$$\delta(r,m,n-\gamma+1) = \frac{1}{4\pi T}\int_0^1 \frac{e^{-\frac{\phi R_{rm}^2}{4T(n-\tau)}}}{(n-\tau)}\,d\tau \quad (4.7)$$

R_{lm} = distance between well l and well m

R_{rm} = distance between canal reach r and well m.

These kernel coefficients for non-homogeneous aquifer are to be obtained numerically.

In the water logged area, the drawdown should be larger than a specified limite, say, $S_{min}(l,n)$ and the maximum limit of drawdown so as not to cause dry wells is assumed to be $S_{max}(l,n)$ Hence,

$$S_{max}(l,n) \geq S(l,n) \geq S_{min}(l,n) \quad (4.8)$$

Once the kernel coefficients have been calculated and saved, then the generation of different sets of drawdowns corresponding to different pumping patterns is easily obtained numerically and one can choose the pumping pattern for the required range of drawdown which can also be suboptimized in the problem. In this manner, $Q_p(l,k,\gamma)$ can be obtained assuming that the $Q_R(l,k,\gamma)$ are known for the canal system.

(iv) Area under all the crops of k^{th} zone should not exceed the total area of k^{th} zone (A_k)

$$\sum_i A_{ik} \leq A_k \quad \text{for all } k \tag{4.9}$$

(v) To meet the food requirements, one may put a constraint on growing each crop to a minimum amount. The area under a particular crop summed over all zones must not be less than the minimum area assigned to i^{th} crop in the project (A_i^{min})

$$\sum_k A_{ik} \geq A_i^{min} \tag{4.10}$$

Further, to calculate crop water requirement b_{ij} of i^{th} crop during j^{th} period, there are several methods available. The method recommended by Ministry of Agriculture (1971), Govt. of India using pan evaporation and the crop coefficient is considered most suitable. The b_{ij} is estimated as follows:

(i) The crop growing period, mid months, and maximum crop factor are used to calculate weighted monthly consumptive use coefficients on the basis of assumptions conerning the probable distribution of planting and harvesting over the respective periods.

(ii) These coefficients are multiplied by the class A pan evaporation figures for the region in question to give consumptive use in depth units.

(iii) Preplanting requirements are estimated and added to the appropriate months' consumptive use. Similarly, end of season soil moisture credit can also be estimated and deducted from the appropriate months' consumptive use.

(iv) If the resulting water requirement exceeds the effective rainfall, the net irrigation requirement is the difference between these two, otherwise it is equal to zero.

(v) The field irrigation requiremnt and the gross irrigation requirement are calculated by dividing the net irrigation requirement by field efficiency, and the field irrigation requirement by conveyance distribution efficiency respectively.

The problem can be easily solved on a computer by simplex algorithm, as it is a linear programming problem and the optimal cropping pattern can be determined.

5.0 REMARKS

Steps that are required in such analyses are, constructing an approach to determine the nature of the problem to be studied and determining the interactions of most importance. Situations where data are limited or very costly, may require the use of coarser spatial and temporal scales than the analyst would prefer. This, in turn, nears that the policy to be implemented will be conservative, reflecting lack of knowledge of flow movement and system dynamics. The economic value of data dictates levels of resolution used for future investigations.

A major problem confronting the analyst remains in his choice of a detailed modelling aparoach. An important direction for research effort would be to examine the general nature of mathematical response surfaces for conjunctive use problems. If they are relatively flat, it may be possible to use simulation approaches, through which considerable system detail can be handled, rather than direct optimization procedures. This would overcome many of the problems of dimensionality that presently limit the use of optimization models and would at the same time provide realistic operating schedules for conjunctive use systems.

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