TR (BR) 145

STUDY OF 2-DIMENSIONAL FLOW BEHAVIOUR OF RIVER USING FESWMS - 2DH MODEL

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INDIA
1995-96

PREFACE

In many surface water flow problems of practical engineering concern, the three dimensional nature of the flow is of secondary importance, particularly when the width-to-depth ratio of the water body is large. The horizontal distribution of flow quantities play the vital role, and two-dimensional flow approximations in such cases are of great economic advantage. FESWMS-2DH, developed by Federal Highway Administrative of US department of transport(Frohlich, 1988), is a two-dimensional model which computes depth averaged horizontal velocities, water depths and time derivatives of these quantities if a time-dependent flow is modelled. The model has been found suitable to simulate flow in water bodies that have irregular topography and geometrical features. The equations that govern depth averaged surface water flow account for the effects of bed friction, wind induced stress at the water surface, fluid stresses caused by turbulence, and the effect of the Earth's rotation.

The study is an attempt to demonstrate the FESWMS-2DH model for different real life conditions. Different hypothetical cases having relevance to the field problems in broad range of flow conditions, namely; flow in a V-shaped channel, strongly curved channel, island area, meandering channel, flood plain area and single bridge opening, had been studied and reported as in hand information.

The report has been prepared by Sri Ramakar Jha, Scientist'C' with supporting inputs from Sri N.C.Ghosh, Scientist'E' and Dr.K.K.S.Bhatia, Scientist'F'. Sri R.K.Jaiswal, SRA, and Sri Atm Prakash, R.A., provided the necessary assistance.

S.M.SETH

(DIRECTOR)

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Notations

Zw	Water surface elevation
Zb	Bed surface elevation above horizontal datum
H	Water depth
u,v,w	Velocities in x,y and z directions
τ	Reynolds stress
β	Momentum correction coefficient
Ω	Coriolis parameter
ϕ	Latitude
g	Acceleration due to gravity
Rc	Radius of curvature
b	Channel width
Hs	Super-elevation
V	Average velocity
CFS	cubic ft./sec.

ABSTRACT

Two dimensional(surface water analysis) models also simulate unsteady, irregular, non-uniform and complex flow behaviour and features of flood plain areas, rivers, tributaries and coastal areas at desired precision level. Finite Element Surface Water Modelling System(FESWMS-2DH), which is a modular set of computer programs developed by Federal Highway Administrative of US-Department of Transport(Frohlich,1988), simulates steady and unsteady surface water flow which are essentially two-dimensional in a horizontal plane.

The FESWMS-2DH model uses the St.Venant equations which describe the conservation of momentum and conservation of mass. The model computes Frictional losses using the Chezy's and Manning's equation and the turbulence using the Boussinesq eddy viscosity concept. The kinematic eddy viscosity was evaluated from the shear velocity and flow depth. The effects of wind stress and the Coriolis force included in the model, are optional. The Galerkin finite element method was considered in the model formation to solve the resulting system of differential equations.

In the present study, the FESWMS-2DH model was applied to hypothetical cases having relevance to the real life problems,

namely; simple channel, strongly curved channel, island area, flood plain area, meandering channel and single bridge opening, to simulate the significant features of flow . The effect of changes in model parameters for different flow conditions were also studied. The results obtained by the model for different flow conditions showed good agreement with the results obtained for similar flow conditions. The two-dimensional model was able to reproduce the water surface elevations, velocity field and energy gradients for different flow conditions with desired satisfaction.

1.0 INTRODUCTION

For the analysis and modelling of river flow behaviour, where the geometry and hydraulic roughness changes gradually, one dimensional models are used. These models are also applied to irregular and complex flow conditions for lack of suitable alternative methods. Techniques available for analysing one dimensional flow includes the standard step method for solving steady gradually varied flow and finite element and finite difference techniques for solving unsteady flow. One dimensional method ignores the variation of flow velocity across the cross-sections and assumes the water surface constant along the cross-section. The properties of the cross-section, area of flow, hydraulic radius, and hydraulic roughness, are reduced to a single value representing the carrying capacity or conveyance of the cross-section. Only the average velocity and a single water surface elevation is solved for each cross-section. The distribution of the flow may be estimated from the incremental conveyance along the cross-section, although this prediction is insensitive to upstream or downstream conditions. Since it is assumed that the flow lines are parallel, the lateral component of the velocity is neglected. While a one-dimensional model may provide acceptable results where the geometry and hydraulic roughness are regular, such methods are not reliable under irregular or complex conditions found near bridge contractions, encroachments, irregular flood plains, or multiple channels.

Two-dimensional models are well suited to analyse irregular or complex flow conditions. It allows the water surface to vary along or across the study area and, also, allows the flow to follow any horizontal orientation. To illustrate where a two dimensional model may provide a more accurate representation of flow in a river, consider steady flow in a prismatic channel. The geometry and hydraulic roughness do not change along the channel, hence the flow depth, the velocity, and the distribution of the flow do not change along the channel. Although the magnitude of the longitudinal velocities may vary from bank to bank, the lateral velocities are zero, hence the lateral component of the bottom friction is zero. In the absence of other external forces, such as the Coriolis force or surface stresses, the water surface will tend to be constant from bank to bank. Under these conditions, the flow is essentially one-dimensional so that one-dimensional simulation and corresponding two-dimensional simulation will produce equivalent results.

The results of the two-dimensional simulation of flow in a simple channel, in an island area, flood plain area, strongly curved channel, in a meandering river and in a single bridge opening indicate that away from perturbations, the water surface of the flow tends to be constant from bank to bank, although the velocity may vary from bank to bank. This observation provides an important insight into one-dimensional modelling when considering the distribution of the total energy of the flow. The local total energy may be evaluated from the sum of the local velocity head and local water surface elevation. Given a constant water surface elevation from bank to bank, the total energy must vary from bank to bank as the velocity head varies. This contradicts the one-dimensional methods assumption that the total energy head is the same for all points in a cross-section.

To continue the illustration of where a two-dimensional model may provide a more accurate representation of flow in a river, consider a similar regular channel, with the exception that at some point the flow is perturbed by a slight asymmetrical change in geometry or hydraulic roughness. As a result of this slight change, the distribution of the flow changes, some lateral flow occurs, and there is a slight bank to bank variation in the water surface. As long as the change is gradual, a one-dimensional simulation and a comparable two-dimensional simulation will produce similar results with minor variations. Under more severe conditions, as may be found in an irregular flood plain or near a bridge contraction, the lateral flows and the bank to bank variation of the water surface may be large enough to cause a significant local discrepancy between the results of a one-dimensional simulation and a comparable two-dimensional simulation. Under irregular or complex flow conditions, the assumptions of one-dimensional flow are no longer valid and a two-dimensional simulation will provide a more complete and more accurate representation of the flow.

Two-dimensional modelling has progressed rapidly in coastal and estuarine applications, but there have been limited applications to riverine studies. In general, two dimensional models may consider flow in either a vertical or horizontal plane. For river flow, a depth averaged model with flow in a horizontal plane is appropriate. The most common approaches to two-dimensional river modelling are looped or linked one dimensional models(quasi-two-dimensional models), finite difference model and finite element models. The finite element method is particularly well suited to represent the irregular geometry and special boundary conditions associated with river flow.

Cunge et al.(1980) advanced the use of looped or linked one-dimensional models where storage cells and connecting channels form a horizontal network. This quasi-two-dimensional flow is best applied where there are definite flow boundaries. Zanobetti et al.(1970) used this method to model flood flows in the Mekong Delta. More recent applications of looped models include the work of Gurule(1982) who modelled the Rio Grande Del Loiza Delta in Puerto Rico.

Abbott et al.(1973) advanced the use of finite difference method for two-dimensional modelling. Finite difference methods have been applied extensively to coastal and estuarine problems and less frequently to river flow. The Danish Hydraulic Institute has developed a generalised finite difference program called System 21 as reported by Abbott and Warren(1981).

Bodine (1982) presented a solution scheme for modelling flows in a complex flood plain in which channels were embedded in a grid network. This scheme was applied to portions of the Buffalo Bayou, Oyster Creek, and Brazos River watersheds. Bodine's method was based on the concept of a model developed by Reid and Bodine(1968).

The work of Norton and King(1973) lead to the development of the generalised finite element model called RMA-2 for the US Army Corps of Engineers. RMA-2 was applied at Gee and McArthur(1982) to model flows on the Columbia river near the McNary Dam. RMA-2 has been incorporated into TABS-2 flow and sedimentation modelling system and the Columbia River Modelling System. While the US Army Corps of Engineers have performed many credible simulations with the TABS-2 modelling system, more accurate and more efficient modelling systems are available. TABS-2 is difficult to use, requiring considerable time and effort for any application. The specification of boundary conditions is awkward and some of the following computational aspects could be improved: (1) high values of the kinematic eddy viscosity are required for the model to converge to a stable solution, (2) the non-conservative form of the governing equations is solved rather than the more accurate conservative form, (3) eight node quadrilateral elements, rather than the more accurate and more stable nine-node quadrilateral elements are used, and (5) much faster solution methods have been developed.

In the present study a two-dimensional river model (Finite Element Surface Water Modelling System-FESWMS-2DH) has been used to study the phenomena & features of two-dimensional flow in fundamental applications. Based on the results, suggestions are made as to the best use of two-dimensional river models, and recommendations are made for the application and further development of two-dimensional river models.

This report contains six sample applications using the Finite Element Surface Water Modelling System (FESWMS-2DH) (Froehlich, 1988); flow in a simple channel, flow in an island area, flow in a strongly curved bend, flow in a meandering channel, flow in a flood plain area and flow in a single opening bridge crossing of a heavily vegetated flood plain.

2.0 DESCRIPTION OF THE MODELLING SYSTEM

FESWMS-2DH(Froehlich, 1988), developed for the Federal Highway Administration by the US Geological Survey, is a modular set of computer programs that simulate surface-water flows which are essentially two-dimensional in the horizontal plane. It consists of three distinct but related programs; **DINMOD**, the data input module, **FLOMOD**, the depth averaged flow analysis module and **ANOMOD**, the output analysis module. While FESWMS-2DH has been developed specifically to analyse flow at bridge crossing, the modelling system may be applied to many complex flow conditions, such as flow around islands and flow over irregular flood plains.

As a pre-processing program, DINMOD checks the input data for errors, generates plots of the finite element network and the ground surface contours, and puts the network data in an appropriate form for subsequent analysis. The program may be used to generate a new network automatically or refine an existing network. A special feature in DINMOD is an element resequencing capability which ensures efficient equation solution, thus reducing the computation time.

The graphics capabilities of FESWMS-2DH are useful tools in the development of the finite element network. Plots of the network may be examined to verify that the overall coverage is adequate, the critical areas such as bridge opening are well represented, and that elements are well formed. Ground surface contour plots may be checked to ensure the ground surface is adequately resolved and to verify that no spurious elevation data has been entered.

The solution of the two-dimensional depth-averaged flow equations is performed by **FLOMOD**. The flow depth and the x & y components of the velocity are evaluated using the St. Venants equations where two equations describe the conservation of momentum and one equation describes the conservation of mass. The Galerkin finite element method is used to solve the above system of differential equations. The fluid density is considered to be uniform and the pressure distribution hydrostatic. The effects of bottom friction and turbulent stresses may be evaluated using the Chezy's & Manning's equation and Boussinesq eddy viscosity concept, respectively. A value for the eddy viscosity may be assigned to an element or calculated by the program using a turbulence model. The effects of wind stress and the Coriolis force may also be included in the simulation.

For a typical subcritical flow simulation, the water surface elevation is specified at the downstream open boundary, slip flow is specified along the closed lateral boundaries, and the total flow is specified at the upstream open boundary. The total flow is apportioned to the nodes along the upstream boundary by the incremental conveyance across that section. For supercritical flow simulation, both the water surface elevation and the total flow are specified along the upstream boundary. Boundary conditions may be essential or natural. Natural boundary conditions are weakened boundary specifications and are generally preferred as the continuity performance at a natural boundary.

The mass continuity performance of a simulation may be assessed generally from flow check lines across the network or locally from the continuity norm of each element. This last feature allows the modeller to identify elements which may require refinement.

The post-processing program, **ANOMOD**, generates plots and printed reports from the network data and the flow field. The solution for the flow depth and velocity at each node generates a large amount of spatially related data. Since, the two-dimensional model allows the water surface to vary along or across the network and allows the flow to follow any horizontal orientation, the water surface and flow field can be very complex. The interpretation of the results of the simulation is greatly facilitated by the plotting capabilities of FESWMS-2DH. A plotted map of the water surface contours is used to describe the three dimensional water surface since discrete water surface elevations or a water surface profile cannot represent these results completely. The difference between the water surface elevations of two simulations may then be represented by lines of equal backwater and drawdown. The velocity or unit discharge at each node may be represented by a plot of the flow field where the direction and magnitude of flow is represented by scaled symbols such as arrows.

The modelling system is capable of simulating flow through single or multiple bridge opening as normal flow, pressure flow, weir flow, or culvert flow. The finite element network can be designed to represent complex geometry as may be found at bridge crossings, training structures, islands, multiple channels, levees, river bends, or irregularly shaped flood plains. The network can accommodate variable roughness as may be found in flood plains where dense vegetation is mixed with open areas. Wetting or drying of parts of the network is resolved by a feature which automatically adjusts boundary conditions enabling elements which are no longer submerged to be removed from the computations.

The flexibility of the finite element method combined with the special features of FESWMS-2DH allows the modelling system to be applied to a wide variety of flow conditions.

3.0 GOVERNING EQUATIONS

3.1 Three Dimensional Equation

The three-dimensional equation of motion, which describes the movement of water in a river, considers the conservation of mass, as expressed by the continuity equation, and the conservation of momentum, as expressed by Newton's second law of motion. These may be written as:

the conservation of mass,

$$\frac{\partial}{\partial_x}U+\frac{\partial}{\partial_y}V+\frac{\partial}{\partial_z}W=0...(1)$$

the conservation of momentum along the x axis,

$$\frac{\partial}{\partial_t}U+\frac{\partial}{\partial_x}(UU)+\frac{\partial}{\partial_y}(UV)+\frac{\partial}{\partial_z}(UW)+g\frac{\partial}{\partial_x}Z_w=0...(2)$$

the conservation of momentum along the y axis,

$$\frac{\partial}{\partial_t}V+\frac{\partial}{\partial_x}(VU)+\frac{\partial}{\partial_y}(VV)+\frac{\partial}{\partial_z}(VW)+g\frac{\partial}{\partial_y}Z_w=0...(3)$$

The velocity components along the x,y and z axis are u,v, and w respectively. The acceleration due to gravity is g. The bed surface elevation above some horizontal datum is z_b, and the water surface elevation, z_w, is equal to z_b plus the water depth, H(**Fig.1**).

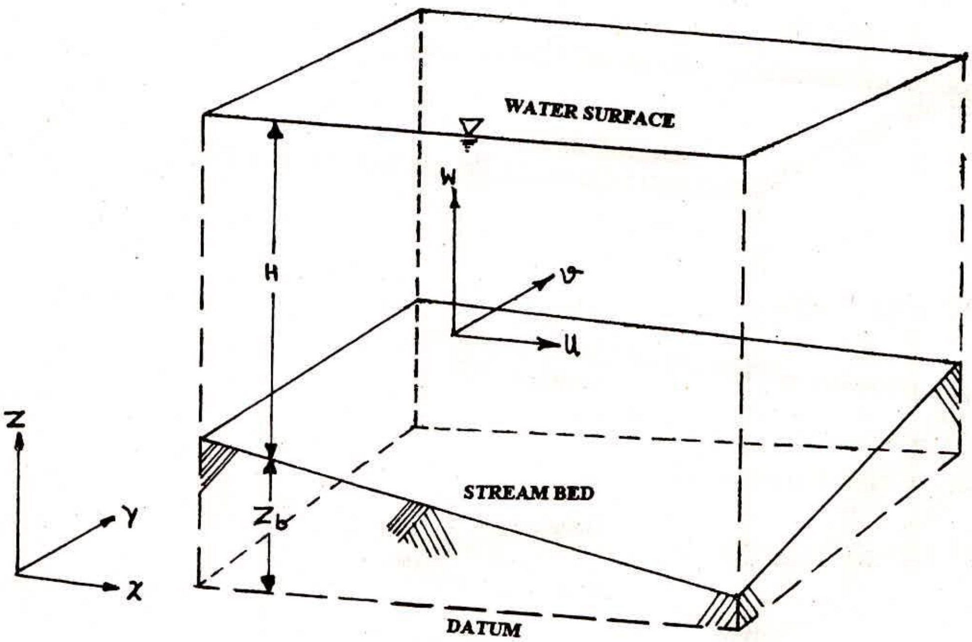


FIG. 1: CO-ORDINATE SYSTEM FOR THREE -DIMENSIONAL FLOW

The equation expressing the conservation of mass is in the form of the continuity equation. This implies that the fluid is incompressible which is a reasonable approximation giving the typical flow depth for rivers.

The equation expressing the conservation of momentum have been simplified. Terms describing bottom stresses, surface stresses, turbulent stresses, and coriolis force are included later in the depth-averaged flow equation. Terms describing the effects of molecular viscosity will not be included as these effects are several orders of magnitude smaller than turbulent effects. Further, the fluid density, is considered constant which may not be the case where suspended sediment, aeration, or differences in the salinity and temperature of the water cause density variation.

The equation describing the conservation of momentum along the z axis is omitted because vertical accelerations are negligible compared to gravity. It follows that the pressure distribution is hydrostatic. This assumption is valid where the vertical curvature of the stream lines may be neglected. This is true in general for rivers, but does not apply to flow over weirs or weirs type structures such as roadway embankments or spillway crests. It also does not apply to rapidly varied flow such as bores, hydraulic jumps, or dam-break flood waves.

The conservation of momentum equations are expressed in the conservative form. The first four terms describe the inertial forces. The first term represents the temporal acceleration which is to be considered when modelling unsteady flow. The second, third and fourth terms represent advective acceleration, as would be experienced by a particle as it moves through a steady flow field. The fifth term represents the pressure gradient resulting from a sloping water surface.

3.2 Effects of Turbulence

River flow is turbulent. While a comprehensive discussion of turbulence is beyond the scope of this report(see Lee and Froehlich, 1986 or Rodi, 1980), a few components are appropriate. The following description of the nature of turbulence is adapted from Rodi.

Turbulence is an eddying motion which at high Reynolds number has a wide spectrum of eddy sizes and corresponding fluctuation frequencies. The largest eddies, associated with the low fluctuations, are determined by the boundary conditions of the flow and their size of the same order as the flow domain. The smallest eddies, associated with high frequency fluctuations, are governed by viscous forces. It is mainly the large scale turbulent motion that transports momentum. As large eddies interact with the mean flow, kinetic energy is extracted from the mean motion and add to the large scale turbulent motion. As the turbulent eddies themselves interact, energy is cascaded to smaller scale eddies until finally dissipated by the smallest eddies through viscous action. The rate at which the mean flow energy is fed into the turbulent motion is determined by the large scale motion; only this energy can be passed on to smaller scales and dissipated. The rate of energy dissipationis determined by the large scale motion although dissipation takes place at the smallest eddies. The molecular viscosity does not determine the rate of energy dissipated, only the scale at which the dissipation takes place(Rodi, 1980).

Turbulent flow is highly irregular; at any point the velocity and pressure components of the flow fluctuate about time mean values u, v, w, and p. At any instant, the respective deviations from the mean u', v', w', and p', may be positive or negative but by definition will be equal to zero when averaged over a sufficient time interval. For rivers, this time interval may be on the order of several minutes. Where the velocity gradients are high the magnitude of the largest velocity deviations may be on the order of the mean horizontal velocity. The instantaneous velocity along the x, y, and z axis respectively may be described in the Reynolds decomposition as:

$$U = \overline{U} + U' \ldots (4)$$
$$V = \overline{V} + V' \ldots (5)$$
$$W = \overline{W} + W' \ldots (6)$$

when the instantaneous velocities are inserted into the equations of motion and a time average is taken, additional unknown terms are introduced into the conservation of momentum equations. Consider the advective term

$$\overline{\frac{\partial}{\partial_x}}(UV) = \overline{\frac{\partial}{\partial_x}}(\overline{U}\overline{V}) + \overline{\frac{\partial}{\partial_x}}(\overline{U}V') + \overline{\frac{\partial}{\partial_x}}(U'\overline{V}) + \overline{\frac{\partial}{\partial_x}}(U'V') \ldots (7)$$

The second and third terms on the right hand side of the equation vanish when time averaged as the mean fluctuation is zero by definition. The fourth term, when multiplied by the density, represents the transport of momentum due to fluctuating motion(Rodi,1980) and can be interpreted as a shear stress between fluid layers called **Reynolds stress.** The shear stress, τ_{xy} acts in the x direction on a plane perpendicular to the y direction and is formed from

$$\tau_{xy} = -\overline{\rho U'V'} \ldots (8)$$

Reynolds stresses are caused by particles of water entering regions of different mean velocity because of turbulent fluctuations. They accelerate or decelerate neighbour particles before returning to their mean values and this has the effect of an apparent shear stress(Jansen et al.,1979). The governing equations of motion shown earlier would contain five other similar terms. As a result of the higher order turbulent terms there are more unknowns than equations. The mean values of the velocity and depth can only be solved when these turbulent terms are resolved to allow closure of the equations of motion restoring to order or mean flow quantities(Rodi, 1980).

3.3 Depth averaged equations of motion

The three dimensional flow structure is not required for most river applications. In many surface water problems of practical engineering concern, the three dimensional nature of the flow is of secondary importance, particularly when the width-to-depth ration of the water body is large. In such a case, the horizontal distribution of flow quantities may be the main interest, and two-dimensional flow approximations can be used to great economic advantage. Neglecting vertical velocities and vertical applications, the depth-averaged velocity may obtained by integrating the horizontal velocity components from the bed elevation to the water surface(for a derivation see Jansen, 1979). An illustration of the depth averaged velocity is shown in **Fig. 2.**

It should be noted that once the flow is averaged over the depth, the vertical component of the velocity as well as the vertical distribution of the horizontal velocities cannot be evaluated directly. The depth averaged velocity along the x axis is U. The depth averaged velocity along the y axis is V. This is given by:

$$U = \frac{1}{H}\int_{Z_b}^{Z_b+H} UdZ \ldots (9) \qquad V = \frac{1}{H}\int_{Z_b}^{Z_b+H} VdZ \ldots (10)$$

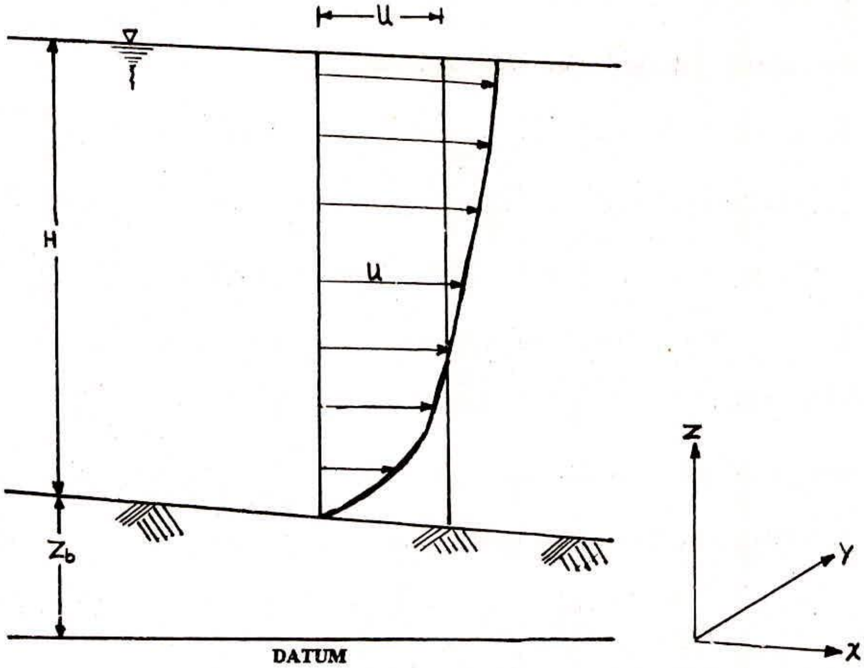


FIG. 2 : ILLUSTRATION OF DEPTH-AVERAGED VELOCITY

The two-dimensional depth-averaged equations of motion (Froehlich, 1988) which describe the movement of water in a river consider the conservation of mass, expressed by the continuity equation, and the conservation of momentum, expressed by Newton's second law. These equations are:

The conservation of mass

$$\frac{\partial}{\partial_t}H + \frac{\partial}{\partial_x}HU + \frac{\partial}{\partial_y}HV = 0 \ldots (11)$$

Conservation of momentum in the x direction

$$\frac{\partial}{\partial_t}HU + \frac{\partial}{\partial_x}(\beta_{uu})HUU + \frac{\partial}{\partial_y}(\beta_{uv})HUV + gH\frac{\partial}{\partial_x}Z_b + \frac{1}{2}g\frac{\partial}{\partial_x}HH - \Omega HV +$$
$$\frac{1}{\rho}[\tau_x^b - \tau_x^s - \frac{\partial}{\partial_x}(H\tau_{xx}) - \frac{\partial}{\partial_y}(H\tau_{xy}) = 0 \ldots (12)$$

and, The conservation of momentum in the y direction

$$\frac{\partial}{\partial_t}HV + \frac{\partial}{\partial_x}(\beta_{vu})HVU + \frac{\partial}{\partial_y}(\beta_{vv})HVV + gH\frac{\partial}{\partial_y}Z_b + \frac{1}{2}g\frac{\partial}{\partial_y}HH + \Omega HU +$$
$$\frac{1}{\rho}[\tau_y^b - \tau_y^s - \frac{\partial}{\partial_x}(H\tau_{yx}) - \frac{\partial}{\partial_y}(H\tau_{yy}) = 0 \ldots (13)$$

The conservation of momentum equations are expressed in the conservative form. The first three terms describe the inertial forces. The first term represents temporal acceleration. The second and third terms represent advective acceleration and include momentum correction coefficient. Momentum correction coefficient B_{uu}, B_{uv}, B_{vu}, and B_{vv} are applied to the advective terms to account for the non-uniform vertical distribution of the horizontal velocities(Froehlich, 1988). The momentum correction coefficient for the second term may be written as

$$\beta_{uv} = \frac{1}{HUV} \int_{Z_b}^{Z_b+H} UVdZ \ldots (14)$$

The fourth and fifth terms express the pressure gradient resulting from sloping water surface. The sixth term represents the Coriolis force which acts perpendicular to the velocity. The Coriolis parameter is given by

$$\Omega = 2W \sin\phi \ldots (15)$$

where W is equal to the angular velocity of the earth(7.27x 10^{-5} radians /s) and ϕ is the latitude. This term may not be significant for confined rivers since a velocity of 4 ft/s at a latitude of 30° induces a water surface slope of only 1x10^{-5} ft/ft perpendicular to the direction of flow.

The seventh and eighth terms represent bottom stresses and surface stresses respectively. These are discussed more completely by Froehlich, 1988 and Lee and Froehlich, 1986. Bottom stresses are modelled using the Manning's or Chezy's equation. Surface stresses due to wind are most significant in broad, shallow bodies of water.

The ninth and tenth terms represent the effects of the Reynolds stress. The Boussinesq eddy viscosity concept is used where the momentum transfers are proportional to the mean velocity gradients. This may be written as

$$\tau_{xy} = \rho \hat{v}_{xy} (\frac{\partial}{\partial_y} U + \frac{\partial}{\partial_x} V) \ldots (16)$$

The generation and dissipation of turbulent energy is assumed to be in local equilibrium. The eddy viscosity ν is a function of the flow and varies throughout the study area. Some finite element schemes require high and unrealistic values of the eddy viscosity to damp spurious oscillations. High values of the eddy viscosity distort the velocity distribution and the water surface elevation. A uniform value of the eddy viscosity does not account for the local turbulence structure. Numerical models which use this approach cannot in general describe the details of the mean flow correctly. Where the turbulence is mainly bed generated, as in open channel flow, the eddy viscosity can reasonably be well correlated with the shear velocity and flow depth(Froehlich, 1988; Lee and Froehlich, 1986).

3.4 Non-Conservative Form of the Governing Equations

Many authors express the conservation of momentum in a non-conservative form. Considering flow along the x-axis, the inertial terms in the conservation of mass may be expanded and expressed as

$$\frac{\partial}{\partial_t}HU + \frac{\partial}{\partial_x}HUU + \frac{\partial}{\partial_y}HUV$$

$$= H\frac{\partial}{\partial_t}U + U\frac{\partial}{\partial_t}H + UH\frac{\partial}{\partial_x}U + UU\frac{\partial}{\partial_x}H + HU\frac{\partial}{\partial_x}U + HV\frac{\partial}{\partial_y}U + HU\frac{\partial}{\partial_y}V + UV\frac{\partial}{\partial_y}H$$

$$= H[\frac{\partial}{\partial_t}U + U\frac{\partial}{\partial_x}U + V\frac{\partial}{\partial_y}V] + U[\frac{\partial}{\partial_t}H + \frac{\partial}{\partial_x}HU + \frac{\partial}{\partial_y}HV] \ldots (17)$$

The non-conservative form of the governing equations has the disadvantages of being less accurate than the conservative form of the equations since it is assumed that continuity is preserved exactly. The last three terms on the right hand side of this equation are similar to the continuity equation multiplied by a velocity. The non-conservative form of the conservation of momentum assumes these terms may be summed to zero. While this simplification is analytically exact for the continuum, it may not be appropriate for discrete numerical procedures. In the case of finite element methods, locally the continuity equation is equal to a residual which is not required to equal zero. The residual tends to be very high where the flow is strongly advecting, so that the non-conservative form of the governing equations would be less accurate near a bridge opening in particular. The non-conservative form of the equations is less exact when mixed with interpolation is used since the variables are approximated at some nodes, rather than solved. For much the difference scheme the continuity equation may not be exactly satisfied at computational nodes(Froehlich, 1988).

Under many conditions, the conservative form of the conservation of momentum equations is more exact than the non-conservative form, particularly where the flow is strongly advecting. It follows from this point that FESWMS-2DH, which uses the conservative form of the equations, is more accurate than the TABS-2 or RMA-2 modelling systems which use the non-conservative forms of the equations.

3.5 Finite Element Method

The finite element method is a numerical procedure for solving differential equations. The finite element method divides the study area, or domain, into sub-domains or elements which are defined by a finite number of nodes located along the element boundary or interior. Values of a dependent variable are approximated within each element using values defined at the element's nodes and a set of piecewise continuous interpolated functions.

A fundamental premise of the finite element method is that a region of arbitrary shape can be modelled by an assemblage of elements. The element shapes may be geometrically simple, and the most commonly used element shapes are triangles and quadrilaterals. In general, the finite element network need not be regular allowing the study area to be represented by elements of varying size. More elements may be used in a region of interest or complex conditions.

A number of finite element techniques have been developed, and the most commonly used techniques employ the Galerkin method of weighted residuals(Lee and Froehlich, 1988). Following this method a general functional behaviour of the dependent variables which approximately satisfies the boundary conditions is assumed. In general, the governing equations are not exactly satisfied when these assumed values are substituted which results in an error or residual. To ensure that error is minimized, the residual is required to vanish, in an average sense, when multiplied by a weighted function and summed at every point in the solution domain. In Galerkin's method, the weighting functions are chosen to be the same as the interpolation functions. A set of simultaneous equations result when the weighting functions are specified. All the element(local) equations are assembled to obtain a complete(global) set of equations. These equations are then solved to obtain the parameters of the functional representation of dependent variable.

A more complete presentation of the finite element method may be found in Froehlich(1988), Lee and Froehlich(1986), Zienkiewicz(1971), Pinder and Gray(1977), Koutitas(1983), Reddy(1984) and references contained therein.

4.0 MODEL APPLICATIONS

4.1 Flow in a Simple Channel

The following case deals with the flow in a short reach of a regular channel. The purpose of this example is to demonstrate the effect of the kinematic eddy viscosity on the velocity distribution. The results of a model with a high value of eddy viscosity are comparable to the results of a similar model with a lower value of the eddy viscosity.

The Boussinesq eddy viscosity concept simulates the turbulent transfer of momentum as a function of the velocity gradient and the kinematic eddy viscosity. An appropriate value of the eddy viscosity should reflect the nature of the flow and the geometry of the watercourse among other conditions.

The eddy viscosity tends to damp variation in the velocity and so may be used to mask numerical instabilities. Vreugdenhill and Wijbenga(1982) discuss the importance of the value of the eddy viscosity in obtaining meaningful results.

4.1.1 Model description

The finite element network and the cross section are shown in **Fig.3**. The study area was 40 ft. long and 20 ft. wide. The centre line of the channel was at 0.0 ft elevation, the elevation of the banks was at 2.0 ft elevation(**Fig.4**). The Mannings roughness coefficient was 0.30. The water surface

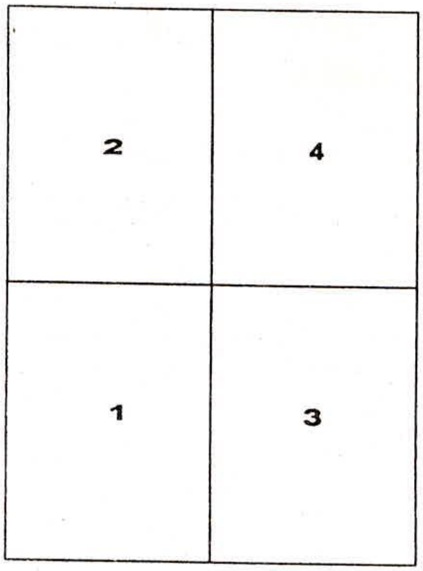


FIG.3: NETWORK DESIGN OF A V-SHAPED CHANNEL

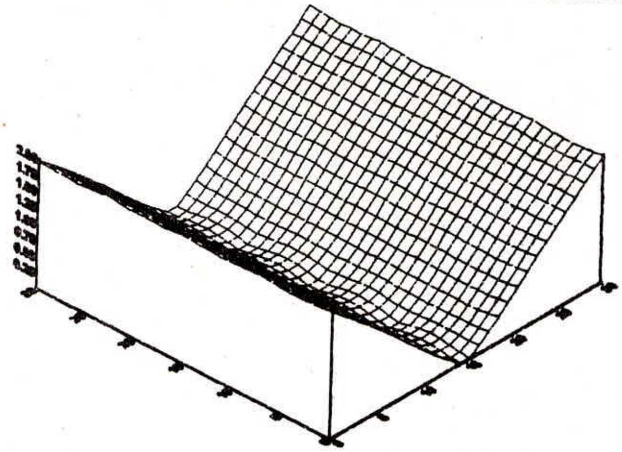


FIG.4: 3-D GROUND SURFACE ELEVATIONS OF V-SHAPED CHANNEL

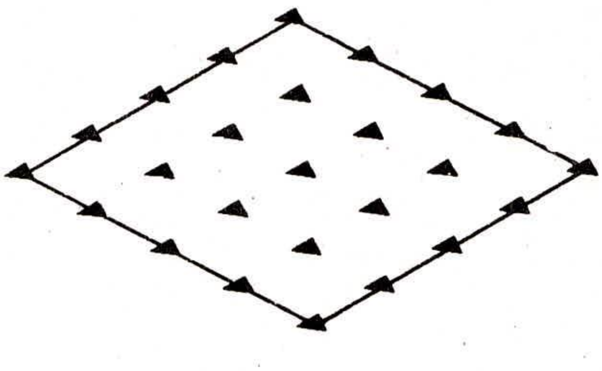


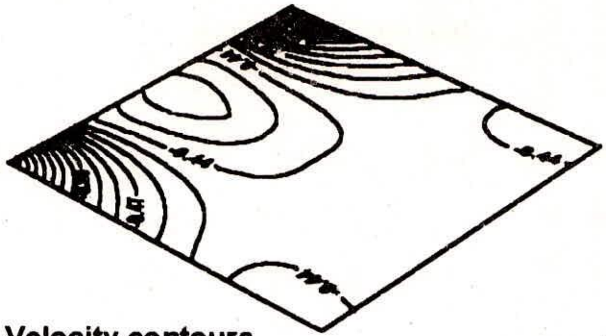
FIG.5: VELOCITY VECTORS FOR THE CHANNEL

elevation specified along the downstream boundary was 2.5 ft. The total flow rate specified along the upstream boundary was 20 cfs.

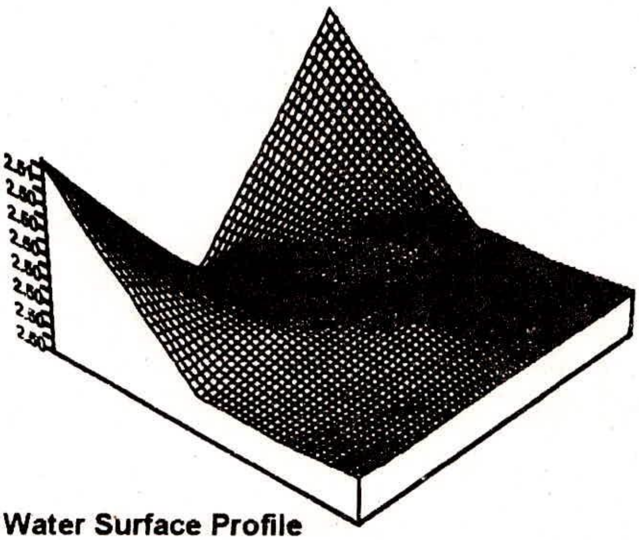
The eddy viscosity was used as the experimental parameter for the three simulations. For simulations, a constant value of 10 sq.ft/s, 5 sq.ft/s and 1. sq.ft/s were used..

4.1.2 Results

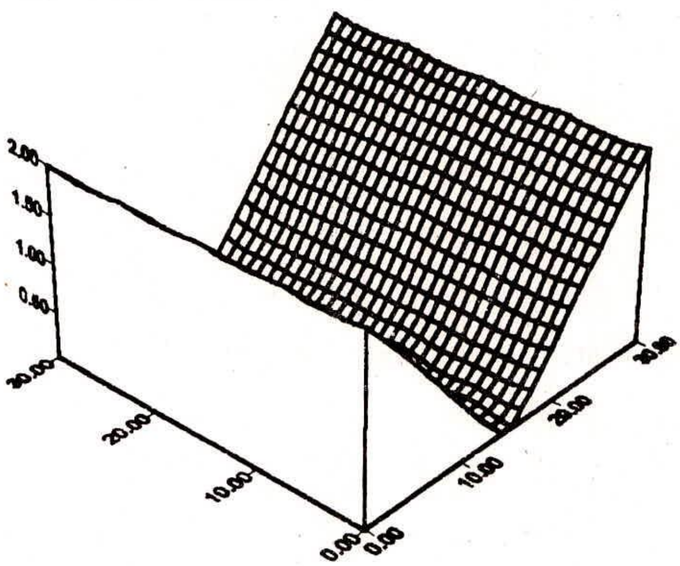
The results of the investigation are shown in terms of velocity vector plots, water surface contours, velocity field and energy gradients(**Figs.5,6,7,8 and 9**). At the upstream boundary the flow is distributed as a function of the conveyance and the boundary effects can be seen some distance downstream. It is essential that open boundaries be located sufficiently far from an area of interest. In the first simulation where a constant eddy viscosity of 10 sq.ft/s is used, the velocity is uniform throughout the network. The velocity should infact increase with the flow depth. The flow behaves as if it was stiff, with as excessive resistance to lateral shear. In other simulations(eddy viscosity 5 sq.ft./s and 1 sq.ft./s) the lower eddy viscosity allows much more variation in the velocity distribution. As shown in Fig.9 the velocity is higher at the bank of the upstream and decreases gradually. But, in case of eddy viscosities equal to 5



(A) Velocity contours

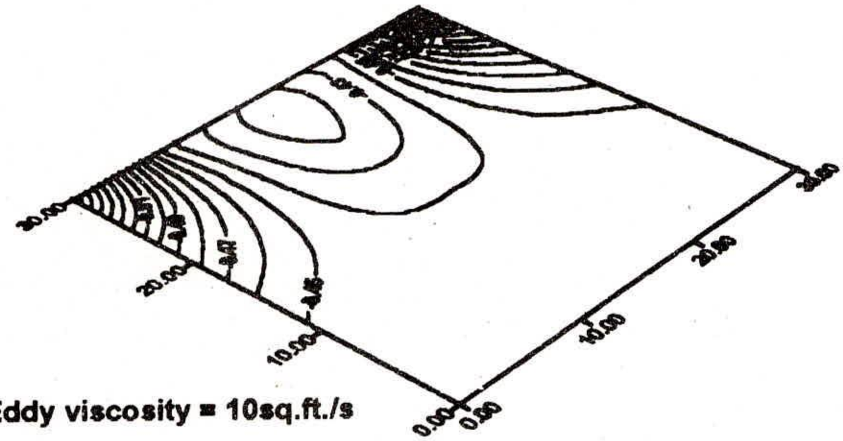


(B) Water Surface Profile

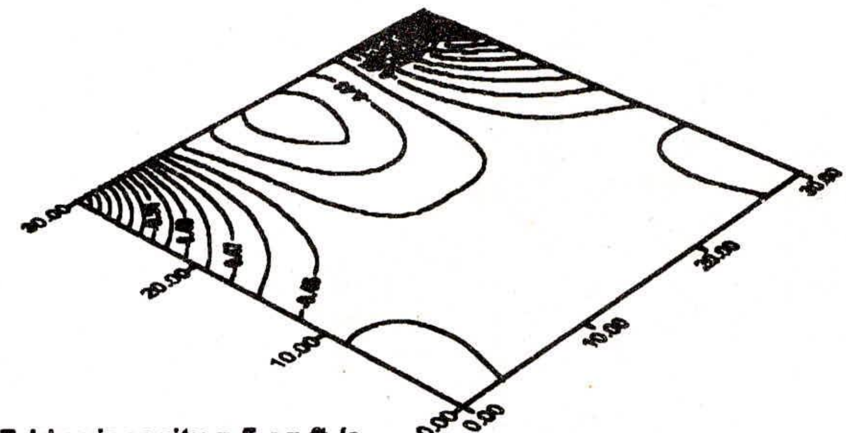


(C) Ground Surface Elevations

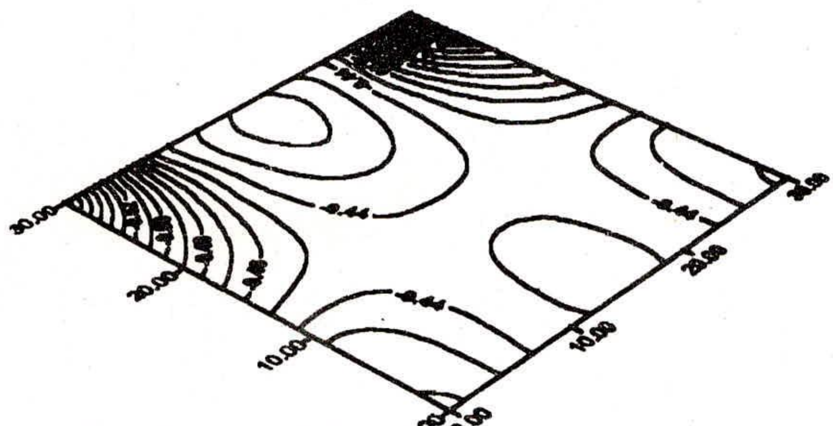
FIG.6: VELOCITY, WSEL AND GROUND SURFACE PROFILES (Eddy Viscosity = 5 sq.ft./s)



(A) Eddy viscosity = 10sq.ft./s

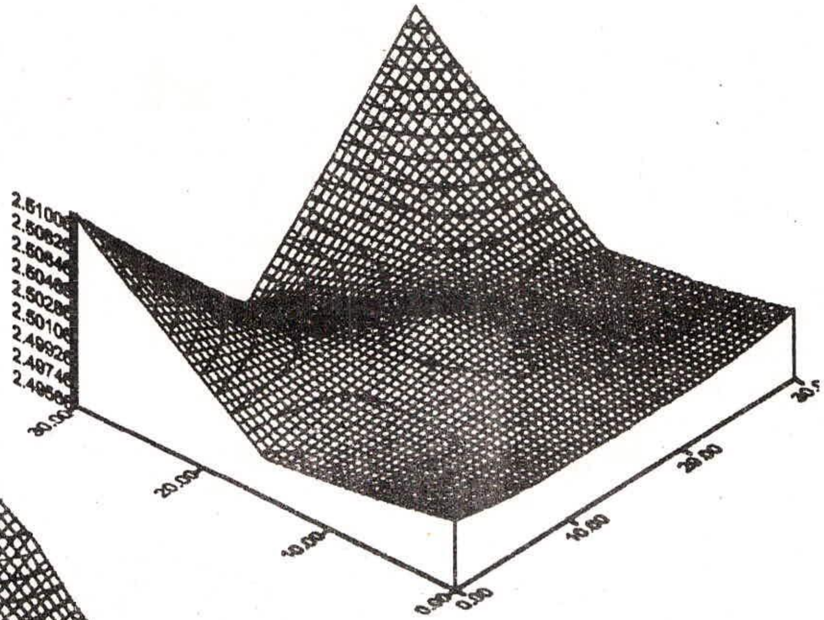


(B) Eddy viscosity = 5 sq.ft./s

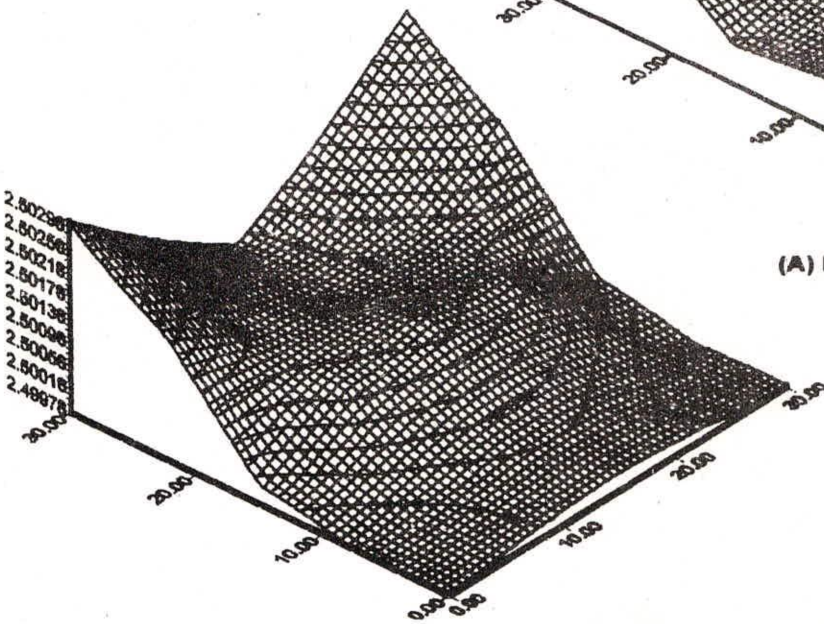


(C) Eddy viscosity = 1 sq.ft./s

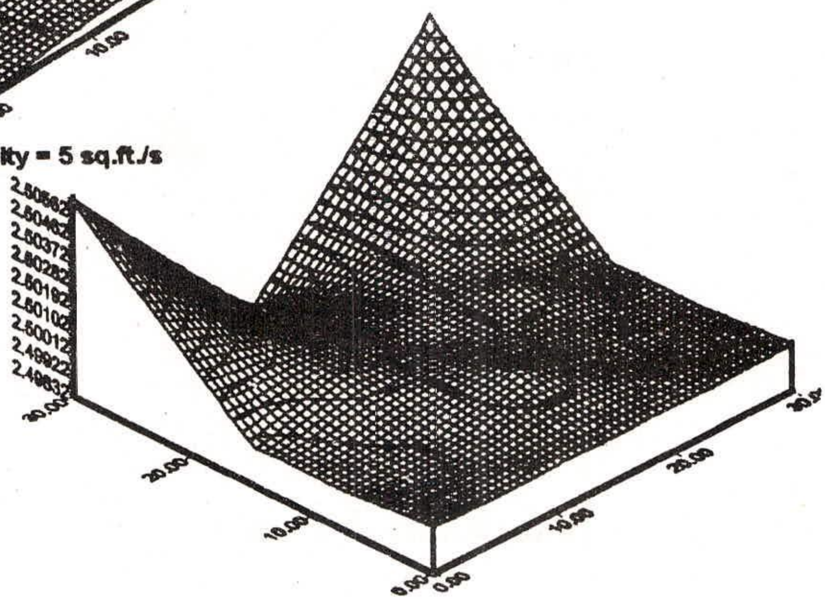
FIG.7: AFFECT OF EDDY VISCOSITY ON VELOCITY PATTERN OF THE V-SHAPED CHANNEL



(A) Eddy Viscosity = 10 sq.ft./s



(B) Eddy viscosity = 5 sq.ft./s



(C) Eddy viscosity = 1 sq.ft./s

FIG.8: WATER SURFACE ELEVATIONS FOR DIFFERENT VALUES OF EDDY VISCOSITY

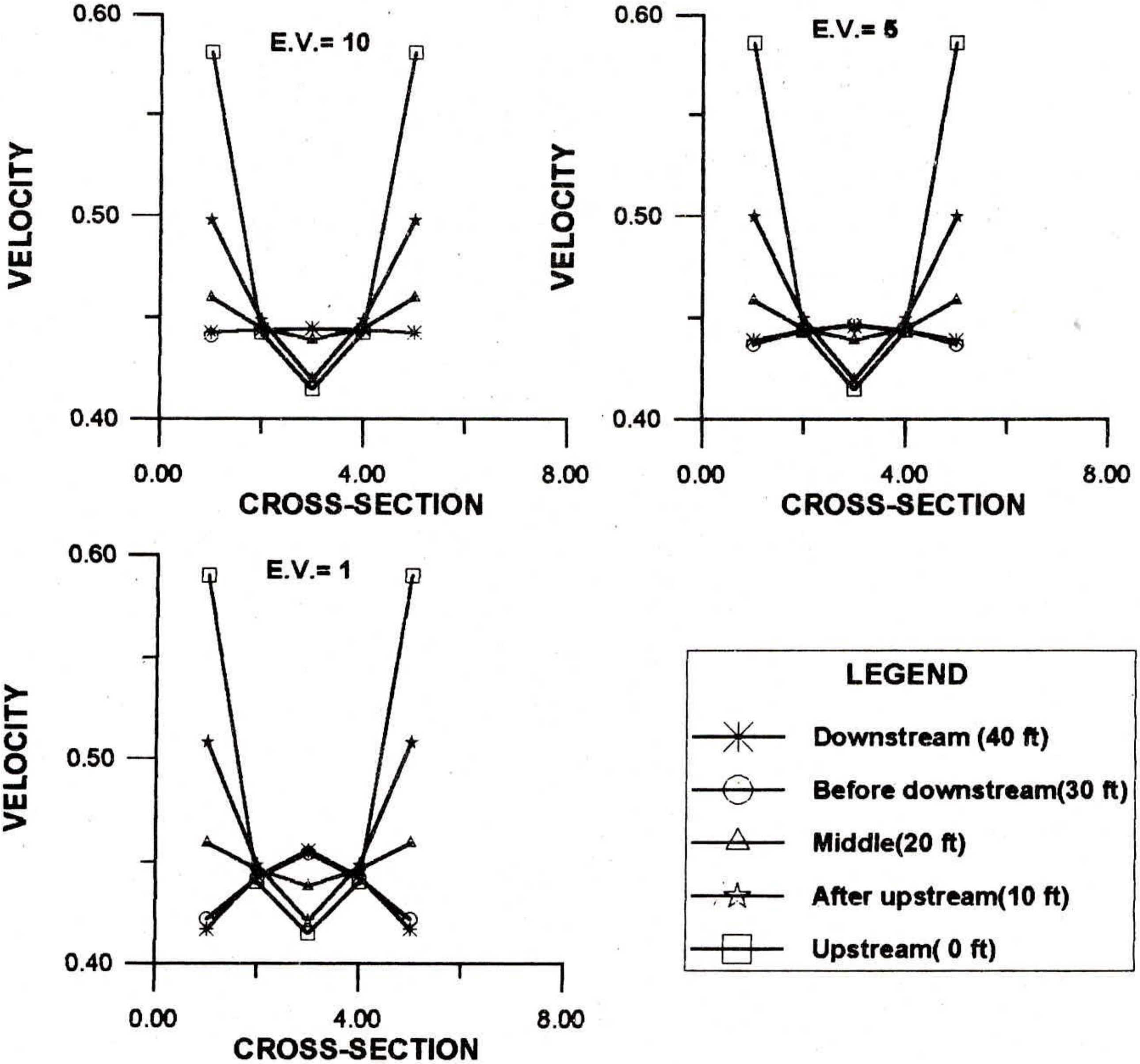


FIG.9: VARIATION IN VELOCITY PROFILES AT VARIOUS CROSS-SECTIONS

sq.ft./s & 1 sq.ft./s, the velocity is higher at the centre position of the channel, which signifies the turbulence in the flow.

4.1.3 Summary

A high value of the kinematic eddy viscosity distorts the velocity distribution by suppressing the variation in the velocity. Lower values of the eddy viscosity allow the velocity to vary from point to point. Appropriate values of the eddy viscosity are necessary to produce physically meaningful results.

4.2 Flow in a Strongly Curved Channels

The behaviour of flow in a strongly curved channel bend has been studied by many investigators, see De Vriend(1981), Chang(1984b), Steffler et al.(1985), Anwar(1986) and references contained therein. Rozovskii(1961) provides a thorough investigation of the theory of flow in channel bends, the longitudinal and transverse velocity distribution at bends, and a comparison of theoretical and experimental data for laboratory and field investigations.

Leschziner and Rodi(1979), Shukry(1950), and others have investigated flow around strongly curved bends. Shukry's results are familiar to many engineers as they are reproduced in Open Channel Hydraulics(Chow, 1959).

The behaviour of the flow in a strongly curved bend differs from the behaviour of flow in mild bends(central angle less than 180^0). It may be observed from Shukry's data that highest ve ocities occurred along the inside of the bend and the water surface elevation along the outside of the bend is much greater than along the inside of the bend. In mild bends, secondary current tend to shift the highest velocities towards the outside bank.

The following application simulates flow through a strongly curved bend. Flow features such as the super-elevation and velocity distribution are of particular interest. An estimate of the super-elevation, H_S, in a strongly curved bend can be obtained by letting the centripetal force due to the flow around the channel curvature balance the pressure gradient due to the inclined water surface. It follows that

$$H_S - \frac{V^2 b}{g R_c} \quad ...(18)$$

wnere the average velocity is v, channel width is b, and the radiu f curvature is R_C. The above equation has been used with some success, despite the exclusion of a velocity correction term which would tend to increase the predicted superelevation. Since the effect of secondary current are not considered, this equation should not be used where the radius of curvature is much greater than channel width(Yen and Yen, 1971).

4.2.1 Model description

The model consisted of two parallel channels with vertical walls connected by a 180 degree bend. The finite

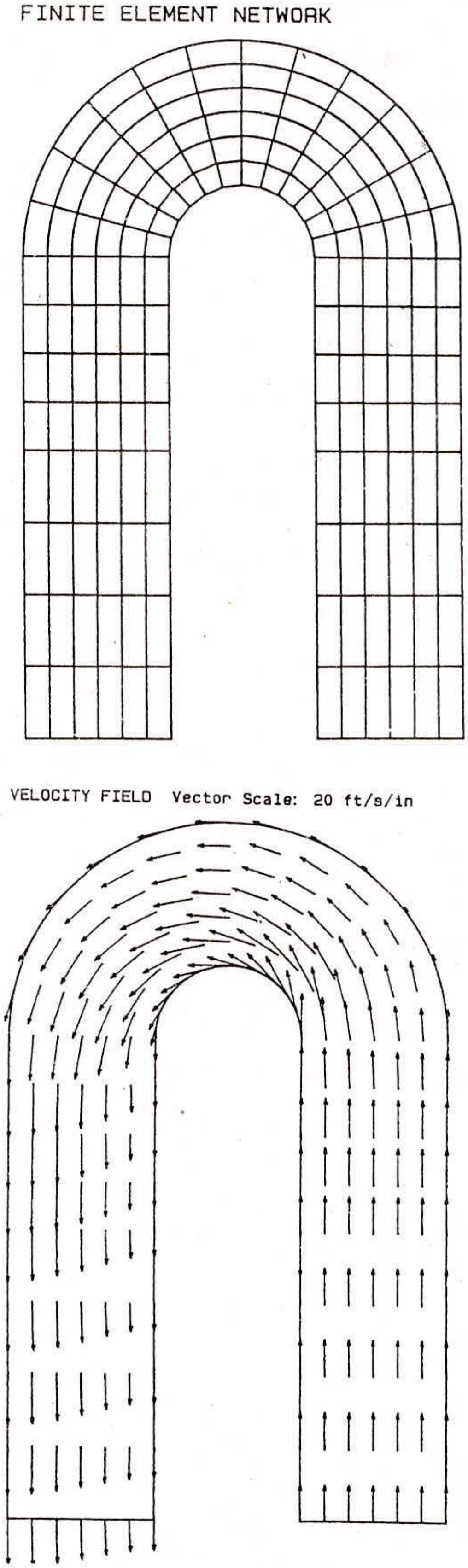


FIG.10: NETWORK DESIGN AND VELOCITY FIELD OF A STRONGLY CORVED CHANNEL

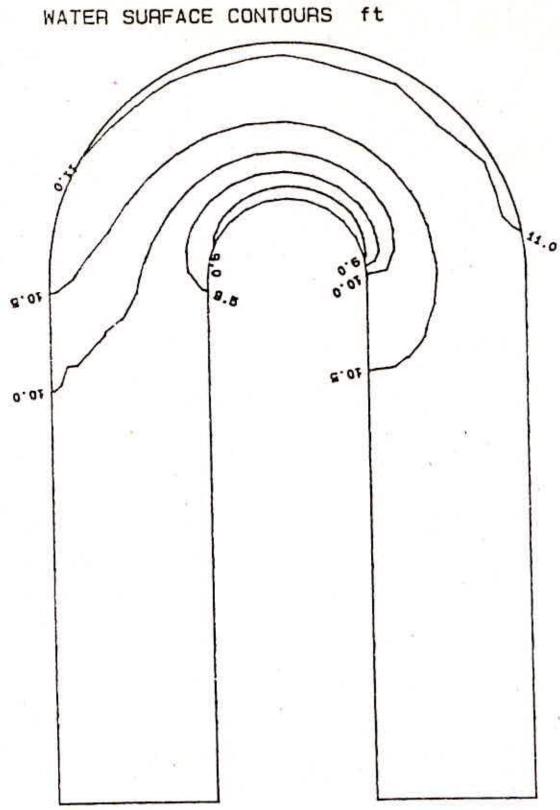
element network, shown in **Fig.10**, contained 203 nodes and 42 elements. The width of the channel was 30 ft and the elevation of the channel bottom was 0.0 ft. The centreline radius of the channel bend was 30 ft. A Mannings roughness coefficient of 0.025 was used. The total flow specified at the upstream boundary was 2440 cfs and the water depth specified at the downstream boundary was 9.9 ft. The Froude number in the exit channel was 0.46. The effects of secondary current could not be simulated by the model since the velocity is averaged over the depth.

4.2.2 Results

The results of the simulation are presented in terms of the velocity field(**Fig.10**), water surface contours and velocity contours(**Figs.11 and 12**). The results obtained for changing eddy viscosities are promising.

The water surface along the inside wall begins to decrease before the bend and reaches a minimum before the midpoint of the bend. The water surface elevation along the outside wall begins to increase before the bend and reaches a maximum before the midpoint of the bend.

The velocity field should be viewed in conjunction with the velocity contours. As flow enters the bend, the velocity is higher along the inside wall with the maximum velocity occurring before the midpoint of the bend. As flow exits the bend the higher velocities shift towards the outside wall while very low velocities occur along the inside wall. Energy head is also varying at different locations.



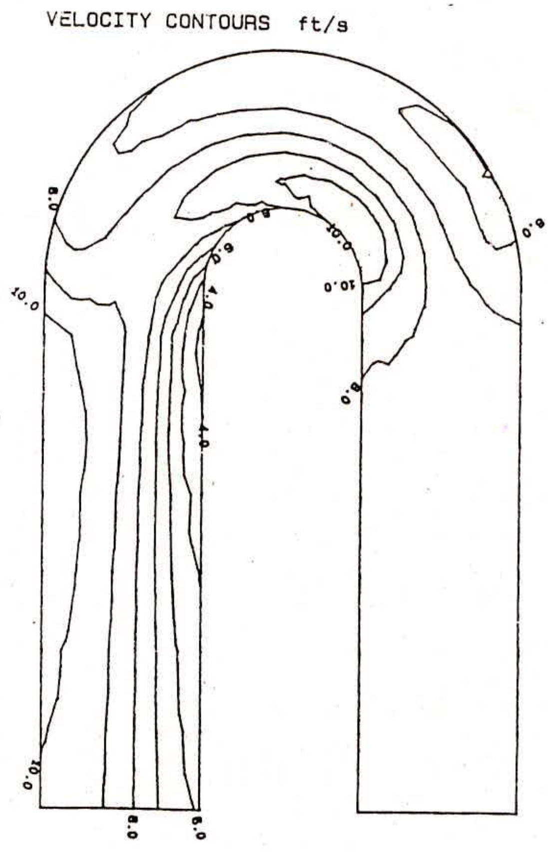


FIG.11: WATER SURFACE AND VELOCITY CONTOURS OF A STRONGLY CURVED CHANNEL

(a) E.. V. =100 Sq. ft./Sec.

(b) E.. V. =10 Sq. ft./Sec

(C) E.. V. =1 Sq. ft./Sec

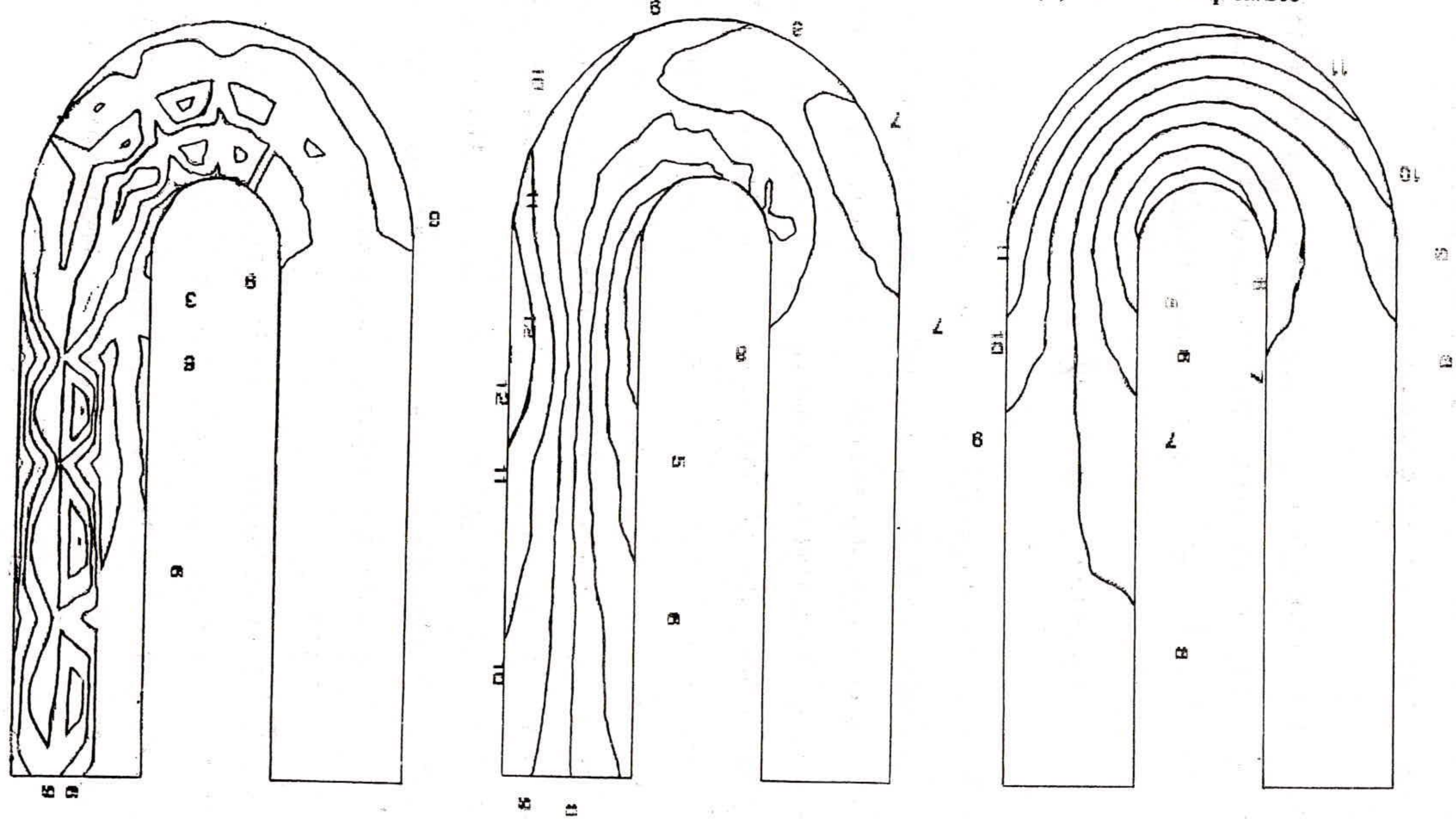


FIG. 12: VELOCITY PROFILES OF A STRONGLY CURVED CHANNEL

The changes in eddy viscosity shows variation & affect of turbulence the strongly curved channel.

4.2.3 Summary

The simulated water surface elevations and velocities showed good qualitative results and agreement with Shukry's experimental results and other experimental results.

4.3 Flow in an Island Area

The following investigates flow in a reach of an island area of a channel(also the braided river). The purpose of this example is to demonstrate the effect of the Mannings roughness coefficient(bottom stresses), changes of flow rates and kinematic eddy viscosity on the velocity distribution. The results of a model with a high value of eddy viscosity are comparable to the results of a similar model with a lower value of the eddy viscosity. Also change in roughness coefficients affects the velocity pattern. In the present study different locations of an island are of the channel three roughness coefficients were considered and velocity magnitudes and water surface elevations were computed.

4.3.1 Model description

The finite element network and the cross section are shown in **Fig.13**. The study area was 990 ft. long and 100 ft. wide. At the upstream, ground surface elevations of the study area was considered to be 108 ft and at downstream 102 ft

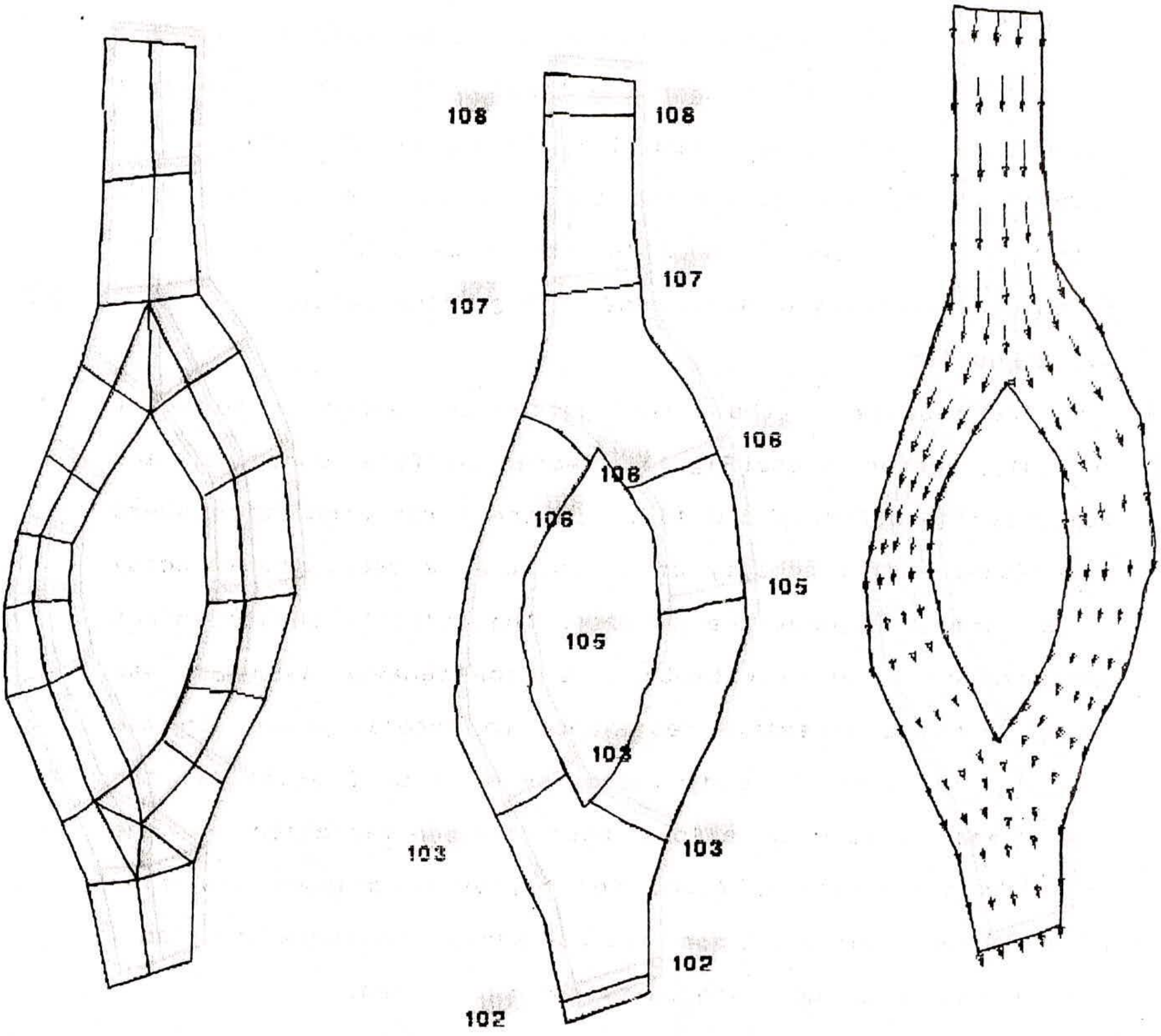


FIG. 13: NETWORK DESIGN, GROUND SURFACE CONTOURS AND VELOCITY VECTORS OF AN ISLAND

(Fig.13). The Mannings roughness coefficients were 0.35, 0.040 and 0.045. The water surface elevation specified along the downstream boundary was 112 ft. The total flow rate specified along the upstream boundary was 1000 cfs and 2000 cfs.

The eddy viscosity was used as the experimental parameter for the two simulations. For the first simulation, a constant value of 10 sq.ft/s was used. For the second simulations, the eddy viscosity was considered to be 1 sq.ft/s for the flow rate 1000 cfs and 2 sq.ft/s for flow 2000 cfs as the turbulence was very high in case of high flow rates.

4.3.2 Results

The results of the investigation are shown in terms of velocity vector plots(**Fig.13**), water surface contours, and velocity field(**Fig.14 and 15**). In the first simulation where a constant eddy viscosity of 10 sq.ft/s is used, the velocity is uniform throughout the network. The velocity should infact increase with the flow depth. The flow behaves as if it was stiff, with as excessive resistance to lateral shear. In the second simulation for eddy viscosity of 1 or 2 sq.ft./s, the lower eddy viscosity allows much more variation in the velocity distribution. Also, for higher discharges the flow becomes very turbulent in island areas and,therefore, an accurate value of eddy viscosity should be used.

4.3.4 Summary

The changes in Mannings roughness coefficient and kinematic eddy viscosity affects the flow hydraulics of an

(a) E.. V. = 10 Sq. ft./Sec.

(b) E.. V. = 1 Sq. ft./Sec

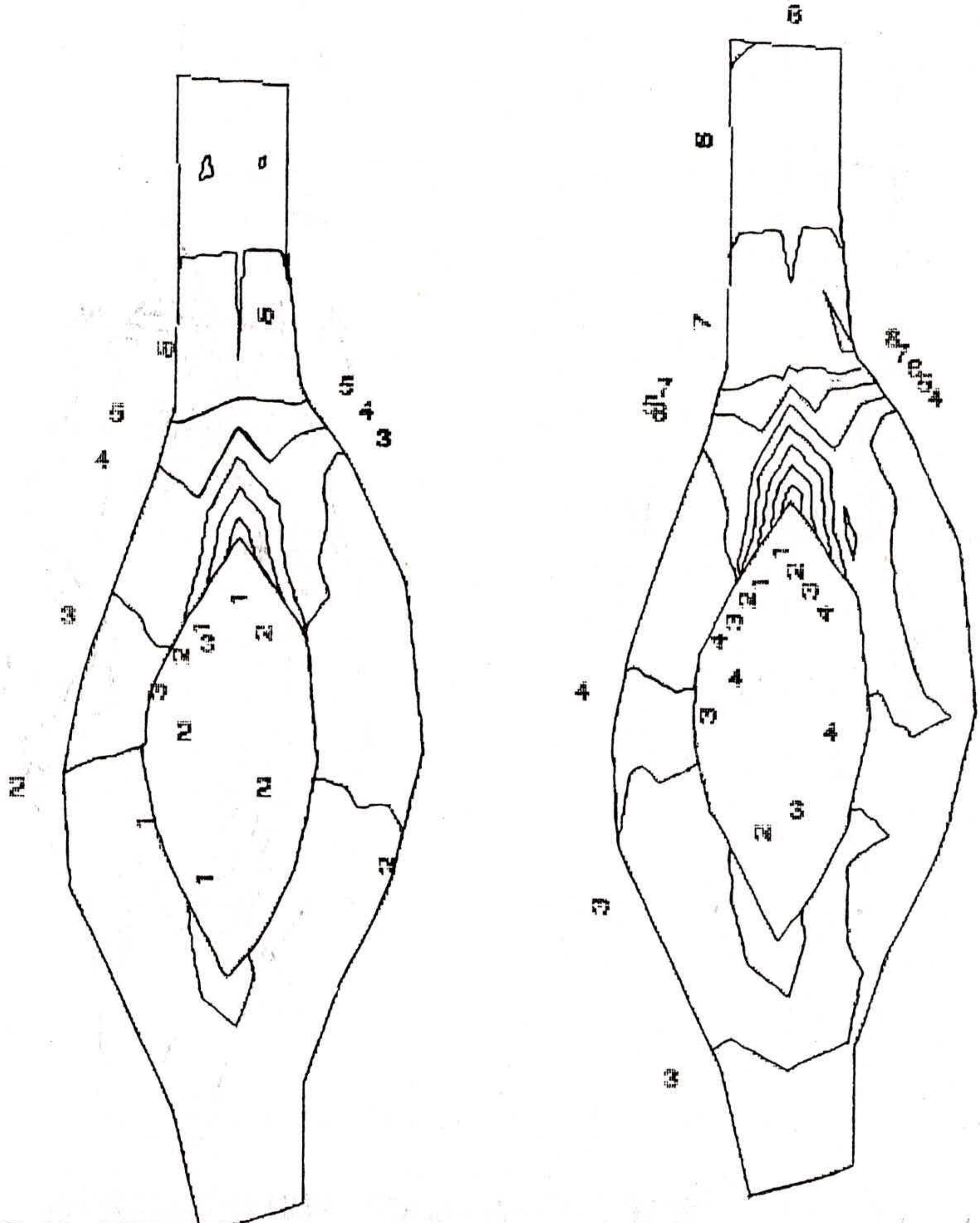


FIG. 14: VELOCITY PROFILES OF AN ISLAND FOR Q=1000 Cft/Sec.

(a) E.. V. = 10 Sq. ft./Sec.

(b) E.. V. = 2 Sq. ft./Sec

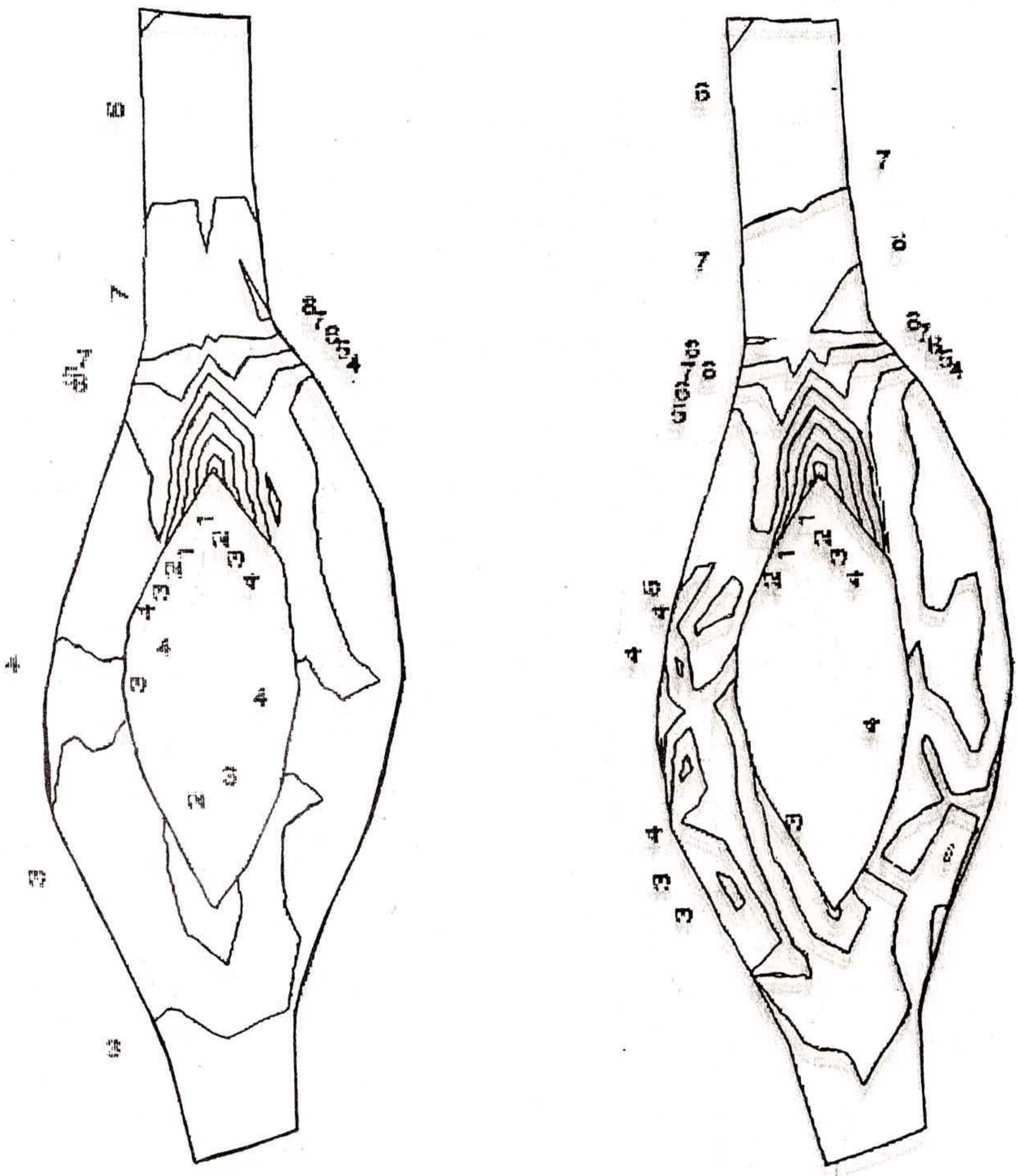


FIG. 15: VELOCITY PROFILES OF AN ISLAND FOR Q=2000 Cft/Scc.

island area. Specifically for unsteady high flows, the appropriate values of the Mannings roughness coefficient and eddy viscosity are necessary to produce physically meaningful results.

4.4 Flow in a Meandering Channel

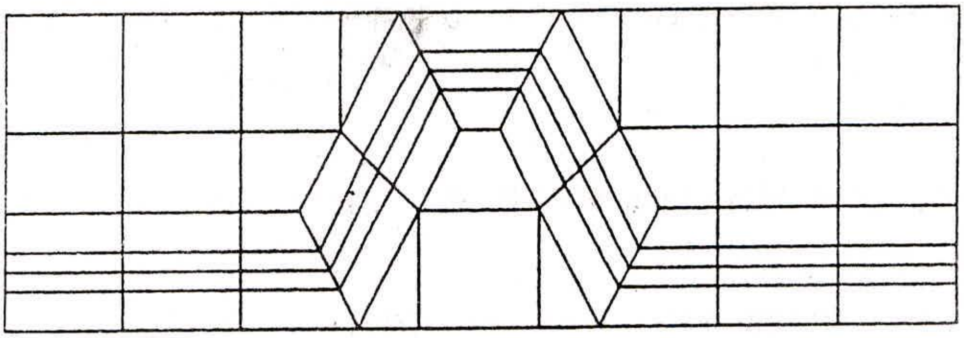
There is considerable interest attached to flow in a winding stream. Rozovskii(1961) provides a thorough investigation of open channel flow in bends. The velocity distribution of flow in short river bends has been investigated by DeVriend and Geldof(1983). Odgaard and Mosconi(1987) investigated flow in river bends particularly in reference to streambank protection using Iowa vanes. Toebes and Sooky (1966) investigated the hydraulics of meandering rivers with flood plains. Chang(1984) investigated the geometry of river meanders. Flood plain and main channel flow interaction has been investigated by Bhowmik and Demissie(1982), Knight and Demtriou(1983), Keller and Rodi(1984), Wormleaton and Hadjipanos(1985), Meyers(1987), and Dracos and Hardegger(1987).

The following case deals with flow in a meandering channel and adjacent overbanks. The flow velocities and cross stream variation in the water surface are of particular interest.

4.4.1 Model Description

A three-dimensional depiction of the ground elevations is shown in **Fig.16**. The study area is 1600 ft wide and 4800 ft lo

(a) NETWORK DESIGN



(b) GROUND SURFACE ELEVATION

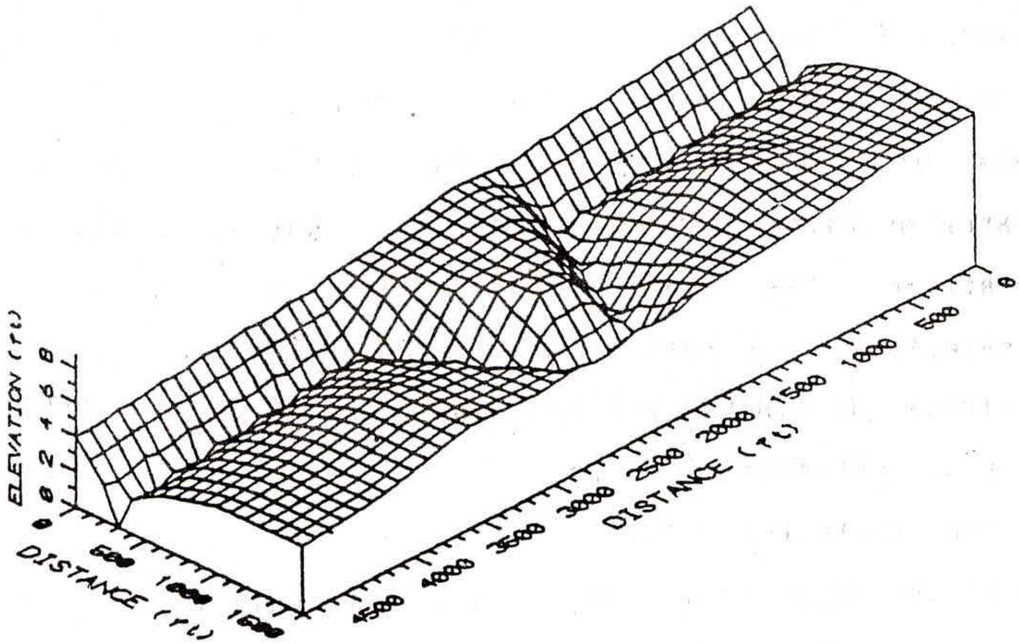


FIG. 16: NETWORK DESIGN AND GROUND SURFACE ELEVATIONS OF A MEANDERING CHANNEL

ng with a downstream slope of 0.001 ft/ft. The channel is 200 ft wide and the radius of curvature of the meander varies from 500 to 700 ft. The Mannings roughness coefficient of the channel is 0.025 and the roughness coefficient of the overbanks is 0.040.

The finite element network, shown in **Fig.16**, contained 271 nodes and 62 elements. The total flow rate specified at the upstream boundary was 13200 cfs. The normal flow depth, 6.0 ft, was specified as the downstream boundary condition.

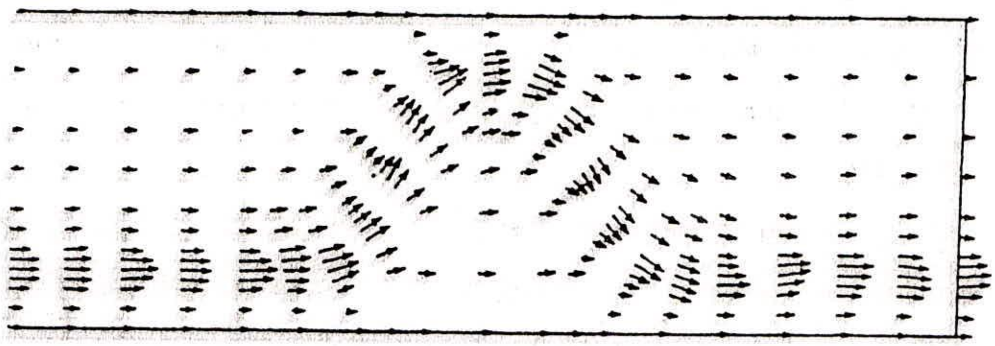
4.4.2 Results

The results of the simulation are shown in terms of the velocity field, the water surface contours and the three-dimensional water surface **Fig.17**. The velocity along the centre line of the channel approaching the meander is 5.0 ft/s. The centreline velocity drops to 3.21 ft/s in the channel bend. The velocities in the overbanks range from 3.2 ft/s across the meander to less than 2.0 ft/s away from the channel. The one-dimensional modelling technique of distributing the flow on the basis of the incremental conveyance of the section would not reflect the inertial effects influencing this variation in the channel and overbank velocity.

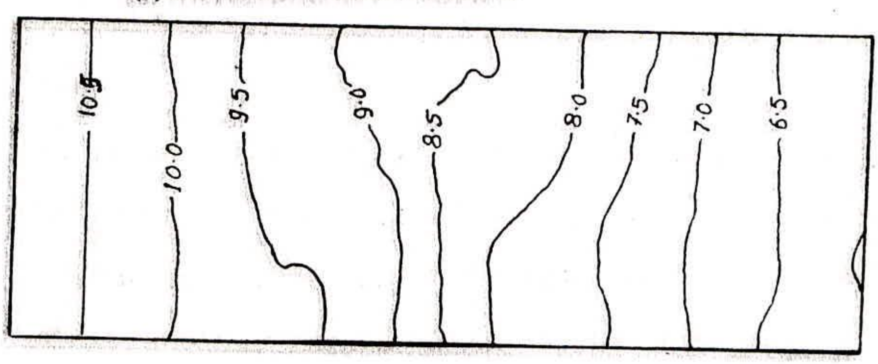
The water surface elevation of three-dimensional surface is presented in **Fig.17**. The three-dimensional surface shows qualitatively the decrease in the water surface along the

VELOCITY FIELD Vector Scale: 20 ft/s/in

(a) VELOCITY PROFILE



(b) WATERSURFACE PROFILE



(c) 3-D VIEW OF WATER SURFACE PROFILE

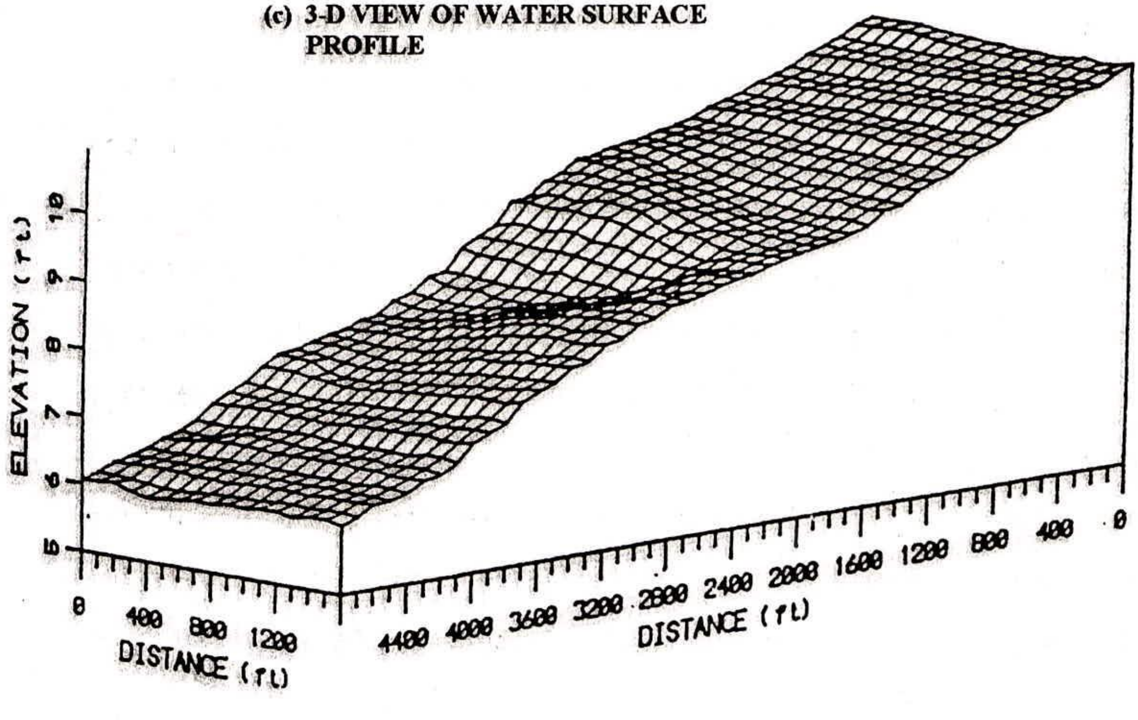


FIG. 17 : VELOCITY AND WATER SURFARCE PROFILE OF THE MEANDERING CHANNEL

right bank downstream of the meander and the increase in the water surface along the right bank upstream of the meander. The water surface contours show the right bank to left bank variation in the water surface exceeds -0.5 ft downstream of the meander and exceeds 0.5 ft upstream of the meander.

4.4.3 Summary

The simulation shows the extent of the cross-stream variation of the water surface and the variation of the velocity in the channel and the overbanks through a meander. The simulation does not consider the effects of secondary current which would tend to shift the higher velocities in the channel towards the outside bank. These results should not be extrapolated beyond this simulation since the extent of the variation of the velocity and the water surface elevation are affected by the stage and discharge of the flow.

4.5 Flow in a Flood Plain Area

The following case deals with flow in a flood plain area(overland flow). The flow velocities and cross stream variation in the water surface are of particular interest.

4.5.1 Model Description

The study area is 3000 ft wide and 2550 ft long with a downstream slope of 9 ft(217ft to 208 ft). The Mannings roughness coefficient of the channel is 0.025 and the roughness coefficient of the flood plain area are 0.150 and 0.08. The eddy viscosity is 25 sq.ft./s. The simulations was carried for three flow conditions i.e. q=1000 cfs, 2000 cfs and 3000 cfs.

The finite element network, shown in **Fig.18**, contained 335 nodes and 93 elements. The total flow rate specified at the upstream boundary was 30000 cfs. The normal flow depth, 215.0 ft(only 15 ft above bed surface), was specified as the downstream boundary condition. The ground surface elevations are also shown in **Fig.18**.

4.5.2 Results

The results of the simulation are shown in terms of the velocity field, the water surface and velocity contours(**Figs.19 and 20**). The velocity field and contours in x- and y-directions shows the flow as maximum in the middle part of the channel in flood plain area.

4.5.3 Summary

The simulation shows the extent of the cross-stream variation of the water surface and the variation of the velocity in the channel and in the flood plain area. The simulation does not consider the effects of secondary current which would tend to shift the higher velocities in the channel towards the outside bank. These results should not be extrapolated beyond this simulation since the extent of the variation of the velocity and the water surface elevation are affected by the stage and discharge of the flow.

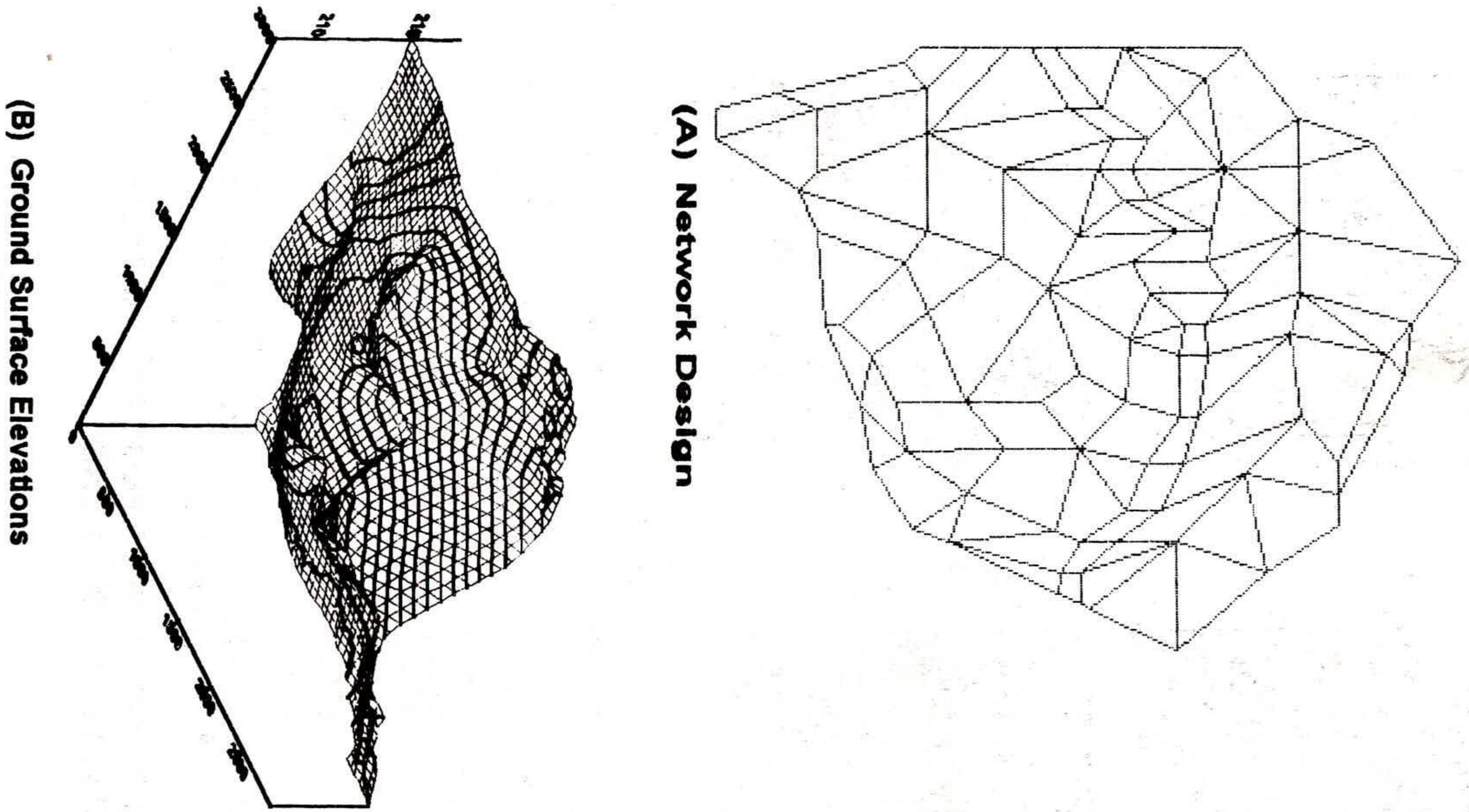


FIG.18: NETWORK AND GROUND SURFACE MAP OF A FLOOD PLAIN AREA

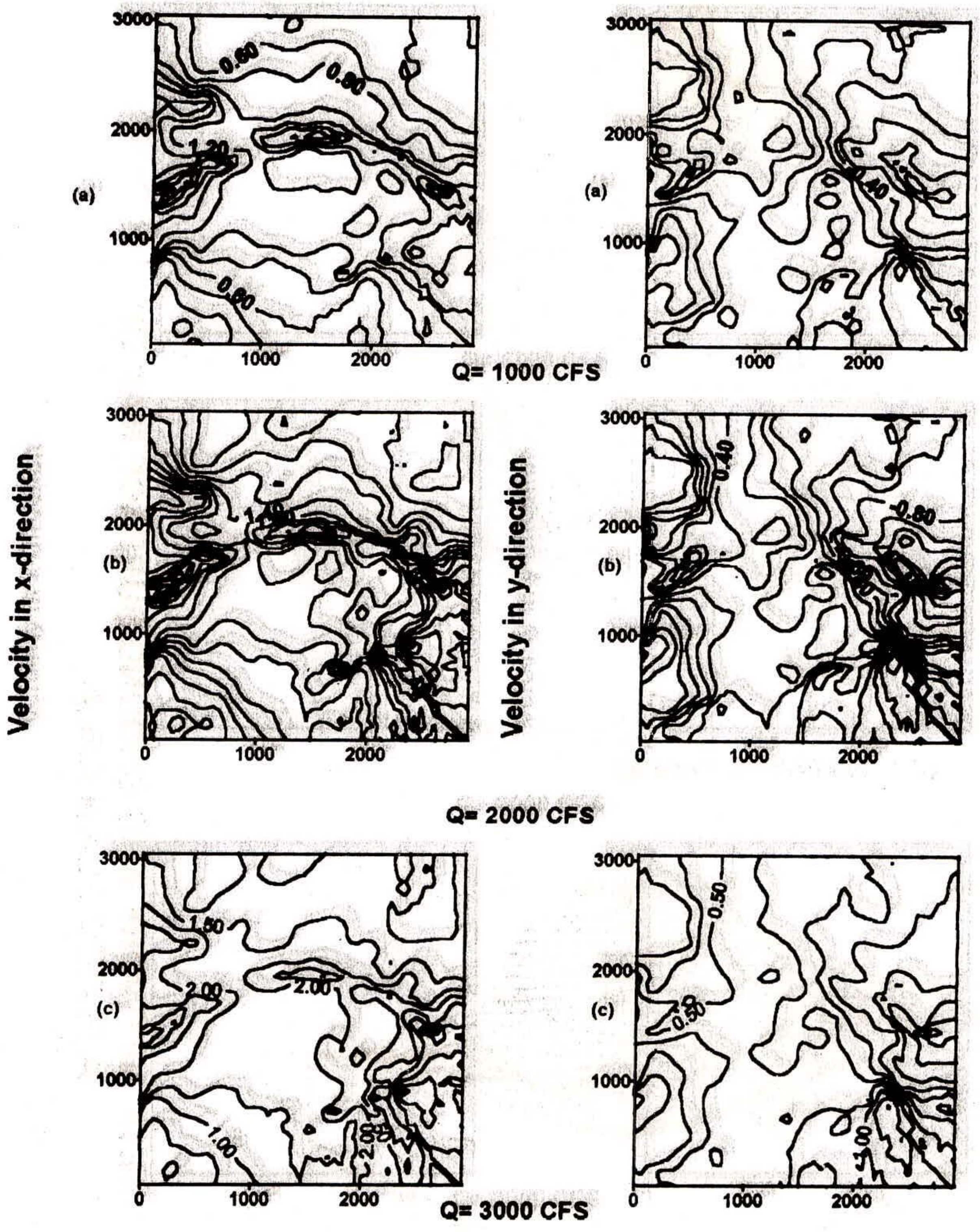
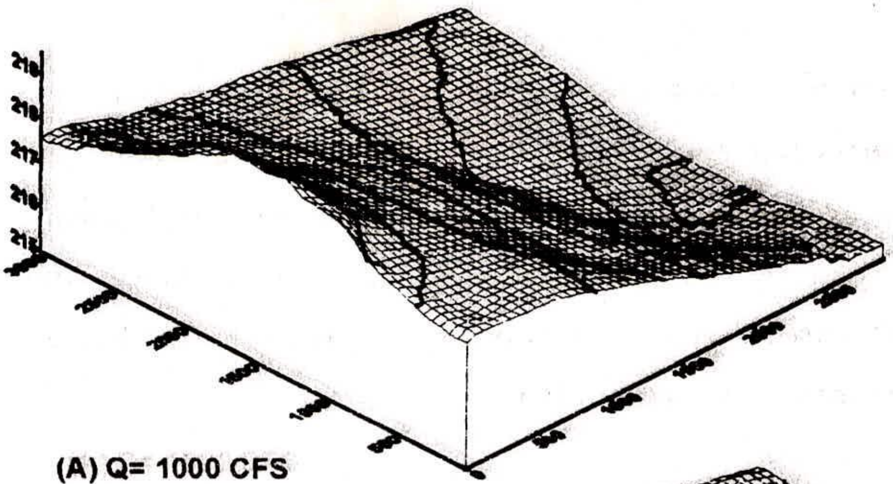
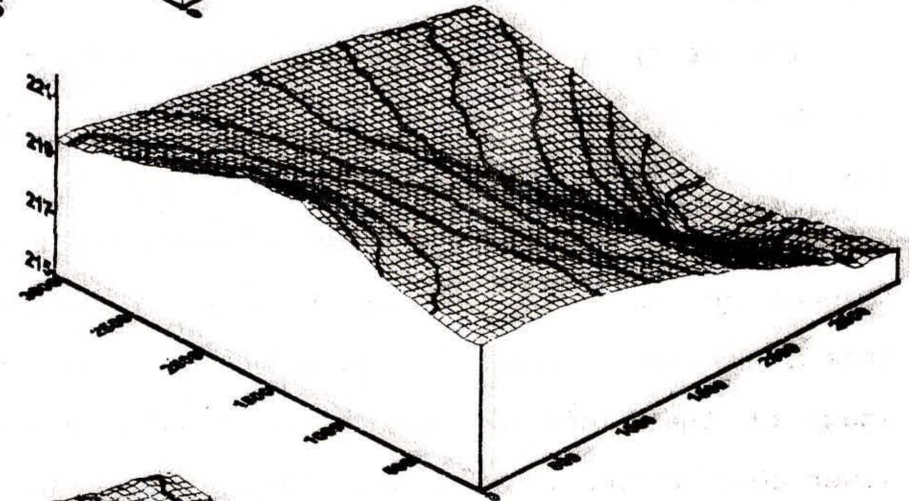


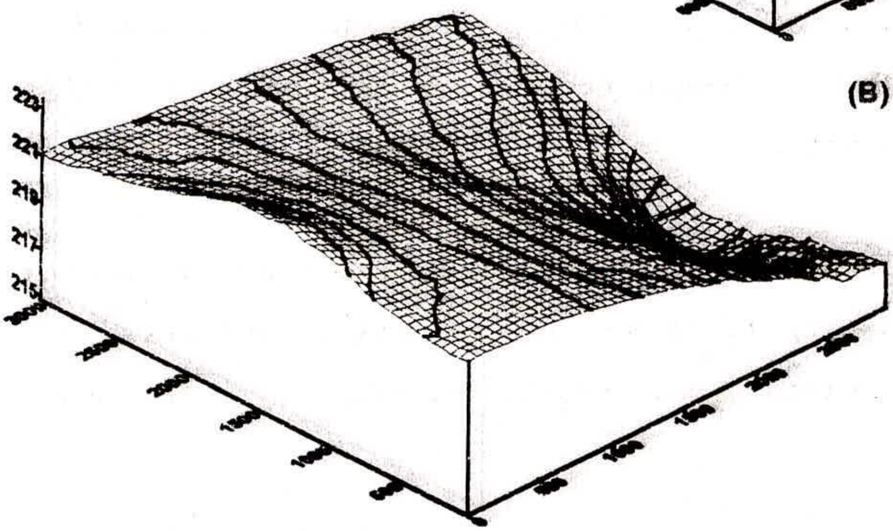
FIG.19: VELOCITY PROFILES OF FLOOD PLAIN AREA FOR VARIOUS FLOW CONDITIONS



(A) Q= 1000 CFS



(B) Q= 2000 CFS



(C) Q= 3000 CFS

FIG.20: WATER SURFACE PROFILES OF FLOOD PLAIN AREA FOR VARIOUS FLOW CONDITIONS

4.6 Flow in a Single Bridge Openiing

A single opening bridge crossing of heavily vegetated flood plain is selected for the study. The following describes the results of a study which compared the observed high water marks with the results of a one-dimensional simulation using HEC-2 and a two-dimensional simulation using FESWMS-2DH as presented by Thompson and James(1988).

4.6.1 Model description

The study area was approximately 3000 ft long with a maximum valley width of 900 ft. The finite element network for the two-dimensional model is shown in **Fig.21** contained 804 nodes and 199 elements. Smaller elements were used along the channel and near the bridge opening. The event analyzed in this study had a peak flow rate of 9000 cfs. Since the peak stage of the flood was below the roadway elevation and the lower cord of the bridge, weir flow and pressure flow was not simulated. Mannings roughness coefficients 0.035 and 0.055 were used for the model.

4.6.2 Results

The results of the two-dimensional model are shown in terms of the velocity field and water surface contours. The results of the two-dimensional model, the results of the one-

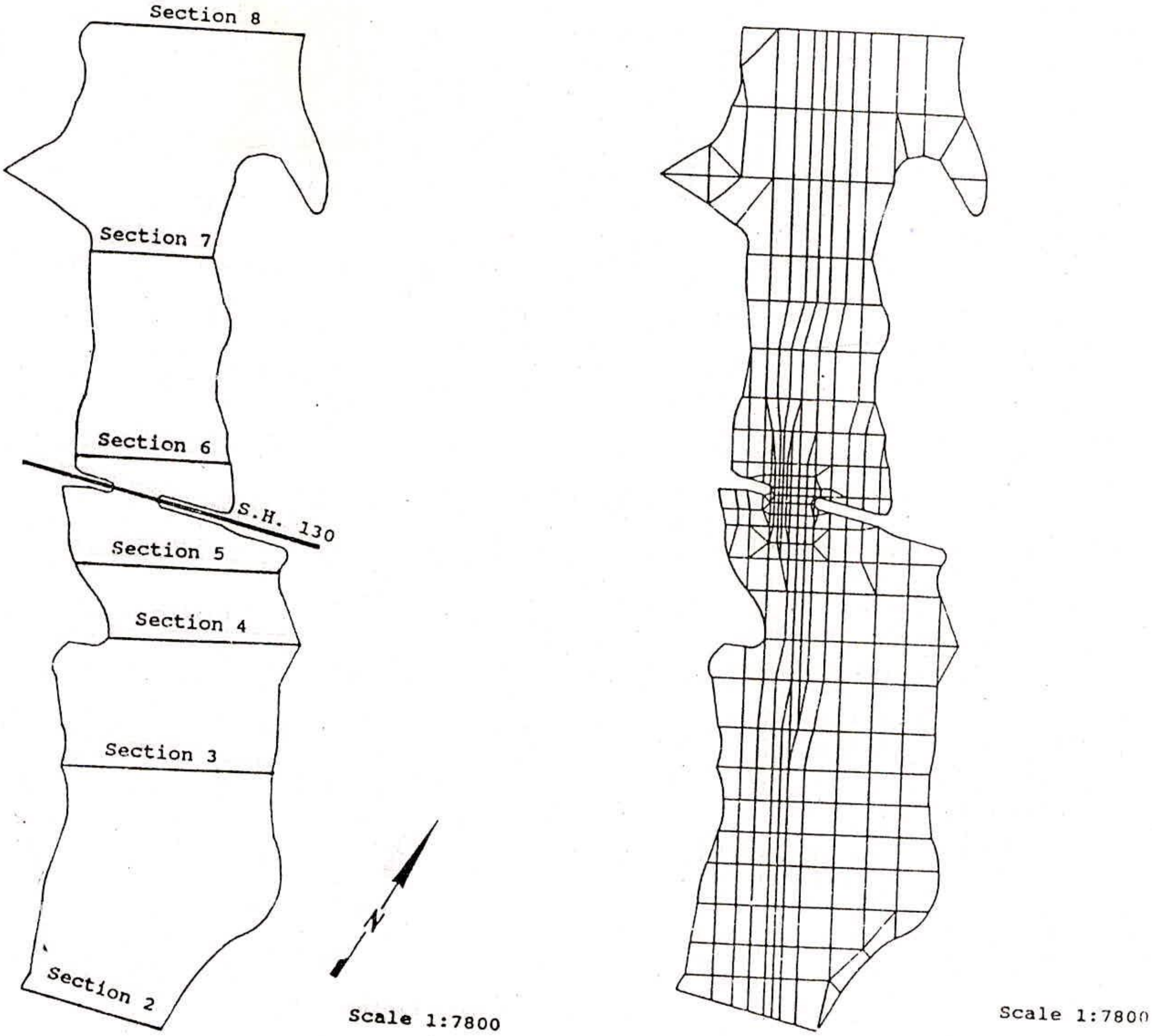


FIG.21: SCETIONS AND NETWORK DESIGN OF A SINGLE BRIDGE OPENING

dimensional model, and the observed high water marks are compared in a plot of the water surface profiles(Fig 22).

The velocity filed shows lateral flow within a distance of approximately one bridge opening upstream and one valley width downstream of the crossing, while the water surface tends to be constant from bank to bank throughout most, except where lateral flow occurs. The assumptions of one-dimensional flow are exceeded near the bridge opening in particular.

4.6.3 Summary

The two-dimensional model, FESWMS-2DH, is able to reproduce the significant features of the observed water surface elevations evaluate the two-dimensional velocity field near the bridge opening. In essence, the two dimensional model performed more efficiently and more accurately than the one-dimensional model near the bridge opening and throughout the flood plain.

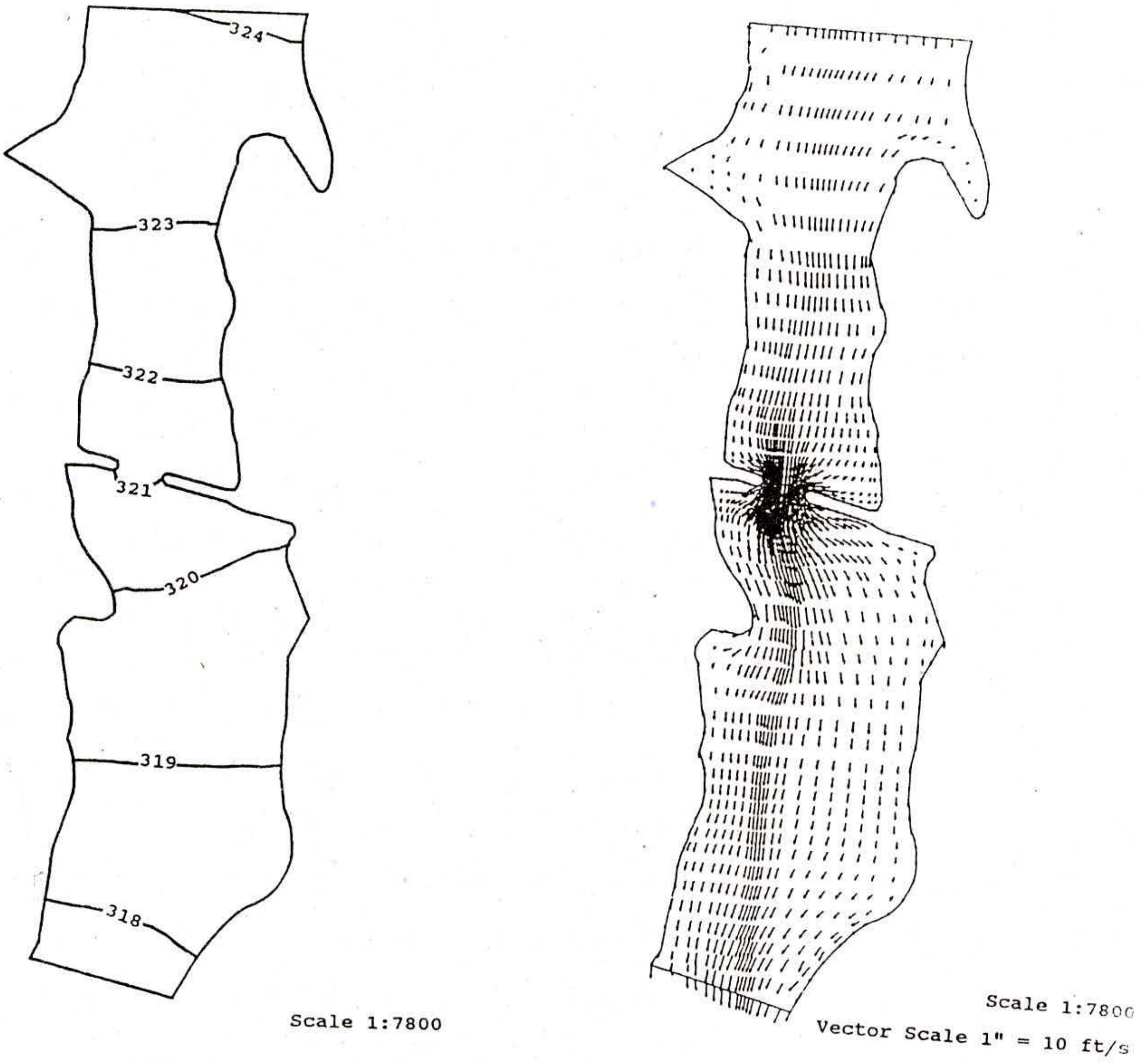


FIG.22: VELOCITY FIELD AND WSEL OF A SINGLE BRIDGE OPENING

5.0 CONCLUSIONS

One-dimensional modelling is suitable for many riverine applications but is most appropriate where there are only gradual variations in the geometry and hydraulic roughness along a water course. One-dimensional models assume that the water surface is constant at all points along the model cross-sections and assumes the flow is perpendicular to the cross-section. That is, water surface elevation tends to be constant from bank to bank and the flow tends to follow a uniform direction. For irregular sections, the one dimensional flow assumptions do not satisfy the conditions, hence, a two-dimensional approach representing the flow velocities and water surface elevations is mandatory.

FESWMS-2DH model, developed by Federal Highway Administrative of US Department of Transport(Frohlich,1988) has been found very effective for two-dimensional flow computations. The values of kinematic eddy viscosity and roughness coefficient have been found to be most characterising parameters to precise the velocity vectors and water surface profiles.

Based on the results presented here, it can be summarised that, FESWMS-2DH is an appropriate model for simple or complex two-dimensional river modelling.

6.0 SCOPE FOR FURTHER STUDY

While FESWMS-2DH is highly sophisticated, it may produce specious results if used properly. It is suggested that the network design be performed very precisely.

Turbulence has a significant effect on flow in a river. Proper modelling of turbulence is essential for a reliable two-dimensional simulation. There is considerable on-going research in turbulence modelling, and it is recommended that FESWMS-2DH be periodically updated to reflect the state-of-the-art in turbulence modelling.

Sediment transport is not included in FESWMS-2DH. Given the large sums of money spent on dredging projects, there is a clear need for a complimentary module which would allow the deposition and erosion of sediment to be predicted.

The micro-computer version of FESWMS-2DH allows widespread use of the two-dimensional modelling system at a low operating cost. The current micro-computer version FESWMS-2DH is limited to 500 elements and 1500 nodes. While this is sufficient for many applications, it would be advantageous to expand the capacity of the modelling system so that larger simulation may be run on a microprocessor.

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