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MODELLING OF SPRING FLOW IN DIFFERENT GEOHYDROLOGICAL CONDITIONS

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NATIONAL INSTITUTE OF HYDROLOGY
JALVIGYAN BHAWAN
ROORKEE - 247 667 (INDIA)

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PREFACE

Springs usually supply good quality water and could serve as a dependable source of water in remote and hilly areas. Springflow are reported to be dwindling and being drying up due to various man made activities in the recharge area of the springs. As such hydrological studies are required to predict their flow and to suggest ameliorative measures to rejuvenate and improve their flow. Most of the existing springflow prediction models are based on the assumption that the springflow is linearly proportional to the dynamic storage in the springflow domain. In an earlier study conducted in the Institute, it was observed that the springflow during recession period is not strictly linearly related to the dynamic storage even for a linear aquifer system of semi infinite nature.

Consequently, a study was conducted to verify the linearity assumption for different geohydrological conditions of spring flow domain. It has been found that linearity assumption between spring flow and dynamic storage during recession is strictly valid only for a flow domain of finite areal extent.

This report entitled "Modelling of spring flow in different geohydrological conditions" is part of the work programme of the Groundwater Assessment Division of this Institute for the year 1994-95. The study was carried out by Shri A.K. Bhar, Scientist-E, under the guidance of Dr. G.C.Mishra, Scientist-F of the Division.

(S.M.Seth)

Director

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ABSTRACT

Most of the existing hydrological models are based on the assumption that the spring discharge is linearly proportional to the dynamic storage in the springflow domain. The validity of such assumption is yet to be verified. In an earlier study, an attempt has been made to verify this for a spring flow domain which is infinite in one side. But it is found that the relationship between spring discharge during the recession period and the dynamic storage of the spring is not linear even for a linear aquifer system.

In the present study the relationship between a springflow and the dynamic storage will be verified for linearity for a flow domain of finite areal extent. The solution for an unsteady flow from a well in a confined aquifer of finite areal extent is available. The same solution has been used to find out the relationship between springflow and dynamic storage. The relationship between the dynamic storage and the springflow become linear for the finite flow domain.

For a single springflow domain the method of image has been used and for multiple images, the non dimensional springflow has been plotted with non dimensional time in log log paper and the plot has been compared with the plot given by Glover. It matches with the values of non dimensional discharge and non dimensional time (3000)given by him for any position of the spring between the recharge zones. But for large values of non dimensional time more than 3000) the respective plot varies from each other for different position of the spring in the flow domain especially when the distance between the recharge zones is large.

1.0 Introduction

Discharge of a spring does not remain constant with time. Fluctuation of spring discharge are due to the variations in rate of recharge and the prevailing hydrologic and geologic conditions. The discharge of a spring depends on the difference between the elevations of water table (or piezometric head) in the aquifer in the vicinity of the spring, and the elevation of the spring threshold. A typical portion of a spring hydrograph is shown in Fig.1. The shape of the springflow hydrograph is manifestation of the dynamic storage built up into the spring's flow domain because of recharge.It will vary from spring to spring and could vary for a spring for different years. Significant information about the dynamic storage in the spring's flow domain can be ascertained by analysing the recession portion of the springflow hydrograph.

Most of the existing springflow models are based on the assumption that the discharge of a spring is linearly proportional to the dynamic storage in the spring flow domain. Based on this assumption the springflow equation has been derived as:

$$Q(t+\Delta t) = Q(t) \exp(-\Delta t/t_o) \qquad \ldots(1)$$

where t_o is known as the depletion time which is a parameter of the spring. It is the time that will be taken to empty the live storage of the spring at the present flow rate, i.e., the dynamic storage at any time t is equal to $Q(t)\, t_o$.The variation of spring discharge with time during recession portion can be plotted in a semilog paper (discharge being in log scale). Due to the linearity assumption stated above such a plot provides a straight line and slope of the straight line is the depletion time, t_o(Fig.2). It is likely that the slope of the discharge time plot will vary from spring to spring. Any change in the slope of the line from year to year or within a year is an indication of interference in the groundwater system. A progressive flattening of the slope indicates the replenishment of the aquifer storage in the

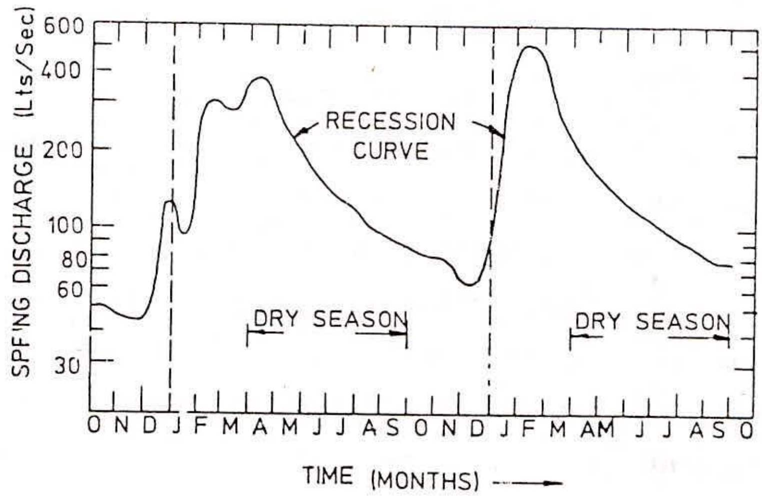

FIG. 1 A TYPICAL SPRING HYDROGRAM WITH SEASONAL FLUCTUATION (AFTER BEAR, 1979)

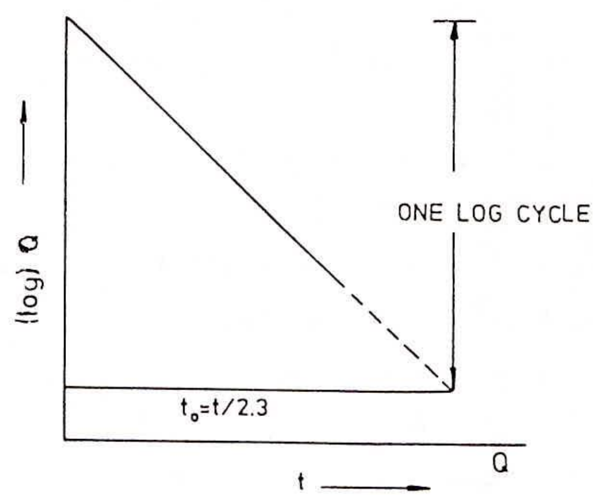

FIG.2 DEPLETION LINE ANALYSIS

supposedly dry season (probably due to return flow of irrigation/urban effluent or seepage from reservoirs) and steepening of the slope indicates groundwater abstraction from the aquifer or reduction in recharge. Occurrence of earthquake can have effect on spring discharge and on the slope of the time discharge line considerably. The models suggested by Maillet (1905 vide Singh, 1989) Mero (1963), Bear (1979) and Mandel and Shiftan (1981) are based on the linearity assumptions mentioned above and provide equations similar to Eq.(1) for springflow for a lumped recharge.

2.0 Existing springflow models

2.1 Model based on non-linear discharge-storage relationship

Boussinesq (1904 vide Singh, 1989) developed the following equation for flow from an unconfined aquifer to a fully penetrating stream having negligible depth of water in it (Fig.3).

$$Q(t) = Q_o/(1+ct)^2 \qquad \ldots(2)$$

where c is a constant. This equation has been extensively used in Europe for estimating spring discharge.

Eq.(4) has been derived by solving the equation ds/dt=-Q for a non-linear storage-discharge relation

$$Q = a\ s^n \ ;\ n \neq 1 \qquad \ldots(3)$$

with an initial condition $Q(o) = Q_o$. This yields

$$Q = Q_o(1+ct)^{n(1-n)} \qquad \ldots(4)$$

where $c = (1-a)a^{1/n}/Q_o^{\,n(1-n)}$, a constant. n = 2 yields equation (2).

It has been observed that hydrograph of a karstic spring exhibits non-linear behaviour due to development of secondary porosity. A hyperbola provided by equation with an exponent n gives a better fit of the springflow from karstic rock. The exponent n usually lies in the range of 0.5 to 2.

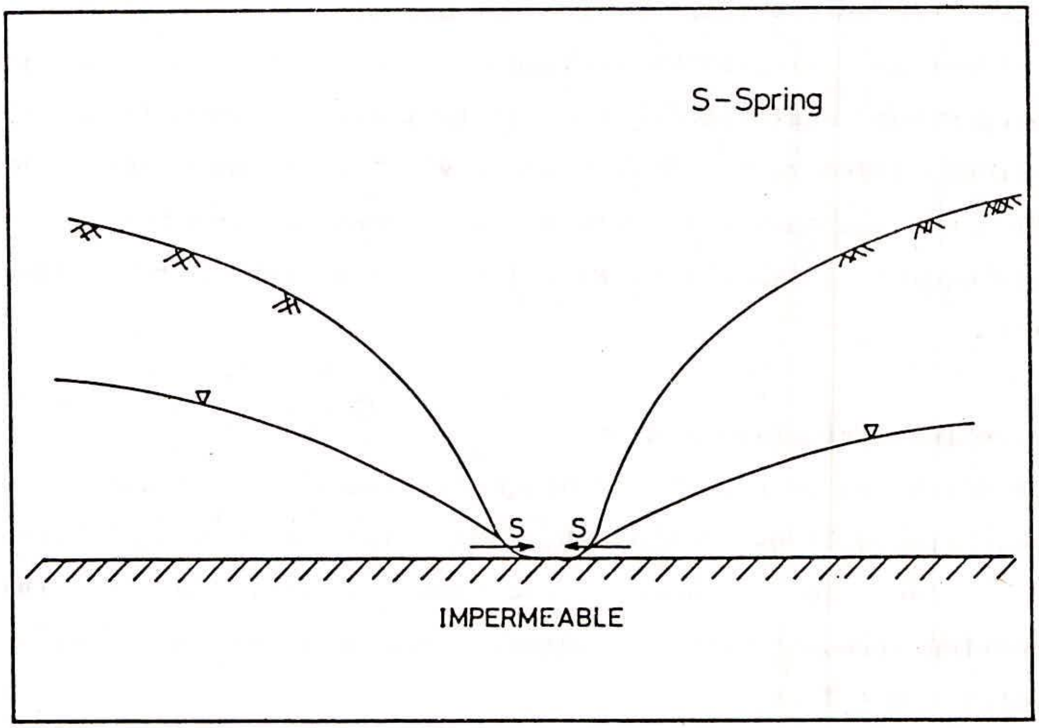

FIG. 3-MODEL CONFIGURATION FOR BOUSSINESQ EQUATION (1904)

2.2 Models based on linear discharge storage relationship

Existing mathematical models of the springflow have been developed on the assumption that the spring outflow is linearly proportional to the dynamic storage in the spring flow domain and guiding equation for the springflow is as given in Eqn.(1).

Under recession condition, the baseflow component Q(t) of a river partially penetrating an aquifer is equal to the total release from groundwater storage and can be estimated from the two-dimensional equation of ground water flow. Further,with Dupuit's assumption of negligible vertical flow, Nutbrown (1975 vide Nutbrown and Downing, 1976) has shown

$$Q(t) = \Sigma A_i \; K_i^t \qquad \ldots(5)$$

where, $K_1, K_2, \ldots$ recession constant of groundwater and $A_1, A_2 \ldots$ are the constant coefficients. Although theoretically the sum in eq. 5 extends to infinity, in practice, only a small number of terms will dominate at any particular time. This property of the expression in eq.5 has led to plots of log Q(t) being fitted with a succession of straight lines of decreasing slope. The implication of eq. 5 is that the interpretation of these successive straight line segments does not necessarily be in a complex aquifer structure. The aquifer may be perfectly uniform, with no particularly unusual features, and still exhibit the behaviour implied by eq. 5 in its baseflow contribution to the flow of a contiguous river.

It has been noted in the analysis of baseflow recession curves for many streams in U.K.,that the semi-log plot of baseflow against time is not a straight line but rather a curve, even for uniform values of aquifer parameters. Nutbrown et.al., inferred that deviation of the plot from a single straight line is due to the dynamics of ground water flow and normally not due to the complex hydrological structure. It is quite normal that the average catchment value of storage coefficient could fluctuate markedly with baseflow decline even for a simple aquifer. Only when same effect persist year after year, under widely different hydrology conditions, can this type of behaviour have implications concerning the aquifer structure.

At NIH(1993) a rigorous mathematical model has been suggested for predicting spring flow from a group of springs for a unsteady flow. Schematic and idealized flow domain for the springs are shown in Fig.4 Method of image and unit response function coefficients have been used in the model. The basic solution for rise in piezometric surface due to recharge from a rectangular basin given by Hantush has been used in the analysis. Duhamal's integration has been used to account for time variant recharge to the spring flow domain. The expression for spring discharge has been obtained in terms of response function coefficients. Any of the springs gets activated when the piezometric surface tends to rise above its threshold. The analysis assumes that once the piezometric surface touches the spring's threshold there is no further rise in the piezometric surface at the location of the spring. The model was tested for the storage-discharge relationship for a simple case of one spring emerging out of the flow domain. A recharge of 20 cm is assumed to occur in a span of 120 days continuously at a uniform rate of (1/600) m/day through a recharging area having W=250m and L=2000m. Out of 1×10^5 cu.m of total recharge to the aquifer, only 0.25×10^5 cu.m appears as springflow during 240 days after the commencement of recharge. It is also found that out of the total recharge, only 0.40×10^5 cu.m of water appears as springflow and the remaining 0.60×10^5 cu.m never appears as springflow.

The springflow during recession has been expressed as

$$q(t+\Delta t) = q(t) \exp(-\Delta t/t_o),$$

where t_o is known as depletion time. The depletion time is a

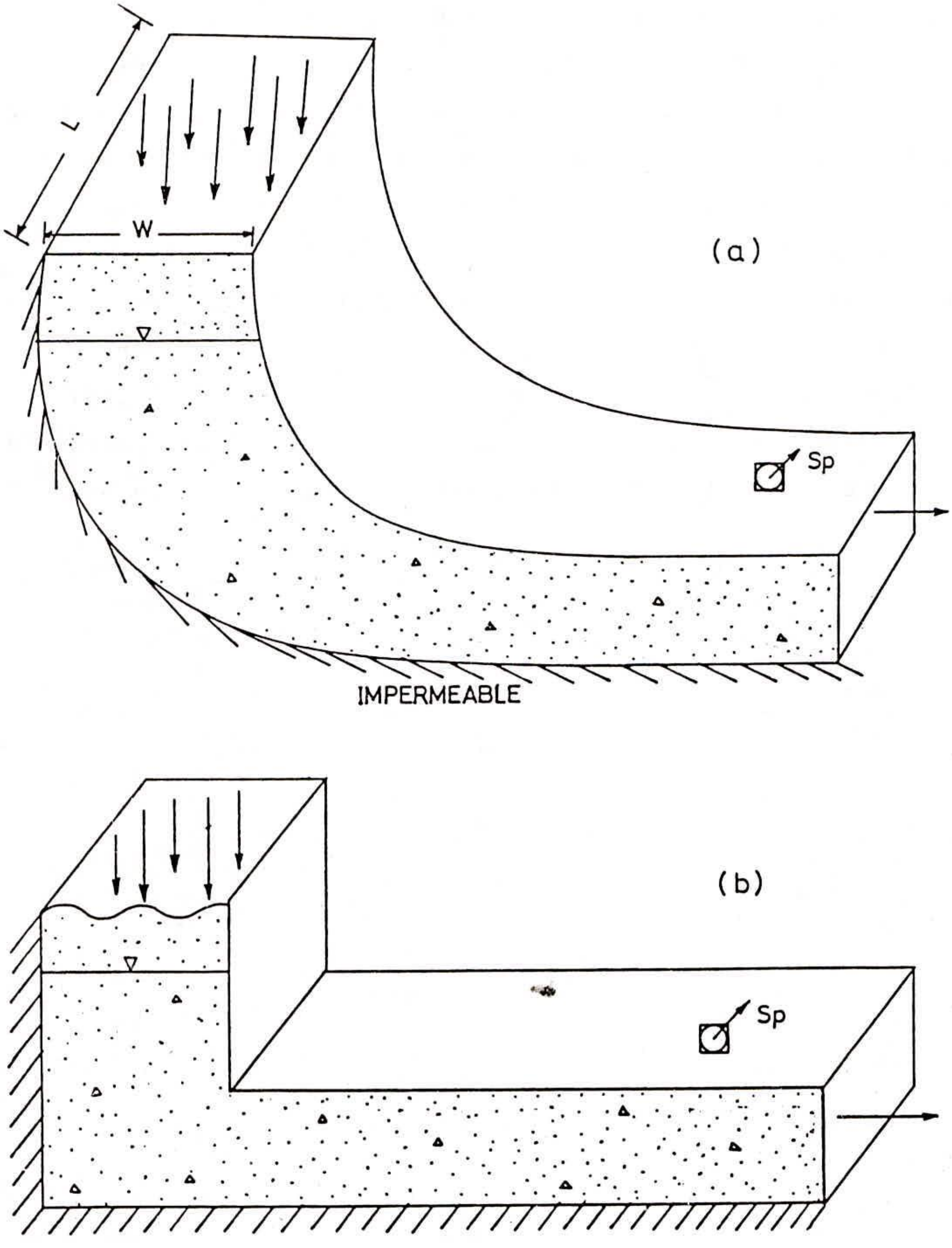

FIG. 4 a & b – SCHEMATIC AND IDEALISED FLOW DOMAIN OF A SPRING parameter of the spring and is the time that will be required to empty the live storage of the spring at the present flow rate, i.e., the dynamic storage at any time t is equal to $q(t).t_o$.

Let S_o is the part of the total recharge which never appears as springflow. S_o has been ascertained from the plot of cumulative recharge and cumulative discharge. The dynamic storage at any time which will subsequently appear as springflow is equal to the difference between the total recharge and the summation of S_o and cumulative spring discharge up to that time. Hence, using eq(1) as the expression for springflow and rewriting the same in term of t_o

$$t_o = [R- \{ \sum_{\gamma=1}^{n} q(\gamma)+S_o \}]/q(n) \qquad ...(6)$$

The values of depletion time at various time steps (days) during recession have been evaluated and are given in Table 1.

Table 1: Variation of depletion time

Days after the cessation of recharge (days)	Depletion time (days)
80	220
90	221
100	224
110	229
120	234
130	240
140	245
150	251
160	258
170	264
180	271
190	278
200	285
210	288
220	294
230	302

Perusal of the values of depletion time in Table 1 shows that the depletion time which has been assumed as constant varies with time and as such the springflow during the recession period does not follow strictly the exponential decay curve and the springflow is not truly linearly proportional to the dynamic storage which will subsequently appear as springflow. From the semilog plot of recession portion of springflow the value of depletion time is 274. So, using this value of depletion time, the dynamic storage of the spring will be overestimated.

So, the relationship between the spring discharge during the recession period and the dynamic storage of the spring is not linear even for a linear system. Though the portion of groundwater flow going to regional groundwater flow has been accounted for testing the linearity, the chosen flow domain is infinite in one side. As such, it would be expedient to test the model for a finite flow domain for linearity verification.

3.0 Model of springflow for finite flow domain

Using the discrete kernel approach, an analytical solution for an unsteady flow from a well in a confined aquifer of finite areal extent is available(Chandra and Mishra,1987). The configuration of the flow domain is given in Fig.5 and same could be used for evaluating springflow.

The level of the spring's threshold is at a height H_2 above the datum. The spring will be active when the piezometric level will be more than the height H_1. The time is measured from the instant the spring becomes active. The discharge of the spring at various time step and the quantity of water that remain in the dynamic storage of the flow domain of the spring need to be determined.

Assumptions are:-i.)the time parameter is discrete and within each time step, the discharge of the spring is constant but it varies from time step to time step, ii)though the aquifer is unconfined at the outcrop, the entire aquifer has been assumed to be confined and the position of the no flow boundary is assumed to be fixed.

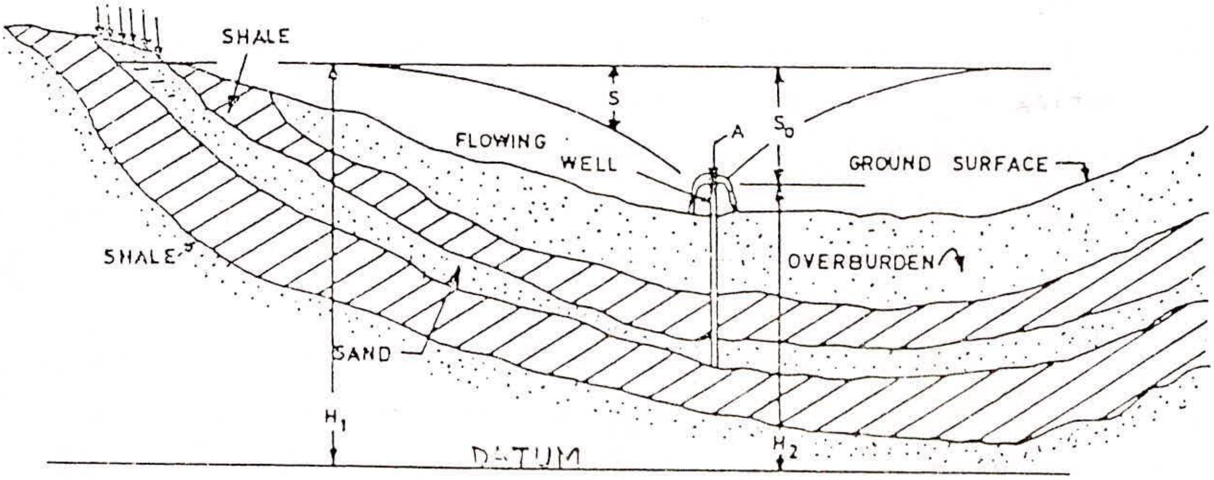

Fig.5 SECTION SHOWING A FLOWING WELL

The solution of the differential equation (Eq.7) for radial unsteady groundwater flow needs to satisfy the following initial and boundary conditions for the flowing spring,

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} = \frac{\phi}{T}\frac{\partial s}{\partial t} \qquad \ldots(7)$$

$$s(r,0)=0$$

$$s(r_w,t)= H_1 - H_2$$

$$\left.\frac{\partial s}{\partial r}\right|_{r=a} = 0 \qquad \ldots(8)$$

where 'a' is the radial distance to the impermeable boundary.

let K(t) be the drawdown in piezometric surface of a confined aquifer of finite areal extent at a radial distance r from the flowing spring due to a unit step excitation. Expression for K(t) can be obtained from the solution of the differential equation (7) solved by Muscat(1937) by substituting flow term Q by 1. Let $\delta_r(N)$ is the discrete kernel and the same has the following relation with unit step response(Morel Seytoux,1975),

$$\delta_r(N) = K(N)-N(N-1) \qquad \ldots(9)$$

Substituting for K(N) and K(N-1) in the above equation and simplifying, the expression for discrete kernel for drawdown $\delta_r(N)$for a confined aquifer could be obtained. The relation between the drawdown at the spring and the spring discharge is

$$s(r_w,I)= \sum_{\gamma=1}^{I} Q(\gamma)\delta_{rw}(I-\gamma+1) \qquad \ldots(10)$$

where $Q(\gamma),\gamma = 1,2,\ldots I$ are the discharge of the spring during different time steps. Since the drawdown at the flowing spring is $(H_1 - H_2)$, so the Eq.(10) can be rewritten in term of springflow Q(I),

$$Q(I)=\frac{1}{\delta(1)}\left[H_1-H_2 - \sum_{\gamma=1}^{I-1} Q(\gamma)\delta_{rw}(I-\gamma+1)\right] \qquad \ldots(11)$$

Q(I) can be determined in succession starting from time step 1.

The model has been tested for the linearity relationship between dynamic storage and springflow with following values of head difference, spring size, aquifer characteristics etc.

Head difference = $H_1 - H_2$ = 1 m,

Size of the opening of the spring=0.1m

Radial distance between the spring and the impermeable boundary = 1000m

Transmissivity = 100sq.m/day

Storage coefficient= 0.001

Time since the head difference is 1m during recession period = 500 days

The plots of variation of spring discharge with time and dynamic storage are given as Figs.6 and 7. A perusal of these plots reveal that the dynamic storage relation with springflow is linear during recession and conforms with the linearity assumption made in springflow prediction models.

4.0 Results and discussions

For a finite flow domain the linearity assumption of the springflow and storage is verified. The time variant flow from a spring emerging from a strip aquifer with following parameters and dimensions have been obtained and plotted in a semilogarithm paper. The multiple images of the flowing spring in the flow domain and the convolution technique have been used in the analysis.

Distance from western boundary = 2000m

Distance from eastern boundary = 1000m

Opening size of the spring = 1m

Transmissivity = 500Sq.m/day

Storativity = 0.001,

Pressure head in the beginning = 5m

Images considered = 50

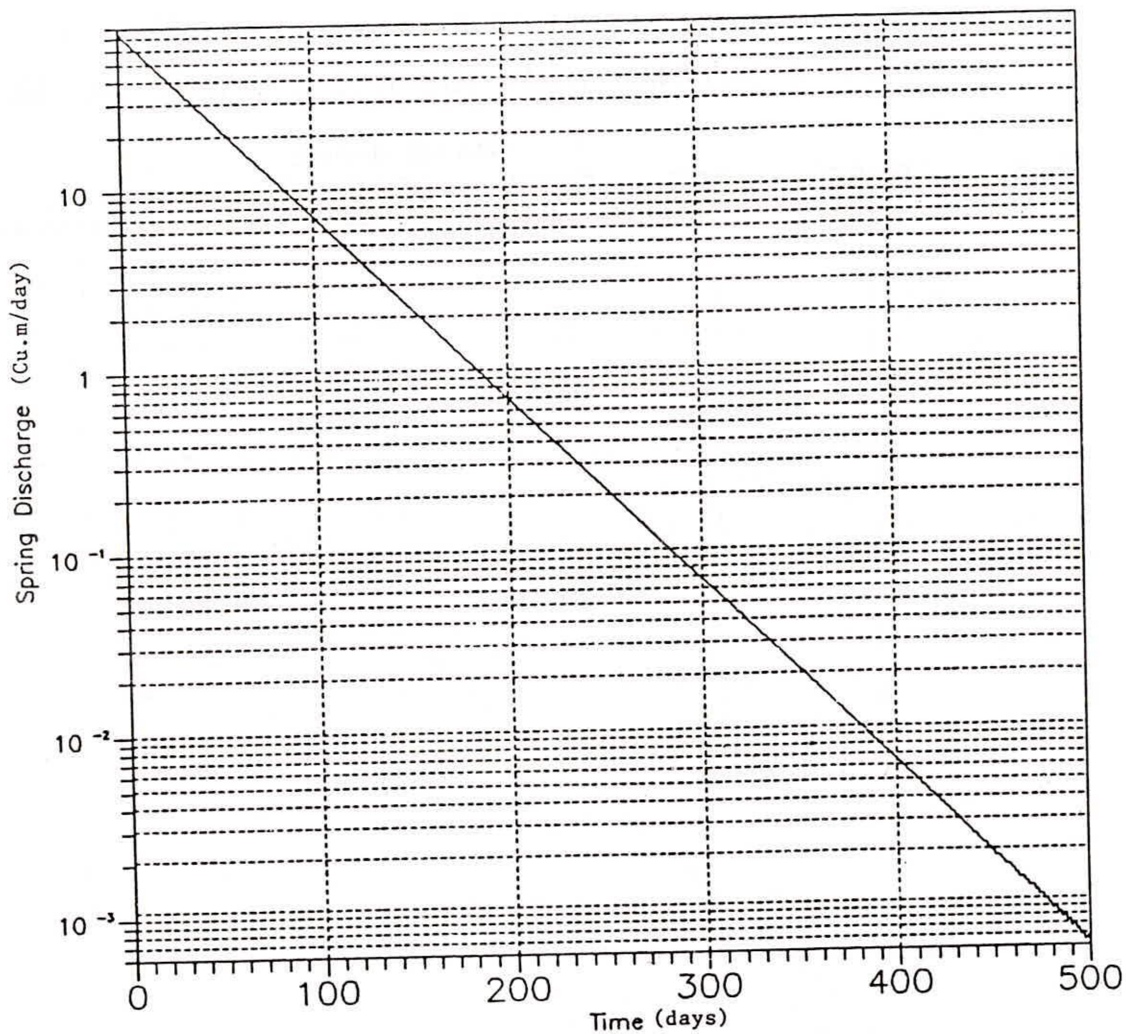

FIG.6 : VARIATION OF DISCHARGE WITH TIME

(H_0 = 1m, r_w = 0.1 m, r_a = 1000 m, T = 100 Sq.m/day, ϕ = 0.001)

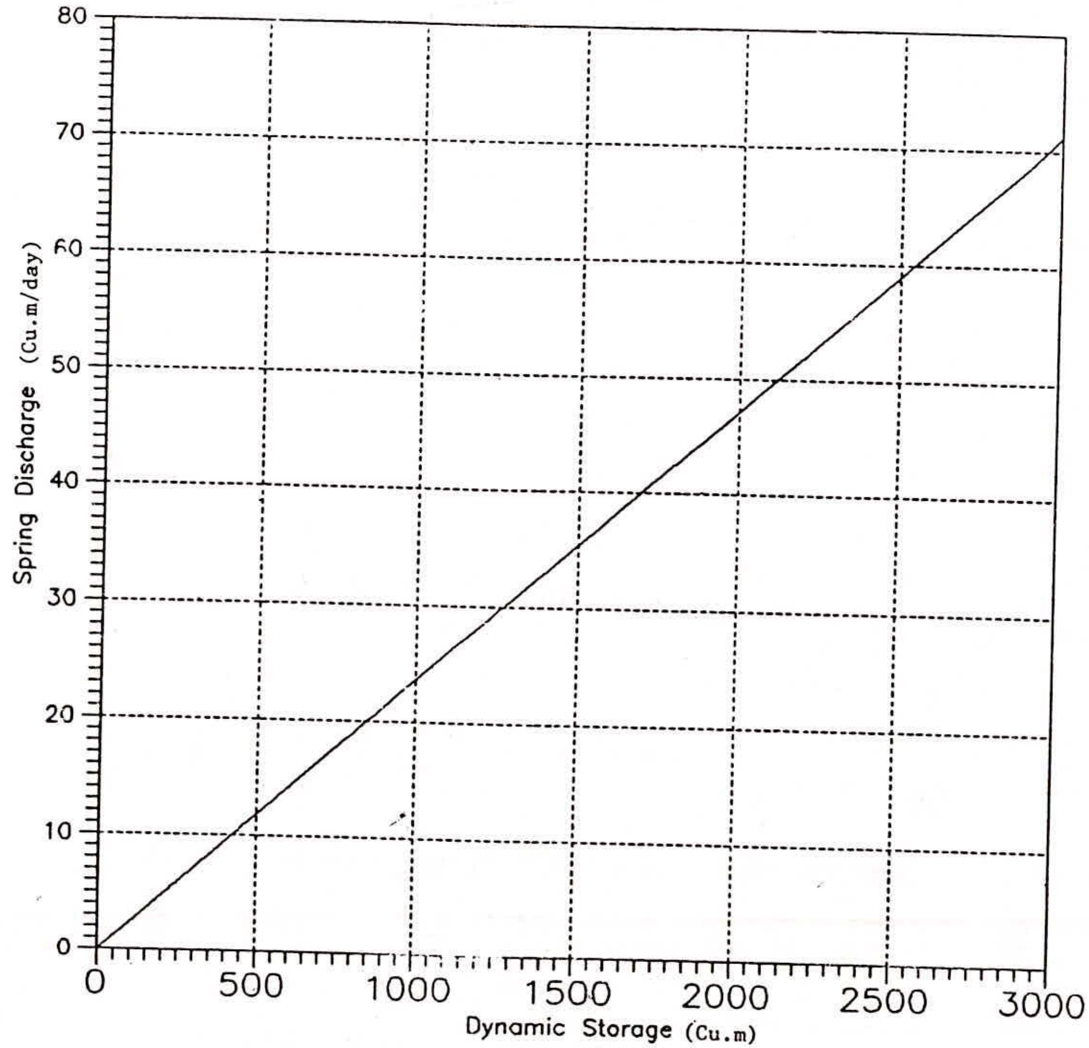

FIG.7 : VARIATION OF SPRING DISCHARGE WITH DYNAMIC STORAGE

(H_0 = 1m, r_w = 0.1m, r_a = 1000 m, T = 100 Sq.m/day, ϕ = 0.001)

The flow domain is not closed in northern and southern boundary but has impermeable boundaries at eastern and western side. The plot is furnished in Fig.8 and the the plot is non linear. This is because the system is not closed i.e., is an open system. The inflow of groundwater coming from the northern and southern open boundaries has made the relation non linear. Glover(1978) provided a solution for a similar flow domain and for a well whose drawdown is constant as is in the case of oil producing well from an extensive sandy layer. The required solution of flow from such a well has been obtained by a number of investigators. But the solution is difficult as the same is in the form of an infinite integral. Glover suggested an alternative method for such a flow domain with a constant head flowing well where the outer boundary is infinitely remote. He worked in terms of a finite outer radius from the well point to the outer boundary of the flow domain. The solution behaves as an infinitely remote outer boundary case until the disturbance produced flow from the well of given radius reaches the outer boundary of the flow domain. By using a sequence of increasingly remote outer boundaries, he computed a limited number of terms of the series solution and to extend the outer boundary to as remote a location as may be desired. A table/plot has been provided by him between non dimensional time and non dimensional discharge.

The flow from a spring for different position in the flow domain with respect to the outer boundary has been obtained by taking multiple images (100) on both the sides of the outer boundary with the help of convolution technique (Fig.9)and the non dimensional discharge from the spring with non dimensional time has been plotted in log log paper(Fig.10 and 11). The plots are same as that given by Glover for the non dimensional time upto 2000 for any east- west distances between the outer boundary and for any position of the spring in the flow domain between the

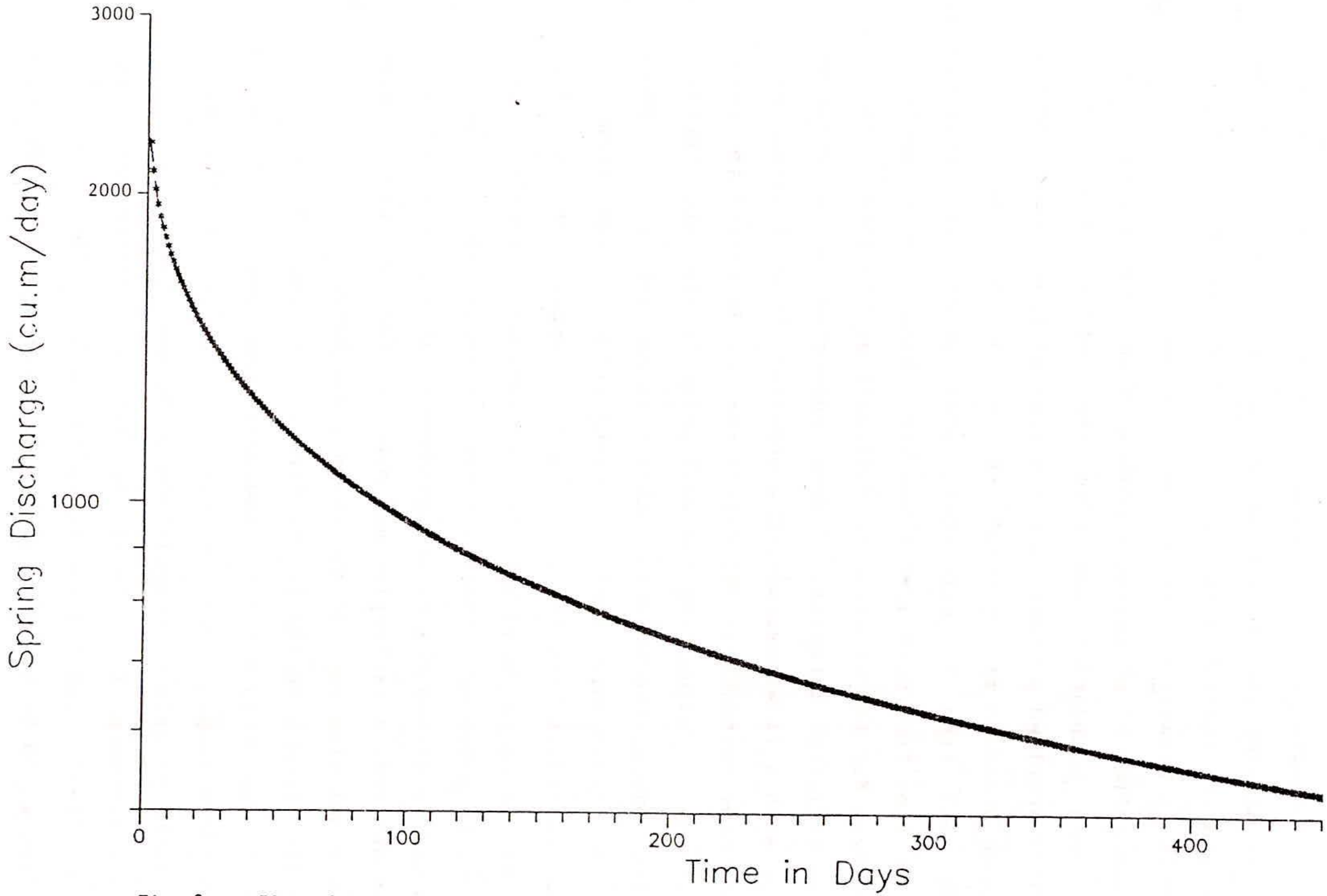

Fig. 8 : Plot of Springflow
(d_1 = 2000 m, d_2 = 1000 m, r_w = 1.0 m, T = 500 Sq.m./day, Ø = 0.001, H = 5.0 m, IMAX = 50)

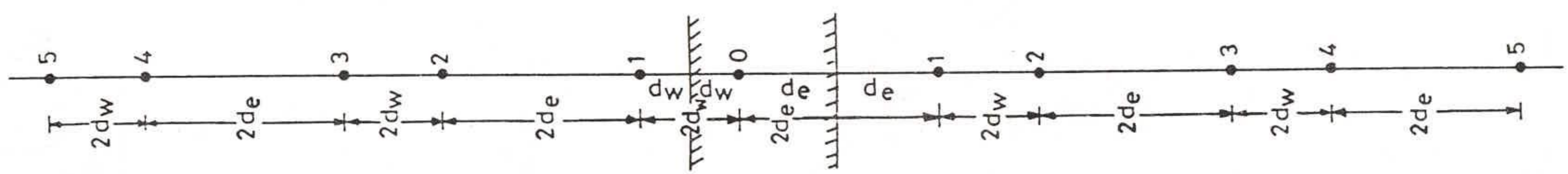

FIG. 9 SPRING AND ITS IMAGES.

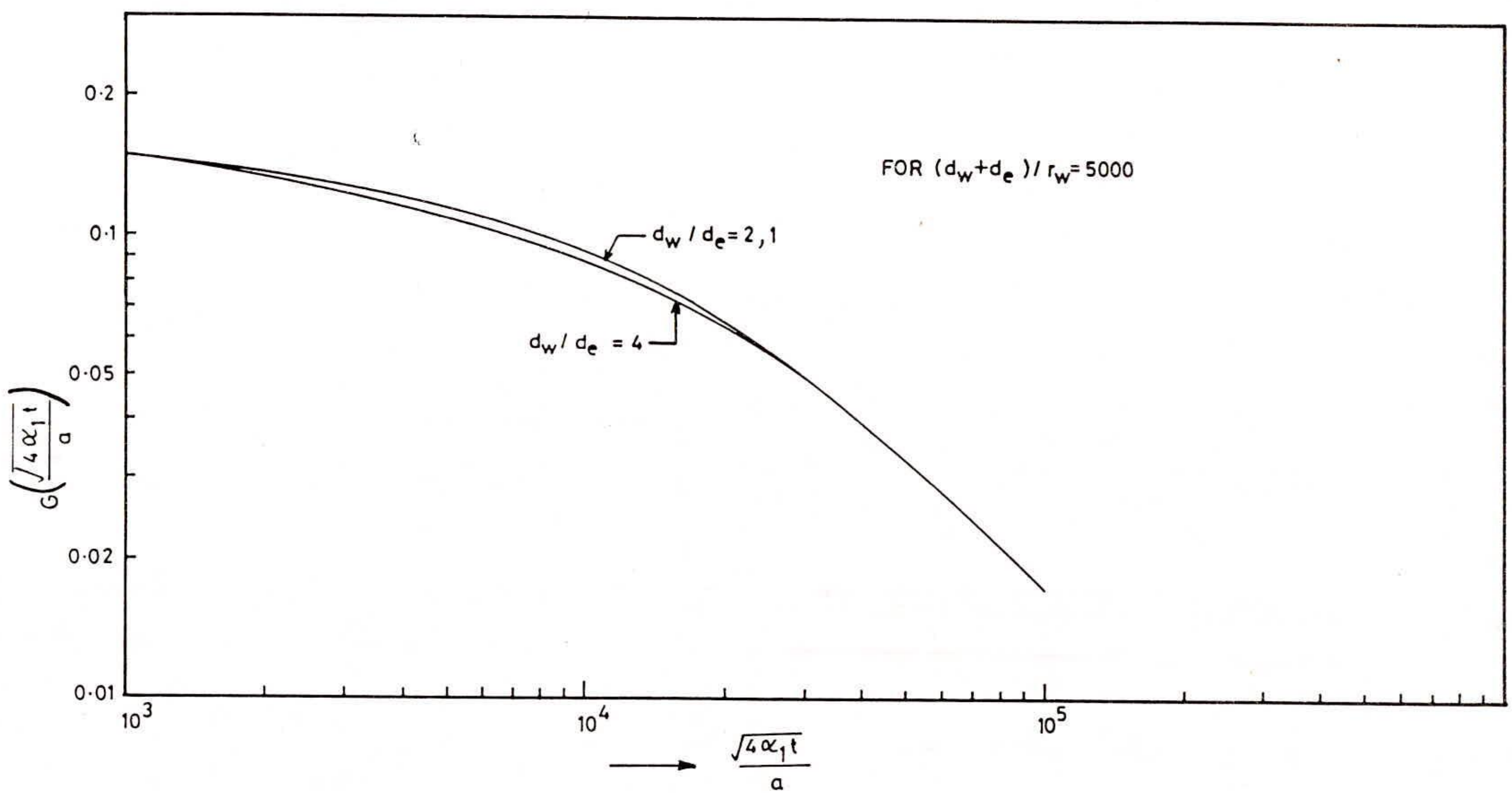

FIG. 10 - NON DIMENSIONAL PLOT OF SPRING DISCHARGE WITH TIME

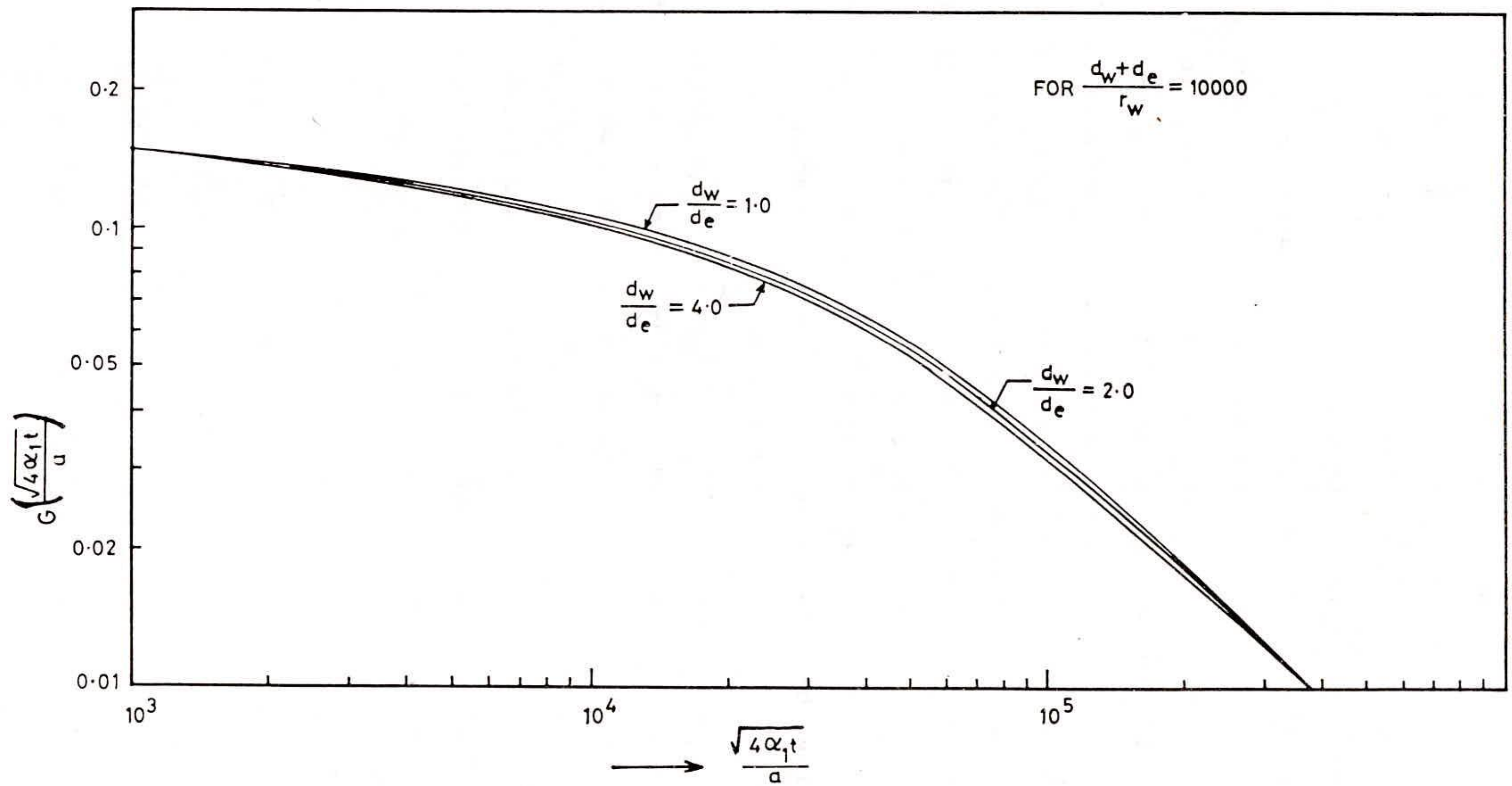

FIG. 11 - NON DIMENSIONAL PLOT OF SPRING DISCHARGE WITH TIME

boundary. But for nondimensional time beyond 2000 the plots differ from each other marginally. For a relatively small flow domain having east -west boundary distance as 5000m(d_w +d_e), and with $(d_w+d_e)/r_w=5000$ $d_w/d_e=1,2,4$, the plot has been given in Fig. 10. For, $d_w/d_e=1,2$ the plots are same and do not have any variation. For $d_w/d_e=4$, the plot differ at non dimensional time 3000 but and merges with the plot of $d_w/d_e=1,2$ at non dimensional time at 30000 implying that the spring dry out at that time and the boundary has no impact on it. Similar is the case for $d_w+ d_e=10000$m and with $(d_w+d_e)/r_w=10000$ $d_w/d_e=1,2,4$, the plot has been given in Fig. 11. For, $d_w/d_e=1,2,4$ the plots are same and do not have any variation upto non dimensional time 3000 but all of these three non dimensional flow vs non dimensional time plot branches off at non dimensional time 3000 and merges after considerable time beyond 100000 implying that the spring dries up after a much longer time than the first case because the flow domain distance east-west is double in the second case than the first one.

So, from the studies made regarding verification of the linearity assumption in most of the existing models of springflow, it could be inferred that the linearity assumption between springflow and the dynamic storage in the flow domain during the recession period is strictly valid for a spring whose flow domain is of finite areal extent.

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DIRECTOR : S.M.SETH

STUDY GROUP : A.K.BHAR
G.C.MISHRA