DP-3

# BEST FIT DISTRIBUTION USING NORMALIZATION PROCEDURES AND CHI-SQUARE CRITERION

S RAMASESHAN
DIRECTOR

STUDY GROUP

S M SETH
N K GOEL

NATIONAL INSTITUTE OF HYDROLOGY
JAL VIGYAN BHAVAN
ROORKEE-247 667(UP)
INDIA

1982-83

CONTENTS

# ABSTRACT

The documentation of the computer programme for best fit distribution using normalization procedures and chi-square criterion includes listing of source programme, data file and output file with test data and example calculations. This also gives input specifications and output description. Various normalization procedures which have been used in the programme are: inverse Pearson type III transformation, log-normal transformation for which the parameters are estimated on the basis of theoretical relationships, log transformation, inverse log Pearson type III transformation, and square root transformation.

## 1.0 INTRODUCTION

Hydrologic data in its original form are rarely normally distributed. Many of the hydrologic data are positively skewed and so the application of normal distribution to the original data is not appropriate. However, transformation methods are available to transform the data to normal distribution. Various normalization procedures are available in practice. Many transformation procedures require the use of computer as the transformation procedure involved is cumbersome to apply with a calculator. Since the sample size of the hydrologic data generally available is limited, it is desirable to transform the data to normality using transformation procedures and select the best fitting normalization procedure based on some fitting criterion.

Log normal transformation, inverse Pearson type III transformation, square root transformation, inverse log Pearson type III transformation and Box-Cox (power) transformation etc. are some of the commonly used transformations for normalization.

Chi-square test, Kolmogorov-Smirnov test and Cramer-Von Mises test are some of the well known tests to judge the goodness of fit.

A computer programme developed by Dr M Krishnaswamy at I.I.T.,Kanpur has been modified and improved after incorporating suitable changes. This report documents the modified computer programme.

## 2.0 PURPOSE OF THE PROGRAMME

The purpose of the programme is to find out best fitting distribution after testing various normalization procedures on the basis of chi-square statistic for different seasons/ months of the year. The programme compares the following normalization procedures:

a. Normal distribution by method of moments

b. Inverse Pearson type III transformation

c. Log normal transformation ( parameters are obtained on the basis of theoretical relationships)

d. Log transformation ( parameters are estimated by method of moments)

e. Inverse log Pearson type III transforamtion

f. Square root transformation

The programme calculates the number of degrees of freedom and can be applied to data sets other than weekly/ monthly/seasonal time series.

## 3.0 METHOD USED

i. The programme first sorts out the data for a particular season. In the case of time series of monthly data the programme sorts out the data for each month while in the case of weekly data, the data for each week is separated.

ii. The programme arranges the seasonal data in descending order.

iii. Various transformations are tested on this arranged data, which are as follows:

a. Normal distribution by method of moments:
The data as such is compared with the normal distribution.

b. Inverse Pearson type III transformation:
The following equation is used to normalize the data:

$$Y = \left\{ \left( \frac{C_s}{2} \left( \frac{X - \mu}{\sigma} \right) + 1 \right)^{1/3} - 1 \right\} \frac{6}{C_s} + \frac{C_s}{6} \qquad \ldots(1)$$

where,

$X$ : Original series

$\mu$ : Mean of the original series

$\sigma$ : Standard deviation of original series

$C_s$ : Coefficient of skewness of original series

Y : Pearson type III transformed series

c. Log normal distribution ( parameters estimation on the basis of theoretical relationships):

Parameters of the log transformed series are calculated on the basis of following theoretical relationships:

$$\mu_y = \log(\mu_x) - 0.5 \log \{ ( \sigma_x / \mu_x)^2 + 1 \} \quad \ldots(2)$$

$$\sigma_y = \{ \log (( \sigma_x / \mu_x)^2 + 1)\}^{\frac{1}{2}} \quad \ldots(3)$$

where,

$\mu_x$ : Mean of the original series

$\sigma_x$ : Standard deviation of the original series

$\mu_y$ : Mean of log transformed series

$\sigma_y$ : Standard deviation of the log transformed series

The parameters so obtained are used for the calculation of chi-square statistic.

d. Log transformation:

$$Y = \log X \quad \ldots(4)$$

where,

Y : Log transformed series

X : Original series

e. Inverse log Pearson type III transformation:

In inverse log Pearson type III transformation log transformed series is used instead of original series. Rest of the procedure for transformation is similar to that for inverse Pearson type III transformation.

f. Square root transformation:

In this procedure square root of the original series is used as transformed series.

iv. After computing the statistical parameters of the transformed series, the programme calculates the chi-square statistic for the transformation. Number of degrees of freedom is also calculated.

v. The chi-square value so obtained can be compared with critical chi-square value for desired significance level and calculated number of degrees of freedom. The transformation giving the least chi-square value is considered to be the best for that season/month/ week data set.

vi. Steps i to v are repeated for other seasons.

## 4.0 COMPUTER PROGRAMME

The computer programme for best fit distribution consists of one main routine and ten subroutines. The subroutines have been described below:

i. SUBROUTINE NDTRI (P, X, C, IER)

The purpose of this subroutine is to calculate standard normal variate corresponding to a given probability. The calling arguments are:

P : Input probability

X : Output argument such that P = Y = the probability that the random variable is less than or equal to 0.0

C : Output density function F(X)

IER: Output error code

IER = -1; if P is not in the interval (0,1)

IER=0; if there is no error

ii. SUBROUTINE MSS (X,AMEAN,STDEV, SKEW)

This subroutine calculates mean, standard deviation and coefficient of skewness of the given series. Various calling arguments are:

X : Given series

AMEAN: Mean of the series

STDEV: Standard deviation of the series

SKEW : Coefficient of skewness of the series

iii. SUBROUTINE CSS (X, AMEAN, STDEV, SKEW, THF, T)

This subroutine calculates chi-square and degrees of freedom for a transformation. Various calling arguments are:

X : Given series

AMEAN: Mean of the series, calculated from subroutine MSS

STDEV: Standard deviation of the series, calculated from subroutine MSS

SKEW : Coefficient of skewness of the series calculated from subroutine MSS

THF : Theoretical frequency of each class

T : Standard normal variate

iv. SUBROUTINE NORMAL ( CX, L, THF, T, AMEAN, STDEV, SKEW)

This subroutine analyses for normal distribution. The calling arguments are:

CX : Input series

L : Number of years

THF : Theoretical frequency

T : Standard normal variate

AMEAN: Mean

STDEV: Standard deviation

SKEW : Coefficient of skewness

v. SUBROUTINE PT3 ( CX, L, THF, T, AMEAN, STDEV, SKEW)

This subroutine analyses inverse Pearson type III transformation.

vi. SUBROUTINE LNC (CX, L, THF, T, AMEAN, STDEV, SKEW)

This subroutine analyses log normal distribution for which parameters are calculated on the basis of theoretical relationships.

vii. SUBROUTINE LN (CX, L, THF, T, AMEAN, STDEV, SKEW)

This subroutine analyses log transformation. The parameters are calculated by method of moments.

viii. SUBROUTINE LP3 (CX, L, THF, T, AMEAN, STDEV, SKEW)

This subroutine analyses inverse log Pearson type III transformation.

ix. SUBROUTINE SQRTT (CX, L, THF, T, AMEAN, STDEV, SKEW)

This subroutine analyses square root transformation.

x. SUBROUTINE SORTX (N, X)

This subroutine sorts out the data in descending order. The calling arguments are:

N : Total number of observations in the series

X : Series to be arranged in descending order

Note: Various calling arguments in subroutines PT3, LNC LN, LP3, AND SQRTT are same.

## 5.0 INPUT SPECIFICATIONS, OUTPUT DESCRIPTION AND RESTRICTIONS ON USE

### 5.1 Input Specifications

Input cards/lines have been divided in two parts:

a. Job cards

b. Data cards.

#### 5.1.1 Job cards

| Card | Variable | Description | Format |
|---|---|---|---|
| FIRST | TITLE | Title of the problem | A |
| SECOND | N | Total number of observations | Free |
| | NS | Number of seasons in a year | |
| | NCLAS | Number of classes for the calculation of chi-square | |
| THIRD | N1 | Option code for normal distribution | Free |
| | N2 | Option code for inverse Pearson type III transformation | |
| | N3 | Option code for log normal distribution (parameters on the basis of theoretical relationships) | |
| | N4 | Option code for log transformation | |

N5 Option code for inverse log Pearson type III transformation

N6 Option code for square root transformation

If any of the transformation is not required 0 is given corresponding to its option code, otherwise 1 is given.

5.1.2 Data cards

Observations are punched till end in the free format.

5.2 Output Description

The following statistics are printed for the desired transformation(s) for all the seasons:

| Statistics | Format |
|---|---|
| Mean of the transformed series | F8.3 |
| Standard deviation of the transformed series | F8.3 |
| Coefficient of skewness of the transformed series | F8.3 |
| Chi-square | F8.3 |
| Number of degree of freedom | I5 |

5.3 Restrictions on Use

1. The number of classes should be chosen in such a way that at least 5 observations are there in each class.

2. The number of data points should not be more than 500. However, if the number of data points is more, the dimension statements of the programme can be changed accordingly.
3. The data should be continuous, without any gap.
4. The observations for a season should be independent.
5. The deficiencies of chi-square criterion are associated with the programme too.

## 6.0 TEST DATA

The programme was run on 38 years monthly data of river Lakshmanatirtha for different options. The values of N, NS, NCLAS, N1, N2, N3, N4, N5 and N6 are given as described below:

N = 38 x 12 = 456

NS = 12 ( since data is monthly)

NCLAS = 6 ( for NCLAS = 6, the theoretical frequency of each class will be 5.83)

N1 = 1

N2 = 1

N3 = 1

N4 = 1

N5 = 1

N6 = 1

(The best fitting distribution is required as such values of N1, N2, N3, N4, N5, N6 have been given as 1).

## 7.0 EXAMPLE CALCULATIONS

The statistical parameters of inverse log Pearson type III transformed series and square root transformed series for season 1 are given under:

(I) Inverse log Pearson type III transformation:

| | | |
|---|---|---|
| Mean | = | 0.001 |
| Standard deviation | = | 1.010 |
| Coefficient of skewness | = | −0.173 |
| Chi-square | = | 4.000 |
| No. of degrees of freedom | = | 3 |

(II) Square root transformation:

| | | |
|---|---|---|
| Mean | = | 3.652 |
| Standard deviation | = | 2.326 |
| Coefficient of skewness | = | 2.606 |
| Chi-square | = | 20.7368 |
| No. of degrees of freedom | = | 3 |

(Inverse log Pearson type III transformation and square root transformations have been chosen just to show the calculations of transformed series).

The chi-square values for normal distribution, Pearson type III distribution, log normal distribution ( parameters on the basis of theoretical relationships), log normal distribution, log Pearson type III distribution and square root distribution are 69.0526, 2.7368, 2.1053, 3.3684, 4.0000, 20.7368 respectively. From standard tables, the critical value of chi-square at 95% probability level and for 3 degrees of freedom is 7.81. On the basis of chi-square statistic the data for month 1 can be assumed to fit Pearson type III distribution or log normal distribution or log Pearson type III distribution. However, the chi-square is minimum in case of log normal distribution ( for which the parameters are estimated on the basis of theoretical relationships), it can be assumed that log normal distribution is best fitting distribution for data of month 1.

## 8.0 APPLICATION, SAMPLE INPUT AND SAMPLE OUTPUT

The programme has been run on 38 years monthly data of river Lakshmanatirtha for various transformations. The listings of the source programme,data file and output file have been given in Appendix I, Appendix II and Appendix III. On VAX-11/780 computer system the run time statistics for the programme is as follows:

a. Compilation time = 12.7 seconds

b. Linking time = 2.47 seconds

c. Run time = 3.60 seconds

## 9.0 RECOMMENDATIONS

The programme for best fit distribution using normalisation procedures and chi-square criterion can be used for any type of seasonal hydrologic data e.g. daily, pentad, ten daily, monthly or annual. It can also be applied to other data sets. The data should be continuous without any gap. The observations across the year should be independent. The programme has been implemented and tested on VAX-11/780 computer system. The programme can be run on any other system also with simple FORTRAN instructions and with minor or no modifications.

REFERENCES

1. Krishnaswamy, M (1979), 'Computer Programme for Best Fit Distribution Using Normalization Procedures and Chi-square Criterion', Indian Institute of Technology, Kanpur ( unpublished).

2. Haan, C.T. (1977), 'Statistical Methods in Hydrology' Iowa State University Press, Ames, IOWA.

## APPENDIX I

### PROGRAMME FOR BEST FIT DISTRIBUTION USING NORMALISATION PROCEDURES AND CHI-SQUARE CRITERION

```
C       PROGRAM TO TEST VARIOUS NORMALIZATION PROCEDURES ON THE
C       BASIS OF CHI SQUARE
C       N1 STANDS FOR NORMAL DISTRIBUTION
C       N2 STANDS FOR PEARSON TYPE 3 DISTRIBUTION
C       N3 STANDS FOR LOGNORMAL DISTRIBUTION PARAMETERS ARE ESTIMATED
C       ON THE BASIS OF THEORETICAL RELATIONS
C       N4 STANDS FOR LOGNORMAL DISTRIBUTION
C       N5 STANDS FOR LOGPEARSON DISTRIBUTION
C       N6 STANDS FOR SQUARE ROOT DISTRIBUTION
C       IF ANY OF THE TRANSFORMATION IS NOT REQUIRED GIVE 0 CORROS
C       PONDING TO THAT TRANSFORMATION
        DIMENSION TITLE(80),KX(500),THF(100),FL(100),T(100),CX(500)
        1,CX1(500),CX2(500)
        COMMON/BL1/L
        COMMON/BK1/N1,N2,N3,N4,N5,N6
        COMMON/BL2/NCLAS
        REAL KX
        OPEN(UNIT=1,FILE='GOEL.DAT',STATUS='OLD')
        OPEN(UNIT=2,FILE='GOEL.OUT',STATUS='NEW')
 80     FORMAT(80A1)
 81     FORMAT(1X,80A1)
 14     FORMAT(1X,12F7.1)
1001    FORMAT(' TOTAL NO. OF VALUES=',I6/' NO.OF SEASONS PER YEAR=',I6
        1/' NCLAS=',I6)
1002    FORMAT(11H INPUT DATA/12(1H*)/)
        READ(1,80) TITLE
        WRITE(2,81) TITLE
118     READ(1,*) N,NS,NCLAS
C       N IS THE TOTAL NO. OF OBSERVATIONS
C       NS IS THE NO. OF SEASONS PER YEAR
C       NCLAS IS THE NO. OF CLASSES REQUIRED FOR CHI
C       SQUARE CALCULATION;  THIS DEPENDS UPON THE NO.
C       OF VALUES PER YEAR
        READ(1,*)N1,N2,N3,N4,N5,N6
        WRITE(2,1001) N,NS,NCLAS
        READ(1,*) (KX(I),I=1,N)
        WRITE(2,1002)
        WRITE(2,14) (KX(I),I=1,N)
        NPT=NCLAS+1
        L=N/NS
        FL(1)=0.0
        FL(NPT)=0.9999
        DO 5 I=2,NCLAS
 5      FL(I)=FLOAT(I-1)/FLOAT(NCLAS)
C       CALCULATE STANDARISED VARIATES CORRESPONDING TO LIMITS OF
C       SELECTED CLASS INTERVALS
        DO 555 I=1,NPT
        FFL=FL(I)
        CALL NDTRI(FFL,AX,C,IER)
 555    T(I)=AX
```

```
C       CALCULATE THEORETICAL FREQUENCY FOR EACH CLASS
        DO 68 I=1,NCLAS
68      THF(I)=FLOAT(L)/FLOAT(NCLAS)
        J=1
 100    DO 15 I=1,L
        II=(I-1)*NS+J
        CX(I)=KX(II)
        CX2(I)=CX(I)
 15     CX1(I)=CX(I)
        WRITE(2,40)J
40      FORMAT(20X,'ANALYSIS FOR SEASON',4X,I5/20X,19(1H*)/)
        WRITE(2,10)(CX(I),I=1,L)
 10     FORMAT(3X,10F7.1)
        CALL SORTX(L,CX)
        WRITE(2,20)
20      FORMAT(3X,'SHORTED RECORDED DATA')
        WRITE(2,10)(CX(I),I=1,L)
        CALL NORMAL(CX,L,THF,T,AMEAN,STDEV,SKEW)
        CALL PT3(CX,L,THF,T,AMEAN,STDEV,SKEW)
        CALL LNC(CX1,L,THF,T,AMEAN,STDEV,SKEW)
        CALL LN(CX1,L,THF,T,AMEAN,STDEV,SKEW)
        CALL LP3(CX1,L,THF,T,AMEAN,STDEV,SKEW)
        CALL SQRTT(CX2,L,THF,T,AMEAN,STDEV,SKEW)
        J=J+1
        IF(J.GT.NS)GO TO 200
        GO TO100
 200    STOP
        END
C       NORMAL DISTRIBUTION
        SUBROUTINE NORMAL(CX,L,THF,T,AMEAN,STDEV,SKEW)
        DIMENSION CX(500),THF(100),T(100)
        COMMON/BK1/N1,N2,N3,N4,N5,N6
        IF(N1)500,100,500
500     WRITE(2,1003)
1003    FORMAT(33H ANALYSIS FOR NORMAL DISTRIBUTION)
        CALL MSS(CX,AMEAN,STDEV,SKEW)
        CALL CSS(CX,AMEAN,STDEV,SKEW,THF,T)
100     RETURN
        END
C       PEARSON TYPE 3 DISTRIBUTION BY BEARD NORMALIZATION
        SUBROUTINE PT3(CX,L,THF,T,AMEAN,STDEV,SKEW)
        COMMON/BK1/N1,N2,N3,N4,N5,N6
        DIMENSION CX(500),THF(100),T(100)
        IF(N2.EQ.0)GOTO 100
        IF(N1.EQ.0)GO TO 200
        GO TO 300
200     CALL MSS(CX,AMEAN,STDEV,SKEW)
300     WRITE(2,25)
25      FORMAT(1X,'PEARSON TYPE 3 DISTRIBUTIN')
        DO 725 J=1,L
```

```
      CX(J)=(CX(J)-AMEAN)/STDEV
      CX(J)=(SKEW*CX(J))/2.0+1.0
      IF(CX(J))507,507,508
507   CX(J)=(-1.)*(ABS(CX(J)))**(1./3.)
      GO TO 509
508   CX(J)=CX(J)**(1./3.)
509   CX(J)=(6./SKEW)*(CX(J)-1.)+(SKEW)/6.
725   CONTINUE
      CALL MSS(CX,AMEAN,STDEV,SKEW)
      CALL CSS(CX,AMEAN,STDEV,SKEW,THF,T)
100   RETURN
      END
C     LOGNORMAL DISTRIBUTION PARAMETERS ESTIMATED BY CHOW'S METHOD
      SUBROUTINE LNC(CX,L,THF,T,AMEAN,STDEV,SKEW)
      COMMON/BK1/N1,N2,N3,N4,N5,N6
      DIMENSION CX(500),THF(100),T(100)
      IF(N3.EQ.0)GO TO 100
      IF(N1.EQ.0)GO TO 300
      IF(N2.EQ.1)GO TO 300
      GOTO 500
300   CALL MSS(CX,AMEAN,STDEV,SKEW)
500   WRITE(2,25)
25    FORMAT(1X,'CHOW METHOD FOR LOGNORMAL DISTRIBUTION')
      VAR=(STDEV/AMEAN)**2+1.0
      AMEAN=ALOG(AMEAN)-0.5*ALOG(VAR)
      STDEV=ALOG(VAR)
      STDEV=SQRT(STDEV)
      DO 60 I=1,L
      IF(CX(I).EQ.0.0)CX(I)=1.
60    CX(I)=ALOG(CX(I))
      CALL MSS(CX,AMEAN1,STDEV1,SKEW1)
      CALL CSS(CX,AMEAN,STDEV,SKEW1,THF,T)
100   RETURN
      END
C     LOG TRANSFORMATION
      SUBROUTINE LN(CX,L,THF,T,AMEAN,STDEV,SKEW)
      COMMON/BK1/N1,N2,N3,N4,N5,N6
      DIMENSION CX(500),THF(100),T(100)
      IF(N4.EQ.0)GO TO 100
      IF(N3.EQ.0)GO TO 200
      GO TO 500
200   DO 20 I=1,L
      IF(CX(I).EQ.0.)CX(I)=1.
20    CX(I)=ALOG(CX(I))
500   WRITE(2,1012)
1012  FORMAT(1X,19H LOG TRANSFORMATION)
      CALL MSS(CX,AMEAN,STDEV,SKEW)
      CALL CSS(CX,AMEAN,STDEV,SKEW,THF,T)
100   RETURN
      END
```

```
C       LOG PEARSON TYPE 3 DISTRIBUTION
        SUBROUTINE LP3(CX,L,THF,T,AMEAN,STDEV,SKEW)
        COMMON/BK1/N1,N2,N3,N4,N5,N6
        DIMENSION CX(500),THF(100),T(100)
        IF(N5.EQ.0)GO TO 100
        IF(N3.EQ.0.AND.N4.EQ.0)GOTO 200
        GO TO 500
200     DO 20 I=1,L
        IF(CX(I).EQ.0.)CX(I)=1.0
20      CX(I)=ALOG(CX(I))
        CALL MSS(CX,AMEAN,STDEV,SKEW)
500     DO 728 J=1,L
       CX(J)=(CX(J)-AMEAN)/STDEV
        CX(J)=(SKEW*CX(J))/2.0+1.0
        IF(CX(J))607,607,608
607     CX(J)=(-1.)*(ABS(CX(J)))**(1./3.)
        GO TO 609
608     CX(J)=CX(J)**(1./3.)
609     CX(J)=(6./SKEW)*(CX(J)-1.)+(SKEW)/6.
728     CONTINUE
        WRITE(2,1023)
 1023   FORMAT(1X,'LOG PEARSON TYPE 3 DISTRIBUTION')
        CALL MSS(CX,AMEAN,STDEV,SKEW)
        CALL CSS(CX,AMEAN,STDEV,SKEW,THF,T)
 100    RETURN
        END
C       SQUARE ROOT TRANSFORMATION
        SUBROUTINE SQRTT(CX,L,THF,T,AMEAN,STDEV,SKEW)
        COMMON/BK1/N1,N2,N3,N4,N5,N6
        DIMENSION CX(500),THF(100),T(100)
        IF(N6)500,100,500
500     DO 123 I=1,L
        IF(CX(I).LE.0.)CX(I)=0.1
123     CX(I)=SQRT(CX(I))
        WRITE(2,1020)
1020    FORMAT(26H SQUAREROOT TRANSFORMATION)
        CALL MSS(CX,AMEAN,STDEV,SKEW)
        CALL CSS(CX,AMEAN,STDEV,SKEW,THF,T)
100     RETURN
        END
C       CALCULATE AND TYPE SEASONAL STATISTICAL PARAMETERS
        SUBROUTINE MSS(X,AMEAN,STDEV,SKEW)
        DIMENSION X(500)
        COMMON/BL1/L
        SUM=0.
        DO 2 I=1,L
 2      SUM=SUM+X(I)
        AMEAN=SUM/L
        SUM1=0.
        SUM2=0.
```

```
      DO 4 I=1,L
      SUM1=SUM1+(X(I)-AMEAN)**2
4     SUM2=SUM2+(X(I)-AMEAN)**3
      STDEV=SQRT(SUM1/(L-1))
      SKEW=(L*SUM2)/((L-1)*(L-2)*STDEV*STDEV*STDEV)
      RETURN
      END
C     CALCULATE CHI SQUARE AND NO. OF DEGREE OF FREEDOM
      SUBROUTINE CSS(X,AMEAN,STDEV,SKEW,THF,T)
      COMMON/BL2/NCLAS
      COMMON/BL1/L
      DIMENSION X(500),THF(100),T(100),FREQ(100)
      DO 78 I=1,NCLAS
78    FREQ(I)=0.
      DO 59 I=1,L
      AMINP=AMEAN+STDEV*T(2)
      M=1
61    IF(X(I)-AMINP)62,62,63
62    FREQ(M)=FREQ(M)+1.0
      GO TO 59
63    IF(M-NCLAS)501,62,501
501   M=M+1
      AMINP=AMEAN+STDEV*T(M+1)
      GO TO 61
59    CONTINUE
C     CALCULATE CHI-SQUARE STATISTIC AND NO. OF DEGREES OF FREEDOM
      CHIS=0.
      DO 69 I=1,NCLAS
69    CHIS=CHIS+((ABS(FREQ(I)-THF(I))))**2/THF(I)
      NDF=NCLAS-3
      WRITE(2,1004)AMEAN,STDEV,SKEW,CHIS,NDF
 1004 FORMAT(7X,'MEAN',2X,'STD. DEVIATION',2X,'COEF.OF SKEW',2X
     1,'CHI SQUARE',2X,'DEGREE OF FREEDOM'/5X,F9.3,2X,F9.3
     2,6X,F8.3,5X,F8.4,4X,I5)
      RETURN
      END
C
C.................................................................
C
C     SUBROUTINE NDTRI
C
C     PURPOSE
C       COMPUTES X=P**(-1)(Y), THE ARGUMENT X SUCH THAT Y=P(X)
C       =THE PROBABILITY THAT THE RANDOM VARIABLE U,DISTRIBUTED
C       NORMALLY(0,1), IS LESS THAN OR EQUAL TO X. F(X), THE
C       ORDINATE OF THE NORMAL DENSITY, AT X, IS ALSO COMPUTED.
C
C     USAGE
C       CALL NDTRI(P,X,C,IER)
C
```

```
C       DESCRIPTION OF PARAMETERS
C         P   -INPUT PROBABILITY
C         X   -OUTPUT ARGUMENT SUCH THAT P=Y=THE PROBABILITY THAT
C              THE RANDOM VARIABLE IS LESS THAN OR EQUAL TO X
C         C   -OUTPUT DENSITY,F(X)
C         IER -OUTPUT ERROR CODE
C             =-1  IF P IS NOT IN THE INTERVAL (0,1),INCLUSIVE
C            X=C=.99999E+37 IN THIS CASE
C             =C  IF THERE IS NO ERROR
C                   SEE REMARKS BELOW
C       REMARKS
C         MAXIMUM ERROR IS 0.00045
C         IF P=0,X IS SET TO -(10)**74.D IS SET TO C
C         IF P=1,X IS SET TO  (10)**74.D IS SET TO C
C       SUBROUTINES AND SUBPROGRAMS REQUIRED
C         NONE
C       METHOD
C          BASED ON APPROXIMATIONS IN C.HASTINGS,''APPROXIMATIONS
C          FOR DIGITAL COMPUTERS'',PRICETON UNIV.PRESS,PRINCETON,
C          N.J.,1955.SEE EQUATION 26.2.23,HAND BOOK OF MATHEMAICAL
C          FUNCTIONS,ABRAMOWITZ AND STEGUN,DOVER PUBLICATIONS,INC.,
C          NEW YORK.
        SUBROUTINE NDTRI(P,X,D,IE)
        IE=C
        X=.99999E+37
        D=X
        IF(P)1,4,2
1       IE=-1
        GO TO 12
2       IF(P-1.0)7,5,1
4       X=-0.999999E+37
5       D=0.0
        GO TO 12
7       D=P
        IF(D-0.5)9,9,8
8       D=1.0-D
9       T2=ALOG(1.0/(D*D))
        T=SQRT(T2)
        X=T-(2.515517+0.802853*T+0.010328*T2)/(1.0+1.432788*T+
        10.189269*T2+0.001308*T*T2)
        IF(P-0.5)10,10,11
10      X=-X
11      D=0.3989423*EXP(-X*X/2.0)
12      RETURN
        END
        SUBROUTINE SORTX (N,X)
C   SORTS IN DECREASING ORDER, X(I)=LARGEST
        DIMENSION X(100)
        K=N-1
        DO 2 L=1,K
```

```
      M=N-L
      DO 2 J=1,M
      IF (X(J)-X(J+1)) 1,1,2
1     XT=X(J)
      X(J)=X(J+1)
      X(J+1)=XT
2     CONTINUE
      RETURN
      END
```

## APPENDIX II

## TEST INPUT

MONTHLY STREAMFLOW DATA OF RIVER LAKSHMANATIRTHA

456,12,6

1,1,1,1,1,1

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 8 | 10 | 11 | 12 | 16 | 7 | 962 | 630 | 808 | 135 | 4 | 2 |
| 7 | 6 | 4 | 6 | 4 | 437 | 1537 | 867 | 132 | 138 | 49 | 6 |
| 4 | 17 | 10 | 8 | 9 | 2 | 1534 | 813 | 23 | 169 | 19 | 4 |
| 4 | 9 | 4 | 7 | 9 | 347 | 1006 | 299 | 105 | 475 | 2 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1625 | 1178 | 248 | 316 | 72 | 2 |
| 6 | 10 | 6 | 6 | 101 | 747 | 2764 | 1496 | 195 | 125 | 923 | 74 |
| 52 | 23 | 13 | 7 | 18 | 641 | 2207 | 1527 | 290 | 626 | 9 | 51 |
| 6 | 15 | 5 | 15 | 59 | 270 | 2878 | 1462 | 425 | 160 | 4 | 4 |
| 36 | 9 | 7 | 3 | 180 | 375 | 3529 | 520 | 329 | 601 | 379 | 20 |
| 13 | 14 | 20 | 13 | 16 | 22 | 2209 | 884 | 63 | 167 | 97 | 11 |
| 5 | 14 | 23 | 10 | 3 | 8 | 1956 | 718 | 133 | 156 | 5 | 1 |
| 9 | 2 | 1 | 3 | 5 | 535 | 2017 | 2962 | 840 | 253 | 679 | 240 |
| 39 | 28 | 92 | 23 | 14 | 2 | 1128 | 873 | 748 | 803 | 10 | 25 |
| 14 | 10 | 3 | 13 | 52 | 773 | 1569 | 3203 | 434 | 62 | 40 | 17 |
| 6 | 7 | 5 | 9 | 15 | 290 | 1004 | 1003 | 349 | 98 | 8 | 5 |
| 5 | 8 | 8 | 6 | 5 | 54 | 2210 | 1134 | 1036 | 286 | 30 | 12 |
| 13 | 16 | 17 | 17 | 30 | 186 | 1287 | 629 | 47 | 240 | 16 | 16 |
| 17 | 18 | 18 | 16 | 16 | 137 | 620 | 903 | 666 | 362 | 14 | 14 |
| 17 | 16 | 12 | 12 | 12 | 206 | 3416 | 3121 | 249 | 482 | 33 | 14 |
| 5 | 14 | 13 | 7 | 25 | 418 | 2142 | 1636 | 192 | 479 | 13 | 24 |
| 17 | 8 | 7 | 7 | 257 | 609 | 560 | 160 | 328 | 451 | 93 | 3 |
| 8 | 8 | 3 | 2 | 15 | 414 | 2921 | 1390 | 162 | 559 | 355 | 15 |
| 14 | 13 | 20 | 9 | 411 | 642 | 2822 | 1193 | 118 | 186 | 292 | 20 |
| 12 | 7 | 7 | 39 | 366 | 171 | 4513 | 1330 | 905 | 360 | 147 | 33 |
| 13 | 14 | 14 | 14 | 44 | 889 | 8746 | 1760 | 1700 | 373 | 188 | 70 |
| 10 | 17 | 27 | 46 | 113 | 197 | 2014 | 1433 | 92 | 50 | 315 | 15 |
| 9 | 3 | 16 | 24 | 428 | 1316 | 15019 | 4112 | 1256 | 1128 | 566 | 293 |
| 188 | 113 | 69 | 140 | 726 | 37 | 3653 | 2305 | 1010 | 2171 | 258 | 310 |
| 76 | 73 | 55 | 24 | 32 | 154 | 1726 | 1158 | 272 | 257 | 20 | 25 |
| 11 | 17 | 11 | 16 | 40 | 51 | 1712 | 6246 | 592 | 831 | 291 | 30 |
| 28 | 36 | 39 | 41 | 44 | 13 | 2463 | 239 | 26 | 9 | 5 | 13 |
| 5 | 7 | 12 | 5 | 15 | 1 | 634 | 394 | 101 | 118 | 83 | 20 |
| 19 | 16 | 4 | 3 | 15 | 598 | 2041 | 1560 | 77 | 27 | 4 | 4 |
| 2 | 0 | 0 | 3 | 9 | 94 | 1819 | 1398 | 104 | 55 | 3 | 7 |
| 2 | 0 | 0 | 0 | 26 | 18 | 1880 | 1251 | 320 | 31 | 46 | 14 |
| 5 | 4 | 4 | 8 | 26 | 280 | 712 | 891 | 69 | 1069 | 205 | 21 |
| 18 | 11 | 9 | 9 | 41 | 1362 | 763 | 424 | 200 | 123 | 21 | 5 |
| 3 | 4 | 4 | 5 | 331 | 72 | 1723 | 292 | 212 | 453 | 54 | 55 |

## APPENDIX III

## TEST OUTPUT

```
 MONTHLY STREAMFLOW DATA OF RIVER LAKSHMANATIRTHA
TOTAL NO. OF VALUES=   456
NO.OF SEASONS PER YEAR=    12
NCLAS=     6
INPUT DATA
************
```

```
   8.0   10.0   11.0   12.0   16.0    7.0  962.0  630.0  808.0  135.0    4.0    2.0
   7.0    6.0    4.0    6.0    4.0  437.0 1537.0  867.0  132.0  138.0   49.0    6.0
   4.0   17.0   10.0    8.0    9.0    2.0 1534.0  813.0   23.0  169.0   19.0    4.0
   4.0    9.0    4.0    7.0    9.0  347.0 1006.0  299.0  105.0  475.0    2.0    0.0
   1.0    0.0    0.0    0.0    0.0    0.0 1625.0 1178.0  248.0  316.0   72.0    2.0
   6.0   10.0    6.0    6.0  101.0  747.0 2764.0 1496.0  195.0  125.0  923.0   74.0
  52.0   23.0   13.0    7.0   18.0  641.0 2207.0 1527.0  290.0  626.0    9.0   51.0
   6.0   15.0    5.0   15.0   59.0  270.0 2878.0 1462.0  425.0  160.0    4.0    4.0
  36.0    9.0    7.0    3.0  180.0  375.0 3529.0  520.0  329.0  601.0  379.0   20.0
  13.0   14.0   20.0   13.0   16.0   22.0 2209.0  884.0   63.0  167.0   97.0   11.0
   5.0   14.0   23.0   10.0    3.0    8.0 1956.0  718.0  133.0  156.0    5.0    1.0
   9.0    2.0    1.0    3.0    5.0  535.0 2017.0 2962.0  840.0  253.0  679.0  240.0
  39.0   28.0   92.0   23.0   14.0    2.0 1128.0  873.0  748.0  803.0   10.0   25.0
  14.0   10.0    3.0   13.0   52.0  773.0 1569.0 3203.0  434.0   62.0   40.0   17.0
   6.0    7.0    5.0    9.0   15.0  290.0 1004.0 1003.0  349.0   98.0    8.0    5.0
   5.0    8.0    8.0    6.0    5.0   54.0 2210.0 1134.0 1036.0  286.0   30.0   12.0
  13.0   16.0   17.0   17.0   30.0  186.0 1287.0  629.0   47.0  240.0   16.0   16.0
  17.0   18.0   18.0   16.0   16.0  137.0  620.0  903.0  666.0  362.0   14.0   14.0
  17.0   16.0   12.0   12.0   12.0  206.0 3416.0 3121.0  249.0  482.0   33.0   14.0
   5.0   14.0   13.0    7.0   25.0  418.0 2142.0 1636.0  192.0  479.0   13.0   24.0
  17.0    8.0    7.0    7.0  257.0  609.0  560.0  160.0  328.0  451.0   93.0    3.0
   8.0    8.0    3.0    2.0   15.0  414.0 2921.0 1390.0  162.0  559.0  355.0   15.0
  14.0   13.0   20.0    9.0  411.0  642.0 2822.0 1193.0  118.0  186.0  292.0   20.0
  12.0    7.0    7.0   39.0  366.0  171.0 4513.0 1330.0  905.0  360.0  147.0   33.0
  13.0   14.0   14.0   14.0   44.0  889.0 8746.0 1760.0 1700.0  373.0  188.0   70.0
  10.0   17.0   27.0   46.0  113.0  197.0 2014.0 1433.0   92.0   50.0  315.0   15.0
   9.0    3.0   16.0   24.0  428.0 1316.015019.0 4112.0 1256.0 1128.0  566.0  293.0
 188.0  113.0   69.0  140.0  726.0   37.0 3653.0 2305.0 1010.0 2171.0  258.0  310.0
  76.0   73.0   55.0   24.0   32.0  154.0 1726.0 1158.0  272.0  257.0   20.0   25.0
  11.0   17.0   11.0   16.0   40.0   51.0 1712.0 6246.0  592.0  831.0  291.0   30.0
  28.0   36.0   39.0   41.0   44.0   13.0 2463.0  239.0   26.0    9.0    5.0   13.0
   5.0    7.0   12.0    5.0   15.0    1.0  634.0  394.0  101.0  118.0   83.0   20.0
  19.0   16.0    4.0    3.0   15.0  598.0 2041.0 1560.0   77.0   27.0    4.0    4.0
   2.0    0.0    0.0    3.0    9.0   94.0 1819.0 1398.0  104.0   55.0    3.0    7.0
   2.0    0.0    0.0    0.0   26.0   18.0 1880.0 1251.0  320.0   31.0   46.0   14.0
   5.0    4.0    4.0    8.0   26.0  280.0  712.0  891.0   69.0 1069.0  205.0   21.0
  18.0   11.0    9.0    9.0   41.0 1362.0  763.0  424.0  200.0  123.0   21.0    5.0
   3.0    4.0    4.0    5.0  331.0   72.0 1723.0  292.0  212.0  453.0   54.0   55.0
                    ANALYSIS FOR SEASON        1
                    *****************
```

```
   8.0     7.0     4.0     4.0     1.0     6.0    52.0     6.0    36.0    13.0
   5.0     9.0    39.0    14.0     6.0     5.0    13.0    17.0    17.0     5.0
```

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 17.0 | 8.0 | 14.0 | 12.0 | 13.0 | 10.0 | 9.0 | 188.0 | 76.0 | 11.0 |
| 28.0 | 5.0 | 19.0 | 2.0 | 2.0 | 5.0 | 18.0 | 3.0 | | |

SHORTED RECORDED DATA

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 188.0 | 76.0 | 52.0 | 39.0 | 36.0 | 28.0 | 19.0 | 18.0 | 17.0 | 17.0 |
| 17.0 | 14.0 | 14.0 | 13.0 | 13.0 | 13.0 | 12.0 | 11.0 | 10.0 | 9.0 |
| 9.0 | 8.0 | 8.0 | 7.0 | 6.0 | 6.0 | 6.0 | 5.0 | 5.0 | 5.0 |
| 5.0 | 5.0 | 4.0 | 4.0 | 3.0 | 2.0 | 2.0 | 1.0 | | |

ANALYSIS FOR NORMAL DISTRIBUTION

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| 18.605 | 31.924 | 4.428 | 69.0526 | 3 |

PEARSON TYPE 3 DISTRIBUTIN

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| 0.274 | 0.840 | -0.096 | 2.7368 | 3 |

CHOW METHOD FOR LOGNORMAL DISTRIBUTION

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| 2.237 | 1.171 | 0.474 | 2.1053 | 3 |

LOG TRANSFORMATION

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| 2.310 | 1.028 | 0.474 | 3.3684 | 3 |

LOG PEARSON TYPE 3 DISTRIBUTION

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| 0.001 | 1.010 | -0.173 | 4.0000 | 3 |

SQUAREROOT TRANSFORMATION

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| 3.652 | 2.326 | 2.606 | 20.7368 | 3 |

ANALYSIS FOR SEASON 2
******************

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 10.0 | 6.0 | 17.0 | 9.0 | 0.0 | 10.0 | 23.0 | 15.0 | 9.0 | 14.0 |
| 14.0 | 2.0 | 28.0 | 10.0 | 7.0 | 8.0 | 16.0 | 18.0 | 16.0 | 14.0 |
| 8.0 | 8.0 | 13.0 | 7.0 | 14.0 | 17.0 | 3.0 | 113.0 | 73.0 | 17.0 |
| 36.0 | 7.0 | 16.0 | 0.0 | 0.0 | 4.0 | 11.0 | 4.0 | | |

SHORTED RECORDED DATA

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 113.0 | 73.0 | 36.0 | 28.0 | 23.0 | 18.0 | 17.0 | 17.0 | 17.0 | 16.0 |
| 16.0 | 16.0 | 15.0 | 14.0 | 14.0 | 14.0 | 14.0 | 13.0 | 11.0 | 10.0 |
| 10.0 | 10.0 | 9.0 | 9.0 | 8.0 | 8.0 | 8.0 | 7.0 | 7.0 | 7.0 |
| 6.0 | 4.0 | 4.0 | 3.0 | 2.0 | 0.0 | 0.0 | 0.0 | | |

ANALYSIS FOR NORMAL DISTRIBUTION

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| 15.711 | 20.446 | 3.653 | 34.9474 | 3 |

PEARSON TYPE 3 DISTRIBUTIN

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| 0.077 | 1.121 | -0.781 | 20.4211 | 3 |

CHOW METHOD FOR LOGNORMAL DISTRIBUTION

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| 2.259 | 0.995 | -0.408 | 5.5789 | 3 |

LOG TRANSFORMATION

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| 2.283 | 1.016 | -0.408 | 5.8947 | 3 |

LOG PEARSON TYPE 3 DISTRIBUTION

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| -0.001 | 1.011 | 0.242 | 8.7368 | 3 |

SQUAREROOT TRANSFORMATION

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| 3.476 | 1.932 | 1.595 | 18.5263 | 3 |

ANALYSIS FOR SEASON 3

******************

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 11.0 | 4.0 | 10.0 | 4.0 | 0.0 | 6.0 | 13.0 | 5.0 | 7.0 | 20.0 |
| 23.0 | 1.0 | 92.0 | 3.0 | 5.0 | 8.0 | 17.0 | 18.0 | 12.0 | 13.0 |
| 7.0 | 3.0 | 20.0 | 7.0 | 14.0 | 27.0 | 16.0 | 69.0 | 55.0 | 11.0 |
| 39.0 | 12.0 | 4.0 | 0.0 | 0.0 | 4.0 | 9.0 | 4.0 | | |

SHORTED RECORDED DATA

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 92.0 | 69.0 | 55.0 | 39.0 | 27.0 | 23.0 | 20.0 | 20.0 | 18.0 | 17.0 |
| 16.0 | 14.0 | 13.0 | 13.0 | 12.0 | 12.0 | 11.0 | 11.0 | 10.0 | 9.0 |
| 8.0 | 7.0 | 7.0 | 7.0 | 6.0 | 5.0 | 5.0 | 4.0 | 4.0 | 4.0 |
| 4.0 | 4.0 | 3.0 | 3.0 | 1.0 | 0.0 | 0.0 | 0.0 | | |

ANALYSIS FOR NORMAL DISTRIBUTION

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| 15.079 | 19.242 | 2.655 | 28.0000 | 3 |

PEARSON TYPE 3 DISTRIBUTIN

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| 0.026 | 1.055 | -0.662 | 10.9474 | 3 |

CHOW METHOD FOR LOGNORMAL DISTRIBUTION

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| 2.230 | 0.983 | -0.132 | 1.4737 | 3 |

LOG TRANSFORMATION

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| 2.143 | 1.118 | -0.132 | 3.0526 | 3 |

LOG PEARSON TYPE 3 DISTRIBUTION

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| -0.001 | 1.000 | -0.005 | 3.6842 | 3 |

SQUAREROOT TRANSFORMATION

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| 3.333 | 2.022 | 1.228 | 4.6316 | 3 |

ANALYSIS FOR SEASON 4

******************

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 12.0 | 6.0 | 8.0 | 7.0 | 0.0 | 6.0 | 7.0 | 15.0 | 3.0 | 13.0 |
| 10.0 | 3.0 | 23.0 | 13.0 | 9.0 | 6.0 | 17.0 | 16.0 | 12.0 | 7.0 |
| 7.0 | 2.0 | 9.0 | 39.0 | 14.0 | 46.0 | 24.0 | 140.0 | 24.0 | 16.0 |
| 41.0 | 5.0 | 3.0 | 3.0 | 0.0 | 8.0 | 9.0 | 5.0 | | |

SHORTED RECORDED DATA

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 140.0 | 46.0 | 41.0 | 39.0 | 24.0 | 24.0 | 23.0 | 17.0 | 16.0 | 16.0 |
| 15.0 | 14.0 | 13.0 | 13.0 | 12.0 | 12.0 | 10.0 | 9.0 | 9.0 | 9.0 |
| 8.0 | 8.0 | 7.0 | 7.0 | 7.0 | 7.0 | 6.0 | 6.0 | 6.0 | 5.0 |
| 5.0 | 3.0 | 3.0 | 3.0 | 3.0 | 2.0 | 0.0 | 0.0 | | |

ANALYSIS FOR NORMAL DISTRIBUTION

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| 15.474 | 23.373 | 4.388 | 40.0000 | 3 |

```
PEARSON TYPE 3 DISTRIBUTIN
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
       0.204       0.975         -0.428       6.5263          3
CHOW METHOD FOR LOGNORMAL DISTRIBUTION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
       2.145       1.090          0.093       4.6316          3
 LOG TRANSFORMATION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
       2.210       1.008          0.093       9.0526          3
LOG PEARSON TYPE 3 DISTRIBUTION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
       0.000       1.000         -0.030       5.8947          3
SQUAREROOT TRANSFORMATION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
       3.393       2.018          2.112       8.7368          3
                   ANALYSIS FOR SEASON         5
                   *******************

     16.0    4.0    9.0    9.0    0.0  101.0   18.0   59.0  180.0   16.0
      3.0    5.0   14.0   52.0   15.0    5.0   30.0   16.0   12.0   25.0
    257.0   15.0  411.0  366.0   44.0  113.0  428.0  726.0   32.0   40.0
     44.0   15.0   15.0    9.0   26.0   26.0   41.0  331.0
  SHORTED RECORDED DATA
    726.0  428.0  411.0  366.0  331.0  257.0  180.0  113.0  101.0   59.0
     52.0   44.0   44.0   41.0   40.0   32.0   30.0   26.0   26.0   25.0
     18.0   16.0   16.0   16.0   15.0   15.0   15.0   15.0   14.0   12.0
      9.0    9.0    9.0    5.0    5.0    4.0    3.0    0.0
ANALYSIS FOR NORMAL DISTRIBUTION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
      92.842     159.090          2.484      38.7368          3
PEARSON TYPE 3 DISTRIBUTIN
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
       0.174       0.714          1.685      37.4737          3
CHOW METHOD FOR LOGNORMAL DISTRIBUTION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
       3.846       1.171          0.306      15.3684          3
 LOG TRANSFORMATION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
       3.408       1.515          0.306       3.6842          3
LOG PEARSON TYPE 3 DISTRIBUTION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
       0.001       0.999          0.022       5.8947          3
SQUAREROOT TRANSFORMATION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
       7.343       6.323          1.610      30.5263          3
                   ANALYSIS FOR SEASON         6
                   *******************

      7.0  437.0    2.0  347.0    0.0  747.0  641.0  270.0  375.0   22.0
      8.0  535.0    2.0  773.0  290.0   54.0  186.0  137.0  206.0  418.0
```

```
   609.0  414.0  642.0  171.0  889.0  197.0 1316.0   37.0  154.0   51.0
    13.0    1.0  598.0   94.0   18.0  280.0 1362.0   72.0
  SHORTED RECORDED DATA
  1362.0 1316.0  889.0  773.0  747.0  642.0  641.0  609.0  598.0  535.0
   437.0  418.0  414.0  375.0  347.0  290.0  280.0  270.0  206.0  197.0
   186.0  171.0  154.0  137.0   94.0   72.0   54.0   51.0   37.0   22.0
    18.0   13.0    8.0    7.0    2.0    2.0    1.0    0.0
ANALYSIS FOR NORMAL DISTRIBUTION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
      325.658    351.967          1.401      25.1579          3
PEARSON TYPE 3 DISTRIBUTIN
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
        0.034      0.916          0.627       2.4211          3
CHOW METHOD FOR LOGNORMAL DISIRIBUTION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
        5.399      0.880         -1.010      18.2105          3
 LOG TRANSFORMATION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
        4.698      2.045         -1.010      16.3158          3
LOG PEARSON TYPE 3 DISTRIBUTION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
       -0.019      0.943         -0.544       5.8947          3
SQUAREROOT TRANSFORMATION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
       15.028     10.125          0.320       1.7895          3
                    ANALYSIS FOR SEASON          7
                    *******************

   962.0 1537.0 1534.0 1006.0 1625.0 2764.0 2207.0 2878.0 3529.0 2209.0
  1956.0 2017.0 1128.0 1569.0 1004.0 2210.0 1287.0  620.0 3416.0 2142.0
   560.0 2921.0 2822.0 4513.0 8746.0 2014.015019.0 3653.0 1726.0 1712.0
  2463.0  634.0 2041.0 1819.0 1880.0  712.0  763.0 1723.0
  SHORTED RECORDED DATA
 15019.0 8746.0 4513.0 3653.0 3529.0 3416.0 2921.0 2878.0 2822.0 2764.0
  2463.0 2210.0 2209.0 2207.0 2142.0 2041.0 2017.0 2014.0 1956.0 1880.0
  1819.0 1726.0 1723.0 1712.0 1625.0 1569.0 1537.0 1534.0 1287.0 1128.0
  1006.0 1004.0  962.0  763.0  712.0  634.0  620.0  560.0
ANALYSIS FOR NORMAL DISTRIBUTION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
     2455.816   2536.997          3.839      32.4211          3
PEARSON TYPE 3 DISTRIBUTIN
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
        0.047      1.160         -0.630      32.1053          3
CHOW METHOD FOR LOGNORMAL DISTRIBUTION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
        7.443      0.852          0.662      12.5263          3
 LOG TRANSFORMATION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
        7.541      0.677          0.662       4.0000          3
LOG PEARSON TYPE 3 DISTRIBUTION
```

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| 0.000 | 1.018 | -0.202 | 4.0000 | 3 |

SQUAREROOT TRANSFORMATION

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| 46.097 | 18.434 | 2.342 | 10.6316 | 3 |

ANALYSIS FOR SEASON 8
*****************

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 630.0 | 867.0 | 813.0 | 299.0 | 1178.0 | 1496.0 | 1527.0 | 1462.0 | 520.0 | 884.0 |
| 718.0 | 2962.0 | 873.0 | 3203.0 | 1003.0 | 1134.0 | 629.0 | 903.0 | 3121.0 | 1636.0 |
| 160.0 | 1390.0 | 1193.0 | 1330.0 | 1760.0 | 1433.0 | 4112.0 | 2305.0 | 1158.0 | 6246.0 |
| 239.0 | 394.0 | 1560.0 | 1398.0 | 1251.0 | 891.0 | 424.0 | 292.0 | | |

SHORTED RECORDED DATA

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 6246.0 | 4112.0 | 3203.0 | 3121.0 | 2962.0 | 2305.0 | 1760.0 | 1636.0 | 1560.0 | 1527.0 |
| 1496.0 | 1462.0 | 1433.0 | 1398.0 | 1390.0 | 1330.0 | 1251.0 | 1193.0 | 1178.0 | 1158.0 |
| 1134.0 | 1003.0 | 903.0 | 891.0 | 884.0 | 873.0 | 867.0 | 813.0 | 718.0 | 630.0 |
| 629.0 | 520.0 | 424.0 | 394.0 | 299.0 | 292.0 | 239.0 | 160.0 | | |

ANALYSIS FOR NORMAL DISTRIBUTION

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| 1405.105 | 1189.506 | 2.330 | 16.9474 | 3 |

PEARSON TYPE 3 DISTRIBUTIN

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| -0.145 | 1.396 | -1.220 | 9.0526 | 3 |

CHOW METHOD FOR LOGNORMAL DISTRIBUTION

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| 6.978 | 0.735 | -0.226 | 4.3158 | 3 |

LOG TRANSFORMATION

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| 6.963 | 0.783 | -0.226 | 3.3684 | 3 |

LOG PEARSON TYPE 3 DISTRIBUTION

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| -0.001 | 1.001 | 0.027 | 5.2632 | 3 |

SQUAREROOT TRANSFORMATION

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| 34.958 | 13.712 | 1.092 | 7.7895 | 3 |

ANALYSIS FOR SEASON 9
*****************

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 808.0 | 132.0 | 23.0 | 105.0 | 248.0 | 195.0 | 290.0 | 425.0 | 329.0 | 63.0 |
| 133.0 | 840.0 | 748.0 | 434.0 | 349.0 | 1036.0 | 47.0 | 666.0 | 249.0 | 192.0 |
| 328.0 | 162.0 | 118.0 | 905.0 | 1700.0 | 92.0 | 1256.0 | 1010.0 | 272.0 | 592.0 |
| 26.0 | 101.0 | 77.0 | 104.0 | 320.0 | 69.0 | 200.0 | 212.0 | | |

SHORTED RECORDED DATA

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1700.0 | 1256.0 | 1036.0 | 1010.0 | 905.0 | 840.0 | 808.0 | 748.0 | 666.0 | 592.0 |
| 434.0 | 425.0 | 349.0 | 329.0 | 328.0 | 320.0 | 290.0 | 272.0 | 249.0 | 248.0 |
| 212.0 | 200.0 | 195.0 | 192.0 | 162.0 | 133.0 | 132.0 | 118.0 | 105.0 | 104.0 |
| 101.0 | 92.0 | 77.0 | 69.0 | 63.0 | 47.0 | 26.0 | 23.0 | | |

ANALYSIS FOR NORMAL DISTRIBUTION

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| 390.947 | 391.405 | 1.584 | 33.0526 | 3 |

```
PEARSON TYPE 3 DISTRIBUTIN
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
       0.052      0.883         0.799       4.6316          3
CHOW METHOD FOR LOGNORMAL DISTRIBUTION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
       5.621      0.833        -0.205       7.4737          3
 LOG TRANSFORMATION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
       5.475      1.068        -0.205       2.1053          3
LOG PEARSON TYPE 3 DISTRIBUTION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
      -0.001      0.999        -0.062       2.1053          3
SQUAREROOT TRANSFORMATION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
      17.630      9.072         0.787       4.3158          3
                   ANALYSIS FOR SEASON        10
                   ******************

    135.0  138.0  169.0  475.0  316.0  125.0  626.0  160.0  601.0  167.0
    156.0  253.0  803.0   62.0   98.0  286.0  240.0  362.0  482.0  479.0
    451.0  559.0  186.0  360.0  373.0   50.0 1128.0 2171.0  257.0  831.0
      9.0  118.0   27.0   55.0   31.0 1069.0  123.0  453.0
  SHORTED RECORDED DATA
   2171.0 1128.0 1069.0  831.0  803.0  626.0  601.0  559.0  482.0  479.0
    475.0  453.0  451.0  373.0  362.0  360.0  316.0  286.0  257.0  253.0
    240.0  186.0  169.0  167.0  160.0  156.0  138.0  135.0  125.0  123.0
    118.0   98.0   62.0   55.0   50.0   31.0   27.0    9.0
ANALYSIS FOR NORMAL DISTRIBUTION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
      378.526    409.130        2.633      27.0526          3
PEARSON TYPE 3 DISTRIBUTIN
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
      -0.158      1.359        -0.864       7.7395          3
CHOW METHOD FOR LOGNORMAL DISTRIBUTION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
       5.549      0.880        -0.642       2.4211          3
 LOG TRANSFORMATION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
       5.423      1.132        -0.642       1.7895          3
LOG PEARSON TYPE 3 DISTRIBUTION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
      -0.002      1.005         0.065       0.8421          3
SQUAREROOT TRANSFORMATION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
      17.317      8.988         1.031       4.0000          3
                   ANALYSIS FOR SEASON        11
                   ******************

      4.0   49.0   19.0    2.0   72.0  923.0    9.0    4.0  379.0   97.0
      5.0  679.0   10.0   40.0    8.0   30.0   16.0   14.0   33.0   13.0
```

```
    93.0  355.0  292.0  147.0  188.0  315.0  566.0  258.0   20.0  291.0
     5.0   83.0    4.0    3.0   46.0  205.0   21.0   54.0
  SHORTED RECORDED DATA
   923.0  679.0  566.0  379.0  355.0  315.0  292.0  291.0  258.0  205.0
   188.0  147.0   97.0   93.0   83.0   72.0   54.0   49.0   46.0   40.0
    33.0   30.0   21.0   20.0   19.0   16.0   14.0   13.0   10.0    9.0
     8.0    5.0    5.0    4.0    4.0    4.0    3.0    2.0
ANALYSIS FOR NORMAL DISTRIBUTION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
      140.842    209.713         2.183       43.4737          3
PEARSON TYPE 3 DISTRIBUTIN
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
        0.116      0.792         1.211       36.8421          3
CHOW METHOD FOR LOGNORMAL DISTRIBUTION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
        4.363      1.081         0.018       22.6316          3
 LOG TRANSFORMATION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
        3.750      1.734         0.018        3.0526          3
LOG PEARSON TYPE 3 DISTRIBUTION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
        0.000      1.000         0.010        3.0526          3
SQUAREROOT TRANSFORMATION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
        9.205      7.591         1.120       21.6842          3
                     ANALYSIS FOR SEASON        12
                     ******************

     2.0    6.0    4.0    0.0    2.0   74.0   51.0    4.0   20.0   11.0
     1.0  240.0   25.0   17.0    5.0   12.0   16.0   14.0   14.0   24.0
     3.0   15.0   20.0   33.0   70.0   15.0  293.0  310.0   25.0   30.0
    13.0   20.0    4.0    7.0   14.0   21.0    5.0   55.0
  SHORTED RECORDED DATA
   310.0  293.0  240.0   74.0   70.0   55.0   51.0   33.0   30.0   25.0
    25.0   24.0   21.0   20.0   20.0   20.0   17.0   16.0   15.0   15.0
    14.0   14.0   14.0   13.0   12.0   11.0    7.0    6.0    5.0    5.0
     4.0    4.0    4.0    3.0    2.0    2.0    1.0    0.0
ANALYSIS FOR NORMAL DISTRIBUTION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
       39.342     74.327         3.026       44.7368          3
PEARSON TYPE 3 DISTRIBUTIN
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
        0.271      0.621         2.134       23.5789          3
CHOW METHOD FOR LOGNORMAL DISTRIBUTION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
        2.913      1.233         0.247        9.6842          3
 LOG TRANSFORMATION
      MEAN  STD. DEVIATION  COEF.OF SKEW  CHI SQUARE  DEGREE OF FREEDOM
        2.682      1.382         0.247   -    4.9474          3
LOG PEARSON TYPE 3 DISTRIBUTION
```

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| 0.001 | 1.000 | -0.005 | 12.5263 | 3 |

SQUAREROOT TRANSFORMATION

| MEAN | STD. DEVIATION | COEF.OF SKEW | CHI SQUARE | DEGREE OF FREEDOM |
|---|---|---|---|---|
| 4.863 | 4.015 | 2.126 | 20.1053 | 3 |