NUMBER OF OBSERVATION WELLS AND THEIR LOCATIONS FOR AN AQUIFER TEST IN DIFFERENT GEOHYDROLOGICAL CONDITIONS

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PREFACE

The hydraulic properties of aquifers are determined using pumping or recharge tests and water-level changes reflecting long-term effects of conditions at the aquifer boundaries. Both approaches contain hydrodynamic analysis of the aquifer. The first method is widely applied because it permits evaluation of the aquifer parameters in a relatively short period of time. The second method is the cheapest.

The principle of an aquifer test is that a well is pumped and the effect of this pumping on the water table in the vicinity of the well is observed. The advantage of two or more piezometers placed at different distances from the discharging well is that the drawdowns measured in these piezometers can be analyzed in two ways: by studying both the time-drawdown and distance-drawdown relationship. It is always better to have as many piezometers as conditions permit, while it has been recommended that at least three wells be employed. The locations of piezometers for a pumping test are governed by the type of aquifer, the hydraulic conductivity of the aquifer material, the discharge rate of the pumped well, and the well screen and stratification of the aquifer material.

In the present study the appropriate locations of observation wells for determining aquifer parameters in a stream aquifer system have been suggested. The procedure for determining the transmissivity and storage coefficient has been developed.

This report entitled 'Number of Observation Wells and Their Locations for an Aquifer Test in Different Geohydrological Conditions' is a part of the research activities of Ground Water Assessment division of the Institute. The study has been carried out by Dr. G.C.Mishra, Scientist'F'and Dr.S.K.Jain, Scientist 'E'.

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Abstract

A method has been proposed to estimate the transmissivity and the storage coefficient of an aquifer separately making use of the rise in stream stage and the consequent fluctuation in water level at observation wells located in the vicinity of the stream. A minimum number of three wells are required for the purpose. The first observation well with a small radius should be located near the stream bank. The second observation well with a radius more than 0.5 m should be located within a distance of 50 to 100m from the first observation well. The third observation well with small radius should be located at about 100m from the second observation well. All the three wells should be in a line perpendicular to the stream boundary.

The Laplace transform method proposed is applicable to any type of change in stream stage. Continuous observation of stream stage and water levels at the observation wells would enable accurate determination of aquifer parameters. The applicability and limitation of the methods presented have been verified using published field data. It is found that the water level fluctuations simulated using parameter estimated by the methods presented here are close to the observed values.

The inverse problem has also been solved using Marquardt algorithm and synthetically generated data. The transmissivity and storage coefficient values could be reestablished by the optimization technique.

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NUMBER OF OBSERVATION WELLS AND THEIR LOCATIONS FOR AN AQUIFERTEST IN DIFFERENT GEOHYDROLOGICAL CONDITIONS

INTRODUCTION

The principle of an aquifer test is that a well is pumped and the effect of this pumping on the water table in the vicinity of the well is observed. For this purpose a number of piezometers should be available near the discharging well. The advantage of two or more piezometers placed at different distances from the discharging well is that the drawdowns measured in these piezometers can be analyzed in two ways: by studying both the time-drawdown and distance-drawdown relationship. It is always better to have as many piezometers as conditions permit, while it has been recommended that at least three wells be employed (Kruseman and De Ridder, ILRI, Bulletin-11, 1970).

Sometimes aquifer tests are conducted near a hydrologic boundary. The effect of a nearby hydrologic boundary on a pumping well is to produce water levels which depart from the Theis-type curve. A recharge boundary such as a river will result in a series of drawdown levels less than those predicted by the Theis analysis with observed data below the type curve. Similarly an impermeable barrier, such as a fault, results in a series of drawdown observations greater than those predicted by the Theis analysis. Analysis of flow to a well near hydrologic boundary has been made using method of image and principle of superposition.

Regarding location of piezometer it has been stated that the piezometers should be placed neither too near nor too far from the pumped well. This statement is rather vague and needs further discussion. Exact guidelines for appropriate location of observation well for conducting an aquifer test near a hydrologic boundary are not available in literature.

The locations of piezometers are governed by the type of aquifer, the hydraulic conductivity of the aquifer material, the discharge rate of the pumped well, and the well screen and stratification of the aquifer material.

The hydraulic diffusivity of an aquifer can be determined from the observation of stream stage rise and the consequent fluctuation in water level position in an near by observation well. In the present study the appropriate locations of observation wells for determining aquifer parameters have been suggested.

REVIEW

The hydraulic properties of aquifers are determined using

(a) pumping or recharge tests and

(b) water-level changes reflecting long-term effects of conditions at the aquifer boundaries.

Both approaches contain hydrodynamic analysis of the aquifer. The first method is widely applied because it permits evaluation of the aquifer parameters in a relatively short period of time. The second method is the cheapest(Brown et al,1972).

Location of Observation Well for an Aquifer Test by Pumping:

Groundwater levels (heads) change in response to pumping or recharge and in response to fluctuations of water stage in contiguous bodies of surface water. The magnitude and the timing of the head changes are related to (a) the magnitude and location of the change in flow or water stage, (b) the hydraulic properties of the aquifer such as transmissivity and coefficient of storage, and (c) the shape and size of the aquifer, or more generally the three-dimensional distribution of the hydraulic properties in the sub surface.

An example of changes in head due to nearby pumping is shown schematically in Figure 1. In most aquifers the hydraulic head changes continuously in response to fluctuations in climate. In Figure 1, such variation is indicated as the antecedent trend line, before t = 0. If pumping had not started at t = 0, the graph would have continued along the dashed line marked 'Extrapolated trend'. After pumping starts the effect on head due only to pumping is the difference between observed head and extrapolated trend, identified as the 'drawdown', and measured at time t. Drawdown is the response only due to the change in boundary conditions imposed by a test pumping that started at t = 0. Analysis of the relation between observed drawdown, s, rate of discharge from the pumping well, Q_w, distance from the pumping well at which drawdown is observed, r, and time, t, can produce estimates of the hydraulic properties of the aquifer.

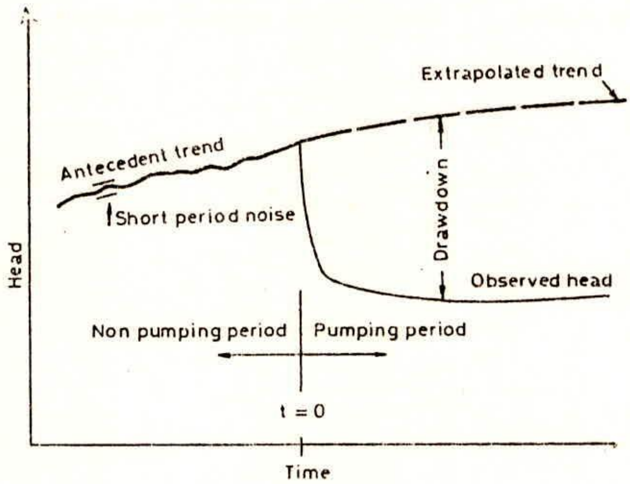

Figure 1-Hypothetical hydrograph at an observation well defining drawdown (Brown et al, 1972)

The Theis type curve is based upon the assumptions that the aquifer is of infinite extent and that the overlying and underlying confining beds are impermeable. If an impermeable boundary such as a fault, or a recharge boundary such as a river, exists within the influence of the discharging well, then the time-drawdown relationship will be different from that given by the Theis-type curve. A recharge boundary, such as a river or leakage downward through a semi-confining bed, will result in a series of drawdown levels less than those in an infinite aquifer predicted by Theis analysis, with observed data below the type curve. Similarly an impermeable barrier such as a fault results in a series of drawdown observations greater than those predicted by the Theis analysis.

Figure 2. shows schematically two cases: the effect of an impermeable barrier such as a fault (upper curve) and of a recharge boundary due to leakage through an overlying semi-permeable confining bed (lower curve) on drawdown that would have occured in an aquifer of infinite areal extent.

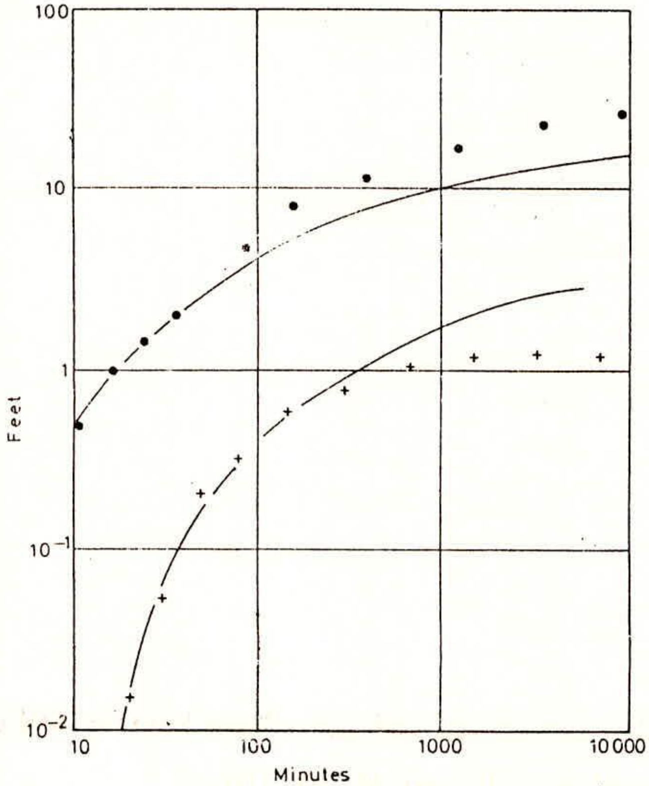

Figure 2- Aquifer boundary conditions as reflected in deperature of data plots from Theis type curves; upper curve shows influence of impermeable boundary; lower curve shows influence of recharge boundary (Brown et al,1972)

The principle of an aquifer test is that a well is pumped and the effect of this pumping on the water table in the vicinity is measured. For this purpose a number of piezometers should be available near the discharging well. Therefore, after the discharging well is completed one has to decide on the number and depth of those piezometers and how far they should be located from the discharging well.

The question of how many piezometers should be employed depends not only on the amount of information desired and the required degree of accuracy, but also on the funds available for the test. It has been shown that the data obtained by measuring the drawdown in a single piezometer often permit calculation of the average hydraulic conductivity and transmissivity of the aquifer and the storage coefficient.

The advantage of two or more piezometers placed at different distances from the discharging well is that the drawdowns measured in these piezometers can be analyzed in two ways: by studying both time-drawdown and the distance-drawdown relationships. Obviously, the results of calculations thus obtained are more accurate and are representative of a larger area.

It is always best to have as many piezometers as conditions permit, while on the other hand it is recommended that at least three be employed.

In confined aquifers a loss of hydraulic head caused by pumping propagates fast because the release of water from storage is entirely due to the compressibility of the aquifer material and that of water. Therefore, the loss of head may still be measurable at great distances, for instance a few hundred meters from the pumped well.

In unconfined or water-table aquifers the propagation of hydraulic head losses is rather slow because the release of water from storage is mostly due to dewatering of the zone through which the water is moving, and only partly due to the compressibility of water and aquifer material in the saturated zone. Unless the period of pumping is extended for several days, the loss of hydraulic head caused by pumping is only measurable within rather short distances of the pumped well, for instance not much farther away than about 100 m.

Semi-confined aquifers have an intermediate position. It depends on the hydraulic resistance of the semi-pervious layer whether the aquifer more closely resembles a confined, or unconfined aquifer.

When the hydraulic conductivity of the aquifer material is high the cone of depression induced by pumping will be wide and flat. When the hydraulic conductivity is low the cone of depression will be steep and narrow. Therefore, in the first case piezometers can be placed further away from the pumped well than they can be in the second case.

If the discharge rate of the pumped well is high the cone of depression induced by pumping will be larger than with a low rate. Therefore, in the first case greater distances between the piezometers are allowed than in the second.

The choice of distances from the pumped well at which piezometers should be installed may be strongly influenced by the length of the well screen in the pumped well. If the discharging well is a fully penetrating one, i.e. a well whose screen penetrates the entire thickness of the aquifer, or at least 80 per cent of it, the flow of water to the pumped well will be horizontal. Therefore, drawdowns measured in piezometers placed even at short distances from the pumped well can be used for the analysis. It is obvious that if the aquifer to be tested is not very thick, it is always best to employ a fully penetrating well.

However, in many cases the aquifer to be tested is thick and conditions may not allow a well screen to be installed over the entire thickness of the aquifer. In such a partially penetrating well, the relatively short length of the well screen will cause a non-uniform distribution of head or drawdown which is most noticeable near the well. So, if the length of the well screen is considerably less than the saturated thickness of the aquifer, a distorted drawdown pattern is induced near the well, due to vertical flow components. Drawdown readings from wells close to such a partially penetrating well may lead to incorrect results, and rather complicated correction methods have to be applied before those readings can be used for the analysis of the test data. These difficulties can be avoided if the piezometers are placed further away from the pumped well, where these abnormal effects do not appear. As a general rule it may be recommended that the nearest piezometers be placed at a distance which is at least equal to the thickness of the aquifer. At such a distance it may be assumed that the flow is horizontal.

From the above it is obvious that several factors are involved in deciding how far from the pumped well the piezometers should be installed. A proper knowledge of the test site, especially of the type of aquifer, its thickness, average hydraulic conductivity, and stratification, will make it easier to choose the proper distances at which the piezometers should be installed.

The ultimate choice for fixing the position of piezometer depends entirely on local conditions and the length of the well screen installed in the pumped well. Placing piezometers about 10 to 100 m from the pumped well will give good results in most cases. The distances must be greater for thick, or stratified confined aquifers and the piezometers should be placed 100 to 250m or more from the pumped well in order to obtain reliable data. It is also useful to have a piezometer outside the radius of influence of the pumped well so that the water table not affected by pumping can be measured. This piezometer should be placed several hundred meters from the well, or in certain cases, as far away as one kilometer or more. If the readings of this piezometer show water table changes during the test, for instance, changes caused by natural discharge or recharge, these data can be used to correct the drawdowns induced by pumping.

Aquifer Test using Changes in Stream Stage and Water Level Fluctuations in an Observation Well:

The transmissivity and storage coefficient of an aquifer are also determined by analyzing its response to natural excitation such as passage of a flood in an adjoining stream. The type of test, in which water is pumped from a well and the consequent changes in piezometric surface are observed, enables determination of both transmissivity and storage coefficient separately. Such aquifer test are of local nature and yields point values of the parameters(Singh and Sagar, 1977). The second type of test, in which the changes in water table in an observation well consequent to the changes in stream stages are recorded and analyzed, is cheaper and requires observation over a long duration. Such test in a stream-aquifer system so far enables determination of the hydraulic diffusivity only; the transmissivity and the storage coefficient can not be estimated individually (Hall and Moench, 1972).

The problem of identifying the hydraulic diffusivity from the observation of stream stage and consequent water table fluctuations in the aquifer is an inverse problem. An inverse problem can not be solved unless the corresponding direct problem has been solved a priori. The stream aquifer interaction problem has been solved by several investigators (Todd, 1955; Cooper and Rorabaugh, 1963; Hall and Moench, 1972; Morel-Seytoux, 1975, Morel-Seytoux and Daly, 1977). By and large the stream-aquifer equations have been applied to the estimation of the aquifer diffusivity by various investigators(Rowe,1960;Ferris et al.,1952; Pinder et al., 1969; Brown et al., 1972; Singh and Sagar, 1977). The equation given by Morel-Seytoux (1975) would enable the determination of tranmissivity and storage coefficient and all other equations can be used for finding aquifer diffusivity.

Parametric observation wells should be located near theboundaries of ground water flow (river, canals, reservoirs) so that water level fluctuations in them are not less than 0.3-0.5 of the amplitude of the water level change at the flow boundary. The distance from such a boundary to the most distant parametric well should be about $0.5\sqrt{(\beta t_d)}$ to $\sqrt{(\beta t_d)}$ where β is the hydraulic diffusivity and t_d is the time over which the most significant change in water level at the flow boundary occurs. In case of a partially penetrating stream the first calculation well should be installed at a distance of about 1.5 times the thickness of the aquifer from the stream (Brown et al, 1972).

From the review it can be inferred that so far no technique is available which enables determination of transmissivity and storage coefficient separately. In the present report an analytical method to determine the transmissivity and storage coefficient separately from the measurement of stream stage and the consequent water level fluctuations in observation wells in the vicinity of the stream has been proposed and the number of observation required for solving the inverse problem has been suggested.

ANALYTICAL DEVELOPMENT

The unit step response function that relates rise in piezometric surface in an initially rest semi-infinite homogeneous and isotropic confined aquifer bounded by a fully penetrating straight stream to a step rise in stream stage is given by (Carslaw and Jaeger, 1959)

$$K(x,t) = 1 - \mathrm{erf}\{\frac{x}{\sqrt{(4\beta t)}}\} = \mathrm{erfc}\{\frac{x}{\sqrt{(4\beta t)}}\} \qquad ...(1)$$

in which

$K(x,t)$ = unit step response function,

x = distance from the bank of the stream,

t = time measured since the onset of change in stream stage,

β = T/ϕ, the hydraulic diffusivity of the aquifer, T = transmissivity,

ϕ = storage coefficient,

erf(x)= error function = $2/\sqrt{\pi}\ \int_0^x \exp(-u^2)du$, and

erfc(x)= complementary error function = 1-erf(x).

Equation(1) is a good approximation for an unconfined aquifer if the changes in water level are small in comparison to the saturated thickness of the aquifer (Cooper and Rorabaugh, 1963).

For varying stream stage, the rise in piezometric surface according to Duhamel's theorem is given by (Pinder et.al, 1969, and Morel-Seytoux, 1978)

$$s(x,t) = \sigma_0\ K(t) + \int_0^t \frac{d\sigma}{d\tau}\ K(t-\tau)d\tau \qquad ...(2)$$

in which σ_0 is the initial sudden rise in the stream stage.

Let the time span be discretised by uniform time-steps of size Δt. Let the rate of change of stream stage ($d\sigma/d\tau$) be a constant within a time step. $d\sigma/d\tau$ may vary from one time step to other. Equation (2) can be rewritten as (Morel-Seytoux, 1978) :

$$s(x,n\Delta t) = \sigma_0[1 - \text{erf}\{ \frac{x}{\sqrt{(4\beta n\Delta t)}}\}]$$

$$+ \sum_{\gamma=1}^{n}[\ \{\frac{\sigma(\gamma\Delta t)-\sigma(\gamma\Delta t-\Delta t)}{\Delta t}\} \int_{(\gamma-1)\Delta t}^{\gamma\Delta t}[1-\text{erf}\{\frac{x}{\sqrt{(4\beta(n\Delta t-\tau))}}\}]d\tau] \qquad ...(3)$$

Let a discrete kernel coefficient $\delta_r(x,m)$ be defined as:

$$\delta_r(x,m) = \frac{1}{\Delta t}\int_0^{\Delta t}[1- \text{erf}(\frac{x}{\sqrt{\{4\beta(m\Delta t-\tau)\}}})]d\tau \qquad ...(4)$$

Making a substitution $\tau = \Delta t\ v$, $d\tau = \Delta t\ dv$ and integrating, the discrete kernel coefficient is found to be

$$\begin{aligned}\delta_r(x,m) = 1 &+ (m-1)\ \text{erf}[x/\sqrt{\{4\ \beta\ \Delta t\ (m-1)\}}]\\ &- m\ \text{erf}\{x/\sqrt{(4\ \beta\ \Delta t\ m)}\}\\ &+ x^2/(2\ \beta\ \Delta t)\ \text{erf}[x/\sqrt{\{4\ \beta\ \Delta t\ (m-1)\}}]\\ &- x^2/(2\ \beta\ \Delta t)\ \text{erf}\{x/\sqrt{(4\ \beta\ \Delta t\ m)}\}\\ &+ x\ \sqrt{\{(m-1)/(\beta\ \Delta t\ \pi)\}}\ \exp[-x^2/\{4\ \beta\ \Delta t\ (m-1)\}]\\ &- x\ \sqrt{\{m/(\beta\ \Delta t\ \pi)\}}\ \exp\{-x^2/(4\ \beta\ \Delta t\ m)\}\end{aligned} \qquad ...(5)$$

The rise in piezometric surface at the end of nth unit time step given by equation (3) is expressed in terms of discrete kernel coefficient as

$$s(x,n\Delta t)=\sigma_0[1-\mathrm{erf}\{\frac{x}{\sqrt{(4\beta n\Delta t)}}\}]+\sum_{\gamma=1}^{n}[\{\sigma(\gamma\Delta t)-\sigma(\gamma\Delta t-\Delta t)\}\delta_r(x,n-\gamma+1)] \quad \ldots(6)$$

The fluctuation in piezometric surface at different locations for a single or several flood events in the stream can be determined using equation (6).

The hydraulic diffusivity, β, can be determined from the observations of changes in stream stage and consequent changes in water level in an observation well which has negligible well storage applying Marquardt algorithm (Marquardt, 1963) on equation(6).

An alternate approach for determining the parameter β is described in the following paragraphs.

Taking Laplace transform of terms on either side of equation (2)

$$\int_0^{\infty} s(x,t)\, e^{-St} dt = \int_0^{\infty} \sigma_0\, K(t) e^{-St}\, dt + \int_0^{\infty} [\int_0^{t} \frac{d\sigma}{d\tau}\, K(t-\tau) d\tau\,]\, e^{-St} dt \quad \ldots(7)$$

Applying Faltung theorem, ie $L\{\int_0^t F_1(t-\tau)F_2(\tau)d\tau\}=L\{F_1(t)\}L\{F_2(t)\}$ equation (7) reduces to

$$\int_0^{\infty} s(x,t)\, e^{-St} dt = \sigma_0\, L\{K(t)\} + L\{\frac{d\sigma}{dt}\}\, L\{K(t)\} \quad \ldots(8)$$

Substituting the Laplace transform of the complementary error function (Abramowitz and Stegun, 1970, p1026) in equation (8)

$$\int_0^{\infty} s(x,t)\, e^{-St} dt = [\sigma_0 + L\{\frac{d\sigma}{dt}\}]\, \exp\{-\sqrt{(Sx^2/\beta)}\}/S \quad \ldots(9)$$

Discretising the time domain by time steps of uniform size Δt and assuming that within a time step the drawdown at the observation well and rate of change of stream stage are separate constants, equation (9) is expressed as

$$\sum_{\substack{\gamma=1 \\ n\to\infty}}^{n} [\bar{s}\{x,\,(\gamma-0.5)\Delta t\} \int_{(\gamma-1)\Delta t}^{\gamma\Delta t} e^{-St}\, dt]$$

$$=[\sigma_0+\sum_{\substack{\gamma=1 \\ n\to\infty}}^{n} \{\frac{\sigma(\gamma\Delta t)-\sigma(\gamma\Delta t-\Delta t)}{\Delta t} \int_{(\gamma-1)\Delta t}^{\gamma\Delta t} e^{-St} dt\, \}]\, \exp\{-\sqrt{(Sx^2/\beta)}\}/S \quad \ldots(10)$$

in which $\bar{s}\{x,(\gamma-0.5)\Delta t\}$ is the average rise in water level at the observation well during time period $(\gamma-1)\Delta t$ to $\gamma\Delta t$ and it is given by $[s\{x,(\gamma-1)\Delta t\}+s(x,\gamma\Delta t)]/2$. Performing the integration appearing in equation (10)

$$\sum_{\substack{\gamma=1\\ n\to\infty}}^{n} [\bar{s}\{x,(\gamma-0.5)\Delta t\}(\frac{e^{-S(\gamma-1)\Delta t}-e^{-S\gamma\Delta t}}{S})]$$

$$=[\sigma_0+\sum_{\substack{\gamma=1\\ n\to\infty}}^{n}\{\frac{\sigma(\gamma\Delta t)-\sigma(\gamma\Delta t-\Delta t)}{\Delta t}(\frac{e^{-S(\gamma-1)\Delta t}-e^{-S\gamma\Delta t}}{S})\}]\exp\{-\sqrt{(Sx^2/\beta)}\}/S \quad ...(11)$$

or

$$\exp\{-\sqrt{(Sx^2/\beta)}\}=\frac{\sum_{\substack{\gamma=1\\ n\to\infty}}^{n}[\bar{s}\{x,(\gamma-0.5)\Delta t\}(e^{-S(\gamma-1)\Delta t}-e^{-S\gamma\Delta t})]}{[\sigma_0+\sum_{\substack{\gamma=1\\ n\to\infty}}^{n}\{\frac{\sigma(\gamma\Delta t)-\sigma(\gamma\Delta t-\Delta t)}{\Delta t}\frac{(e^{-S(\gamma-1)\Delta t}-e^{-S\gamma\Delta t})}{S}\}]} \quad ...(12)$$

Taking logarithm of terms on either side and solving for β

$$\beta=Sx^2/[\log\frac{\sum_{\substack{\gamma=1\\ n\to\infty}}^{n}[\bar{s}\{x,(\gamma-0.5)\Delta t\}(e^{-S(\gamma-1)\Delta t}-e^{-S\gamma\Delta t})]}{\sigma_0+\sum_{\substack{\gamma=1\\ n\to\infty}}^{n}\{\frac{\sigma(\gamma\Delta t)-\sigma(\gamma\Delta t-\Delta t)}{S\Delta t}(e^{-S(\gamma-1)\Delta t}-e^{-S\gamma\Delta t})\}}]^2 \quad ...(13)$$

Making use of equation (13) the hydraulic diffusivity, β, can be computed. If a flood wave follows a definite equation, the computation of β can be further simplified.

Let σ_0 be equal to zero and let the flood wave follow Cooper and Rorabough's equation

$$\sigma(t)=\begin{cases} NH_0(1-\cos\omega t)\,e^{-\delta t} & \text{for } 0\le t\le t_d \\ 0 & \text{for } t>t_d \end{cases} \quad ...(14)$$

where $\sigma(t)$ = the rise above initial water level in the stream, t_d = the duration of the flood wave, $\omega = 2\pi/t_d$, $\delta = \omega\cot(0.5\omega t_c)$, t_c = the time of flood peak, $N = \exp(\delta t_c)/(1-\cos\omega t_c)$ and H_0= height of peak flood stage above initial water level.

From equation (14)

$$\frac{d\sigma}{dt}=-\delta\, NH_0(1-\cos\omega t)e^{-\delta t}+NH_0\omega\sin\omega t\, e^{-\delta t} \quad ...(15)$$

$$L\{\frac{d\sigma}{dt}\}=-\delta NH_0[1-\exp\{-(\delta+S)\ t_d\}]/(\delta+S)$$

$$+\delta NH_0[\frac{\delta+S}{(\delta+S)^2+\omega^2}-\frac{\exp\{-(\delta+S)t_d\}}{(\delta+S)^2+\omega^2}\{(\delta+S)\cos(\omega t_d)-\omega\ \sin(\omega t_d)\}]$$

$$+\omega NH_0[\frac{\omega}{(\delta+S)^2+\omega^2}-\frac{\exp\{-(\delta+S)t_d\}}{(\delta+S)^2+\omega^2}\{(\delta+S)\sin(\omega t_d)+\omega\ \cos(\omega t_d)\}]$$

...(16)

The Laplace transform of $\frac{d\sigma}{dt}$ can be substituted for the denominator in the logarithmic term in equation (13) and β can be determined. Once the hydraulic diffusivity is determined the transmissivity can be known as described below.

Let r_w be the radius of the observation well which is located at a distance l from the stream bank. Let $V_a(\gamma\Delta t)/\Delta t$ be the average rate flow of water leaving the aquifer storage and $V_w(\gamma\Delta t)/\Delta t$ be the average rate of flow of water entering the well storage during the time period from $(\gamma-1)\Delta t$ to $\gamma\Delta t$. $V_a(\gamma\Delta t)/\Delta t$ and $V_w(\gamma\Delta t)/\Delta t$ are equal. $V_a(\gamma\Delta t)$ is the volume of water leaving the aquifer storage and entering the observation well storage during $(\gamma-1)\Delta t$ to $\gamma\Delta t$. The change in height of piezometric surface at the observation well is comprised of two components. One component is the rise in piezometric surface consequent to the rise in stream stage and the other component is the drop in piezometric surface because of the exit of water from aquifer storage to the observation well storage. The rise in piezometric surface in the aquifer at the observation well due to change in river stage is given by equation(6). The drop in piezometric surface due to exit of water from the aquifer to observation well storage is given by

$$s_a(n\Delta t)=\sum_{\gamma=1}^{n}\frac{V_a(\gamma\Delta t)}{\Delta t4\pi T}\int_{(\gamma-1)\Delta t}^{\gamma\Delta t}\frac{e^{-r_w^2/\{4\beta(n\Delta t-\tau)\}}}{(n\Delta t-\tau)}d\tau$$

$$-\sum_{\gamma=1}^{n}\frac{V_a(\gamma\Delta t)}{\Delta t4\pi T}\int_{(\gamma-1)\Delta t}^{\gamma\Delta t}\frac{e^{-(2l^2)/\{4\beta(n\Delta t-\tau)\}}}{(n\Delta t-\tau)}d\tau$$

$$=\sum_{\gamma=1}^{n}V_a(\gamma\Delta t)\ \delta_p(r_w,n-\gamma+1)-\sum_{\gamma=1}^{n}V_a(\gamma\Delta t)\ \delta_p(2l,n-\gamma+1)$$

...(17

in which $\delta_p(r,m)$ is a discrete pumping kernel coefficient (Morel Seytoux, 1975) defined as :

$$\delta_p(r,m) = \frac{1}{\Delta t}\int_0^{\Delta t} \frac{1}{4\pi T(m\Delta t-\tau)} e^{-r^2/\{4\beta(m\Delta t-\tau)\}} d\tau$$

$$= 1/(4\pi T\Delta t)[E_1\{r^2/(4\beta m\Delta t)\}-E_1\{r^2/(4\beta\Delta t(m-1))\}]$$

...(18)

The last term in equation(17) accounts for the presence of the fully penetrating stream boundary.

The resultant rise in piezometric level at the observation well is

$$s(l,n\Delta t)=\sigma_0[1-\mathrm{erf}\{\frac{l}{\sqrt{(4\beta n\Delta t)}}\}]+\sum_{\gamma=1}^{n}[\{\sigma(\gamma\Delta t)-\sigma(\gamma\Delta t-\Delta t)\}\delta_r(l,n-\gamma+1)]$$

$$-\sum_{\gamma=1}^{n}[V_a(\gamma\Delta t)\delta_p(r_w,n-\gamma+1)] + \sum_{\gamma=1}^{n}[V_a(\gamma\Delta t)\delta_p(2l,n-\gamma+1)]$$

...(19)

The rise in water table height in the observation well can be expressed as:

$$s_o(n\Delta t)= \sum_{\gamma=1}^{n}V_w(\gamma\Delta t)/(\pi r_w^2) \qquad ...(20)$$

Since, $s_o(n\Delta t)= s(l,n\Delta t)$, and $V_a(\gamma\Delta t)=V_w(\gamma\Delta t)$, therefore,

$$\sigma_0[1-\mathrm{erf}\{\frac{l}{\sqrt{(4\beta n\Delta t)}}\}]+ \sum_{\gamma=1}^{n}\{\sigma(\gamma\Delta t)-\sigma(\gamma\Delta t-\Delta t)\}\delta_r(l,n-\gamma+1)$$

$$-\sum_{\gamma=1}^{n} V_w(\gamma\Delta t)\delta_p(r_w,n-\gamma+1)+\sum_{\gamma=1}^{n} V_w(\gamma\Delta t)\delta_p(2l,n-\gamma+1)$$

$$= \sum_{\gamma=1}^{n} V_w(\gamma\Delta t)/(\pi r_w^2) \qquad ...(21)$$

Splitting the temporal summation to two parts, one part containing the summation up to (n-1)th time step and the other part containing the nth term, the unknown $V_w(n\Delta t)$ is solved. $V_w(n\Delta t)$ is given by

$$V_w(n\Delta t)=[\ \sigma_0[1-\mathrm{erf}\{\frac{l}{\sqrt{(4\beta n\Delta t)}}\}]+\sum_{\gamma=1}^{n}[\{\sigma(\gamma\Delta t)-\sigma(\gamma\Delta t-\Delta t)\}\delta_r(l,n-\gamma+1)]$$

$$-\sum_{\gamma=1}^{n-1}\{V_w(\gamma\Delta t)\delta_p(r_w,n-\gamma+1)\}+ \sum_{\gamma=1}^{n-1}\{V_w(\gamma\Delta t)\delta_p(2l,n-\gamma+1)\}$$

$$-\sum_{\gamma=1}^{n-1}V_w(\gamma\Delta t)\ /(\pi r_w^2)\ \ ]/[\ (1/(\pi r_w^2)-\delta_p(2l,1)+\delta_p(r_w,1)\ ]$$

...(22)

$V_w(\gamma\Delta t)$, $\gamma = 1,2,3,\ldots,n$; can be solved in succession starting from $\gamma=1$. In particular for $\gamma=1$

$$V_w(\Delta t) = [\ \sigma_0[1-\mathrm{erf}\{\frac{l}{\sqrt{(4\beta\Delta t)}}\}] + [\{\sigma(\Delta t)-\sigma_0\}\delta_r(l,1)]\ ] /$$

$$[(1/(\pi r_w^2)-\delta_p(2l,1)+\delta_p(r_w,1)] \qquad \ldots(23)$$

Once $V_w(\gamma\Delta t)$, $\gamma = 1,2,\ldots,n$, are determined, the drawdowns can be computed using either equation (19) or (20).

In the inverse problem the aquifers parameter β and T are to be determined only from the record of stream stage and water level fluctuations in observation wells. Let two observation wells, one having negligible well storage and the other having appreciable well storage be selected for solving the two parameter inverse problem. Making use of the data of water table rise at the observation well that has negligible well storage the hydraulic diffusivity can be known from equation(13) Once the hydraulic diffusivity is determined the transmissivity can be known making use of the data at the observation well which has appreciable well storage adopting the following procedure:

$V_w(n\Delta t)$ can be known in succession starting from time step 1 using the relation

$$V_w(n\Delta t) = \pi r_w^2\ s(l,n\Delta t) - \sum_{\gamma=1}^{n-1} V_w(\gamma\Delta t) \qquad \ldots(24)$$

Equation(24) also provides solution for $V_a(n\Delta t)$.

Equation(19) can be rewritten as

$$s(l,n\Delta t) = \sigma_0 \mathrm{erfc}\{\frac{l}{\sqrt{(4\beta n\Delta t)}}\} + \sum_{\gamma=1}^{n}[\{\sigma(\gamma\Delta t)-\sigma(\gamma\Delta t-\Delta t)\}\delta_r(l,n-\gamma+1)]$$

$$- \{1/(4\pi T\Delta t)\}\sum_{\gamma=1}^{n}[V_a(\gamma\Delta t)\ \delta_p'(r_w,n-\gamma+1)]$$

$$+ \{1/(4\pi T\Delta t)\}\sum_{\gamma=1}^{n}[V_a(\gamma\Delta t)\ \delta_p'(2l,n-\gamma+1)] \qquad \ldots(25)$$

in which the coefficient $\delta_p'(r,m)$ is only a function of hydraulic diffusivity β and it is given by

$$\delta_p'(r,m) = E_1\{r^2/(4\beta\Delta t m)\} - E_1[r^2/\{4\beta\Delta t(m-1)\}] \qquad \ldots(26)$$

From equation (25) the only unknown transmissivity is found to be

$$T=-\{1/(4\pi\Delta t)\}[\sum_{\gamma=1}^{n} V_a(\gamma\Delta t)\{\delta_p'(r_w,n-\gamma+1)-\delta_p'(2l,n-\gamma+1)\}]/$$

$$[s(l,n\Delta t)-\sigma_0 \mathrm{erfc}\{\frac{l}{\sqrt{(4\beta n\Delta t)}}\}-\sum_{\gamma=1}^{n}\{(\sigma(\gamma\Delta t)-\sigma(\gamma\Delta t-\Delta t)\}\delta_r(l,n-\gamma+1)\}] \quad ...(27)$$

The aquifer parameters T, and β can also be determined by applying the Marquardt algorithm on equation (25). The data at the observation well having appreciable well storage are only required for finding T and β by the Marquardt algorithm.

RESULTS AND DISCUSSION

The proposed methods for finding the aquifer parameters from the response of the stream aquifer system have been tested making use of synthetic data. The synthetic data for rise in piezometric level at observation wells, one having neglible well storage and the other having appreciable well storage, were generated for different sets of T and ϕ and r_w for the following flood wave:

Time of concentration, t_c:	24 hours
Duration of the flood wave, t_d:	120 hours
Maximum rise in stream stage, H_0:	2m
Duration of observation:	200 hours at interval of Δt
Distance of the observation well, l:	50m, and 100m
Radius of the observation well, r_w:	{ 0.1m, 0.3m, 0.5m, and 1.0m }
Time step size, Δt:	{ 1, 0.5, 0.25, 0.125 hour }
Transmissivity, T:	{5.0, 50.0 m^2 per hour}
Storage coefficient, ϕ	{0.00001, 0.2 }

Determination of Hydraulic Diffusivity:

Making use of the synthetically generated stream stage and piezometric level at an observation well with negligible well storage, and applying Marquardt algorithm on equation(6), the parameter β was estimated and is presented in table 1. The objective function was the sum of squares of difference between the observed and the predicted drawdowns over the time of observations. The value of the objective function at the optimum point is given in the column 4 of the table 1. The results presented in table 1 covey that the hydraulic diffusivity can be estimated accurately in the lower range. For very high value of hydraulic diffusivity, the metod is approximate and the error in estimation is of the order 0.3 percent.

Hydraulic diffusivity evaluated by Laplace transform technique is presented in Table2 for different durations of observation, time step sizes and for different values of Laplace transform parameter S. Table 2 shows that a small value of S provides accurate result provided observation for longer duration and smaller time step size have been made use of in the computation. The sensitivity of the hydraulic diffusivity to the time step size, duration of observation and Laplace transform parameter S is presented in Table 3.

Determination of Transmissivity:

Making use of the synthetically generated stream stage and piezometric level at an observation well with appreciable storage, and applying Marquardt algorithm on equations(24) and (25), the parameters T and β were estimated. The estimated parameters are presented in table 4. If the well radius of the observation well is less than 0.5m, the transmissivity values can not be estimated though the hydraulic diffusivity can be esimated approximately. For accurate determination of transmissivity, the observation well should have a radius more than 0.5m.

After finding the hydraulic diffusivity from the synthetically generated data at an observation well of negligible radius, the transmissivity value could be computed accurately making use of the synthetically generated data at an observation well of large radius

Field Verification of the Proposed Method:

A field example is considered next to show the applicability and limitations of the different methods that have been presented in this paper. The field data reported by Reynolds(1987) have been used for this purpose. Reynolds(1987) has calculated aquifer diffusivity for three sites in a glacial-outwash valley aquifer near Cortland, New York, from water-level fluctuation induced by rises in stream stage. The data at site 1 (Elm Street) are suitable to be used for solving the inverse problem as an almost complete response of the aquifer to a single flood wave in a stream is available. The water table fluctuations at the observation well near the stream bank and at the second observation well, read from the graph presented by Reynolds, are given in Table 5. As suggested by Reynolds the water levels in the observation well A, installed at the stream bank, are substituted for stream stage. Approximating the rise at the observation well A to be equivalent to a flood wave which commences with a step rise of 0.457m at 15 hours, the aquifer diffusivity was determined using the recorded water table rise at well A and B and equation 13. The hydraulic diffusivity corresponding to the observed water table rise is found to be 1393.34 m^2/hour. This value has been arrived at using a time step size $\Delta t=5$ hours and Laplace parameter $S=0.02$ $hour^{-1}$. If the simulated rise at observation well B reported by Reynolds is regarded as the response of the aquifer, the corresponding hydraulic diffusivity computed by equation(13) is 2179.5m^2/hour. Reynolds has reported the value of $\beta=2194.0m^2$/hour and has simulated the rise with this value of β.

Making use of equation 6 and the recorded rise at observation wells A and B, the hydraulic diffusivity is found to be 1480.9 m^2/hour by Marquardt algorithm. If the simulated values reported by Reynolds are regarded as the true response of the aquifer, the diffusivity estimated by Marquardt algorithm is 2286.9 m^2/hour. The water level fluctuations at observation well B computed by equation(6) are given in table 5. The peak recorded at observation well B is 1.267m. The height of the peak simulated by Reynolds is 1.508m. The peak simulated by the present method is 1.314m.

The radii of the observation wells reported by Reynolds are 0.0762m. Since the radii of the observation wells are small, the storage coefficient and transmissivity can not be estimated separately form the observations recorded at these wells.

Response of an aquifer at an observation well, which is located at a distance of 152.40m from a stream bank and has a radius = 0.5 m, to a flood wave is shown in Figure 3. The flood

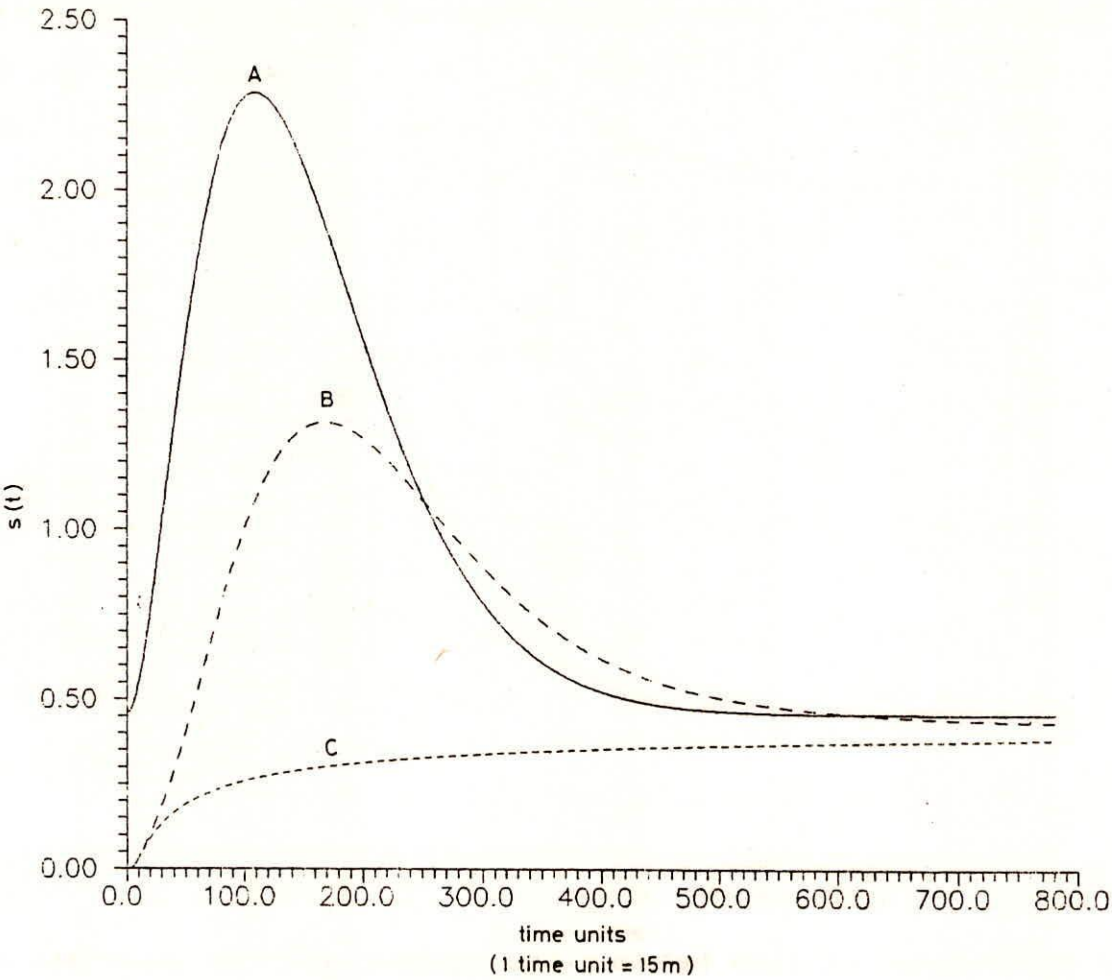

Figure 3-Simulation of water level rise at an observation well.

wave consists of two parts:(i) a sudden rise of 0.4572m in stream stage which takes place at time t=0 and continues indefinitely and (ii) a fluctuating part, that follows the equation proposed by Cooper and Rorabough, having a duration of 195 hours. The peak stage is 2.286m and it occurs at t=27 hours. It is assumed that $\beta=1393.34\ m^2$/hour and $\phi = 0.034$. In the figure the curve A is the flood wave; curve c is the response had there been only a step rise in stream stage of magnitude 0.4572m :curve b is the simulated rise at the observation well. The maximum height of the rise at the observation well is 1.316m and it occurs at 41.75 hours after the on set of flood. The variation of dimensionless exchange of flow, $|V_w(t)/(\Delta t)|/(H_0T)$ that takes place between the aquifer and observation well with dimensionless time parameter, $4\beta t/l^2$, is shown in Figure 4. The absolute values of the flow, $V_w(t)/\Delta t$, have been plotted in the figure. Figure 4 shows three

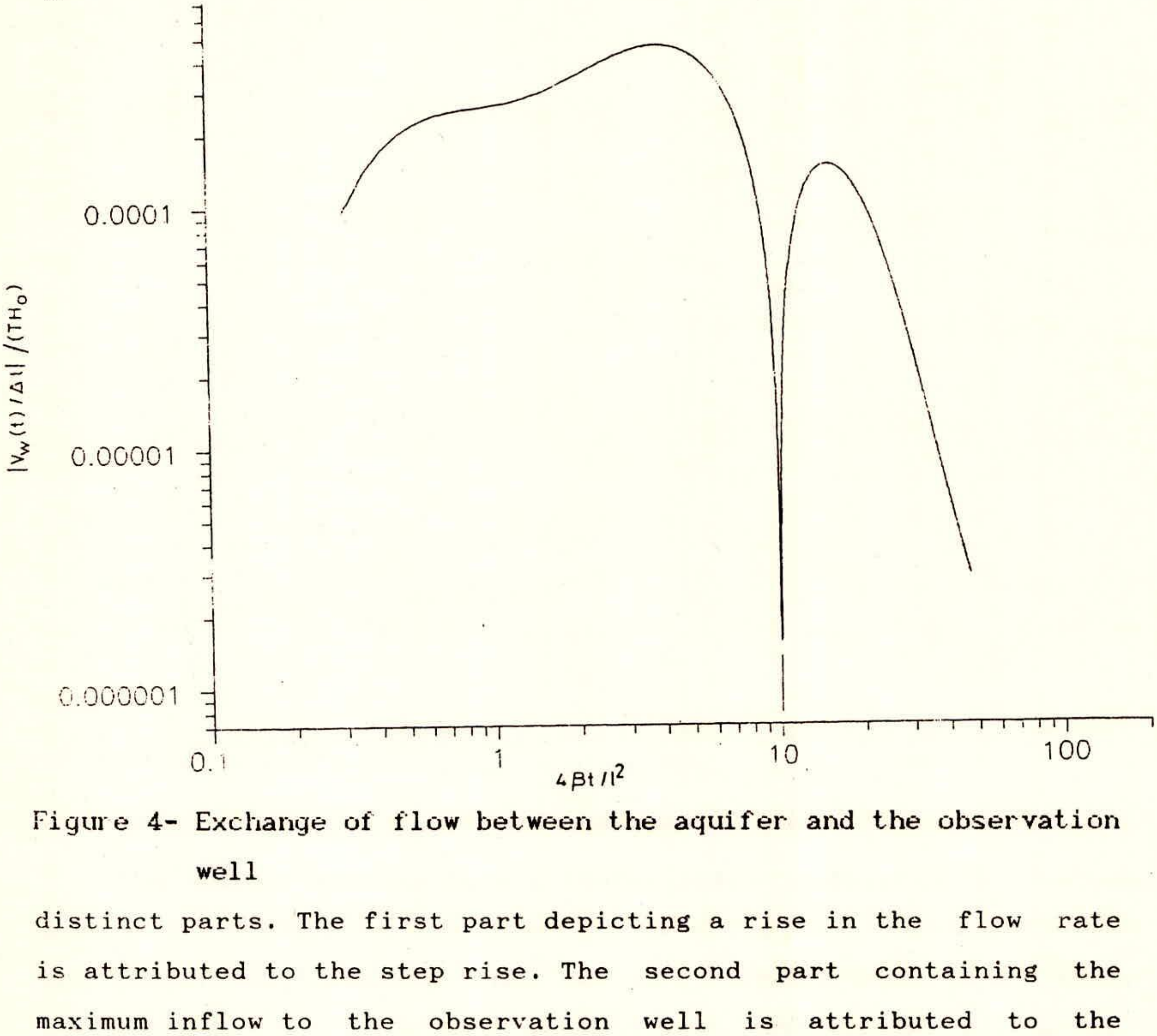

Figure 4- Exchange of flow between the aquifer and the observation well

distinct parts. The first part depicting a rise in the flow rate is attributed to the step rise. The second part containing the maximum inflow to the observation well is attributed to the fluctuating part in stream stage. The peak in flow occurs at t=16.25 hours and the maximum inflow rate is $0.0406m^3$ per hour. The third part of the graph represents the out flow from the well storage. The out flow commences at 42 hours. The maximum out flow rate is 0.0129 m^3 per hour and it takes place at 64 hours.

Appropriate Location of Observation Well:

The variations of dimensionless water level fluctuation, $s(l,t)/H_0$, with dimensionless time parameter, $Kt/(\phi E)$, are presented in Figure 5. The observation wells are assumed to have very small radius. The result has been presented for a flood wave which attains the peak in 48 hours, continues to remain at the peak for 48 hours and then recedes in a duration of 72 hours. Thus the duration of the flood is 7 days. The aquifer diffusivity is $10000 m^2/day$. Figure 5 shows that at the observation well which is located at $l/E=20$, the amplitude of water level change is 47 percent of the amplitude at the flow boundary. Thus for the assumed flood wave the farthest observation well in this aquifer

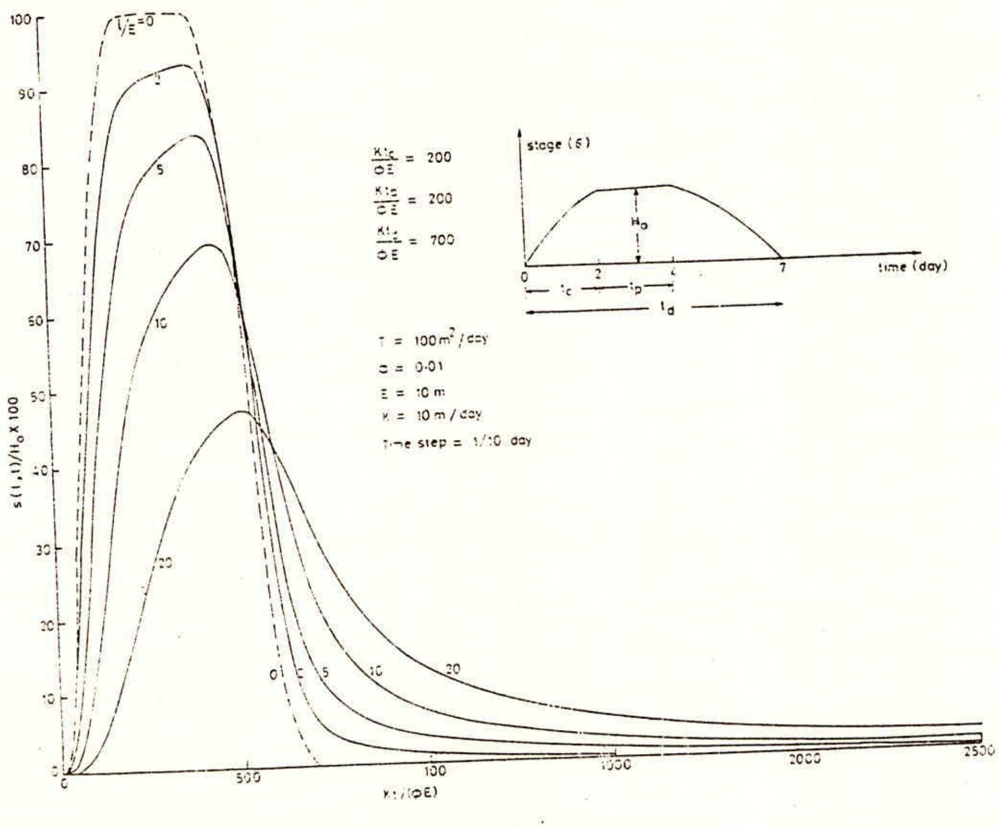

Figure 5- Water level fluctuations at different observation wells for a flood wave with $Kt_c/(\phi E)=200$, $Kt_p/(\phi E)=200$, $Kt_d/(\phi E)=700$ in an aquifer with diffusivity $\beta=1000 m^2/day$

can be located at l/E=20 since the amplitude of fluctuation at this location is more than 0.3 of the amplitude at the flow boundary. For another flood wave which attains peak in two days and which has a duration of 5 days, the water table fluctuations at different distances from the stream in an aquifer whose diffusivity is $5000 m^2$/day, are presented in Figure 6 for $Kt_c/(\phi E)=100$ and $Kt_d/(\phi E)=250$. The amplitude of fluctuation is 0.445 of the amplitude of the flood wave at l/E=10. At l/E=20 , the amplitude of fluctuation is 0.22 of the amplitude of the flood wave and an observation well should not be located at l/E in this aquifer as the amplitude of fluctuation is less than 0.3 of the amplitude at the flow boundary. Thus location of an observation well will depend on the flood wave and the hydraulic diffusivity of the aquifer. The variations of the amplitude of fluctuation with distance for the above mentioned flood waves and aquifer diffusivity are shown in Figures 7 and 8.

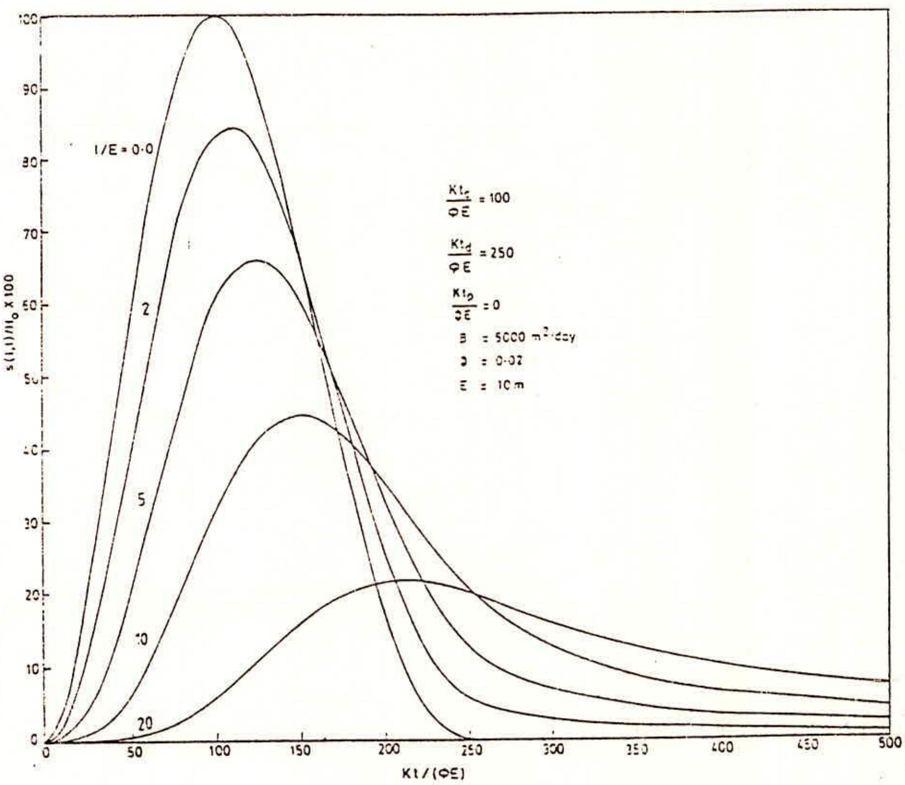

Figure 6- Water level fluctuations at different observation wells for a flood wave with $Kt_c/(\phi E)=100$, $Kt_p/(\phi E)=0$, $Kt_d/(\phi E)=250$ in an aquifer with diffusivity $\beta=5000 m^2$/day

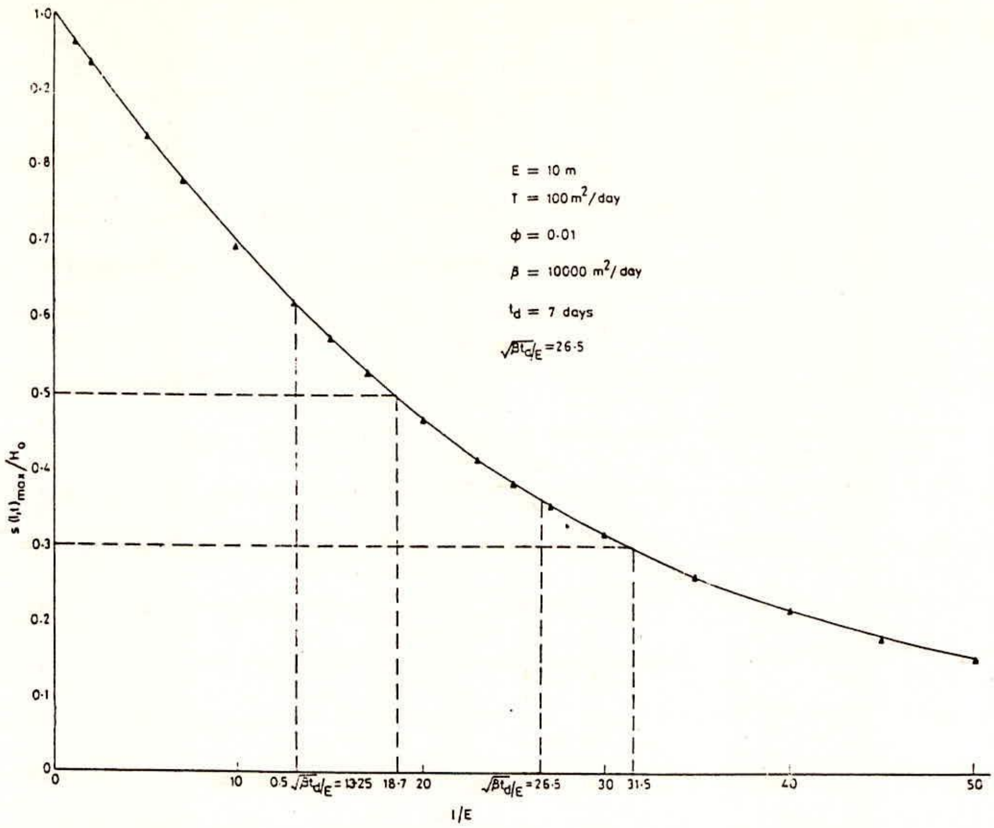

Figure 7- Amplitude of fluctuations at different observation wells for a flood wave with $Kt_c/(\phi E)=200$, $Kt_p/(\phi E)=200$, $Kt_d/(\phi E)=700$ in an aquifer with diffusivity $\beta=10000 m^2/d$

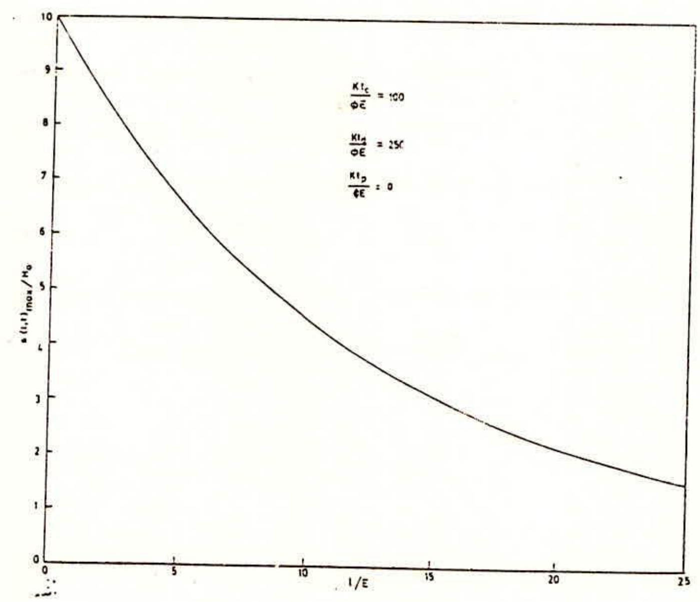

Figure 8- Amplitude of fluctuations at different observation wells for a flood wave with $Kt_c/(\phi E)=100$, $Kt_p/(\phi E)=0$, $Kt_d/(\phi E)=250$ in an aquifer with diffusivity $\beta=5000 m^2/day$

CONCLUSION

A Laplace transform technique has been proposed to find the hydraulic diffusivity from observed stream stage fluctuations and water table rise in an observation well in the vicinity of the stream. The validity of the method has been checked by estimating the parameter using an optimization technique.

A method has also been described to estimate the aquifer parameters T and ϕ from the observation of rise in stream stage and the consequent water table fluctuation in a nearby observation well by taking into account the well storage effect. If the radius of the well is more than 0.5m, the parameters can be estimated correctly. If the radius of the observation well is small, only the hydraulic diffusivity can be estimated. The method described is applicable for any pattern of stream stage fluctuations. The validity of the method has been checked using synthetic data.

The data of field study reported in literature have been used to solve the inverse problem by Laplace transform technique and by optimization method. The simulated water table rise by the method presented is found close to the observed rise.

In order to find the transmissivity and the storage coefficient of the aquifer there should be three observation wells, one close to the stream bank ,a second one at a distance of 100m from the stream bank having a radius more than 0.5m and a third observation well with a small radius of 0.1 m at a distance of 200m from the stream bank. The observation of stream stage and rise in water level in the observation well should be monitered continuously.

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Table 1: Comparison of true and estimated hydraulic diffusivity; the diffusivity has been estimated using Marquardt algorithm

Distance of the observation well from the stream (m)	Hydraulic diffusivity assumed for generating data (m^2/hour)	Estimated Diffusivity (m^2/hour)	Objective Function
50	25.00	25.00	1.4194×10^{-7}
50	5000000.00	5014762.00	3.1668×10^{-7}
100	25.00	24.99	1.0922×10^{-7}
100	5000000.00	5003862.00	3.5664×10^{-6}
500	5000000.00	4994707.00	3.0159×10^{-6}

Table 2: Comparison of true and estimated hydraulic diffusivity; the parameter has been obtained using Laplace transform technique; duration of observation 200 hours.

Distance x (m)	True β (m^2/h)	Time Step Size, Δt (h)	S (h^{-1})	$L\{\frac{d\sigma}{dt}\}$ Analytically	$L\{\frac{d\sigma}{dt}\}$ Numerically	Estimated β (m^2/h)
50	25.0	1	1	0.36226	0.03915	25.840
		0.5			0.03697	25.212
		0.25			0.03641	25.055
		0.125			0.03627	25.100
		0.0625			0.03624	25.168
		1	0.1	0.80383	0.80450	25.026
		0.5			0.80400	25.007
		0.25			0.80387	25.002
		0.125			0.80384	25.000
		0.0625			0.80383	25.000
50	$5x10^6$	1	1			$0.709x10^6$
		0.5				$0.820x10^{10}$
		0.25				$0.899x10^6$
		0.125				$5.739x10^6$
		0.0625				$5.181x10^6$
		1	0.1			$6.518x10^6$
		0.5				$5.333x10^6$
		0.25				$5.081x10^6$
		0.125				$5.020x10^6$
		0.0625				$4.999x10^6$
100	25.0	1	1			24.338
		0.5				31.531
		0.25				32.295
		0.125				34.065
		0.0625				35.826
		1	0.1			25.014
		0.5				25.008
		0.25				25.010
		0.125				25.015
		0.0625				25.029
100	$5x10^6$	1	1			$6.279x10^6$
		0.5				$21.260x10^6$
		0.25				$6.726x10^6$
		0.125				$5.397x10^6$
		0.0625				$5.103x10^6$
		1	0.1			$5.721x10^6$
		0.5				$5.174x10^6$
		0.25				$5.044x10^6$
		0.125				$5.011x10^6$
		0.0625				$5.001x10^6$

Table 3: Sensitivity of estimation of β with respect to duration of observation, Laplace transform factor S and time step size Δt; distance of observation well = 100m.

Duration of observation (h)	True β (m^2/h)	S (h^{-1})	Time Step Size, Δt (h)	$L\{s(x,t)\}$ (mh)	$L\{\frac{d\sigma}{dt}\}$ (m)	Estimated β (m^2/h)
200	25	0.01	1	6.74497	0.57528	21.765
400		0.01	1	7.71794	0.57528	24.783
500		0.01	1	7.76660	0.57528	24.939
800		0.01	1	7.78505	0.57528	24.999
800		0.01	0.5	7.78505	0.57528	24.998
800		0.01	0.25	7.78504	0.57528	24.998
200	$5x10^6$	0.01	1	57.2683	0.57528	487962.2
400		0.01	1	57.2718	0.57528	501409.6
500		0.01	1	57.2718	0.57528	501702.9
800		0.01	1	57.2718	0.56527	501702.9
800		0.01	0.5	57.2710	0.57527	499363.9
800		0.01	0.25	57.2706	0.57270	499019.7

Table 4: Comparison of true and estimated aquifer parameters; time step size, $\Delta t=1$ hour. The parameters have been computed using Marquardt algorithm.

Distance (m)	Radius of Observation Well(m)	True T (m^2/hour)	True ϕ	Estimated β (m^2/hour)	Estimated T (m^2/hour)	Objective Function
50.0	0.1	5	0.2	25.00	5.209	1.286×10^{-9}
		5	0.00001	500824.40	5.229	2.975×10^{-6}
		50	0.2	250.01	59.299	1.533×10^{-6}
		50	0.00001	5038713.00	39.951	3.329×10^{-6}
50.0	0.3	5	0.2	25.00	5.020	1.383×10^{-7}
		5	0.00001	496942.60	5.104	2.985×10^{-6}
		50	0.2	249 98	52.431	1.382×10^{-6}
		50	0.00001	4964911.00	51.564	3.538×10^{-6}
50.0	0.5	5	0.2	25.00	5.028	1.258×10^{-7}
		5	0.00001	505392.60	4.972	3.238×10^{-6}
		50	0.2	250.00	52.107	1.668×10^{-6}
		50	0.00001	5159611.00	46.439	3.161×10^{-6}
50.0	1.0	5	0.2	24.99	5.001	1.343×10^{-7}
		5	0.00001	502681.80	4.985	2.712×10^{-6}
		50	0.2	250.01	49.600	2.411×10^{-6}
		50	0.00001	5096605.00	49.422	3.161×10^{-6}
100.0	0.1	5	0.2	25.00	4.184	1.113×10^{-7}
		5	0.00001	497140.20	5.581	2.817×10^{-6}
		50	0.2	250.02	13.055	1.622×10^{-7}
		50	0.00001	5009649.00	32.978	3.533×10^{-6}
100.0	0.3	5	0.2	25.00	5.001	1.201×10^{-7}
		5	0.00001	500604.50	5.009	2.740×10^{-6}
		50	0.2	250.00	47.484	1.129×10^{-7}
		50	0.00001	4972438.00	51.102	2.910×10^{-6}
100.0	0.5	5	0.2	24.99	5.081	1.155×10^{-7}
		5	0.00001	501059.80	4.979	2.868×10^{-6}
		50	0.2	250.00	49.560	1.191×10^{-7}
		50	0.00001	4992696.00	51.329	3.366×10^{-6}
100.0	1.0	5	0.2	24.99	5.013	1.251×10^{-7}
		5	0.00001	498362.70	5.000	2.483×10^{-6}
		50	0.2	250.00	50.225	1.087×10^{-7}
		50	0.00001	4974755.00	50.362	3.735×10^{-6}

Table 5: Observed and Simulated Water Table Rise

Time (h)	Rise at Well No.1 (m)	Rise at Well No.2 (m)	Rise Simulated at Well No 2 by Reynolds(1989) $\beta(m^2/h)=2194.83$ (m)	Present Method 1480.93	Present Method 2286.97
20	0.670	0.126	0.209	0.114	0.175
25	0.733	0.178	0.314	0.241	0.313
30	1.089	0.251	0.419	0.354	0.445
35	1.843	0.691	0.733	0.548	0.695
40	2.286	1.068	1.047	0.829	1.027
45	2.286	1.183	1.277	1.074	1.276
50	2.282	1.235	1.403	1.225	1.411
55	2.199	1.256	1.487	1.321	1.488
60	2.063	1.267	1.508	1.368	1.511
65	1.843	1.240	1.466	1.369	1.485
70	1.686	1.225	1.403	1.334	1.424
75	1.518	1.173	1.319	1.285	1.354
80	1.434	1.152	1.277	1.229	1.286
85	1.340	1.110	1.214	1.182	1.229
90	1.256	1.078	1.173	1.135	1.174
95	1.152	1.047	1.110	1.087	1.117
100	1.078	1.005	1.047	1.036	1.059
105	1.005	0.985	0.984	0.989	1.006
110	0.942	0.942	0.942	0.943	0.956
115	0.900	0.900	0.900	0.902	0.912
120	0.838	0.848	0.877	0.864	0.870
125	0.785	0.817	0.796	0.825	0.828
130	0.754	0.796	0.775	0.789	0.790
135	0.732	0.775	0.754	0.758	0.760
140	0.701	0.754	0.733	0.733	0.733
145	0.670	0.733	0.691	0.707	0.707
150	0.649	0.712	0.670	0.683	0.682
155	0.607	0.681	0.649	0.660	0.657
160	0.586	0.649	0.628	0.635	0.631
165	0.565	0.639	0.607	0.614	0.609
170	0.555	0.629	0.586	0.595	0.591
175	0.544	0.607	0.565	0.579	0.575
180	0.523	0.597	0.544	0.564	0.560
185	0.502	0.565	0.534	0.548	0.544
190	0.492	0.556	0.523	0.533	0.528
195	0.482	0.534	0.502	0.519	0.515
200	0.471	0.523	0.492	0.507	0.503
205	0.466	0.521	0.482	0.496	0.492
210	0.461	0.518	0.471	0.486	0.483

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